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PROGRAM EDUCATION

البرنامج المميز

Mr. Ahmed Ata
The Featured Program

12 ADVANCED

MATH ENG

CHAPTER 5

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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5 - Integration

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LESSON 5-1

Antiderivatives

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Lesson (5-1)

Antiderivatives

THEOREM 1.1

Suppose that F and G are both antiderivatives of f on an interval I. Then,

$$G(x) = F(x) + c,$$

for some constant c.

DEFINITION 1.1

Let F be any antiderivative of f on an interval I. The **indefinite integral** of $f(x)$ (with respect to x) on I, is defined by

$$\int f(x)\,dx = F(x) + c,$$

where c is an arbitrary constant (the **constant of integration**).

THEOREM 1.2 (Power Rule)

For any rational power $r \neq -1$,

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c.$$

Here, if $r < -1$, the interval I on which this is defined can be any interval that does not include $x = 0$.

THEOREM 1.3

Suppose that $f(x)$ and $g(x)$ have antiderivatives. Then, for any constants, a and b,

$$\int [af(x) + bg(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx.$$

$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c$, for $r \neq -1$ (power rule)	$\int \sec x \tan x\,dx = \sec x + c$
$\int \sin x\,dx = -\cos x + c$	$\int \csc x \cot x\,dx = -\csc x + c$
$\int \cos x\,dx = \sin x + c$	$\int e^x\,dx = e^x + c$
$\int \sec^2 x\,dx = \tan x + c$	$\int e^{-x}\,dx = -e^{-x} + c$
$\int \csc^2 x\,dx = -\cot x + c$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$
$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c$	$\int \frac{1}{\lvert x\rvert\sqrt{x^2-1}}\,dx = \sec^{-1} x + c$

THEOREM 1.4

For $x \neq 0$, $\frac{d}{dx} \ln|x| = \frac{1}{x}$.

COROLLARY 1.1

In any interval not containing 0,

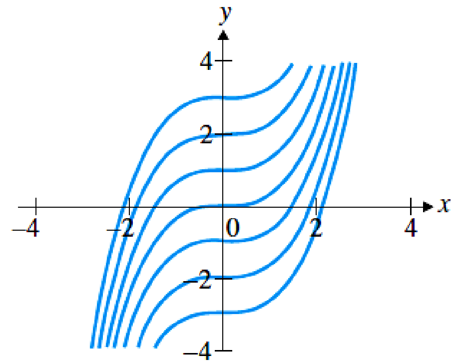
$$\int \frac{1}{x}\,dx = \ln|x| + c.$$

COROLLARY 1.2

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c,$$

in any interval in which $f(x) \neq 0$.

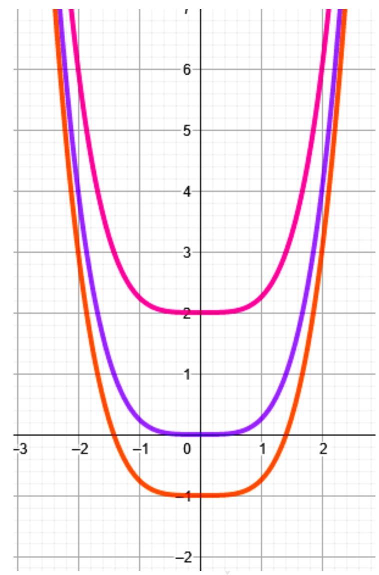
1 **Find an antiderivative of $f(x) = x^2$**



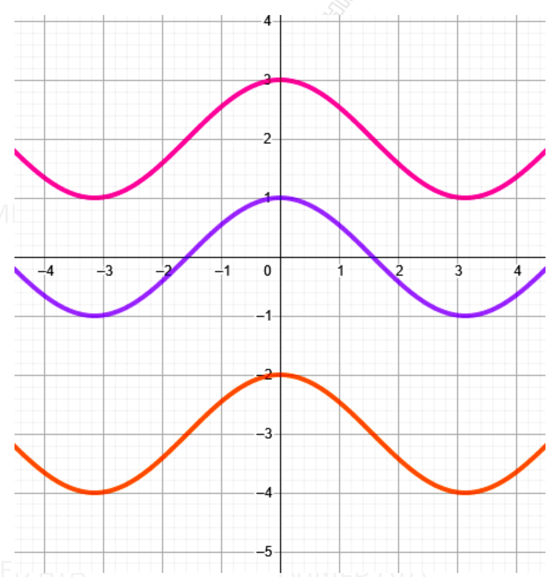
A family of antiderivative curves

Sketch several members of the family of functions defined by the antiderivative.

2 $\int x^3 \, dx$



3 $\int -\sin x \, dx$



find the general antiderivative.

4 $\int (3x^4 - 3x)\,dx$

5 $\int \left(3\sqrt{x} - \frac{1}{x^4}\right)dx$

6 $\int \frac{x^{1/3} - 3}{x^{2/3}}\,dx$

7 $\int (2\sin x + \cos x)\,dx$

8 $\int 2\sec x \tan x\,dx$

9 $\int \frac{4}{\sqrt{1 - x^2}}\,dx$

10 $\int 4\frac{\cos x}{\sin^2 x}\,dx$

11 $\int 5\sec^2 x\,dx$

12 $\int (3e^x - 2)\,dx$

13 $\int (3\cos x - 1/x)\,dx$

14 $\int (2x^{-1} + \sin x)\,dx$

15 $\int \frac{4x}{x^2+4}\,dx$

16 $\int \frac{3}{4x^2+4}\,dx$

17 $\int \frac{\cos x}{\sin x}\, dx$

18 $\int \left(2\cos\ x - \sqrt{e^{2x}}\right) dx$

19 $\int \frac{e^x}{e^x + 3}\, dx$

20 $\int \frac{e^x + 3}{e^x}\, dx$

21 $\int x^{1/4}(x^{5/4} - 4)\, dx$

22 $\int x^{2/3}(x^{-4/3} - 3)\, dx$

23 $\int \left(\sqrt{x^3}+4\right) dx$

24 $\int \frac{3x^2-4}{x^2} dx$

25 $\int \sec^2 x \, dx$

26 $\int \left(\frac{1}{x^2}-1\right) dx$

27 **Find the derivative.**

a $\frac{d}{dx} \ln|\sec x + \tan x|$

b $\frac{d}{dx} \ln|\sin x \cdot 2|$

Find the function $f(x)$ satisfying the given conditions.

28 $f'(x) = 3e^x + x, \quad f(0) = 4$

29 $f'(x) = 4\cos x, \quad f(0) = 3$

30 $f''(x) = 12x^2 + 2e^x, \quad f'(0) = 2, \quad f(0) = 3$

31 $f''(x) = 2 + 2t, \quad f(0) = 2, \quad f(3) = 2$

32 $f''(x) = 4 + 6t, \quad f(1) = 3, \quad f(-1) = -2$

33 $f''(x) = 3 \sin x + 4x^2$

34 Determine the position function if the velocity function is $v(t) = 3 - 12t$ and the initial position is $s(0) = 3$.

35 Determine the position function if the velocity function is $v(t) = 3e^{-t} - 2$ and the initial position is $s(0) = 0$.

36 Determine the position function if the acceleration function is $a(t) = 3\sin t + 1$, the initial velocity is $v(0) = 0$ and the initial position is $s(0) = 4$.

37 Determine the position function if the acceleration function is $a(t) = t^2 + 1$, the initial velocity is $v(0) = 4$ and the initial position is $s(0) = 0$.

38 **Find an antiderivative by reversing the chain rule, product rule or quotient rule**

a $\int xsin2x + x^2cos2x\, dx$

b $\int \frac{2xe^{3x} - 3x^2e^{3x}}{e^{6x}}\, dx$

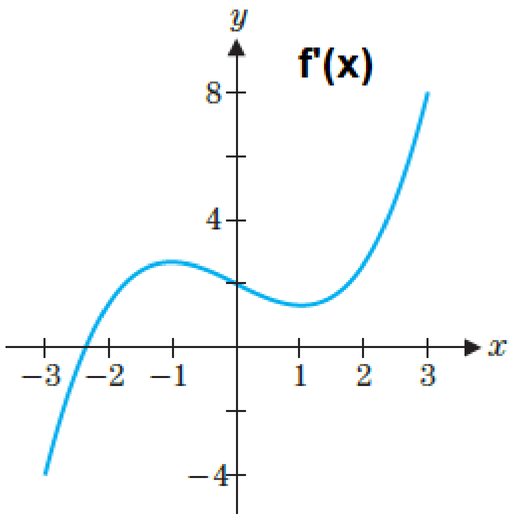
c $\int \frac{xcosx^2}{sinx^2}\, dx$

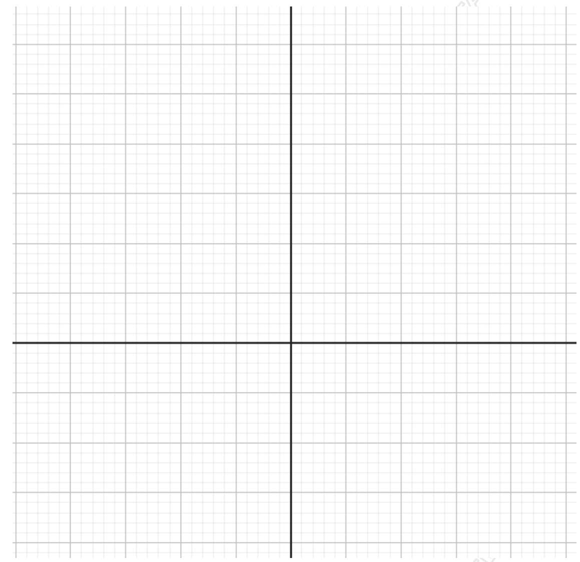
39 Find a function $f(x)$ such that the point $(1,2)$ is on the graph of $y = f(x)$, the slope of the tangent line at $(1,2)$ is 3 and $f''(x) = x - 1$

40 Find a function $f(x)$ such that the point $(-1,1)$ is on the graph of $y = f(x)$, the slope of the tangent line at $(-1,1)$ is 2 and $f''(x) = 6x + 4$.

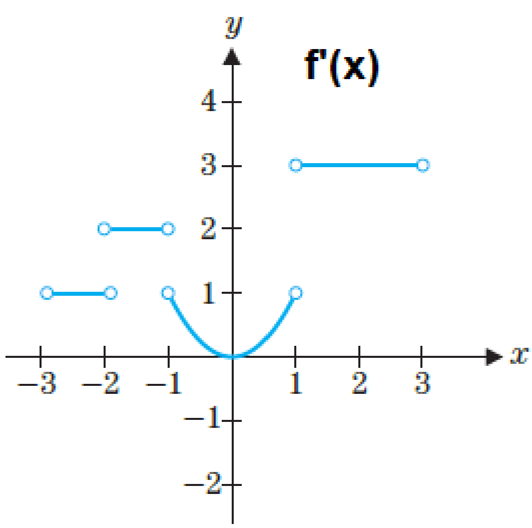
41 If an object's downward acceleration is given by $y''(t) = -9.8\, m/s^2$, find the position function $y(t)$. Assume that the initial velocity is $y'(0) = -30\, m/s$ and the initial position is $y(0) = 30,000\, m$.

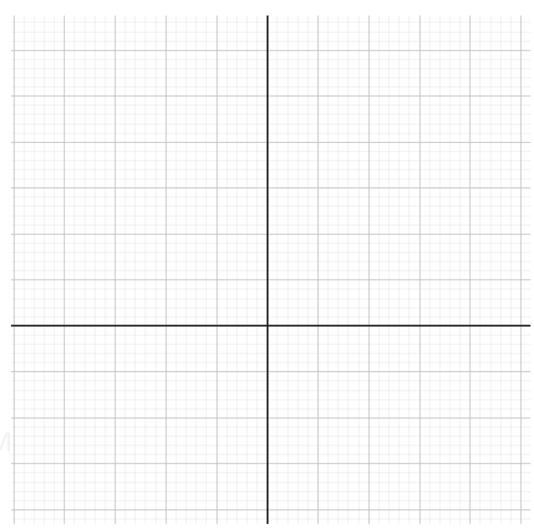
42 Sketch the graph of functions f (x) corresponding to the given graph of y = f ′(x). f (x) Is continuous





43 Sketch the graph of function f (x) corresponding to the given graph of y = f ′(x). f (x) Is continuous





Lesson (5-1) part 2

Find the general antiderivative.

1. $\int \frac{x-4}{\sqrt{x}-2}dx$

2. $\int secx(tanx - secx)dx$

3. $\int \frac{sin^2x + cos^2x}{cosx\ cotx}dx$

4. $\int \frac{2sinx}{cos^2x} + \frac{cosx}{sin^2x}dx$

5 $\int \frac{sec^2 x}{tanx} dx$

6 $\int \frac{x^2 + 2x}{x^3 + 3x^2 - 5} dx$

7 $\int \frac{2x + 5}{2x(x + 5)} dx$

8 $\int \frac{2e^x - e^{-x}}{4e^x + 2e^{-x}} dx$

9 $\int (tanx - cotx)dx$

10 $\int 5e^{7x-1}\, dx$

11 $\int 3e^{x}\,(e^{x} - 5)dx$

12 $\int \frac{1}{5 + e^{-x}} dx$

13 $\int \frac{1}{1+e^x}\,dx$

14 $\int \frac{5x}{2x^3+2x}\,dx$

15 $\int \frac{3}{\sqrt{9-9x^2}}\,dx$

16 $\int \frac{1}{\sqrt{x^4-x^2}}\,dx$

17 $\int cos5x\, dx$

18 $\int \frac{sin^2 x}{1 - cosx} dx$

19 $\int \frac{x + 1}{x^2 + 1} dx$

20 $\int secxdx$

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LESSON 5-2

Sums and Sigma Notation

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Lesson (5-2)

Sums and Sigma Notation

$$\sum_{i=1}^{20} i^2 = 1^2 + 2^2 + 3^2 + \cdots + 20^2$$

to indicate that we add together terms of the form i^2, starting with $i = 1$ and ending with $i = 20$. The variable i is called the index of summation

In general, for any real numbers $a_1, a_2, \ldots\ldots, a_n$, we have

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

Write in summation notation

1. $\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{10}$

2. $3^3 + 4^3 + 5^3 + \cdots + 45^3.$

3 $2(1)^2+2(2)^2+2(3)^2+\cdots+2(14)^2$

4 $\sqrt{2-1}+\sqrt{3-1}+\sqrt{4-1}+\cdots+\sqrt{15-1}$

5 **Write in summation notation: the sum of the first 200 odd positive integers**

6 **The sum of the squares of the first 50 positive integers**

7 **The square of the sum of the first 50 positive integers**

8 The sum of the square roots of the first 10 positive integers.

9 The square root of the sum of the first 10 positive integers

write out all terms and compute the sums

10 $\sum_{i=1}^{8}(2i+1)$

11 $\sum_{i=2}^{6}\sin(2\pi i)$

12 $\sum_{i=4}^{10}5$

13 $\sum_{i=1}^{6} 3i^2$

14 $\sum_{i=3}^{7} (i^2 + i)$

15 $\sum_{i=6}^{10} (4i + 2)$

16 $\sum_{i=6}^{8} (i^2 + 2)$

THEOREM 2.1

If n is any positive integer and c is any constant, then

(i) $\sum_{i=1}^{n} c = cn$ (sum of constants),

(ii) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ (sum of the first n positive integers) and

(iii) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ (sum of the squares of the first n positive integers).

THEOREM 2.2

For any constants c and d,

$$\sum_{i=1}^{n}(ca_i + db_i) = c\sum_{i=1}^{n} a_i + d\sum_{i=1}^{n} b_i.$$

Computing Sums Using Theorems 2.1 and 2.2

17 $\sum_{i=1}^{8}(2i+1)$

18 $\sum_{i=1}^{800}(2i+1)$

19 $\sum_{i=1}^{20} i^2$

20 $\sum_{i=1}^{20} \left(\frac{i}{20}\right)^2$

21 $\sum_{i=1}^{70} (3i - 1)$

22 $\sum_{i=1}^{40} (4 - i^2)$

23 $\sum_{n=1}^{100}(n^2 - 3n + 2)$

24 $\sum_{i=4}^{20}(i - 3)(i + 3)$

25 $\sum_{k=3}^{n}(k^2 - 3)$

26 $\sum_{i=3}^{30}[(i - 3)^2 + i - 3]$

compute sums of the form $\sum_{i=1}^{n} f(x_i)\Delta x$ for the given values of x_i.

27 $f(x) = x^2 + 4x;\quad x = 0.2, 0.4, 0.6, 0.8, 1.0;\quad \Delta x = 0.2;\quad n = 5$

28 $f(x) = 3x + 5;\quad x = 0.4, 0.8, 1.2, 1.6, 2.0;\quad \Delta x = 0.4;\quad n = 5$

29 $f(x) = x^3 + 4;\quad x = 2.05, 2.15, 2.25, 2.35, \ldots, 2.95;\quad \Delta x = 0.1;\quad n = 10$

30 *Sum the values* of $f(x) = x^2 + 3$
Evaluated at $x = 0.1, x = 0.2, \ldots, x = 1.0.$

31 *Sum the values* of $f(x) = 3x^2 - 4x + 2$
evaluated at $x = 1.05, x = 1.15, x = 1.25, \ldots, x = 2.95.$

compute the sum and the limit of the sum as $n \to \infty$

32 $$\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right]$$

33 $\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 - 5\left(\frac{i}{n}\right)\right]$

34 $\sum_{i=1}^{n} \frac{1}{n}\left[4\left(\frac{2i}{n}\right)^2 - \left(\frac{2i}{n}\right)\right]$

35 $\sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{2i}{n}\right)^2 + 4\left(\frac{i}{n}\right)\right]$

36 **Suppose that a car has velocity 50 mph for 2 hours, velocity 60 mph for 1 hour, velocity 70 mph for 30 minutes and velocity 60 mph for 3 hours. Find the distance traveled**

37 **Suppose that a car has velocity 50 mph for 1 hour, velocity 40 mph for 1 hour, velocity 60 mph for 30 minutes and velocity 55 mph for 3 hours. Find the distance traveled**

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LESSON 5-3

Area

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Lesson (5-3)

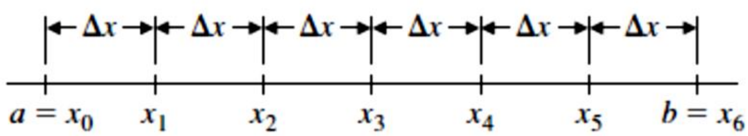
Area

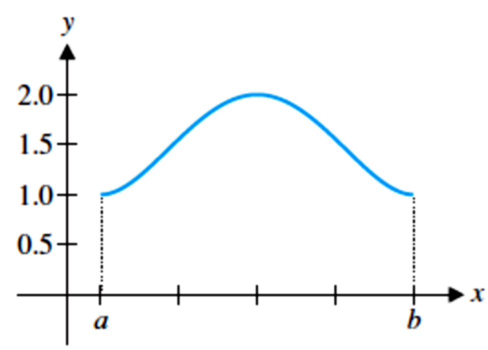
Assume that $f(x) \geq 0$ and f is continuous on the interval $[a, b]$, as in Figure

We start by dividing the interval $[a, b]$ into n equal pieces. This is called a regular partition of $[a, b]$. The width of each subinterval in the partition is then $\frac{b-a}{n}$ which we denote by Δx (Meaning a small change in x). The points in the partition are denoted by

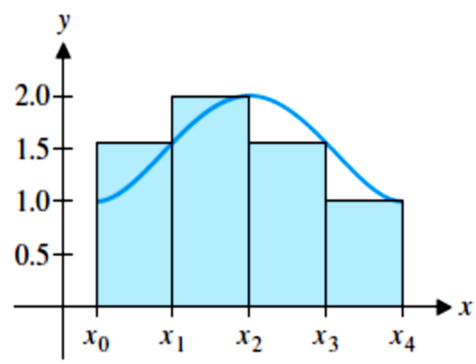
$x_0 = a,\quad x_1 = x_0 + \Delta x,\quad x_2 = x_1 + \Delta x$ and so on. In general,

$$x_i = x_0 + i\Delta x, \qquad for\ i = 1, 2, \ldots, n$$





$$A \approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + f(x_3)\,\Delta x + f(x_4)\,\Delta x = A_4.$$

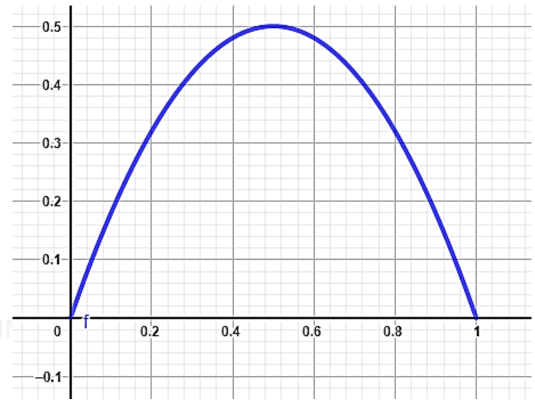


$$A \approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + \cdots + f(x_n)\,\Delta x$$
$$= \sum_{i=1}^{n} f(x_i)\,\Delta x = A_n.$$

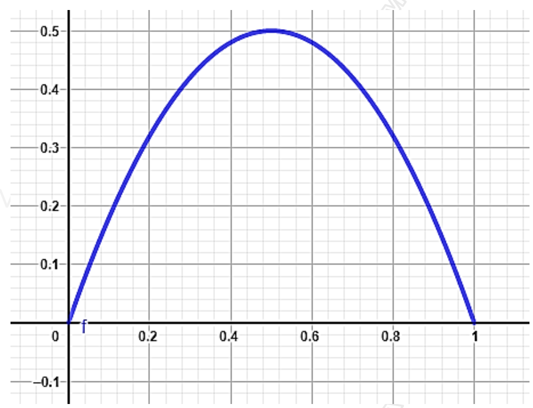
1 Approximate the area under the curve $f(x) = 2x - 2x^2$ on the interval [0, 1], using 5 rectangles and the evaluation rules

(a) right endpoint

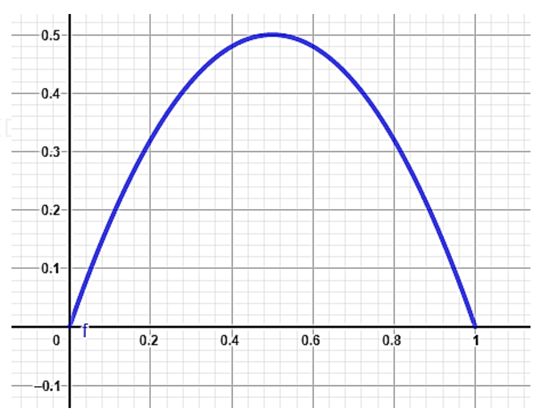
$$A \approx A_5 = \sum_{i=1}^{5} f(x_i)\Delta x =$$

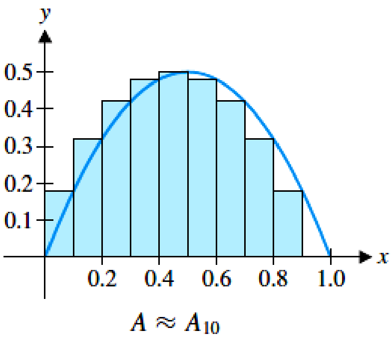


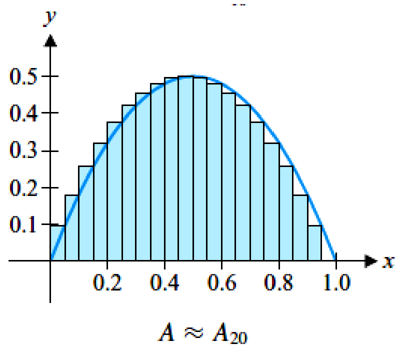
(b) left endpoint

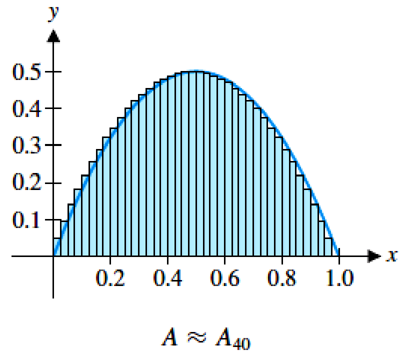


(c) midpoint





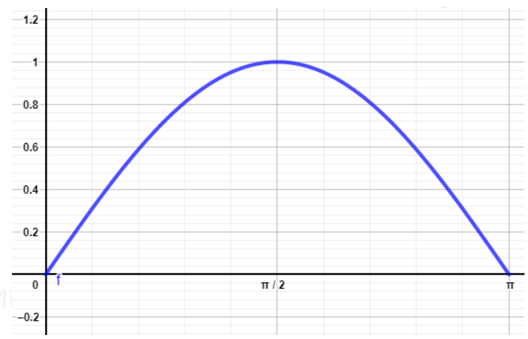




Notice that as n gets larger and larger, A_n seems to be approaching $\frac{1}{3}$

2 Approximate the area under the curve $f(x) = \sin x$ on the interval $[0, \pi], n = 4$

a right endpoint



b left endpoint

c midpoint

use the given function values to estimate the area under the curve using

3

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x)$	2.0	2.4	2.6	2.7	2.6	2.4	2.0	1.4	0.6

a left endpoint

b right-endpoint

4

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
$f(x)$	2.0	2.2	1.6	1.4	1.6	2.0	2.2	2.4	2.0

a left endpoint

b right-endpoint

5 **Approximate the area under the curve on the given interval using n rectangles and the evaluation rules**

$$A \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

$R;\ x_i = a + \Delta x i \qquad L;\ x_i = a + \Delta x(i-1) \qquad M;\ x_i = a + \Delta x\left(i - \frac{1}{2}\right)$

$y = x^2 + 1 \quad on\ [0,1], \quad n = 16$

(a) Right endpoint

(b) Left endpoint

(c) Midpoint

6 $y = \sqrt{x + 2}$ on $[1,4], n = 16$

(a) Right endpoint

(b) Left endpoint

7 $y = \cos x$ on $[0, \frac{\pi}{2}]$, $n = 50$

(a) Right endpoint

(b) Midpoint

DEFINITION 3.1

For a function f defined on the interval $[a, b]$, if f is continuous on $[a, b]$ and $f(x) \geq 0$ on $[a, b]$, the **area** A **under the curve** $y = f(x)$ on $[a, b]$ is given by

$$A = \lim_{n \to \infty} A_n = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\,\Delta x.$$

DEFINITION 3.2

Let $\{x_0, x_1, \ldots, x_n\}$ be a regular partition of the interval $[a, b]$, with $x_i - x_{i-1} = \Delta x = \dfrac{b-a}{n}$, for all i. Pick points $c_1, c_2, \ldots, c_n$, where c_i is any point in the subinterval $[x_{i-1}, x_i]$, for $i = 1, 2, \ldots, n$. (These are called **evaluation points.**) The **Riemann sum** for this partition and set of evaluation points is

$$\sum_{i=1}^{n} f(c_i)\,\Delta x.$$

Use Riemann sums and a limit to compute the exact area under the curve

8 $y = 3x$ *on* $[0, 3]$

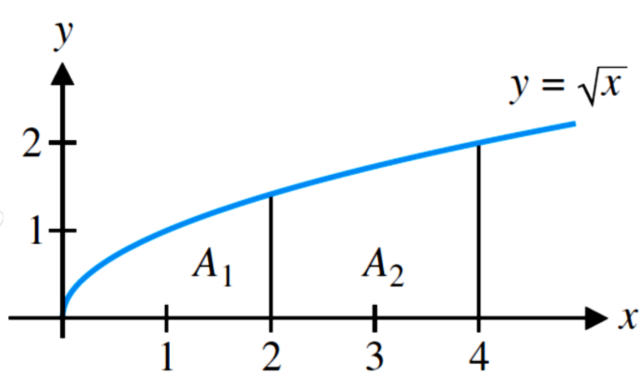
9 $y = x^2 + 1$ *on* $[0, 1]$

10 $y = x^2 + 3x$ on $[0, 2]$

11 $y = 3x^2$ on $[1, 3]$

12 **In the figure**

which area equals $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{2}\sqrt{1+i/n}\,\frac{2}{n}$?



13 Show that the area under $y = ax^2$ for $0 \leq x \leq b$ is $\frac{1}{3}$ of the base times the height

$(\frac{1}{3}b \cdot ab^2)$

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LESSON 5-4

The Definite Integral

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Lesson (5-4)

The Definite Integral

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x,$$

DEFINITION 4.1

For any function f defined on $[a, b]$, the **definite integral** of f from a to b is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x,$$

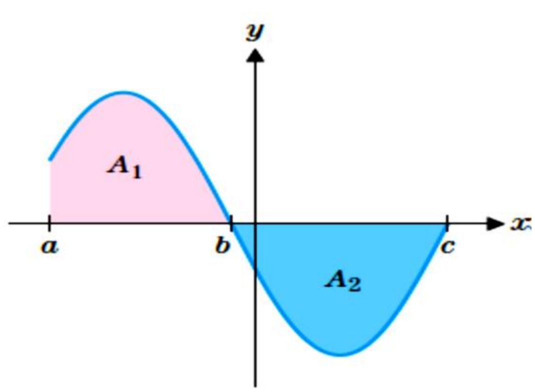
whenever the limit exists and is the same for every choice of evaluation points, $c_1, c_2, \ldots, c_n$. When the limit exists, we say that f is **integrable** on $[a, b]$.

THEOREM 4.1

If f is continuous on the closed interval $[a, b]$, then f is integrable on $[a, b]$.

DEFINITION 4.2

Suppose that $f(x) \geq 0$ on the interval $[a, b]$ and A_1 is the area bounded between the curve $y = f(x)$ and the x-axis for $a \leq x \leq b$. Further, suppose that $f(x) \leq 0$ on the interval $[b, c]$ and A_2 is the area bounded between the curve $y = f(x)$ and the x-axis for $b \leq x \leq c$. The **signed area** between $y = f(x)$ and the x-axis for $a \leq x \leq c$ is $A_1 - A_2$, and the **total area** between $y = f(x)$ and the x-axis for $a \leq x \leq c$ is $A_1 + A_2$. (See Figure 5.16.)



Use the Midpoint Rule with *n* =6 to estimate the value of the integral

1 $\int_0^3 (x^3 + x)\,dx$

Use the Right endpoint Rule with *n* =6 to estimate the value of the integral

2 $\int_0^{\pi} \sin x^2\,dx$

Evaluate the integral by computing the limit of Riemann sums.

3 $\int_0^1 2x\,dx$

4 $\int_1^2 2x\,dx$

5 $\int_0^2 x^2\,dx$

Properties of Definite Integrals

Linearity of Definite Integrals

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b \left(f(x)+g(x)\right)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b \left(f(x)-g(x)\right)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Additive Interval Property of Definite Integrals

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Reversed Interval Property of Definite Integrals

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Simple Maximum and Minimum Values for Definite Integrals

If a function $f(x)$ is continuous and bounded between $y=m$ and $y=M$ on the interval $[a,b]$, i.e. $m \le f(x) \le M$ on the interval, then

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

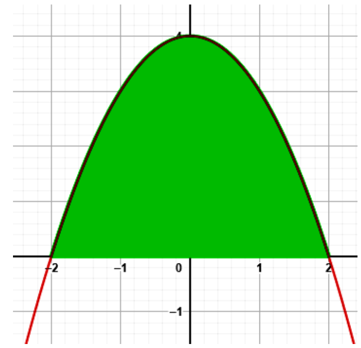
Comparison of Definite Integrals

If $f(x) \le g(x)$ on an interval $[a,b]$, then

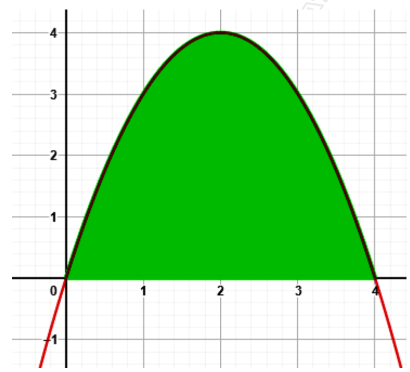
$$\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$$

write the given (total) area as an integral or sum of integrals.

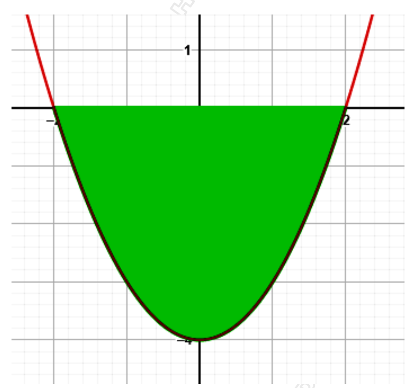
6 The area above the x-axis and below $y = 4 - x^2$



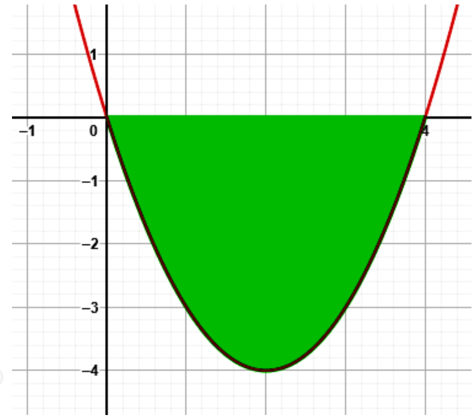
7 The area above the x-axis and below $y = 4x - x^2$



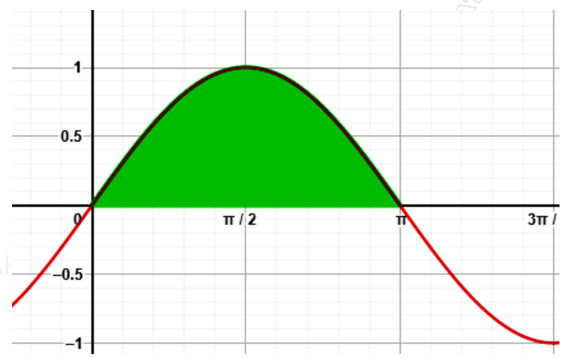
8 The area below the x-axis and above $y = x^2 - 4$



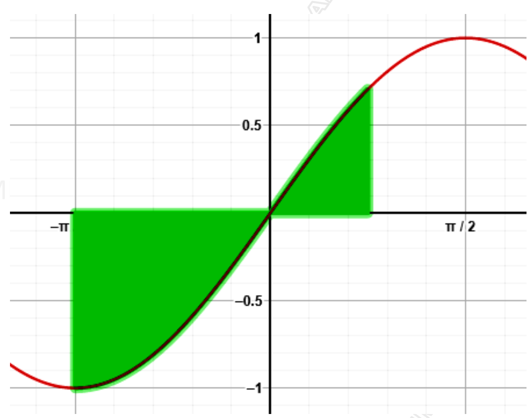
9 The area below the x-axis and above $y = x^2 - 4x$



10 The area between $y = \sin x$ and the $x - axis$ for $0 \le x \le \pi$

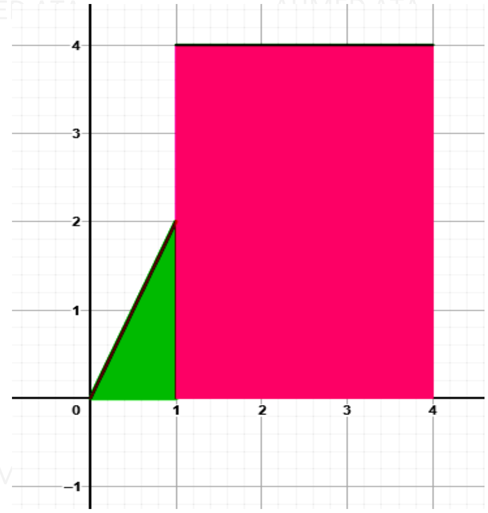


11 The area between $y = \sin x$ and the $x - axis$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{4}$

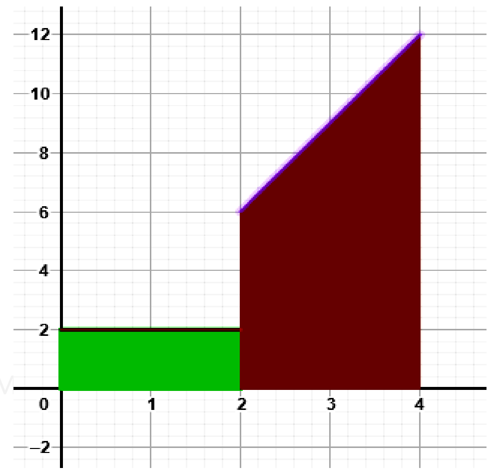


compute $\int_0^4 f(x)\,dx$

12 $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ 4 & \text{if } x \geq 1 \end{cases}$

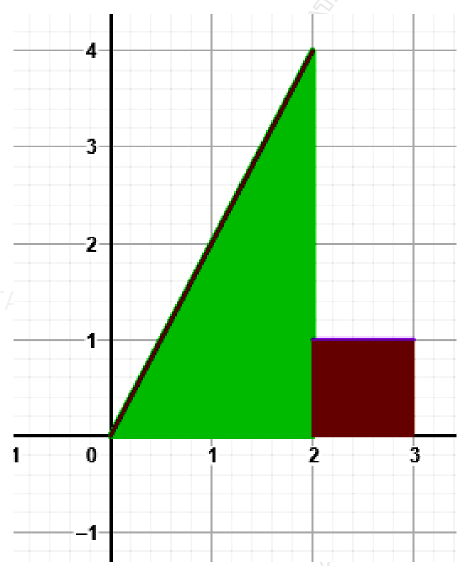


13 $f(x) = \begin{cases} 2 & \text{if } x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$



Evaluate $\int_0^3 f(x)\,dx$, where $f(x)$ is defined by

14 $f(x) = \begin{cases} 2x, & \text{if } x \leq 2 \\ 1, & \text{if } x > 2 \end{cases}$.



An integral representing average value:

$$f_{\text{ave}} = \lim_{n\to\infty}\left[\frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\,\Delta x\right] = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

THEOREM 4.4 (Integral Mean Value Theorem)

If f is continuous on $[a, b]$, then there is a number $c \in (a, b)$ for which

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Compute the average value of the function on the given interval.

15. $f(x) = 2x + 1$, $[0, 4]$

16. $f(x) = x^2 + 2x$, $[0, 1]$

17. $f(x) = x^2 - 1$, $[1, 3]$

Simple Maximum and Minimum Values for Definite Integrals

If a function $f(x)$ is continuous and bounded between $y = m$ and $y = M$ on the interval $[a, b]$, i.e. $m \leq f(x) \leq M$ on the interval, then

$$m(b-a) \leq \int_a^b f(x)\, dx \leq M(b-a)$$

Use the Integral Mean Value Theorem to estimate the value of the integral

18 $\int_0^1 \sqrt{x^2+1}\, dx$

19 $\int_{\pi/3}^{\pi/2} 3\cos x^2\, dx$

20 $\int_0^{1/2} e^{-x^2}\, dx$

Find a value of c that satisfies the conclusion of the Integral Mean Value Theorem

21 $\int_0^2 3x^2\,dx\ (= 8)$

22 $\int_{-1}^1 (x^2 - 2x)\,dx\ (= \frac{2}{3})$

Use properties of definite integrals to write the expression as a single integral

23 $\int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx$

24 $\int_0^3 f(x)\,dx - \int_2^3 f(x)\,dx$

25 $\int_0^2 f(x)\,dx + \int_2^1 f(x)\,dx$

26 $\int_{-1}^2 f(x)\,dx + \int_2^3 f(x)\,dx$

assume that $\int_1^3 f(x)\,dx = 3$ and $\int_1^3 g(x)\,dx = -2$ and find

27 $\int_1^3 [f(x)+g(x)]\,dx$

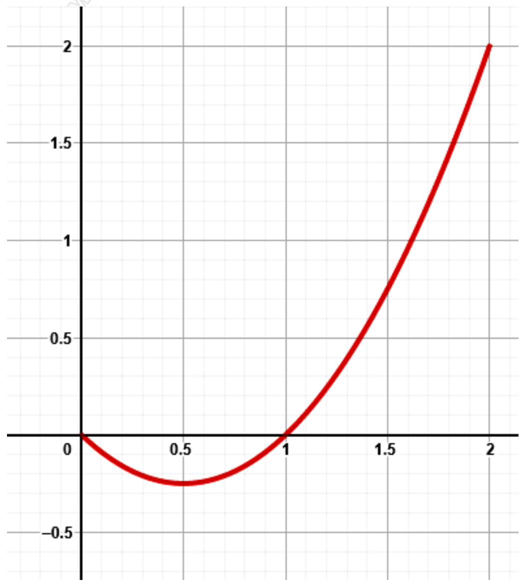
28 $\int_1^3 [2f(x)-g(x)]\,dx$

29 $\int_1^3 [f(x)-g(x)]\,dx$

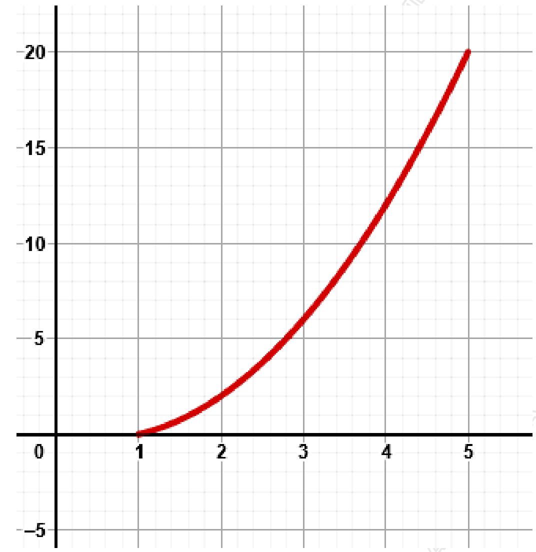
30 $\int_1^3 [4g(x)-3f(x)]\,dx$

Sketch the area corresponding to the integral.

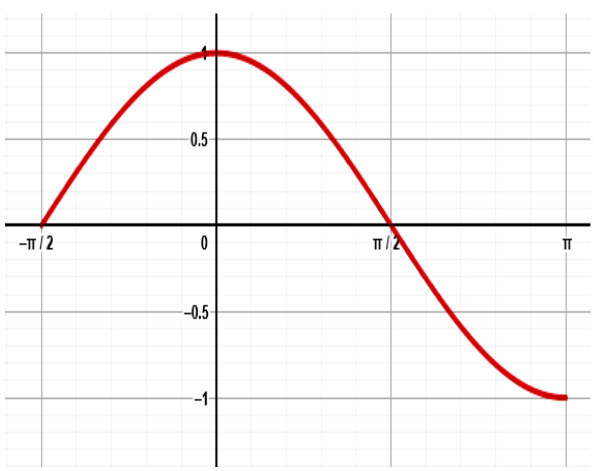
31 $\int_1^2 (x^2 - x)\,dx$



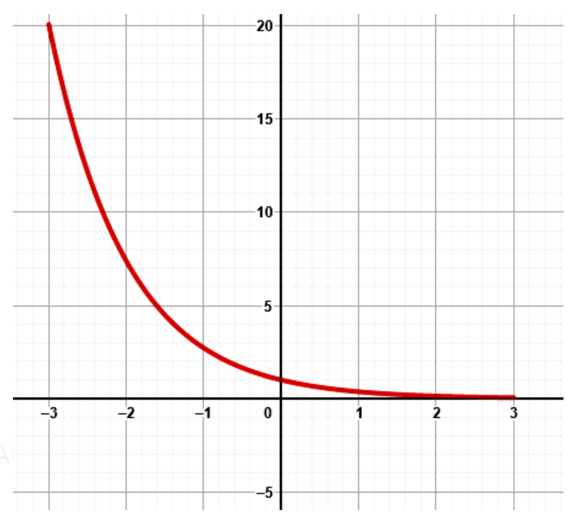
32 $\int_2^4 (x^2 - x)\,dx$



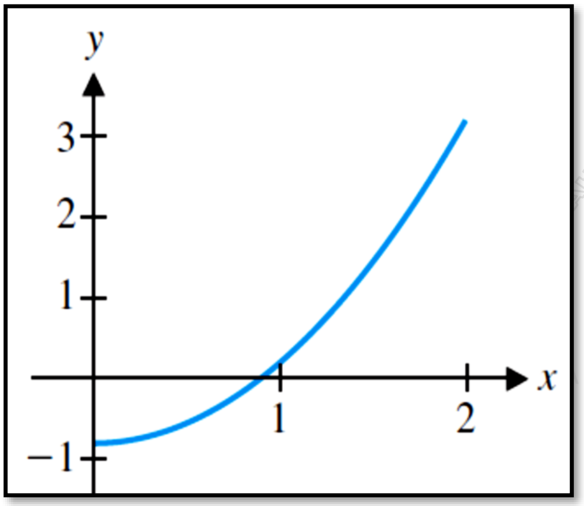
33 $\int_0^{\pi/2} \cos x\,dx$

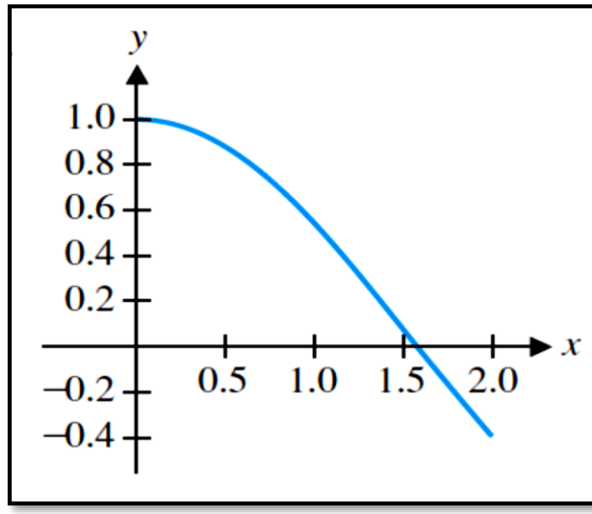


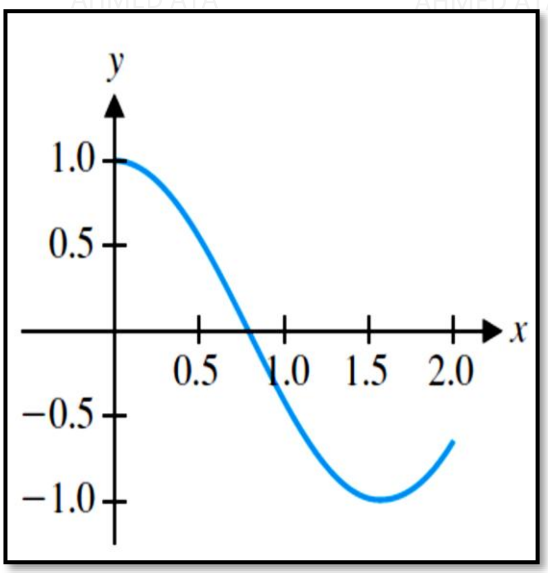
34 $\int_{-2}^{2} e^{-x}\,dx$

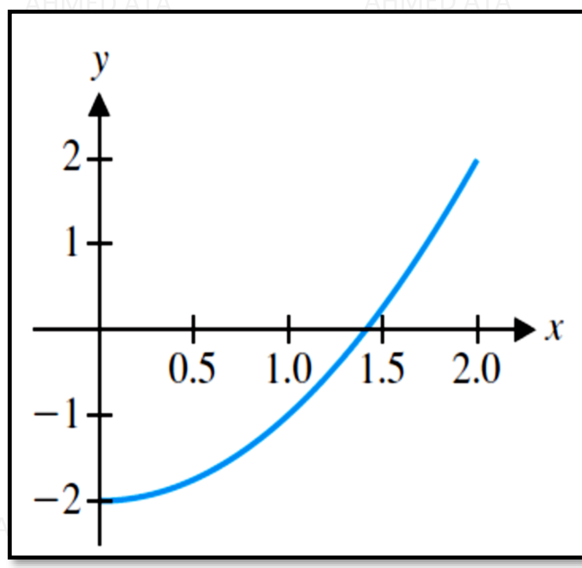


35 Use the graph to determine whether $\int_0^2 f(x)dx$ is positive or negative



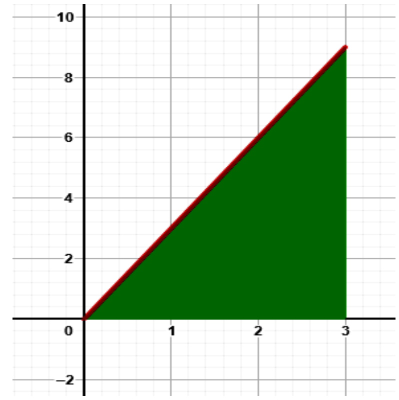




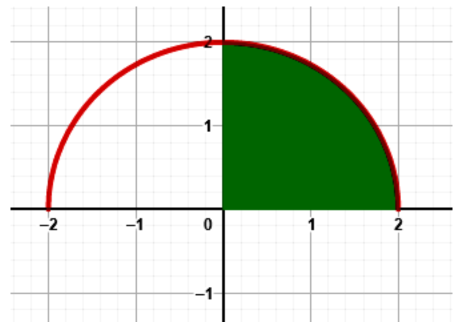


36 **Use a geometric formula to compute the integral.**

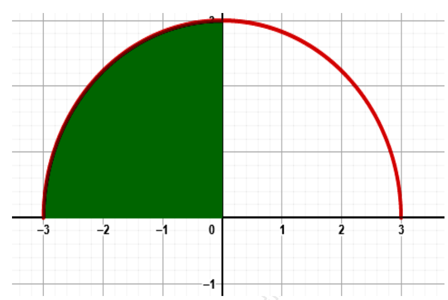
a $\int_0^2 3x\,dx$



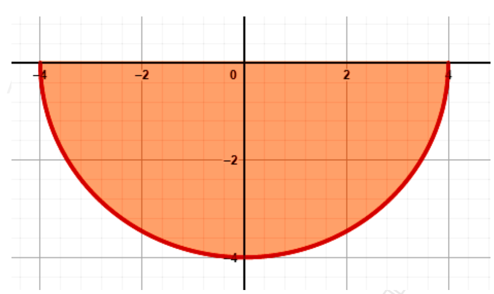
b $\int_0^2 \sqrt{4-x^2}\,dx$



c $\int_{-3}^0 \sqrt{9-x^2}\,dx$



d $-\int_{-4}^4 \sqrt{16-x^2}\,dx$



THE FEATURED PROGRAM EDUCATION

البرنامج المميز

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MATH ENG

LESSON 5-5

The Fundamental Theorem of Calculus

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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Lesson (5-5)

The Fundamental Theorem of Calculus

THEOREM 5.1 **(The Fundamental Theorem of Calculus, Part I)**

If f is continuous on $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a). \tag{5.1}$$

Using the Fundamental Theorem to Compute each integral exactly

1. $\int_0^2 (2x - 3)\,dx$

2. $\int_0^3 (x^2 - 2)\,dx$

3. $\int_{-1}^1 (x^3 + 2x)\,dx$

4 $\int_1^4 \left(\sqrt{x} - \frac{1}{x^2}\right) dx.$

5 $\int_1^4 \left(x\sqrt{x} + \frac{3}{x}\right) dx$

6 $\int_0^1 (6e^{-3x} + 4)\, dx$

7 $\int_{\pi/2}^{\pi} (2\sin x - \cos x)\, dx$

8 $\int_0^{\pi/4} \sec t \tan t \, dt$

9 $\int_{0}^{\pi/4} \sec^2 t \, dt$

10 $\int_{0}^{1/2} \frac{3}{\sqrt{1-x^2}} \, dx$

11 $\int_{-1}^{1} \frac{4}{1+x^2} \, dx$

12 $\int_{1}^{4} \frac{t-3}{t} \, dt$

13 $\int_0^4 t(t-2)\,dt$

14 $\int_0^4 e^{-2x}dx$

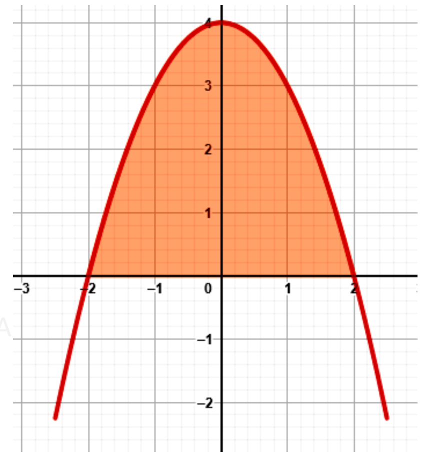
15 $\int_1^x 12t^5 dt.$

16 $\int_0^t (e^{x/2})^2\,dx$

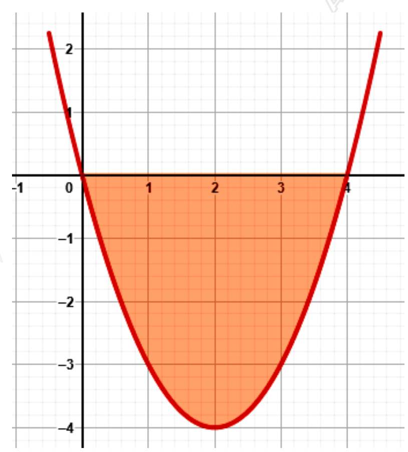
17 $\int_0^t (\sin^2 x + \cos^2 x)\,dx$

Find the given area

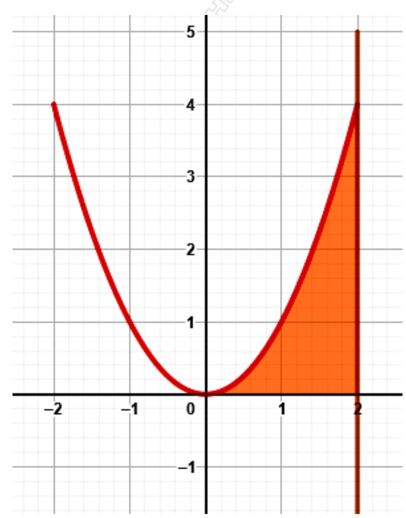
18 The area above the x-axis and below $y = 4 - x^2$



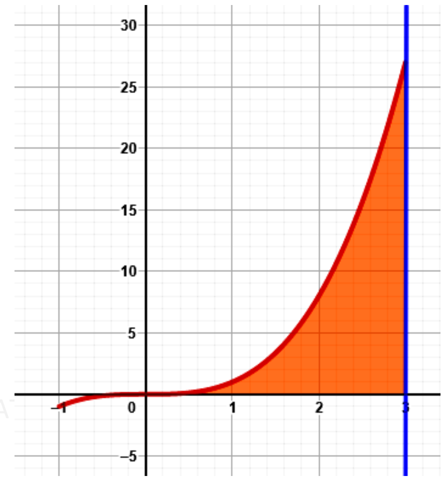
19 The area below the x-axis and above $y = x^2 - 4x$



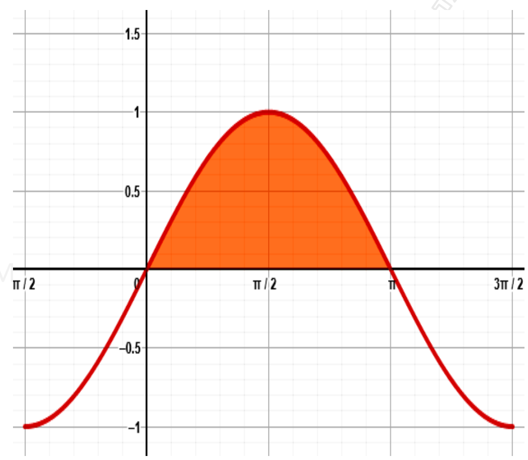
20 The area of the region bounded by $y = x^2, x = 2$ and the $x - axis$



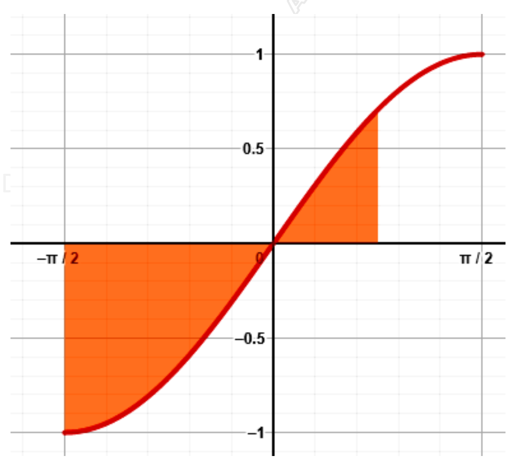
21 The area of the region bounded by $y = x^3, x = 3$ *and the* $x-axis$



22 The area between $y = \sin x$ *and the* $x - axis$ *for* $0 \le x \le \pi$



23 The area between $y = \sin x$ *and the* $x - axis$ *for* $-\frac{\pi}{2} \le x \le \frac{\pi}{4}$



THEOREM 5.2 (The Fundamental Theorem of Calculus, Part II)

If f is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$, on $[a, b]$.

Find the derivative $f'(x)$.

24 $f(x) = \int_0^x (t^2 - 3t + 2)\,dt$

25 $f(x) = \int_2^x (t^2 - 3t - 4)\,dt$

26 $f(x) = \int_0^{x^2} (e^{-t^2} + 1)\,dt$

27 $f(x) = \int_x^2 \sec t\,dt$

28 $f(x) = \int_{e^x}^{2-x} \sin t^2 dt$

29 $f(x) = \int_{2-x}^{xe^x} e^{2t}\,dt$

30 $f(x) = \int_{x^2}^{x^3} \sin(3t)\,dt$

31 $f(x) = \int_{3x}^{\sin x} (t^2 + 4)\,dt$

Find the position function $s(t)$ from the given velocity or acceleration function and initial value(s). Assume that units are feet and seconds

32 $v(t) = 40 - \sin t,\ s(0) = 2$

33 $v(t) = 10e^{-t},\ s(0) = 2$

34 $a(t) = 4 - t,\ v(0) = 8,\ s(0) = 0$

35 $a(t) = 16 - t^2,\ v(0) = 0,\ s(0) = 30$

36 Suppose that the rate of change of water in a storage tank is $f(t) = 10 \sin t$ gallons per minute.

(a) For $0 \leq t \leq 2\pi$, determine when the water level is increasing and when the water level is decreasing.

(b) If the tank has 100 gallons of water at time $t = 0$, determine how many gallons are in the tank at $t = \pi$.

Find an equation of the tangent line at the given value of x.

37 $F(x) = \int_{4}^{x^2} \ln(t^3 + 4)\, dt \qquad at\ x = 2$

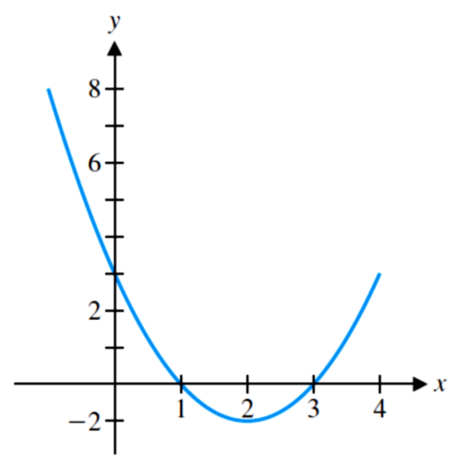
38 $F(x) = \int_{0}^{x} \sin\sqrt{t^2 + \pi^2}\,dt \qquad at\ x = 0$

39 $F(x) = \int_{1}^{x^2} \sqrt{t^2 + 1}\,dt \qquad at\ x = 1$

40 $F(x) = \int_{-1}^{x} \ln(t^2 + 2t + 2)\,dt \qquad at\ x = -1$

41 $F(x) = \int_0^x e^{-t^2+1}dt \qquad at\ x = 0$

42 Use the graph to list $\int_0^1 f(x)dx$; $\int_0^2 f(x)dx$ and $\int_0^3 f(x)dx$ in order, from smallest to largest. For $g(x) = \int_0^x f(t)\,dt$ determine intervals where g is increasing and identify critical points for g.



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MATH ENG

LESSON 5-6

Integration by Substitution

Mr. Ahmed Ata
The Featured Program

2025-2026

البرنامج المميز طريقك للتميز

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Lesson (5-6)

Integration by Substitution

Key Point

To evaluate

$$\int f(g(x))g'(x)\mathrm{d}x$$

substitute $u = g(x)$, and $\mathrm{d}u = g'(x)\mathrm{d}x$ to give

$$\int f(u)\,\mathrm{d}u$$

Integration is then carried out with respect to u, before reverting to the original variable x.

Use the given substitution to evaluate the indicated integral

1. $\int x^2\sqrt{x^3+2}\,dx,\ u = x^3+2$

2. $\int x^3(x^4+1)^{-2/3}dx,\ u = x^4+1$

3 $$\int \frac{(\sqrt{x}+2)^3}{\sqrt{x}}\,dx,\ u=\sqrt{x}+2$$

4 $$\int \sin x \cos x\,dx,\ u=\sin x$$

evaluate the indicated integral.

5 $$\int x^3\sqrt{x^4+3}\,dx$$

6 $\int \frac{\sin x}{\sqrt{\cos x}}\,dx$

7 $\int \sin^3 x \cos x\,dx$

8 $\int t^2 \cos t^3\,dt$

9 $\int xe^{x^2+1}\,dx$

10 $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

11 $\int \frac{\cos(1/x)}{x^2}\,dx$

12 $\int \sec^2 x\sqrt{\tan x}\,dx$

13 $\int \frac{v}{v^2+4}\,dv$

14 $\int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}}\,dx$

15 $\int x^2 \sec^2 x^3 \, dx$

16 $\int \frac{x}{\sqrt{1-x^4}} \, dx$

17 $\int \frac{x^3}{\sqrt{1-x^4}} \, dx$

18 $$\int \frac{x^2}{1+x^6}\,dx$$

19 $$\int \frac{x^5}{1+x^6}\,dx$$

20 $$\int \frac{1+x}{1+x^2}\,dx$$

21 $\int \frac{1+x}{1-x^2}\,dx$

22 $\int \frac{3\sqrt{x}}{1+x^3}\,dx$

23 $\int \frac{2t+3}{t+7}\,dt$

24 $\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx$

25 $\int \frac{t^2}{\sqrt[3]{t+3}}\,dt$

26 $\int_0^2 x\sqrt{x^2+1}\,dx$

27 $\int_{1}^{3} x\sin(\pi x^2)\,dx$

28 $\int_{-1}^{1} \frac{t}{(t^2+1)^2}\,dt$

29 $\int_{0}^{2} \frac{e^x}{1+e^{2x}}\,dx$

30 $\int_{\pi/4}^{\pi/2} \cot x \, dx$

31 $\int_{1}^{e} \frac{\ln x}{x} \, dx$

32 $\int_{1}^{4} \frac{x-1}{\sqrt{x}} \, dx$

33 $\int_0^1 \frac{x}{\sqrt{x^2+1}}\,dx$

34 $\int_0^1 (e^x-2)^2\,dx$

35 $\int_0^{10} (1-e^{-t/4})\,dt$