



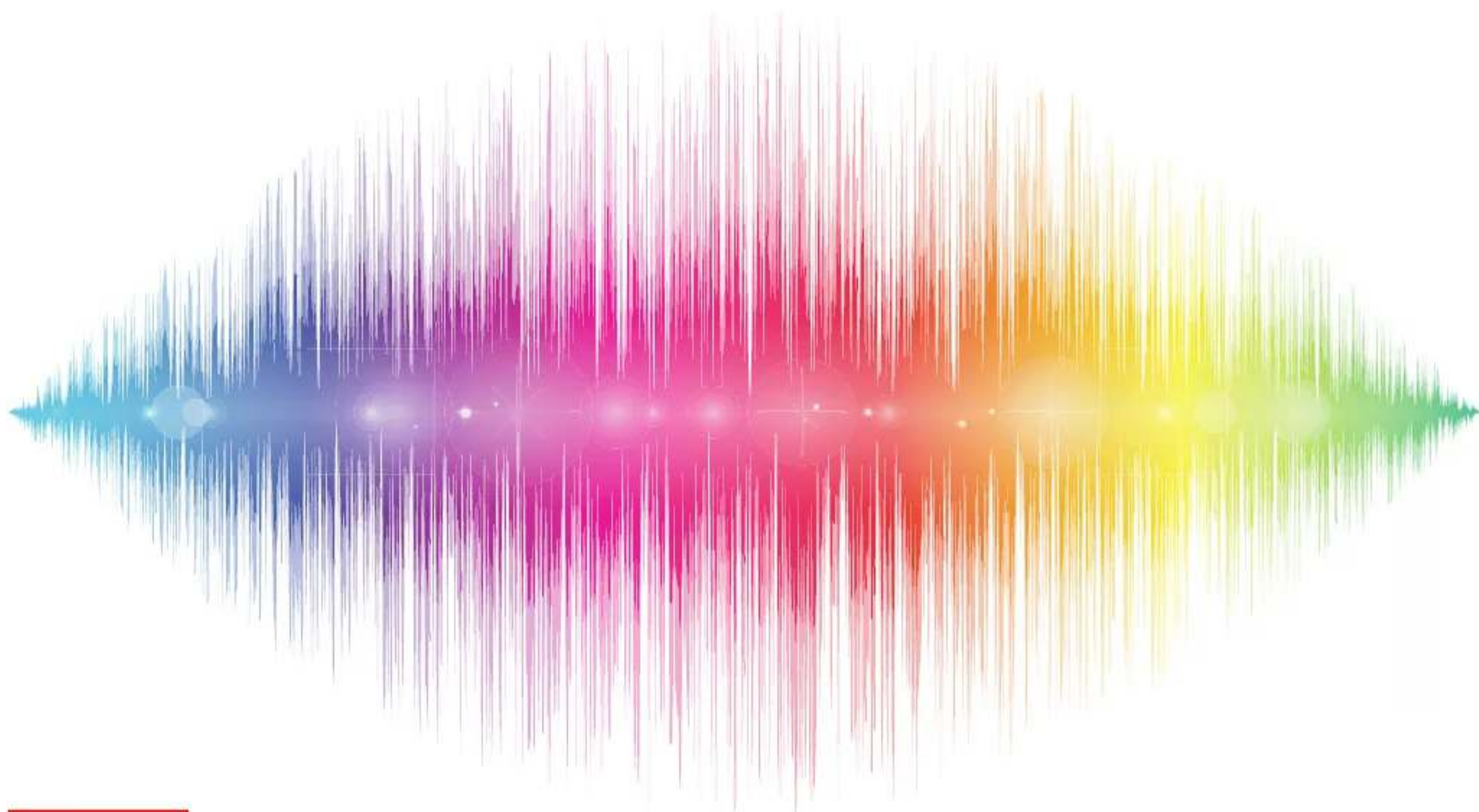
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Inspire Physics

UAE Edition
Grade 11 Advanced
Student Edition



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Physics

United Arab Emirates Edition

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Physics

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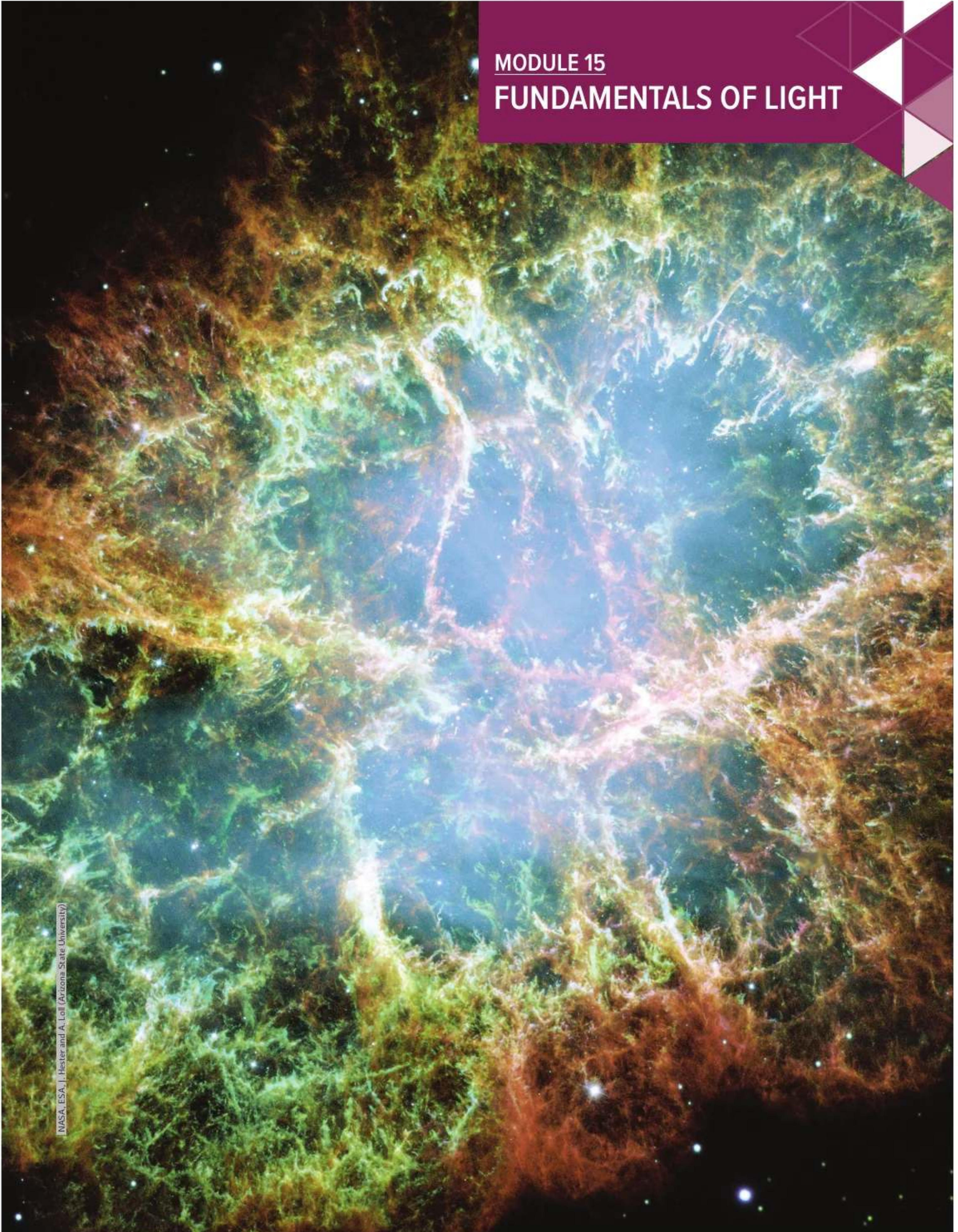
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MODULE 15 FUNDAMENTALS OF LIGHT

NASA, ESA, J. Hester and A. Loll (Arizona State University)



MODULE 15

FUNDAMENTALS OF LIGHT

ENCOUNTER THE PHENOMENON

What does the light from a distant star or supernova tell us about it?



 **GO ONLINE** to play a video about measuring the speed of light.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about what light from a distant star or supernova tells us about it. Explain your reasoning.

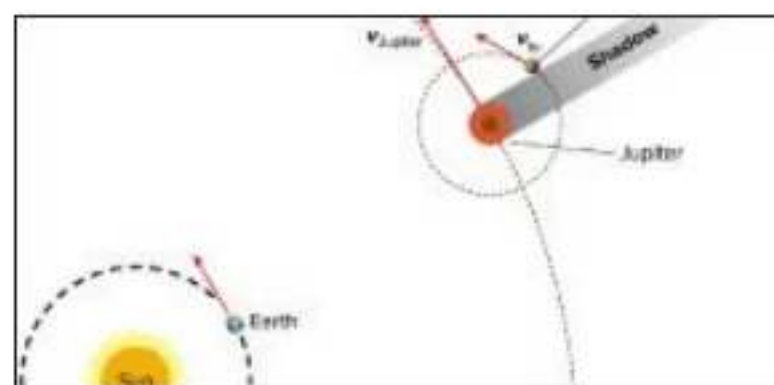
Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

 **GO ONLINE** to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain:
How Light Travels



LESSON 2: Explore & Explain:
Speed of Light



Smithsonian

Additional Resources

LESSON 1 ILLUMINATION

FOCUS QUESTION

How does distance affect how bright a star appears?

Light

Light is electromagnetic radiation, like radio and microwaves. It can be modeled as a wave of changing electric and magnetic fields or as particles called photons. The wave model is useful for explaining many features of electromagnetic radiation, and the particle model explains other features. In this module, you will learn about the wave properties of light. You will learn about photons in a later module.

Light's path How does your body receive information? Many people respond to this question with the five senses, starting with sight and hearing. The sense of sight depends on light from your surroundings reaching your eyes. Did you ever wonder how light travels? Think of how a



Dwight Nadiy/E+/Getty Images

narrow beam of light, such as that of a flashlight or sunlight streaming through a small window, is made visible by dust particles in the air. You see the path of the light as a straight line. When your body blocks sunlight, you see your outline in a shadow, a result of light's straight path.

Figure 1 depicts light's straight path.

Figure 1 Light rays traveling in straight lines are evident in many situations.

Explain how this photo demonstrates the wave properties of light.



3D THINKING

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.



DCI Disciplinary Core Ideas

INVESTIGATE

GO ONLINE to find these activities and more resources.



Quick Investigation: The Speed of Light

Use the relationship among the frequency, wavelength, and speed of a wave to calculate the speed of light.

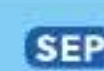


Probeware Lab: Light Intensity and Distance

Analyze data to determine how distance from the source affects the intensity of light.



CCC Crosscutting Concepts



SEP Science & Engineering Practices

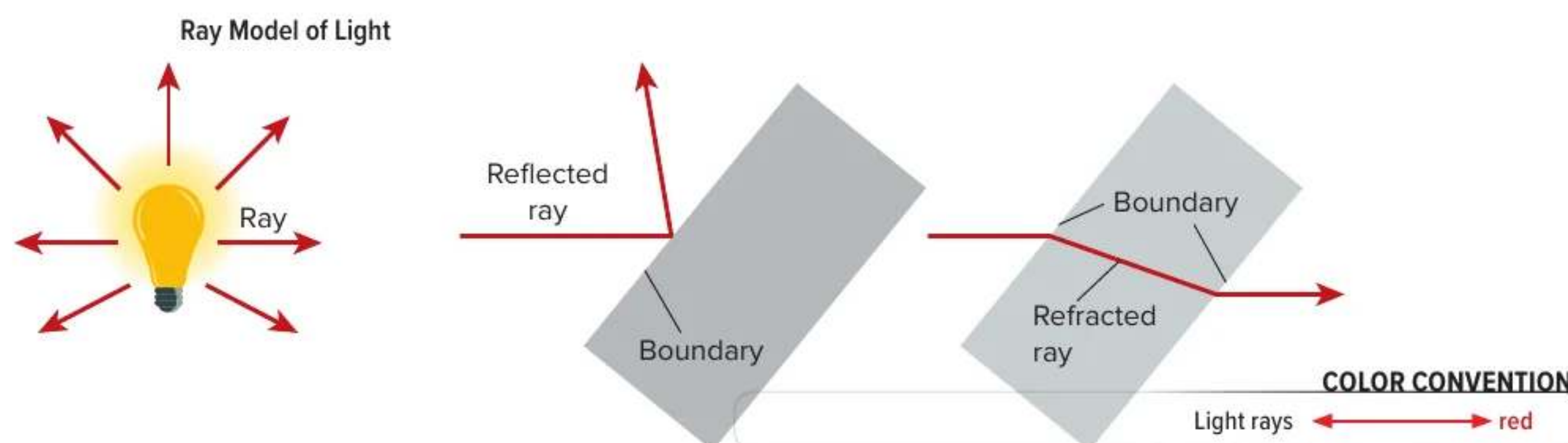


Figure 2 Light's straight-line path is demonstrated by the ray model. Light rays change direction when they are reflected or refracted by matter. In either case, light rays continue in a straight path.

Ray Model of Light

Imagine being in an empty, dark room when a small lightbulb is turned on in the center of the room. You can see around the room, and you can look at the bulb and see it. This must mean the bulb sends light in all directions. You could visualize the light coming from the bulb as an infinite number of arrows traveling straight away from the bulb in all directions. Each arrow represents a ray of light, which travels in a straight path until it reaches a boundary, as shown in **Figure 2**.

After interacting at a boundary, the ray still moves in a straight line, but its direction is changed. These basic principles—that light travels in straight lines and that its direction can be changed by encountering a boundary—constitute the **ray model of light**. The study of light interacting with matter is called ray optics or geometric optics.

Sources of light What is the difference between sunlight and moonlight? For one, sunlight is much brighter. Another important fundamental difference is that the Sun produces and emits its own light, while the Moon is only visible because it reflects the Sun's light. Everything you see fits into one of these two categories. Objects such as the Sun and other stars that emit their own light are **luminous sources**, while those that you see due to light reflecting from them are illuminated sources, like the Moon and planets.

Luminous sources include natural sources such as flames and fireflies and human-made devices such as television screens, computer monitors, lasers, and tiny, light-emitting diodes. In **Figure 3**, the luminous source is fluorescent bulbs. They produce light from electrical energy. The other objects in the room are illuminated when the light from the bulbs is reflected off of them. In a room with no light, it would be impossible to see anything, because there is no light reflecting off of objects into your eyes.



Figure 3 Objects in the room are visible because of reflected light.

Recognize What, if anything, would be visible if the room had no luminous source? Explain.



Figure 4 Light is transmitted through the transparent and translucent candle holders. The candle cannot be seen clearly in the opaque holder because the light is absorbed.

Light and matter Objects can absorb, reflect, or transmit light. Objects that reflect and absorb light but do not transmit it are **opaque**. Many common objects—such as books, people, and backpacks—are opaque. Mediums that transmit and reflect light but do not allow objects to be seen clearly through them are **translucent** mediums. The frosted glass in **Figure 4** is a translucent medium. A **transparent** medium, such as air or glass, transmits most of the light that reaches it. **Figure 4** illustrates objects that are opaque, translucent, and transparent. Transparent mediums transmit light, but they also often reflect some light. For example, you can see the glass of the transparent candle holder. This is possible because light is reflected off the glass.

All three types of objects also absorb some light. There are various factors that determine how much light will be absorbed, but opaque objects usually absorb a greater portion of light than translucent or transparent objects.



Get It?

Explain why it is possible to see a fish through a glass fishbowl and also see the glass of the bowl.

Quantity of Light

If you were to have a flashlight shone at you from across the room, what factors would determine how bright that light would appear to you? Three main factors determine the brightness: the quantity of light the flashlight produces, the distance between the lightbulb and your eye, and the angle at which the light rays hit your eye. In this lesson, you will read about the first two of these factors.

Luminous flux With the ray model of light, a source that is brighter produces more light rays than a less bright source. Imagine again a single lightbulb sending rays in nearly all directions. How could you capture all the light it emits? You would need to construct a surface that completely encloses the bulb, as in **Figure 5**. The rate at which the bulb, a luminous source, produces light energy is called the **luminous flux** (P) and is measured in lumens (lm). The total amount of light that strikes the surface in a given unit of time depends only on the luminous flux of the source.

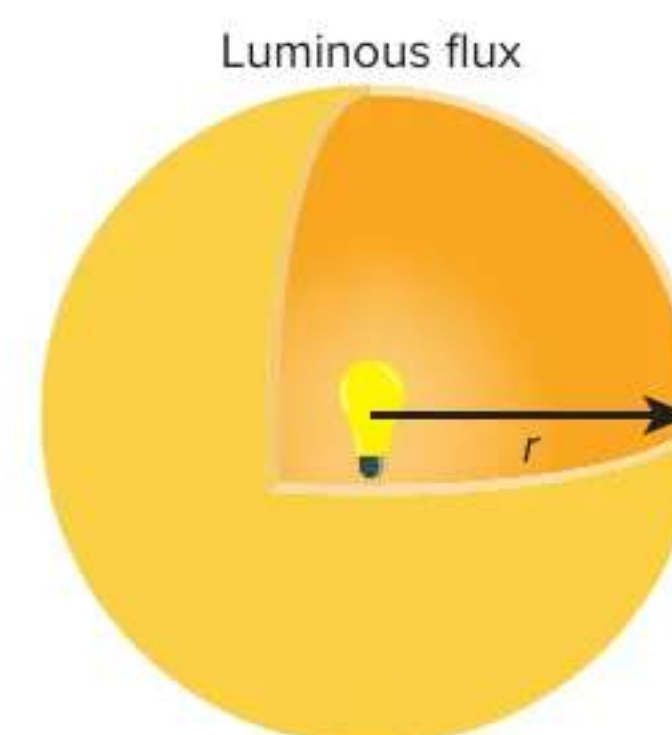


Figure 5 Luminous flux is the rate at which light rays are emitted from a luminous source.

Illuminance Once you know the quantity of light being emitted by a luminous source, you can determine the amount of illumination the source provides to an object, such as a book. The luminous flux falling on a given surface area at any instant is called **illuminance** (E). It is measured in lux (lx), which is equivalent to lumens per square meter (lm/m^2). In this module, we assume, for simplification, that all light sources are point sources.

Consider the setup shown in **Figure 6**. The luminous flux of the source is 1750 lm (typical of a 26-W compact fluorescent bulb). What is the illuminance of the sphere's inside surface at $r = 1$ m? Because all the bulb's luminous flux strikes the surface, divide the luminous flux by the surface area of the sphere, $4\pi r^2$. The surface area is $4\pi(1.00 \text{ m})^2 = 4\pi \text{ m}^2$, so the illuminance is $\frac{1750 \text{ lm}}{4\pi \text{ m}^2} = 139 \text{ lx}$. This mathematical relationship means that at a distance of 1.00 m from the bulb, 139 lm strikes each square meter.

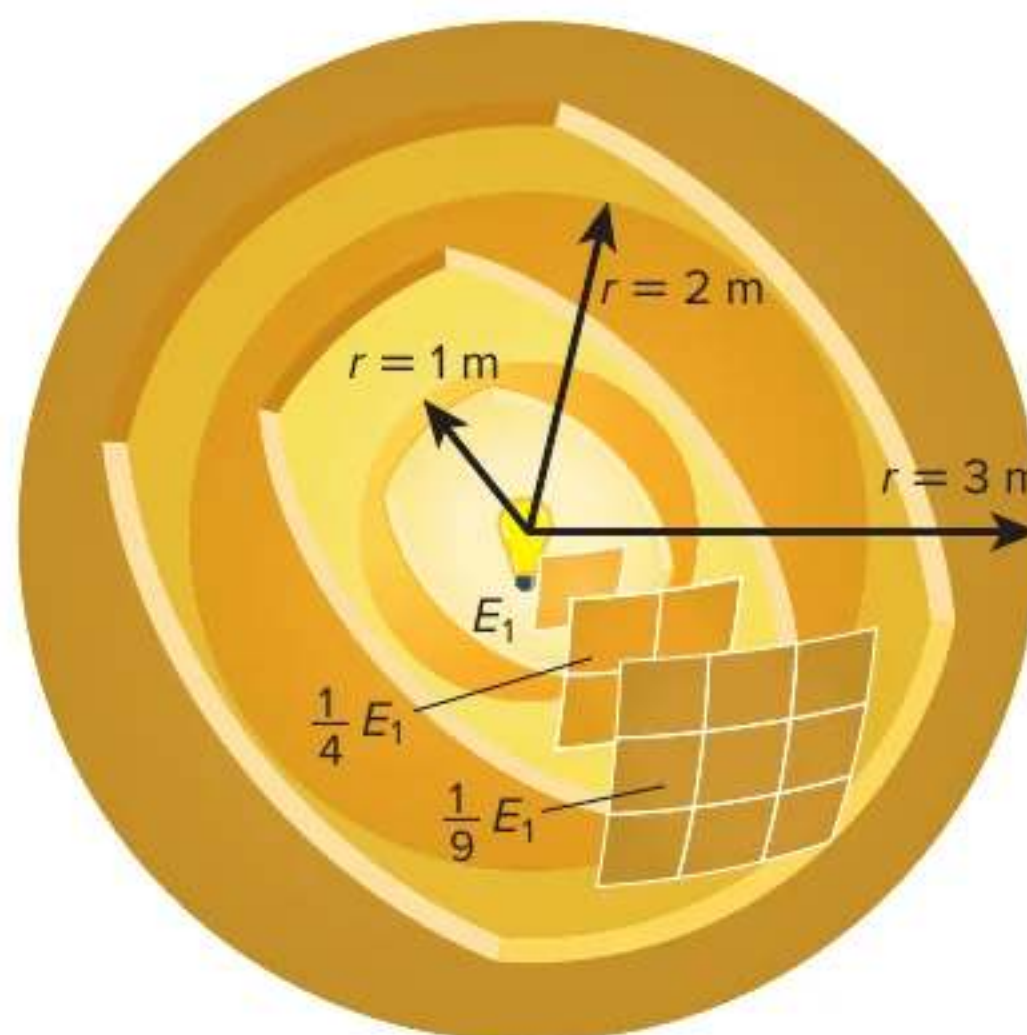


Figure 6 Illuminance (E) is the quantity of light that strikes a surface. As the distance from the luminous source (r) increases, E decreases. E depends on the inverse of r squared.

Illuminance

$$E_1 = \frac{1750}{4\pi} \text{ lx}$$

$$E_2 = \frac{1750}{4\pi 2^2} \text{ lx}$$

$$E_3 = \frac{1750}{4\pi 3^2} \text{ lx}$$



Get It?

Define *illuminance*, and state the units that are used for illuminance.

Inverse-square relationship What if the sphere surrounding the lamp were larger? If the sphere's radius were 2.00 m, the luminous flux still would total 1750 lm because it only depends on the bulb. With a radius of 2.00 m, however, the area of the sphere would now be equal to $4\pi(2.00 \text{ m})^2 = 16.0\pi \text{ m}^2$. The new area is four times larger than that of the 1.00-m sphere, as shown in **Figure 6**. The illuminance of the inside of the 2.00-m sphere is $\frac{1750 \text{ lm}}{(16.0\pi \text{ m}^2)} = 34.8 \text{ lx}$, so 34.8 lm strikes each square meter.

The illuminance on the inside surface of the 2.00-m sphere (E_2) is one-fourth the illuminance on the inside of the 1.00-m sphere. In the same way, the inside of a sphere with a 3.00-m radius has an illuminance only one-ninth $\left[\left(\frac{1}{3}\right)^2\right]$ as large as that of the 1.00-m sphere. **Figure 6** shows that the illuminance produced by a point source is proportional to $\frac{1}{r^2}$: an inverse-square relationship. In the case of the 3.00-m radius, only 15.5 lm strike each square meter inside the sphere. As the light rays spread out in straight lines in all directions from a point source, the number of light rays that illuminate a unit of area decreases as the square of the distance from the point source.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Write a paragraph explaining how mathematical models were used in the study of light. Include information about the system and system boundaries that scientists used.

CONNECTING MATH to Physics

Direct and Inverse Relationships The illuminance provided by a source of light has both a direct and an inverse relationship.

Math	Physics
$y = \frac{x}{az^2}$	$E = \frac{P}{4\pi r^2}$
If z is constant, then y is directly proportional to x . - When x increases, y increases. - When x decreases, y decreases.	If r is constant, then E is directly proportional to P . - When P increases, E increases. - When P decreases, E decreases.
If x is constant, then y is inversely proportional to z^2 . - When z^2 increases, y decreases. - When z^2 decreases, y increases.	If P is constant, then E is inversely proportional to r^2 . - When r^2 increases, E decreases. - When r^2 decreases, E increases.

Luminous intensity Some luminous sources are specified in candelas (cd). A candela is not a measure of luminous flux but of luminous intensity. The luminous intensity of a point source is the luminous flux that falls on 1 m^2 of the inside of a 1-m-radius sphere, so luminous intensity is luminous flux divided by 4π . A bulb with 1750 lm of flux has an intensity of $\frac{1750 \text{ lm}}{4\pi} = 139 \text{ cd}$.

In **Figure 7**, the lightbulb is twice as far away from the screen as the candle. For the bulb to provide the same illuminance on its side of the screen as the candle does on the candle side of the screen, the bulb would have to be four times brighter than the candle. The lightbulb's luminous intensity, therefore, would have to be four times the candle's luminous intensity. If both sources in **Figure 7** had the same luminous intensity, the source at $2r$ would only provide one-quarter the illuminance to the screen. This is consistent with the inverse-square relationship we just developed.

**Get It?**

Describe what luminous intensity is a measure of and what its relationship is to illuminance.

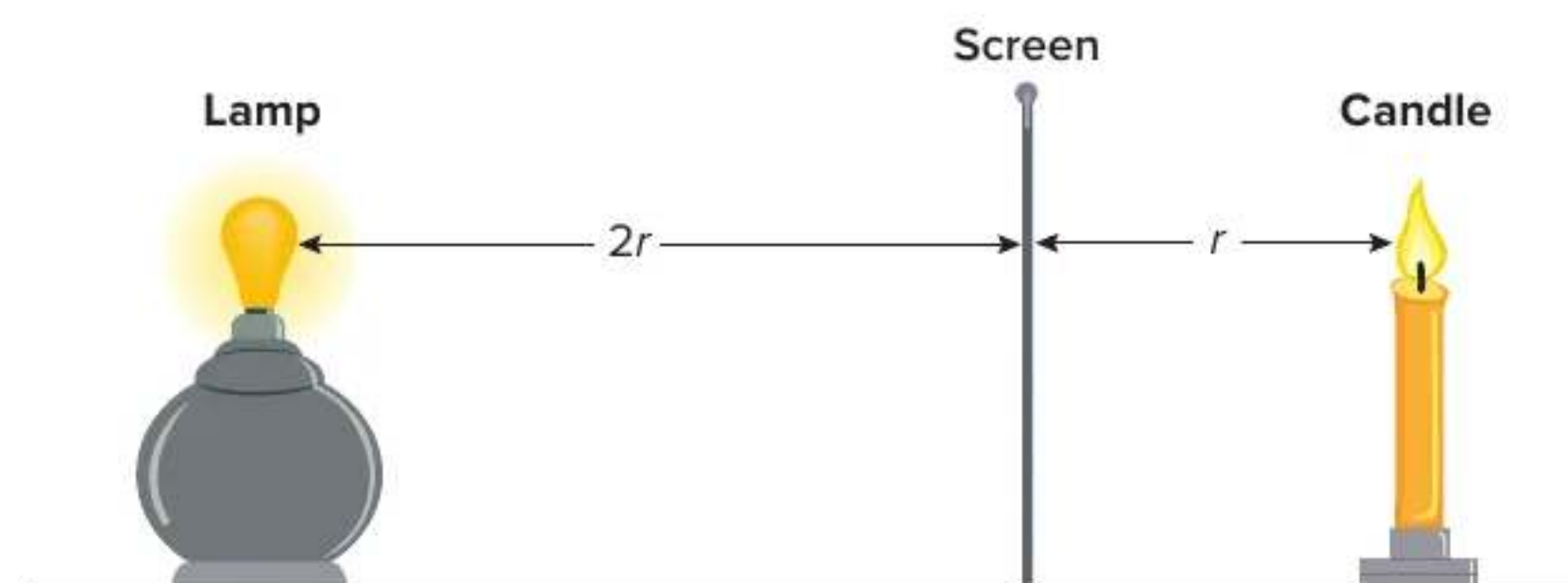


Figure 7 For the lightbulb and the candle to provide the same illuminance to the screen, the luminous intensity of the lightbulb is four times that of the candle.

Surface Illumination

Think again about the scenario in which a flashlight is shining at you from across the room. If the bulb has a small luminous intensity, the light will not be very bright. To increase the brightness, you could use a brighter bulb, thereby increasing the luminous flux, or you could move so that your eyes are closer to the light, decreasing the distance between the light source and your eyes. Following the simplification that we are treating all light sources as point sources, the illuminance and distance will follow the inverse-square relationship. In this case, and in all the cases we will deal with in this book, the illuminance caused by a point light source is represented by the following equation.

Real-World Physics

Illuminated Minds

When deciding how to achieve the correct illuminance on students' desktops in a classroom, architects must consider the luminous flux of the lights as well as the distance of the lights above the desktops. In addition, the efficiencies of the light sources are an important economic factor.

Point-Source Illuminance

If an object is illuminated by a point source of light, then the illuminance at the object is equal to the luminous flux of the light source divided by the surface area of the sphere whose radius is equal to the distance the object is from the light source.

$$E = \frac{P}{4\pi r^2}$$

Remember that the luminous flux of the light source is spreading out in all directions, so only some fraction of the luminous flux is available to illuminate the object. Use of this equation is valid only if the light from the luminous source strikes perpendicular to the surface it is illuminating. It is also only valid if the luminous source is small enough or far enough away to be considered a point source. Thus, the equation does not give accurate values of illuminance for long fluorescent lamps or lightbulbs that are close to the surfaces they illuminate.

Engineers who design lighting systems must understand how the light will be used. If an even illumination is needed to prevent dark areas, the common practice is to evenly space normal lights over the area to be illuminated, as was most likely done with the lights in your classroom. Because such light sources do not produce truly uniform light, however, engineers also design special light sources that control the spread of the light, such that they produce even illuminations over large surface areas. For safety reasons, this is extremely important for automobile headlights, as in **Figure 8**. Automobile engineers must consider these factors when designing headlights.

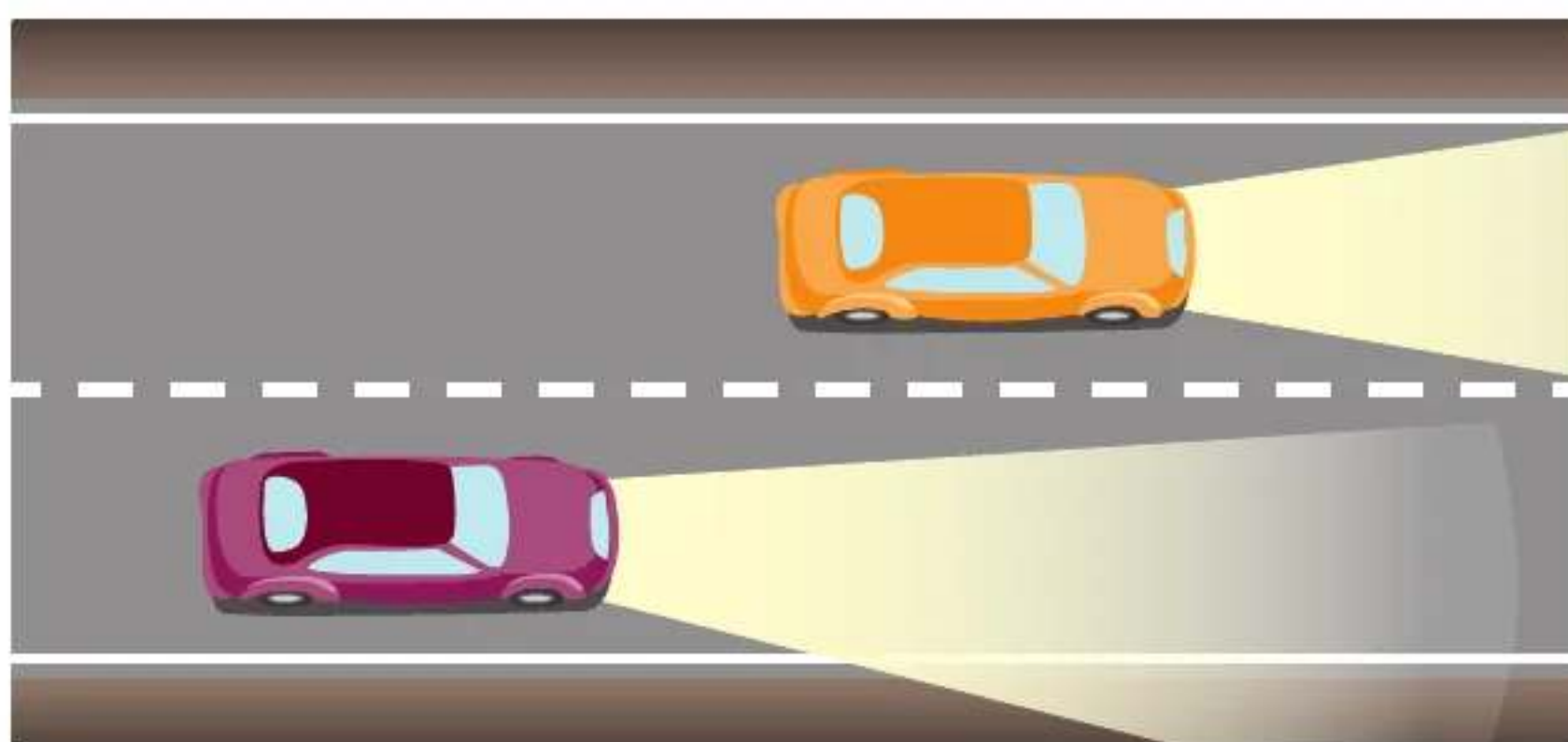


Figure 8 Dark areas can occur if headlights on cars are not set at the correct angles to adequately illuminate the road.

EXAMPLE Problem 1

ILLUMINATION OF A SURFACE What is the illuminance on your desktop if it is lit by a 1750-lm lamp that is 2.50 m above your desk?

1 ANALYZE AND SKETCH THE PROBLEM

- Assume the lightbulb is the point source.
- Diagram the position of the bulb and the desktop. Label P and r .

Known

$$P = 1.75 \times 10^3 \text{ lm}$$

$$r = 2.50 \text{ m}$$

Unknown

$$E = ?$$

2 SOLVE FOR THE UNKNOWN

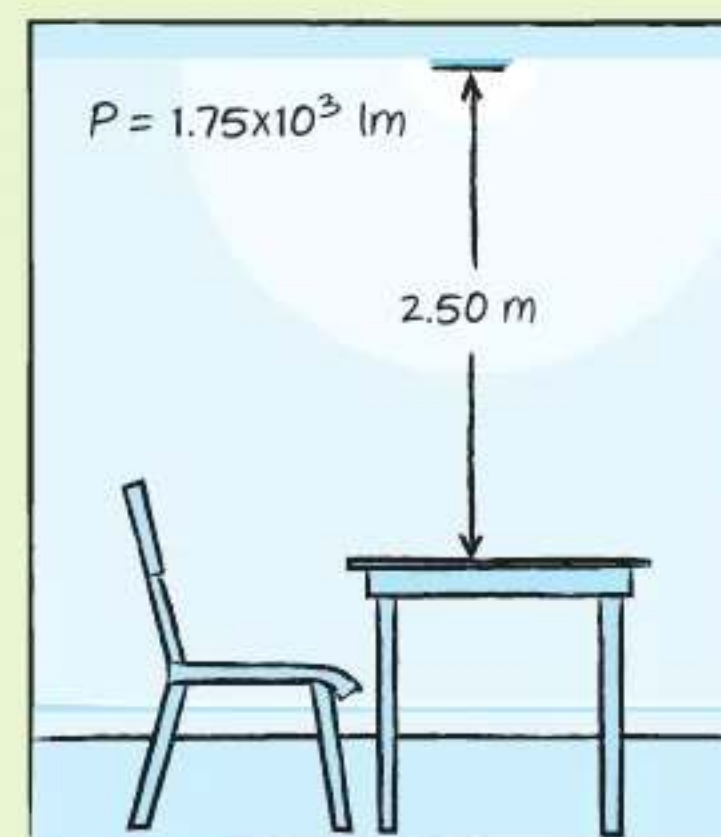
The surface is perpendicular to the direction in which the light ray is traveling, so you can use the point-source illuminance equation.

$$E = \frac{P}{4\pi r^2}$$

$$= \frac{1.75 \times 10^3 \text{ lm}}{4\pi(2.50 \text{ m})^2}$$

$$= 22.3 \text{ lm/m}^2$$

$$= 22.3 \text{ lx}$$

Substitute $P = 1.75 \times 10^3 \text{ lm}$, $r = 2.50 \text{ m}$ **3 EVALUATE THE ANSWER**

- **Are the units correct?** The units of luminance are $\text{lm/m}^2 = \text{lx}$, which the answer agrees with.
- **Do the signs make sense?** All quantities are positive, as they should be.
- **Is the magnitude realistic?** Illuminance from an 1800-lm lamp at a distance of 2 m is about 20 lx.

PRACTICE Problems**ADDITIONAL PRACTICE**

1. A lamp is moved from 30 cm to 90 cm above the pages of a book. Compare the illumination on the book before and after the lamp is moved.
2. Draw a graph of the illuminance produced by a lamp with a luminous flux of 2275 lm at distances from 0.50 m and 5.0 m.
3. A 64-cd point source of light is 3.0 m away from a painting. What is the illumination on the painting in lux?
4. A screen is placed between two lamps so that they illuminate the screen equally, as shown in **Figure 9**. The first lamp emits a luminous flux of 1445 lm and is 2.5 m from the screen. What is the distance of the second lamp from the screen if the luminous flux is 2375 lm?

**Figure 9**

5. What is the illumination on a surface that is 3.0 m below a 150-W incandescent lamp that emits a luminous flux of 2275 lm?
6. A public school law requires a minimum illuminance of 160 lx at the surface of each student's desk. An architect's specifications call for classroom lights to be located 2.0 m above the desks. What is the minimum luminous flux that the lights must produce?
7. **CHALLENGE** Your local public library is planning to remodel the computer lab. The contractors have purchased fluorescent lamps with a rated luminous flux of 1750 lm. The desired illumination on the keyboard surfaces is 175 lx. Assume a single lamp illuminates each keyboard. What distance above the surface should the lights be placed to achieve the desired illumination? If the contractors had also already purchased fixtures to hold the lights that when installed would be 1.5 m above the keyboard surface, would the desired illuminance be achieved? If not, would the illuminance be greater or less than desired? What change in the lamp's luminous flux would be required to achieve the desired illuminance?

The Speed of Light

Arguments that light must travel at a finite speed have existed for more than 2400 years. By the seventeenth century, several scientists had performed experiments that supported the view that light travels at a finite speed, but that this speed is much faster than the speed of sound.

Actually measuring the speed of light was not an easy task in the seventeenth century. As you know from studying motion, if you can measure the time light takes to travel a certain distance, you can calculate the speed of light. However, the time that it takes light to travel between objects on Earth is much shorter than a human's reaction time. How could a seventeenth-century scientist solve this problem?

Clues from Io Danish astronomer Ole Roemer was the first to measure the time it took for light to travel between two points with any success. Between 1668 and 1674, Roemer made 70 measurements of the 1.8-day orbital period of Io, one of Jupiter's moons. He recorded the times when Io emerged from Jupiter's shadow, as shown in **Figure 10**. He made his measurements as part of a project to improve maps by calculating the longitude of locations on Earth. This is an early example of the needs of technology driving scientific advances.

After making many measurements, Roemer was able to predict when the next eclipse of Io would occur. He compared his predictions with the actual measured times and found that Io's observed orbital period increased on average by about 13 s per orbit when Earth was moving away from Jupiter and decreased on average by about 13 s per orbit when Earth was approaching Jupiter. Roemer believed that Jupiter's moons were just as regular in their orbits as Earth's moon; thus, he wondered what might cause this discrepancy in the measurement of Io's orbital period. He considered another variable within the system, the movement and position of Earth relative to Jupiter.

The Eclipse of Io

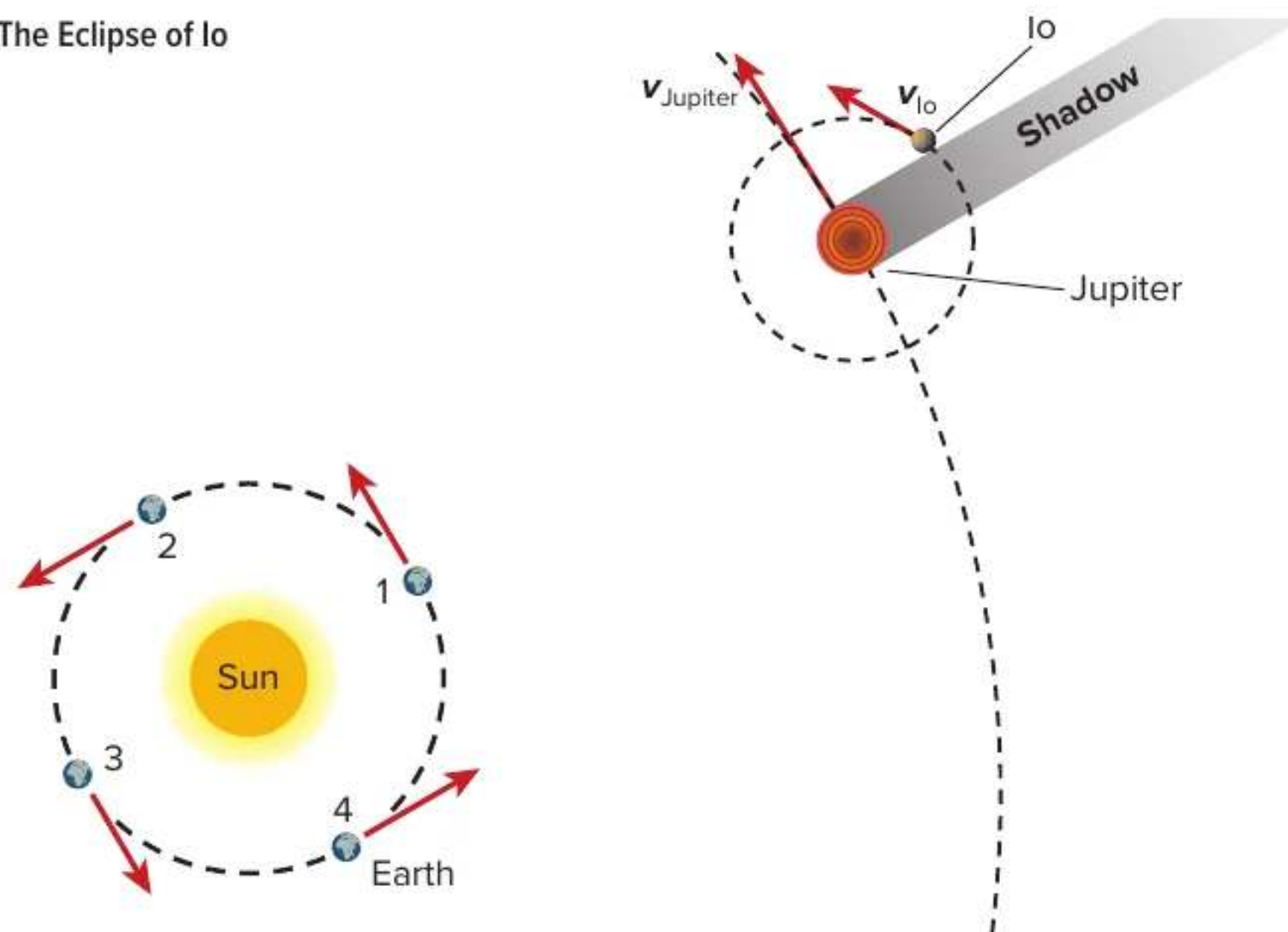


Figure 10 As Earth approaches Jupiter, the light reflected from Io takes less time to reach Earth than when Earth moves away from Jupiter. (Illustration is not to scale.)

Measuring the speed of light Roemer concluded that as Earth moved away from Jupiter, the light from each new appearance of Io took longer to reach Earth because it traveled farther. As Earth moved toward Jupiter, Io's orbital period seemed to decrease. During the 182.5 days it took for Earth to travel from position 1 to position 3, shown in **Figure 10**, there were (182.5 days) (1 Io eclipse/1.8 days) = 1.0×10^2 Io eclipses. Thus, for light to travel the diameter of Earth's orbit, he calculated that it takes $(1.0 \times 10^2 \text{ eclipses})(13 \text{ s/eclipse}) = 1.3 \times 10^3 \text{ s}$, or 22 min.

Using the presently known value of the diameter of Earth's orbit ($2.93 \times 10^{11} \text{ m}$), Roemer's value of 22 min gives a value for the speed of light of $2.9 \times 10^{11} \text{ m}/((22 \text{ min})(60 \text{ s/min})) = 2.2 \times 10^8 \text{ m/s}$. Today, the speed of light is known to be closer to $3.0 \times 10^8 \text{ m/s}$, so light takes 16.5 min, not 22 min, to cross Earth's orbit. Nevertheless, Roemer had successfully proved that light travels at a finite speed.

Michelson's measurements Although many measurements of the speed of light have been made, the most notable were performed by American physicist Albert A. Michelson. Between 1880 and the 1920s, he developed Earth-based techniques to measure the speed of light. In 1926 Michelson measured the time required for light to make a round trip between two California mountains 35 km apart. Michelson's best result was $(2.99796 \pm 0.00004) \times 10^8 \text{ m/s}$. For this work, he became the first American to receive a Nobel Prize in science.

The speed of light in a vacuum has its own special symbol, c . The International Committee on Weights and Measurements has measured and defined the speed of light in a vacuum to be $c = 299,792,458 \text{ m/s}$. For many calculations, the value $c = 3.00 \times 10^8 \text{ m/s}$ is precise enough. At this speed, light travels $9.46 \times 10^{12} \text{ km}$ in a year. This distance is called a light-year.

Check Your Progress

8. **Light** What evidence have you observed that light travels in a straight line?
9. **Light Properties** Why might you choose a window shade that is translucent? Opaque?
10. **Illuminance** Does one lightbulb provide more or less illuminance than two identical lightbulbs at twice the distance? Explain.
11. **Luminous Intensity** Two lamps illuminate a screen equally from distances shown in **Figure 11**. If Lamp A is rated 75 cd, what is Lamp B rated?
12. **Distance of a Light Source** A lightbulb illuminating your computer keyboard provides only half the illuminance that it should. If it is currently 1.0 m away, how far should it be to provide the correct illuminance?
13. **Light and Sound Travel** How far does light travel in the time it takes sound to travel 1 cm in air at 20°C ?
14. **Distance of Light Travel** The distance to the Moon can be found with the help of mirrors left on the Moon by astronauts. A pulse of light is sent to the Moon and returns to Earth in 2.562 s. Using the defined value for the speed of light to the same precision, calculate the distance from Earth to the Moon.
15. **Critical Thinking** The correct time taken for light to cross Earth's orbit is 16.5 min, and the diameter of Earth's orbit is $2.98 \times 10^{11} \text{ m}$. Calculate the speed of light using Roemer's method. Does this method appear to be accurate? Why or why not?

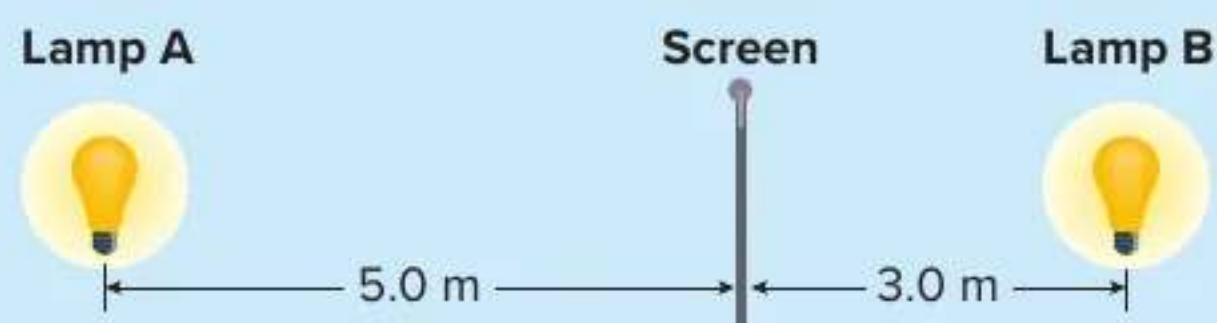


Figure 11

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Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

LESSON 2

THE WAVE NATURE OF LIGHT

FOCUS QUESTION

How do scientists use the Doppler shift to determine how stars and galaxies are moving?

Diffraction and the Wave Model

In 1665 Italian scientist Francesco Maria Grimaldi observed that the edges of shadows are not perfectly sharp. He introduced a narrow beam of light into a dark room and held a rod in front of the light such that it cast a shadow on a white surface. The shadow cast by the rod was wider than the shadow should have been if light traveled in a straight line past the edges of the rod. Grimaldi also noted that the shadow was bordered by colored bands. He determined that both of these observations could be explained if light bent slightly. He called the bending of light as it passes the edge of a barrier **diffraction**.

Huygens' principle In 1678 Dutch scientist Christiaan Huygens used a wave model to explain diffraction. According to Huygens' principle, all the points of a wavefront of light can be thought of as new sources of smaller waves.

These smaller waves, or wavelets, expand in every direction and are in step with one another. A flat, or plane, wavefront of light consists of an infinite number of point sources in a line. **Figure 12** illustrates Huygen's principle.

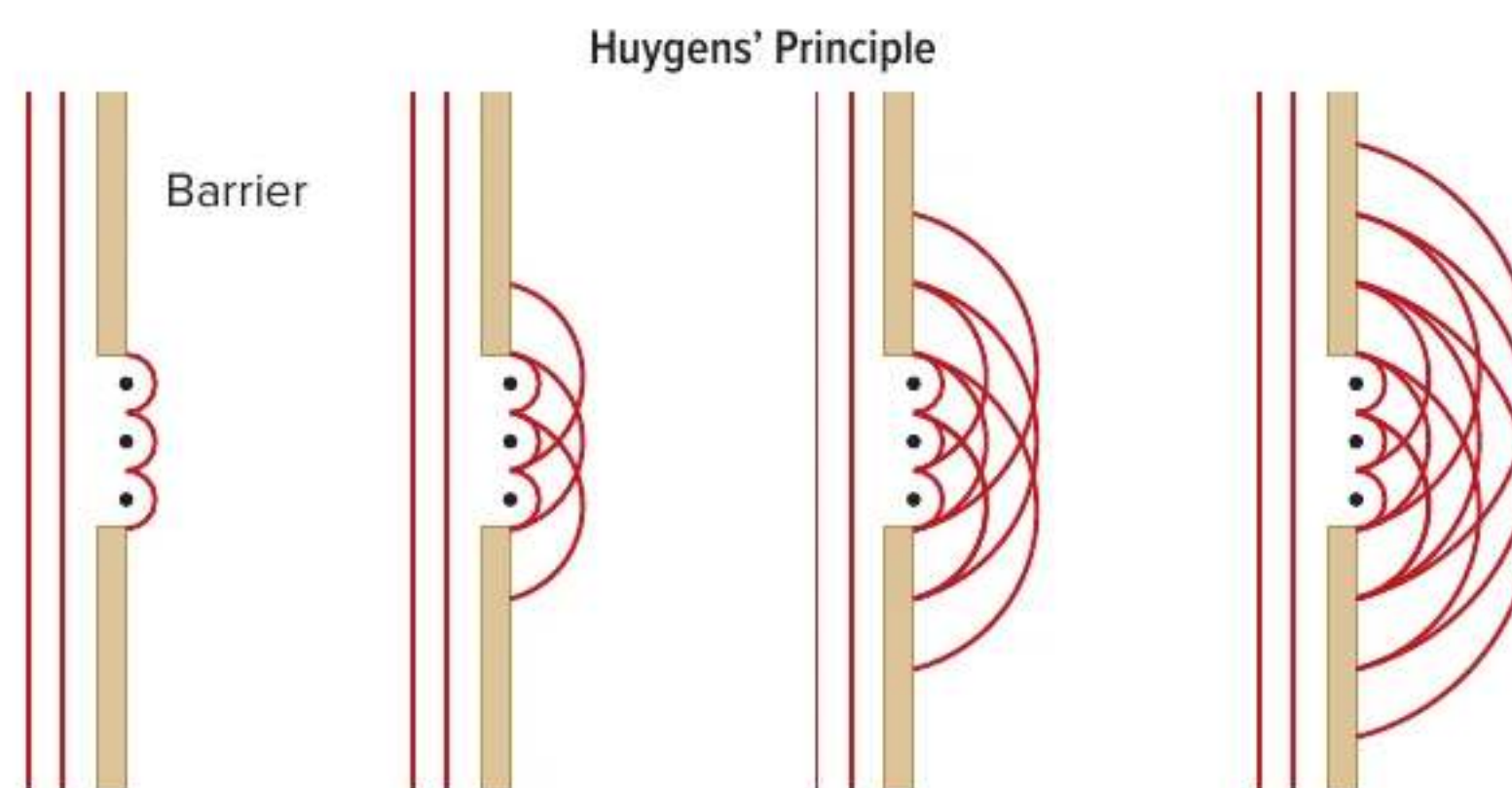


Figure 12 Huygens' wavelets combine to form a straight wavefront, except at the edges of the wave. The wavelets spread out in a circular manner when a barrier creates an edge.



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



PhysicsLAB: Polarization

Carry out an investigation to determine the effect of polarizing filters on light.



Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

As a wavefront passes by an edge, the edge cuts the wavefront such that each circular wavelet generated by each Huygens' point will propagate as a circular wave in the region where the original wavefront was bent. This wave model explained the diffraction Grimaldi.

Color

In 1666 Newton performed experiments on the colors produced when a narrow beam of sunlight passed through a glass prism, as shown in **Figure 13**. Newton called the ordered arrangement of colors a spectrum. Using his later-disproved corpuscle (or particle) model of light, he thought that particles of light interacted with some unevenness in the glass to produce the spectrum.

To test this assumption, Newton allowed the spectrum from one prism to fall on a second prism. If the spectrum was caused by irregularities in the glass, he reasoned that the second prism would increase the spread in colors. Instead, the second prism reversed the spreading of colors and recombined them to form white light. After more experiments, Newton concluded that white light is composed of colors and that a property of the glass other than unevenness caused the light to separate into colors.

Different wavelengths Can the wave model of light explain Newton's observations? For light to be a wave, it must have wavelength and frequency. The work of Grimaldi, Huygens, Newton, and others suggested that the color of light is related to wavelength. Visible light falls within the range of wavelengths from about 400 nm (4.00×10^{-7} m) to 700 nm (7.00×10^{-7} m), as shown in **Figure 13**. The longest visible wavelengths are seen as red light and the shortest as violet.

As white light crosses the boundary from air into glass and back into air in **Figure 13**, its wave nature causes each different color of light to be bent at a different angle. The shorter the wavelength, the more the light is bent. This unequal bending of the different colors causes the white light to be spread into a spectrum.

Color by addition of light White light can be formed from colored light in a variety of ways. For example, when the correct intensities of red, green, and blue light are projected onto a white screen, as in **Figure 14**, white light is formed. This is called the additive color process, which is used in many television screens. A television screen uses three colors—red, green, and blue. Combinations of these produce the colors you see.

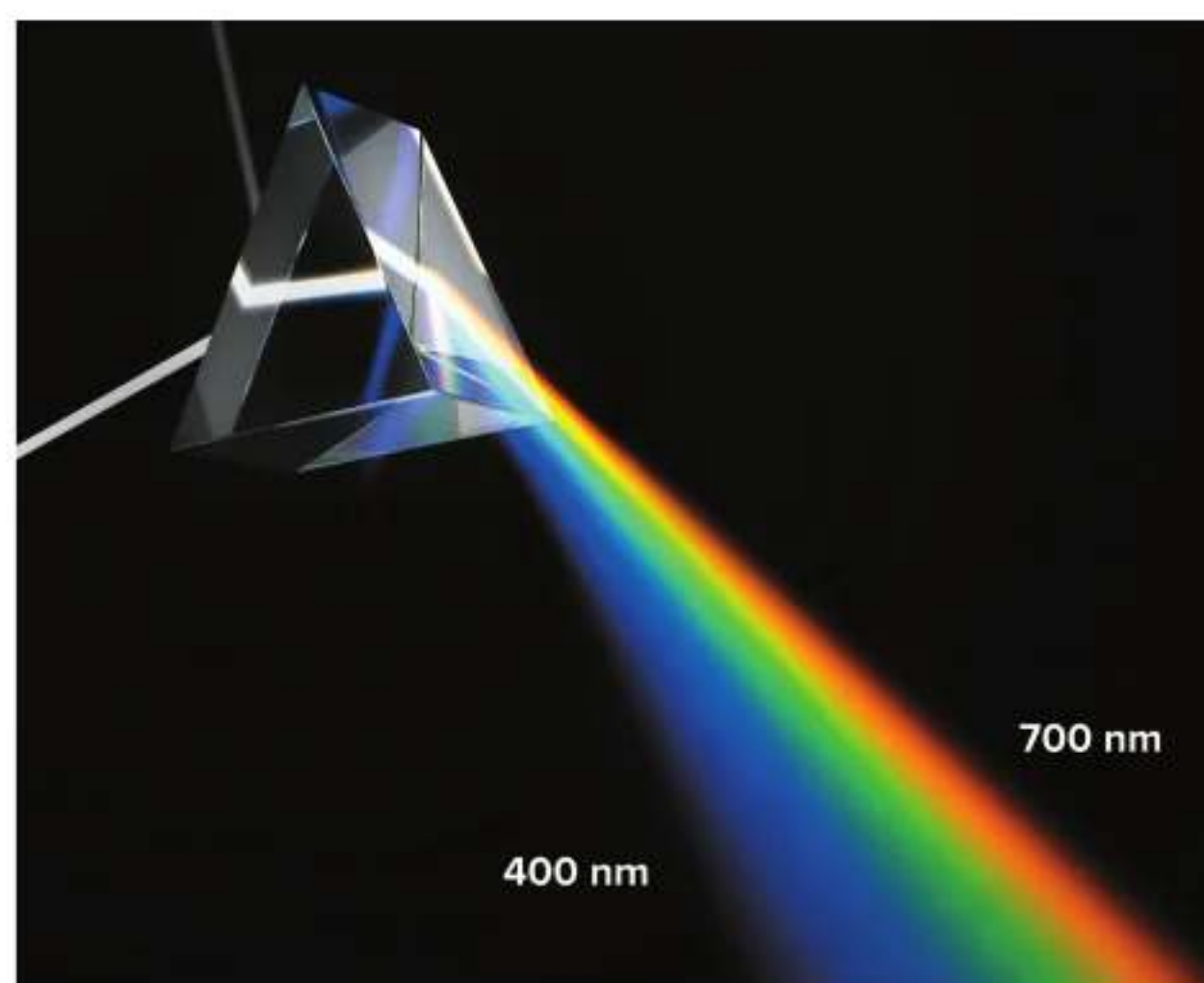


Figure 13 White light is separated into bands of color by a prism. Each color has a different wavelength.

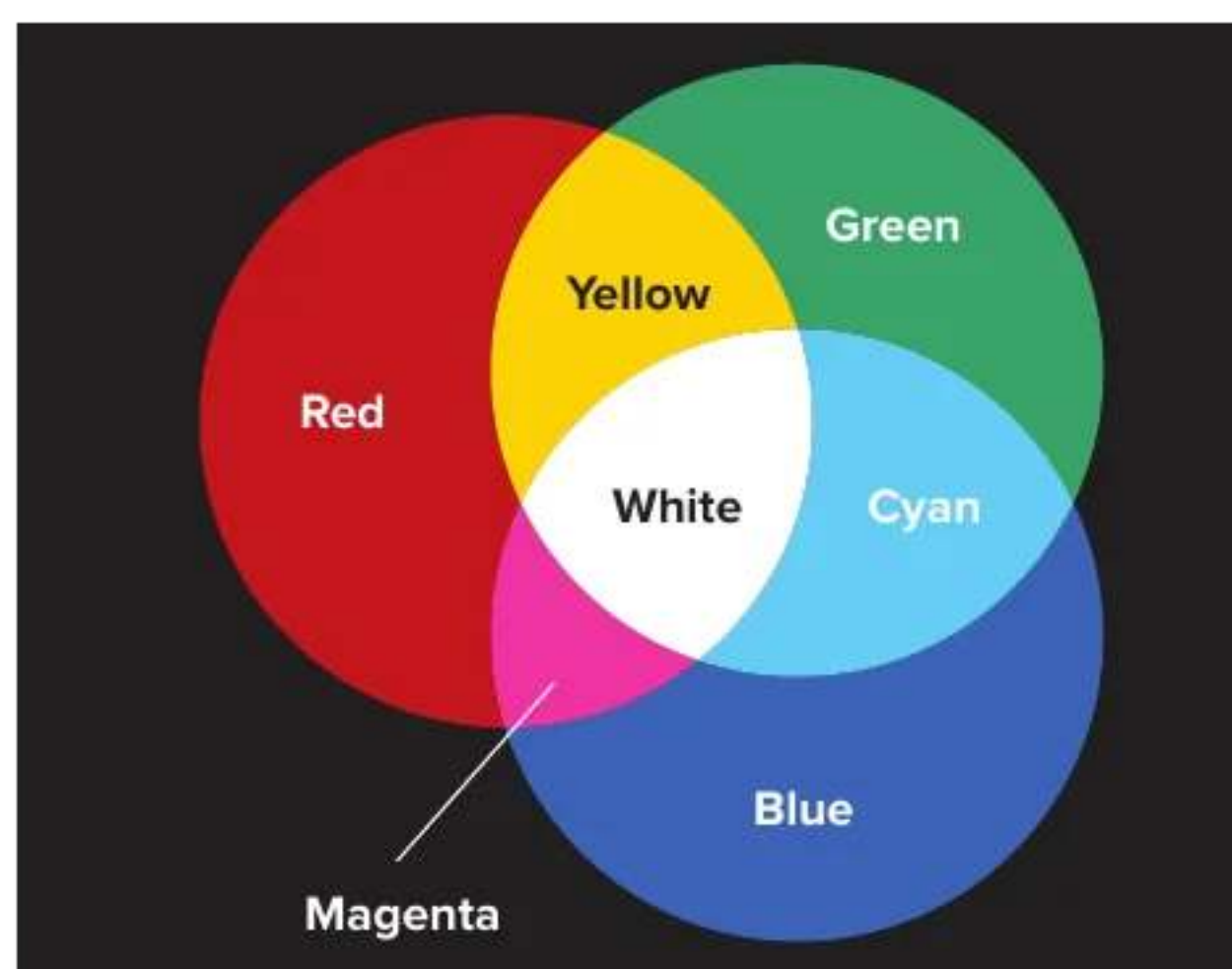


Figure 14 Red, green, and blue light, the primary colors, combine in pairs to produce yellow, cyan, or magenta light. The region where these three colors overlap on the screen appears white.

The correct intensities of red, green, and blue light appear on a screen as white light when combined. Because of this phenomenon, they are called the **primary colors** of light. The primary colors can be mixed in pairs to form three additional colors, as shown in **Figure 14**. Red and green light together produce yellow light, blue and green light produce cyan, and red and blue light produce magenta. The colors yellow, cyan, and magenta are called secondary colors. A **secondary color** is a combination of two primary colors. Note that these are slightly different from the primary and secondary colors you might have learned in art class; the reasons for this will be explained later in this module.

As shown in **Figure 14**, yellow light can be made from red light and green light. If yellow light and blue light are projected onto a white screen with the correct intensities, the surface will appear to be white. **Complementary colors** are two colors of light that can be combined to produce white light. Thus, yellow is a complementary color of blue, and vice versa, because the two colors of light combine to make white light. In the same way, cyan and red are complementary colors. Magenta and green are the other pair of complementary colors. A practical application of this is that yellowish laundry can be whitened with a bluing agent added to detergent.

Color by subtraction of light As you learned in the first lesson of this module, objects can reflect and transmit light. They also can absorb light. The color of an object depends on the wavelengths present in the light that illuminates the object. The color also depends on which wavelengths are absorbed by the object and which wavelengths are reflected. The natural existence or artificial placement of dyes in the material of an object, or pigments on its surface, gives the object color.

Dyes You are probably familiar with dyes that are used to color cloth. Dyes can be made from plant or insect extracts. For example, purple dye can be extracted from the berries of a black mulberry tree. The saffron crocus is a source of yellow dye. One type of red dye is extracted from an insect called a cochineal. A dye is a molecule that absorbs certain wavelengths of light and transmits or reflects others. When light is absorbed, its energy is transferred to the object that it strikes and is transformed into other forms of energy. A red shirt is red because the dyes in it reflect mostly red light to our eyes. When white light falls on the red object shown in **Figure 15**, the dye molecules in the object absorb most of the blue and green light and reflect mostly red light. When only blue light falls on the red object, very little light is reflected and the object appears to be almost black.



Get It?

Distinguish the difference between color by subtraction and color by addition.



Figure 15 The colors of objects we see are determined by which wavelengths of light are absorbed and which are reflected.

Explain why the die that is yellow in the white light appears red in red light.

Pigments The difference between a dye and a pigment is that pigments usually are made of crushed minerals rather than plant or insect extracts. For example, hematite produces a red pigment, and blue pigment can be obtained from azurite. Pigment particles can be seen with a microscope. A pigment that absorbs only one primary color and reflects two from white light is called a **primary pigment**. Yellow pigment absorbs blue light and reflects red and green light. Yellow, cyan, and magenta are the colors of primary pigments.

A pigment that absorbs two primary colors and reflects one color is called a **secondary pigment**. The colors of secondary pigments are red (which absorbs green and blue light), green (which absorbs red and blue light), and blue (which absorbs red and green light). Note that the primary pigment colors are these secondary colors of light. In the same way, the secondary pigment colors are light's primary colors.

The primary and secondary pigments are shown in **Figure 16**. When the primary pigments yellow and cyan are mixed, the yellow absorbs blue light and the cyan absorbs red light. **Figure 16** shows yellow and cyan combining to make green pigment. When yellow pigment is mixed with the secondary pigment blue, which absorbs green and red light, all the primary colors are absorbed, and the result is black. Yellow and blue are complementary pigments. Cyan and red, as well as magenta and green, are also complementary pigments.

CHEMISTRY Connection A color printer uses yellow, magenta, and cyan dots of pigment to make a color image on paper. Often, pigments that are used are finely ground compounds, such as titanium(IV) oxide (white), chromium(III) oxide (green), and cadmium sulfide (yellow). Pigments mix to form suspensions rather than solutions. Their chemical form is not changed in a mixture, so they still absorb and reflect the same wavelengths.

BIOLOGY Connection You can now begin to understand the colors that you see in **Figure 17**. The plants on the mountain look green because of the chlorophyll in them. One type of chlorophyll absorbs mostly red light and the other absorbs mostly blue light, but they both reflect green light. The energy in the red and blue light that is absorbed is used by the plants during photosynthesis to make food.



Figure 16 Magenta, cyan, and yellow are the primary pigments. Secondary pigments, red, green, and blue, are produced from mixing the primary pigments in pairs.



Figure 17 Chlorophyll in green leaves reflects mostly green light, giving the leaves their color.

Explain why the plants are various shades of green.

Polarization by Filtering

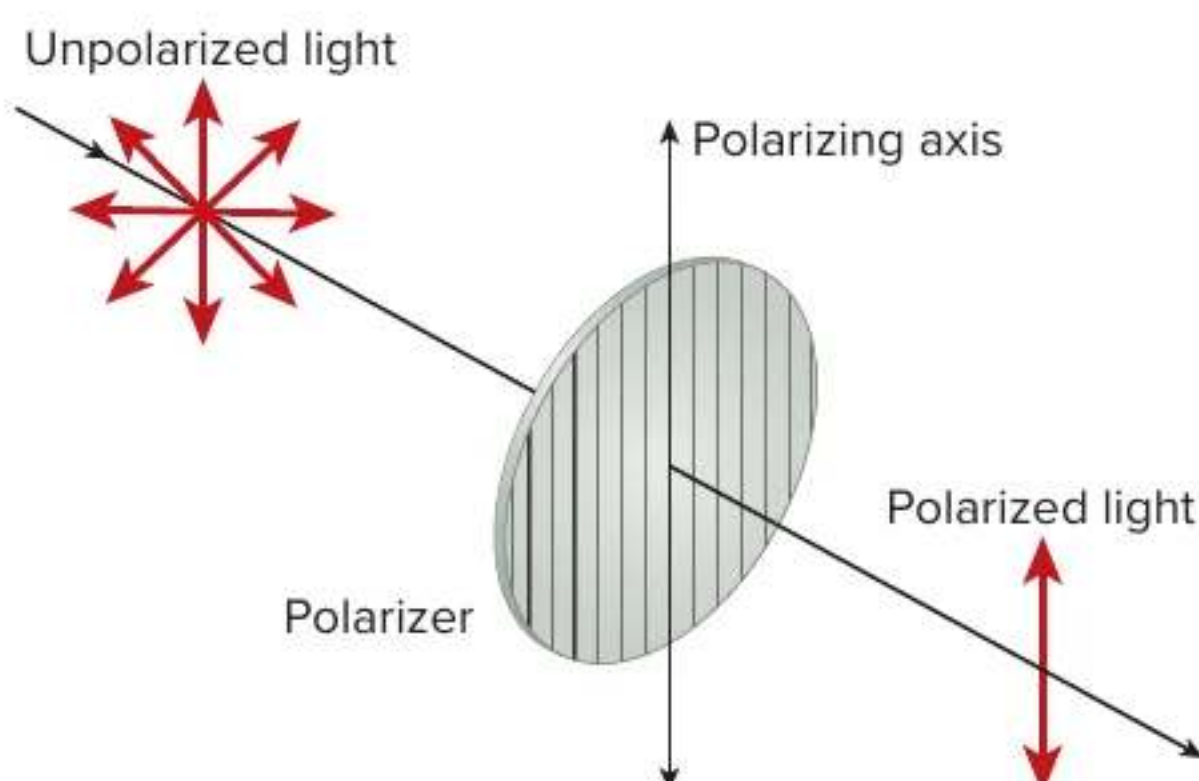


Figure 18 Nonpolarized light rays vibrate randomly in every direction perpendicular to the direction they travel. A polarizing medium blocks light that is not parallel to the polarizing axis.

Polarization of Light

Have you ever looked at light reflected off a road through polarizing sunglasses? If you rotate the glasses, the road first appears to be dark, then light, and then dark again. Light from a lamp, however, changes very little as the glasses are rotated. Why is there a difference? Normal lamp-light is not polarized. However, the light that is coming from the road is reflected and has become polarized. **Polarization** is the production of light with a specific pattern of oscillation.

Recall that light behaves as a transverse wave. For waves on a rope, the oscillating medium is the rope. For light waves, the oscillating medium is the electric field. When this electric field oscillates in random directions, the light is nonpolarized. How can you filter nonpolarized light so that whatever passes through the filter is polarized light?



Get It?

Draw a diagram, with text explanations, showing nonpolarized light.

Polarization by filtering The lines in the polarizer in **Figure 18** represent a polarizing axis. The light with the portion of the electric field that oscillates parallel to these lines passes through. The light with the portion of the electric field that oscillates perpendicular to these lines is absorbed. If a polarizer is placed in a beam of nonpolarized light, only the components of the waves in the same direction as the polarizing axis can pass through. As a result, half of the total light passes through, reducing the intensity of the light by half.



Get It?

Draw a second diagram, with text explanations, showing nonpolarized light passing through a filter as polarized light.

CHEMISTRY Connection

Polarizing mediums contain long molecules in which electrons can oscillate, or move back and forth, all in the same direction. As light travels past the molecules, the electrons absorb light waves that oscillate in the same direction as the electrons. This allows light waves vibrating in one direction to pass, while the waves vibrating in the other direction are absorbed. The direction of a polarizing medium perpendicular to the long molecules is called the polarizing axis. Only waves oscillating parallel to that axis can pass through.

Polarization by reflection When you look through a polarizing filter at the light reflected by a sheet of glass and rotate the filter, you will see the light brighten and dim. The light is partially polarized parallel to the plane of the glass when it is reflected. Polarized reflected light causes glare. Polarizing sunglasses reduce glare from the polarized light reflected off roads. Photographers can use polarizing filters over camera lenses to block reflected light. This result is shown in **Figure 19**.

Malus's law Suppose you produce polarized light with a polarizing filter. What would happen if you place a second polarizing filter in the path of the polarized light? If the polarizing axis of the second filter is parallel to that of the first, the light will pass through. If the polarizing axis of the second filter is perpendicular to that of the first, no light will pass through, as shown in **Figure 20**.

If the light intensity after the first polarizing filter is I_1 and the intensity after the second filter is I_2 , how can you control I_2 ? I_2 depends only on I_1 and the angle between the axes of the filters, θ . If θ is 0° , I_2 equals I_1 ; if θ is 90° , all of the light is blocked, resulting in I_2 being 0. This indicates that the intensity might depend on the cosine of θ . The actual relationship is a that of a cosine squared. The law that explains the reduction of light intensity as light passes through a second polarizing filter is **Malus's law**.

Malus's Law

The intensity of light coming out of a second polarizing filter is equal to the intensity of polarized light coming out of a first polarizing filter multiplied by the cosine, squared, of the angle between the polarizing axes of the two filters.

$$I_2 = I_1 \cos^2 \theta$$



Figure 19 Glare is light that has been polarized by reflection. Photographers use polarizing filters to reduce glare.

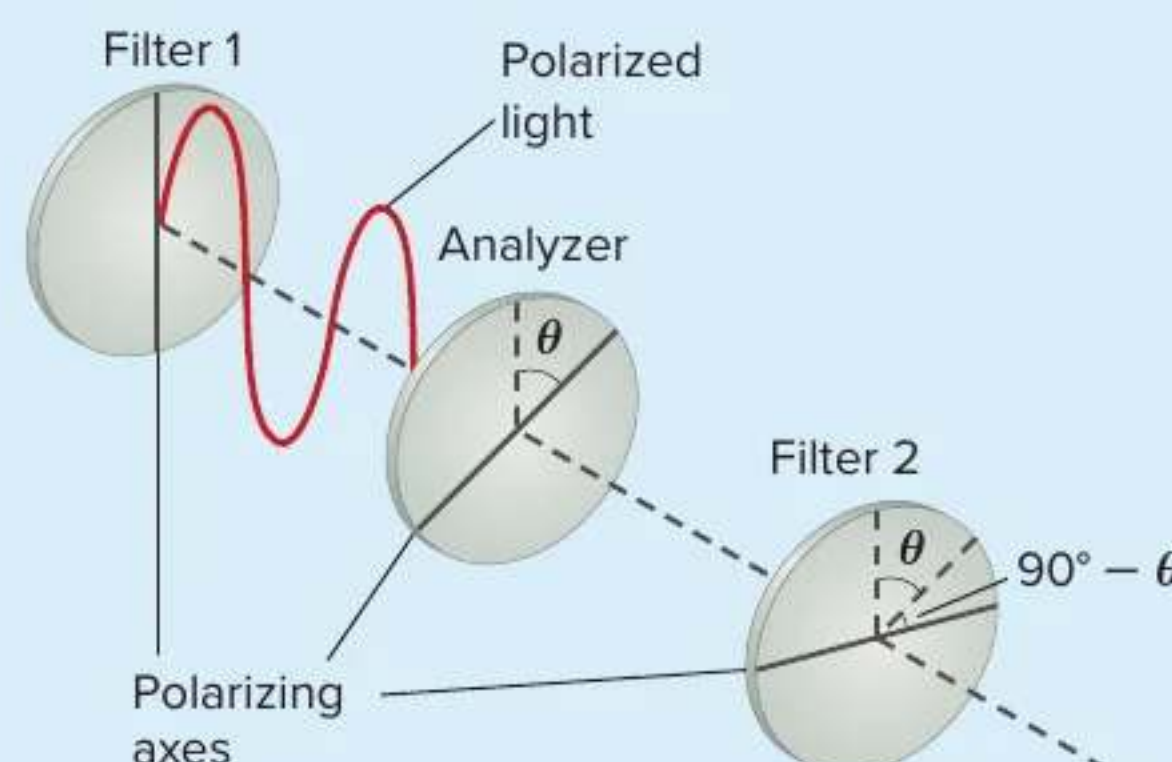


Figure 20 Polarizing filters with their axes parallel will allow the light with the same orientation to pass. With the polarizing axes at a 45° angle, the filters allow some light to pass. If the axes of the filters are perpendicular, the second filter will block the light that has passed through the first filter.

PHYSICS Challenge

You place an analyzer filter between the two cross-polarized filters, such that its polarizing axis is not parallel to either of the two filters, as shown in the figure to the right.

1. You observe that some light passes through filter 2, though no light passed through filter 2 before you inserted the analyzer filter. Why does this happen?
2. The analyzer filter is placed at an angle of θ relative to the polarizing axis of filter 1. Derive an equation for the intensity of light coming out of filter 2 compared to the intensity of light coming out of filter 1.



Speed, Wavelength, and Frequency of Light

As you have learned, the source of a wave determines that wave's frequency (f), and the medium and the frequency together determine the wavelength (λ) of a wave. Because light has wave properties, the same mathematical models used to describe waves in general can be used to describe light. For light of a given frequency traveling through a vacuum, wavelength is a function of the speed of light (c), which can be written as $\lambda_o = c/f$. The development of the laser in the 1960s provided new ways to measure the speed of light. The frequency of light can be measured with extreme precision using lasers and the time standard of atomic clocks. Measurements of wavelengths of light, however, are much less precise.

All colors of light travel at c in a vacuum, though the wavelengths are different. Since $\lambda_o = c/f$, once the frequency of a light wave in a vacuum is measured, the wavelength can be determined.

Relative motion and light What happens if a light source travels toward you or you move toward the source? You have learned that the frequency of a sound heard by a listener changes if either the source or the listener of the sound is moving. The same is true for light. However, when you consider the velocities of a sound source and the observer, you are really considering each one's velocity relative to the medium through which the sound travels. This is not the case for light.

**Get It?**

Explain how the wavelength, frequency, and speed of light are mathematically related.

The Doppler effect The nature of light waves is such that they are not vibrations of the particles of a medium. The Doppler effect for light can involve only the relative velocity between the source and the observer. Remember that the only factors in the Doppler effect are the velocity components along the axis between the source and the observer, as shown in **Figure 21** on the next page.

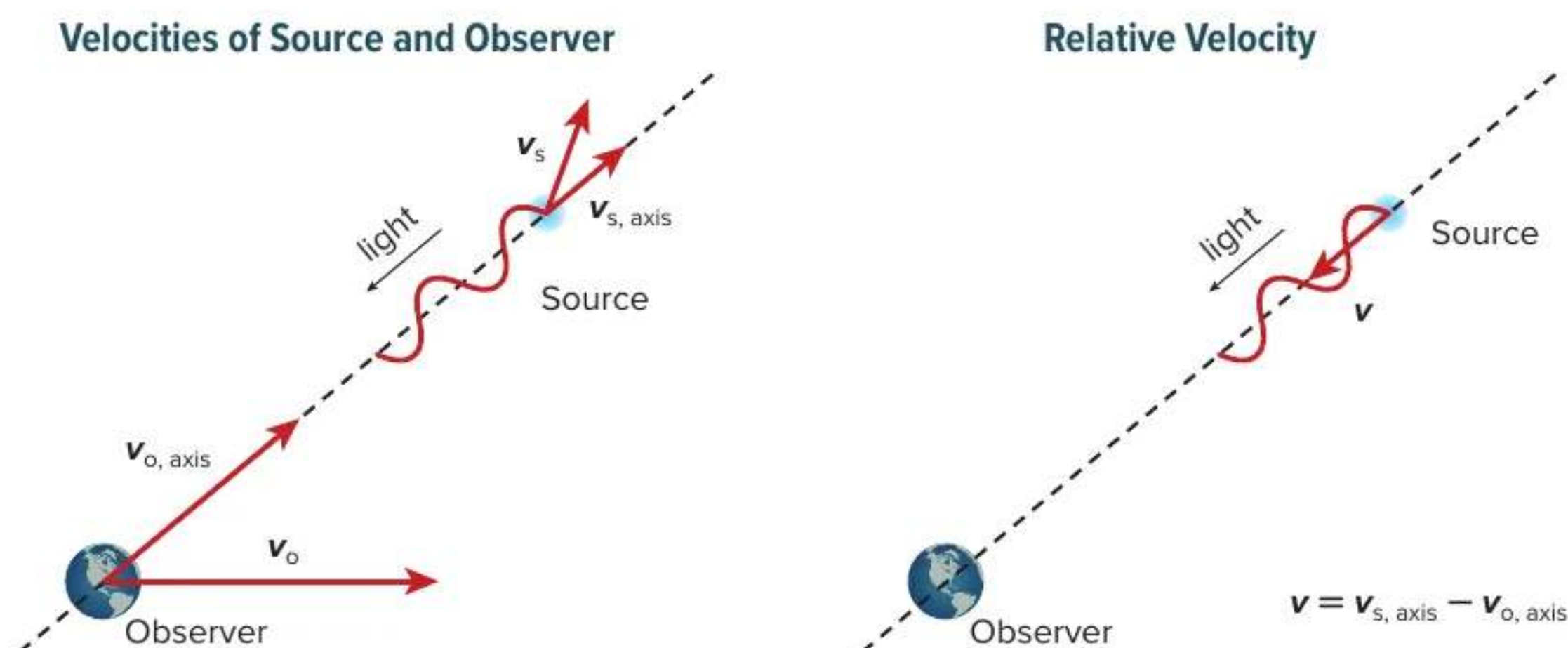


Figure 21 The Doppler effect describes how light frequency changes if an observer and a light source are moving toward or away from each other.

Doppler effect for light problems can be simplified by considering axial relative speeds that are much less than the speed of light ($v \ll c$). This simplification is used to develop the equation for the observed light frequency (f_{obs}), shown below.

Observed Light Frequency

The observed frequency of light from a source is equal to the actual frequency of the light generated by the source, times the quantity 1 plus the relative speed along the axis, divided by the speed of light, between the source and the observer if they are moving toward each other, or 1 minus the relative speed, divided by the speed of light, if they are moving away from each other.

$$F_{\text{obs}} = f \left(1 \pm \frac{v}{c} \right)$$

Applications Most applications of the Doppler effect for light are in astronomy, where phenomena are discussed more in terms of wavelength. Using the relationship $\lambda = c/f$ and the $v \ll c$ simplification, the following equation describes the Doppler shift ($\Delta\lambda$), the difference between the observed and the actual wavelengths.

Doppler Shift

The difference between the observed wavelength of light and the actual wavelength of light generated by a source is equal to the actual wavelength of light generated by the source, times the relative speed of the source and observer, divided by the speed of light.

$$(\lambda_{\text{obs}} - \lambda) = \Delta\lambda = \pm \left(\frac{v}{c} \right) \lambda$$

A positive change in wavelength occurs when the relative velocity of the source is away from the observer. In this case, the observed wavelength is longer than the original wavelength. The light appears closer to the red end of the spectrum than it normally would. We say this light is red shifted. A negative change in wavelength occurs when the relative velocity of the source is in a direction toward the observer. In this case, the observed wavelength is shorter than the original wavelength. This is known as a blue shift.

Because the speed of light is constant, when the wavelength is red shifted, the observed frequency is lower than the original due to the inverse relationship between the two variables. When light is blue shifted, the observed frequency is higher.

ASTRONOMY Connection Astronomers can determine how objects, such as galaxies, are moving relative to Earth by observing the Doppler shift of their light. This is done by observing the spectrum of light coming from stars in the galaxy using a spectrometer, as shown in **Figure 22**. The same elements that are present in the stars of galaxies emit light of specific wavelengths in labs on Earth. By comparing wavelength, astronomers can learn the velocities of objects toward or away from Earth.



Figure 22 The bottom hydrogen emission spectrum is red shifted compared to the laboratory spectrum, indicating the light source is moving away from Earth.

PRACTICE Problems

ADDITIONAL PRACTICE

16. Oxygen can be made to produce light with a wavelength of 513 nm. What is the frequency of this light?
17. A hydrogen atom in a galaxy moving with a speed of 6.55×10^6 m/s away from Earth emits light with a frequency of 6.16×10^{14} Hz. What frequency of light from that hydrogen atom would be observed by an astronomer on Earth?
18. A hydrogen atom in a galaxy moving with a speed of 6.55×10^6 m/s away from Earth emits light with a wavelength of 486 nm. What wavelength would be observed on Earth from that hydrogen atom?
19. **CHALLENGE** An astronomer is looking at the spectrum of a galaxy and finds that it has an oxygen spectral line of 525 nm, while the laboratory value is measured at 513 nm. Calculate how fast the galaxy would be moving toward or away from Earth and how you know.

STEM CAREER Connection

Photographer

Do you like capturing important events with a camera? Photographers must use light to compose a perfect picture or to achieve a desired effect. Understanding the fundamentals of light is an important part of being a good photographer.

In 1929, Edwin Hubble analyzed the light from many galaxies like the one shown in **Figure 23**. He observed that the light produced by familiar elements were at longer wavelengths than he had expected them to be. The light was shifted toward the red end of the spectrum. No matter what area of the sky he observed, almost all the galaxies were sending red shifted light to Earth. What do you think caused the spectral lines to be red shifted?

Hubble concluded that galaxies are moving away from Earth and suggested that the universe is expanding. Additional studies since then have supported this conclusion. As galaxies move, they sometimes collide and merge. The results of the collision depend on the size and speed of the galaxies.

You have learned that some characteristics of light can be explained with a simple ray model of light, whereas others require a wave model of light. You can use both of these models to study how light interacts with mirrors and lenses. There are some aspects of light that can be understood only through the use of the wave model of light.



Figure 23 Edwin Hubble observed that galaxies were sending red shifted light to Earth, indicating that they were moving away from Earth and the universe was expanding.

Check Your Progress

20. **Doppler Effect** Describe the relative motions of objects when light is red shifted and when light is blue shifted. Answer using the term *Doppler effect*.
21. **Addition of Light Colors** What color of light must be combined with blue light to obtain white light?
22. **Light and Pigment Interaction** What color will a yellow banana appear to be when illuminated by each of the following?
 - a. white light
 - b. green and red light
 - c. blue light
23. **Pigment Colors** What are the secondary pigment colors, and why do they give objects the appearance of those colors?
24. **Combination of Pigments** What primary pigment colors must be mixed to produce red? Explain your answer in terms of color subtraction for pigment colors.
25. **Polarization** Describe a simple experiment you could do to determine whether sunglasses in a store are polarizing.
26. **Polarizing Sunglasses** Use **Figure 24** to determine the direction the polarizing axis of polarizing sunglasses should be oriented to reduce glare from the surface of a road: vertically or horizontally? Explain.



Figure 24

27. **Red Light** The speed of red light is slower in air and water than in a vacuum. The frequency, however, does not change when red light enters water. Does the wavelength change? If so, how?
28. **Critical Thinking** Astronomers have determined that our galaxy, the Milky Way, is moving toward Andromeda, a neighboring galaxy. Explain how they determined this. Can you think of a possible reason why the Milky Way is moving toward Andromeda?

SCIENTIFIC BREAKTHROUGHS

Super-Efficient Solar Cells

Solar panels on homes and businesses are an increasingly popular alternative source of energy in today's society. This technology converts energy from the Sun into electricity people can use for the needs of everyday life. Solar energy is an inexhaustible source of clean energy. However, the current technology used to convert solar energy to electricity has limitations. Scientists working to optimize this technology have recently made a breakthrough that could greatly increase its efficiency.



Solar panels convert light waves into electricity.

What is a photovoltaic cell?

A solar panel is made up of many photovoltaic (PV) cells, which convert light waves into electrical current. Incoming light waves strike the semiconductor, usually silicon, and transfer energy to electrons. The electrons flow in one direction through the material, creating an electric current.

Limitations of Current Technology

Energy emitted by the Sun encompasses the entire electromagnetic spectrum. The silicon used in PV cells can absorb only a portion of these waves—from red to violet on the visible light spectrum. The remainder of the Sun's energy cannot be utilized by existing PV cells. As a result, PV cells are very inefficient. For example, most residential solar panels convert about 10 to 20 percent of the incoming energy to electricity, so solar panels need to be very large, especially in areas with cold and cloudy climates.

Hot Solar Cells

Scientists have developed a new solar thermo-photovoltaic (STPV) cell for converting solar energy to electricity. These STPV cells, called “hot solar cells,” couple conventional PV cells with a light concentrator to utilize a greater percentage of the incoming energy from the Sun. The absorbing layer of carbon nanotubes turns energy from the Sun into thermal energy—with temperatures up to 1000°C! The emitting layer of nanophotonic crystals converts this thermal energy back into light that is specifically in the range of wavelengths that can be absorbed by the traditional PV cell. Initial results show that STPV cells create up to twice the amount of electrical energy compared to basic PV cells. This increased energy production can reduce the dependence on nonrenewable energy sources such as fossil fuels.



EVALUATE DESIGN SOLUTIONS

Using the Internet, gather information about the new STPV cell technology and evaluate this design solution. With a partner, discuss how the new technology might impact the cost, safety, reliability, or aesthetics of using solar energy to produce electricity. Summarize your discussion in a written paragraph.

MODULE 15

STUDY GUIDE

 **GO ONLINE** to study with your Science Notebook.

Lesson 1 ILLUMINATION

- Light can be modeled as a ray that travels in a straight path until it encounters a boundary. Mediums can be characterized as being transparent, translucent, or opaque, depending on how light interacts with them.
- The luminous flux of a light source is the rate at which light is emitted. It is measured in lumens (lm). Illuminance is the luminous flux per unit area. Illuminance is measured in lux (lx), or lumens per square meter (lm/m^2). For a point source, illuminance follows an inverse-square relationship with distance and a direct relationship with luminous flux.

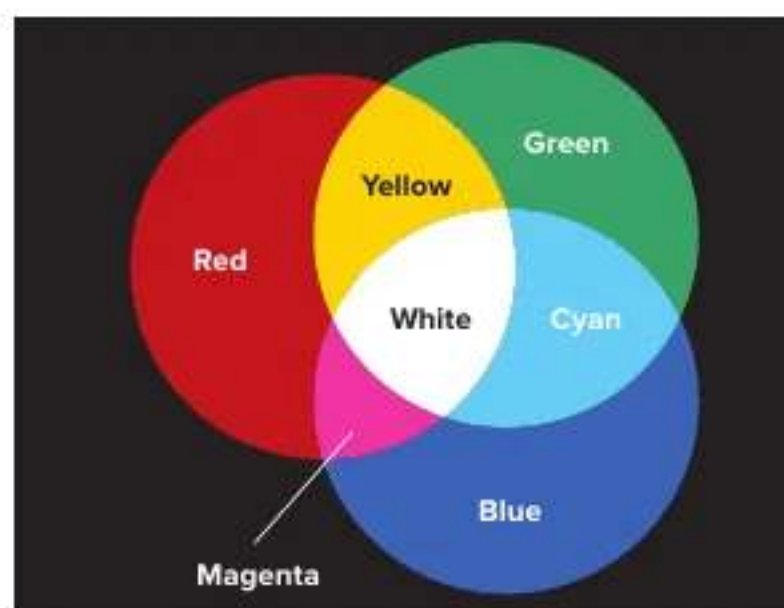
$$E = \frac{P}{4\pi r^2}$$

- Early measurements of the speed of light involved measurement of the time it takes for light to reach Earth from Jupiter's moon Io. Michelson used a land-based technique that involved the distance between two mountains and a set of rotating mirrors. In a vacuum, light has a constant speed of $c = 3.00 \times 10^8 \text{ m/s}$.

- ray model of light
- luminous sources
- opaque
- translucent
- transparent
- luminous flux
- illuminance

Lesson 2 THE WAVE NATURE OF LIGHT

- In the wave model of light, all the points in a wavefront can be thought of as sources of smaller waves. As light travels past an edge, the wavefront is cut and each new wavelet generates a new circular wave.
- Visible light can have wavelengths between 400 and 700 nm. White light is a combination of the spectrum of colors, each color having a different wavelength. Combining the primary colors—red, blue, and green—forms white light. Combinations of two primary colors form the secondary colors, yellow, cyan, and magenta.



The primary pigments, cyan, magenta, and yellow, are used in combinations of two to produce the secondary pigments, red, blue, and green.

- Polarized light consists of waves whose electric fields oscillate with a specific pattern. Often, the oscillation is in a single plane. Light can be polarized with a polarizing filter or by reflection. Light waves traveling through a vacuum can be characterized in terms of frequency, wavelength, and the speed of light. Light waves are Doppler shifted based on the relative speed of the observer and light source along the axis of the observer and the light source.

- diffraction
- primary colors
- secondary color
- complementary colors
- primary pigment
- secondary pigment
- polarization
- Malus's law



THREE-DIMENSIONAL THINKING Module Wrap-Up

REVISIT THE PHENOMENON

What does the light from a distant star or supernova tell us about it?



CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.



STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

Can a traffic light appear to change color?

Suppose you are a traffic officer and you stop a driver for going through a red light. Further suppose the driver draws you a picture and explains that the light looked green because of the Doppler effect when he drove through it. The wavelength for red light is 645 nm and it is 545 nm for a green light.

CER Analyze and Interpret Data

1. **Claim** Would you have given the driver a ticket for running the red light or accepted his explanation?
2. **Evidence and Reasoning** How would you explain your decision to the driver?

REFLECTION AND REFRACTION

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REFLECTION AND REFRACTION

ENCOUNTER THE PHENOMENON

How does light transmit information through a communication network?

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about how light is used to transmit information in a communication network. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

 **GO ONLINE** to access your CER chart and explore resources that can help you collect evidence.



Dentists use concave mirrors to magnify and inspect the teeth of their patients.



Cities often place convex mirrors on busy streets, because the reduced images of the convex mirror allow for a wider field of view.



LESSON 2: Explore & Explain:
Images in Curved Mirrors

LESSON 4: Explore & Explain:
Applications of Lenses

LESSON 1

REFLECTION OF LIGHT

FOCUS QUESTION

Why does light reflected from a mirror make an image while light reflected from a piece of paper does not?

Reflected Images

When you look at the surface of a body of water, you don't always see a clear reflection as in **Figure 1**. Sometimes, the wind causes ripples in the water, or passing boats produce waves. Disturbances on the water's surface prevent the light from reflecting in a manner such that a clear image is visible.

Almost 4000 years ago, Egyptians understood that the type of reflection you see from a still pond requires a smooth surface. They used polished metal mirrors to view their images.

Artisans in sixteenth-century Venice created mirrors by coating the back of a flat piece of glass with a thin sheet of metal. Sharp, well-defined, reflected images were not possible until 1857, however, when Jean Foucault, a French scientist, developed a method of coating glass with silver.

Today, we don't only use mirrors to view our reflections. Mirrors are key components in lasers, telescopes, and other precise optical systems. We use ever-increasing precision to make modern mirrors. Today's mirrors are made by evaporating aluminum or silver onto highly polished glass. The same basic physics principles, however, govern reflection from the smooth surface of a pond, from a sixteenth-century mirror, and from a tiny mirror inside a laser.



Figure 1 Disturbances on the surface of a pond or lake produce a distorted reflected image.



3D THINKING

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.



DCI Disciplinary Core Ideas



CCC Crosscutting Concepts



SEP Science & Engineering Practices

INVESTIGATE

GO ONLINE to find these activities and more resources.



Forensics Lab: A Little Time to Reflect

Carry out an investigation into the relationship between the angles of incident and reflection of light on a plane mirror.



Identify Crosscutting Concepts

Create a table of the crosscutting concepts and fill in examples you find as you read.

The Law of Reflection

Figure 2 shows a light ray striking a reflecting surface. The normal is an imaginary line that is perpendicular to a surface at the location where light strikes the surface. The incident ray, the reflected ray, and the normal are in the same plane, which is perpendicular to the surface. Although light travels in three dimensions, the reflection of light is planar (two-dimensional). Planar relationships are known as the law of reflection.

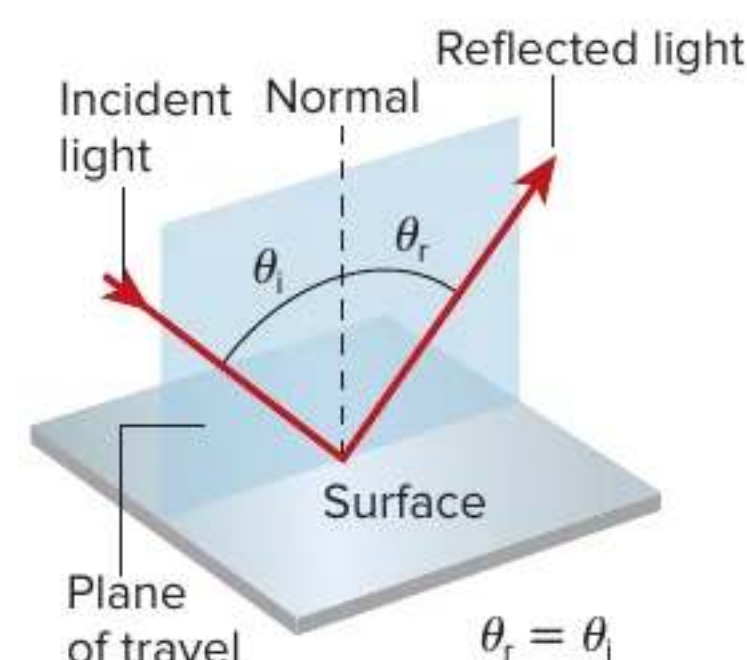


Figure 2 Reflecting light waves have an angle of reflection in the same plane as and equal to the angle of incidence.

Law of Reflection

The angle that a reflected ray makes as measured from the normal to a reflective surface equals the angle that the incident ray makes as measured from the same normal.

$$\theta_r = \theta_i$$

COLOR CONVENTION

light ray ←→ red

Wave model We can describe this law in terms of the wave model of light. **Figure 3** shows a wavefront of light approaching a surface. The wavefront is perpendicular to the light ray. As each point along the wavefront reaches the surface, it reflects off that surface. Because all points are traveling at the same speed, they all travel the same total distance in the same time. Thus, the wavefront as a whole leaves the surface at an angle equal to its incident angle. Note that the wavelength of the light does not affect this process. The surface reflects red, green, and blue light all in the same direction.

Smooth surfaces Consider the light rays shown on the left in **Figure 4** on the next page. All the rays reflect off the surface parallel to one another. This occurs only if the reflecting surface is not rough on the scale of the wavelength of the light. We consider such a surface to be

smooth. A smooth surface, such as a mirror, produces **specular reflection**, in which parallel light rays reflect in parallel.

Rough surfaces What happens when light strikes a sheet of paper? A sheet of paper might appear smooth, but on the scale of the wavelength of light, paper is actually quite rough. The right side of **Figure 4** shows light rays reflecting off a sheet of paper. All of the light rays are parallel before they strike the surface, but the reflected rays are not parallel, as shown. This scattering of light off a rough surface is called **diffuse reflection**.

The law of reflection applies to smooth and rough surfaces. For a rough surface, the angle that each incident ray makes with the normal equals the angle that its reflected ray makes with the normal. On a microscopic scale, however, the normals to the surface locations are not parallel.

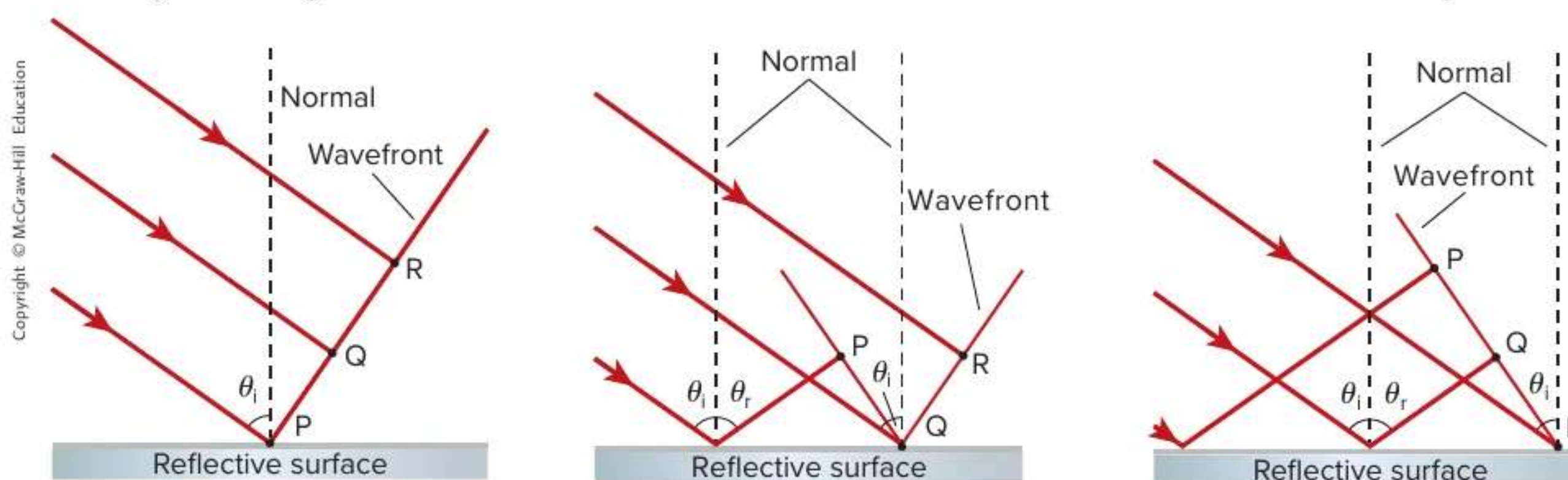


Figure 3 All parts of a wavefront reflect from a surface at the same angle. The angle of incidence equals the angle of reflection.

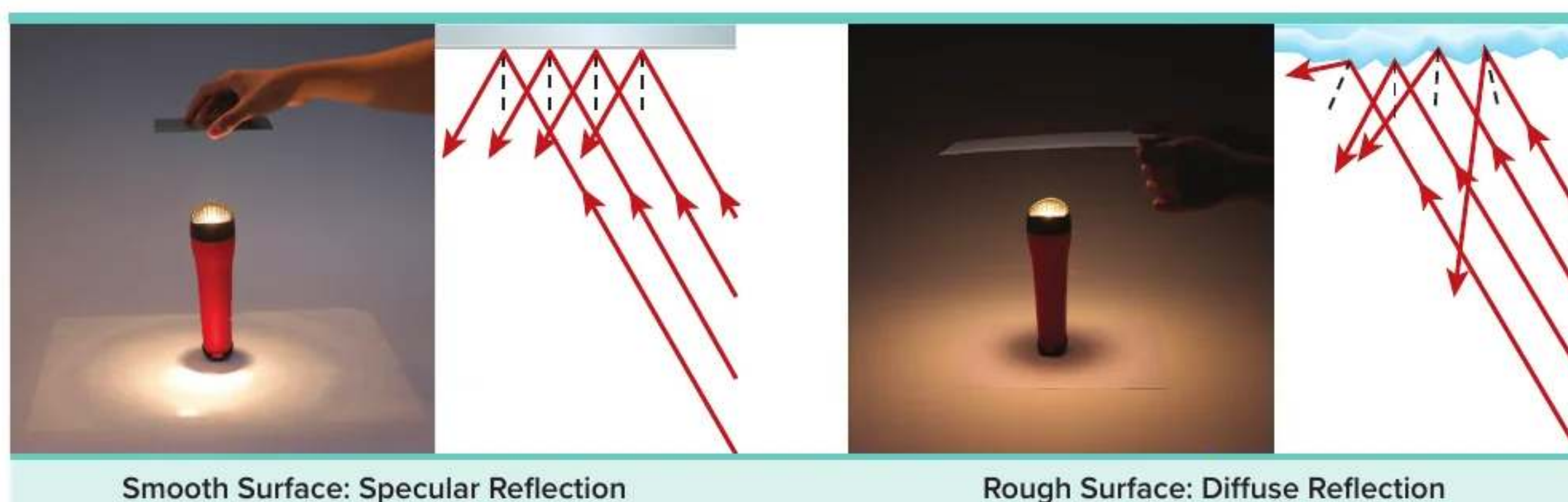


Figure 4 Notice that the image of the lightbulb in the flashlight is reflected on the table by the smooth mirror. The surface of the paper reflects a featureless area of light. Diffuse reflection enables you to read this page from various angles.

Thus, the rough surface prevents the reflected rays from being parallel, so the reflected rays are scattered in different directions. With specular reflection, as with a mirror, you can see your face. But no matter how much light reflects off a wall or a sheet of paper, as shown in **Figure 4**, you will never be able to use them as mirrors.

Objects and Plane-Mirror Images

A **plane mirror**, such as a mirror we use to look into, is a flat, smooth surface from which light is reflected by specular reflection.

When studying illumination, an **object** is either a luminous source of light rays, such as a lightbulb, or an illuminated source of light rays, such as a person. For most of the light sources that you will study in this module, light spreads out from that source in all directions. A mirrored surface can reflect these light rays so an image is visible, as shown in **Figure 5**.

In **Figure 6** light reflects diffusely from all parts of the bird (the object). The girl sees the light that reflects into her eye. Because her brain processes this information as if the rays travel in a straight path, it seems to the girl as if the light follows the dashed lines. The light seems to have come from a point behind the mirror. Just as the rays diverge from the object, they also diverge from the image.

The combination of light rays reflected from the bird in **Figure 6** forms the **image** of the bird. It is a **virtual image**, a



Figure 5 Reflection from the woman to the mirror is diffuse reflection. Reflection from the mirror is specular reflection.

Describe how the woman is illuminated.

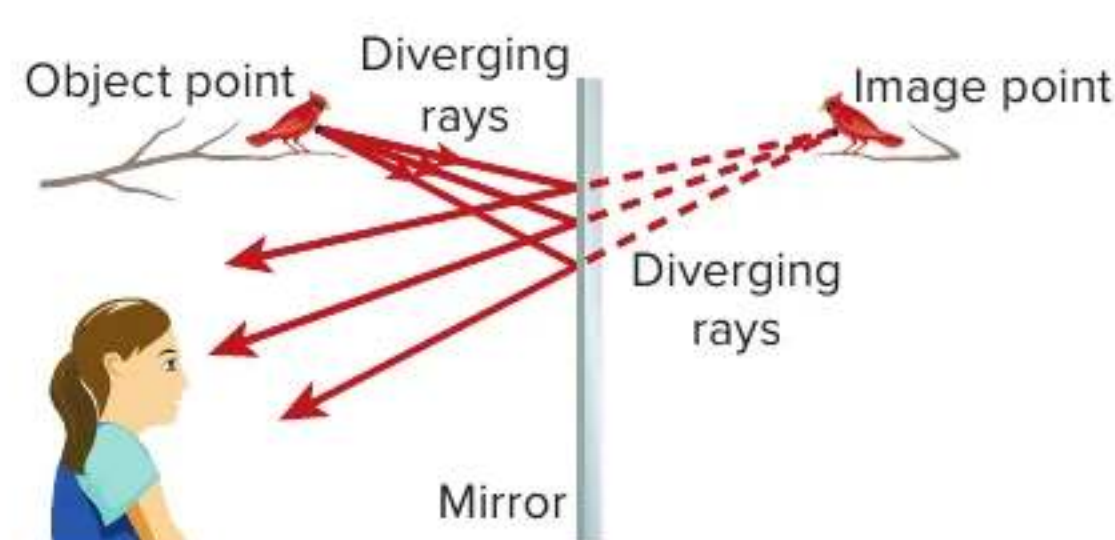


Figure 6 Rays that reflect from the bird disperse in many directions. Only a few that travel toward the mirror are shown. The image is located where multiple light rays from a point on an object seem to converge.

type of image formed by diverging light rays. A virtual image is always on the opposite side of the mirror from the object. The image is virtual because there are no light rays at the image location. Plane mirrors produce only virtual images.

EXAMPLE Problem 1

CHANGING THE ANGLE OF INCIDENCE A light ray strikes a plane mirror at an angle of 52.0° to the normal. The mirror then rotates 35.0° around the point where the ray strikes the mirror so that the angle of incidence of the light ray decreases. The axis of rotation is perpendicular to the plane of the incident and the reflected rays. What is the angle between the initial and final reflected ray?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation before the rotation of the mirror.
- Draw another sketch with the angle of rotation applied to the mirror.
- Draw a third sketch of the reflected rays.

Known

$$\theta_{i, \text{initial}} = 52.0^\circ$$

$$\Delta\theta_{\text{mirror}} = 35.0^\circ$$

Unknown

$$\Delta\theta_r = ?$$

2 SOLVE FOR THE ANGLE DIFFERENCE

For the angle of incidence to reduce, rotate clockwise.

$$\theta_{i, \text{final}} = \theta_{i, \text{initial}} - \Delta\theta_{\text{mirror}}$$

$$= 52.0^\circ - 35.0^\circ$$

$$\text{Substitute } \theta_{i, \text{initial}} = 52.0^\circ, \Delta\theta_{\text{mirror}} = 35.0^\circ.$$

$$= 17.0^\circ \text{ clockwise from the new normal}$$

Apply the law of reflection.

$$\theta_{r, \text{final}} = \theta_{i, \text{final}}$$

$$= 17.0^\circ \text{ counterclockwise from the new normal} \quad \text{Substitute } \theta_{i, \text{final}} = 17.0^\circ.$$

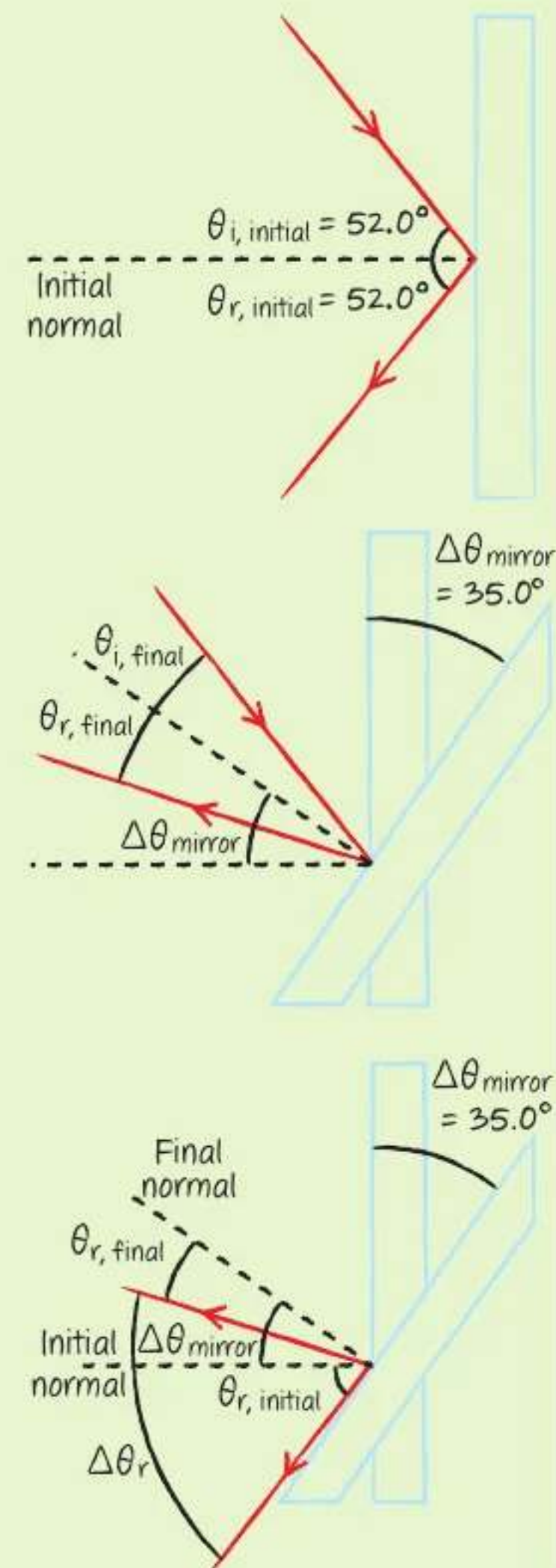
Use the sketches to help determine the angle through which the reflected ray has rotated.

$$\Delta\theta_r = \theta_{r, \text{initial}} + \Delta\theta_{\text{mirror}} - \theta_{r, \text{final}} = 52.0^\circ + 35.0^\circ - 17.0^\circ$$

$$= 70.0^\circ \text{ clockwise from the original angle}$$

3 EVALUATE THE ANSWER

- **Is the magnitude realistic?** Comparing the final sketch with the initial sketch shows that the angle the light ray makes with the normal decreases as the mirror rotates clockwise. It makes sense, then, that the reflected ray also rotates clockwise and rotates through an angle twice as large as that of the mirror rotation.

**PRACTICE Problems****ADDITIONAL PRACTICE**

1. Explain why the reflection of light off ground glass changes from diffuse to specular if you spill water on it.
2. What is the angle of incidence of a light ray reflected off a plane mirror at an angle of 35° to the normal?
3. Suppose the angle of incidence of a light ray is 42° .
 - a. What is the angle of reflection?
 - b. What is the angle the incident ray makes with the mirror?
 - c. What is the angle between the incident ray and the reflected ray?
4. Light from a laser strikes a plane mirror at an angle of 38° to the normal. If the angle of incidence increases by 13° , what is the new angle of reflection?
5. You position two plane mirrors at right angles to each other. A light ray strikes one mirror at an angle of 60° to the normal and reflects toward the second mirror. What is its angle of reflection off the second mirror?
6. **CHALLENGE** You are asked to design a retro-reflector using two mirrors that will reflect a laser beam by 180° independent of the incident direction of the beam. What should be the angle between the two mirrors?

Properties of Plane-Mirror Images

Looking at yourself in a mirror, you can see that your image in the mirror appears to be the same distance behind the mirror as you are in front of the mirror. How could you test this? Place a ruler between you and the mirror. Where does the image touch the ruler? You see that your image is vertically oriented as you are, and it matches your size. This is where the expression *mirror image* originates.

Image position and height The geometric model in **Figure 7** shows why the distances are the same. Two rays from point O at the tip of the candle strike the mirror at points P_1 and P_2 . The mirror reflects both rays according to the law of reflection.

We can extend the reflected rays behind the mirror as sight lines (the dashed lines in the figure). These sight lines converge at point I, which is the image of point O. Ray 1, which strikes the mirror at an angle of incidence of 0° , reflects back on itself, so the sight line is at 90° to the mirror, just as ray 1 is. Ray 2 also reflects at an angle with respect to the normal equal to the angle of incidence, so the sight line is at the same angle to the mirror as ray 2.

This geometric model reveals that line segments OP_1 and IP_1 are corresponding sides of two congruent triangles, $\triangle OP_1P_2$ and $\triangle IP_1P_2$. The object's position with respect to the mirror (x_o) has a magnitude equal to the length of $\overline{OP_1}$. The apparent position of the image with respect to the mirror, which is called the image position (x_i), has a magnitude equal to the length of $\overline{IP_1}$. If image position is negative, indicating that the image is virtual, the following is true.

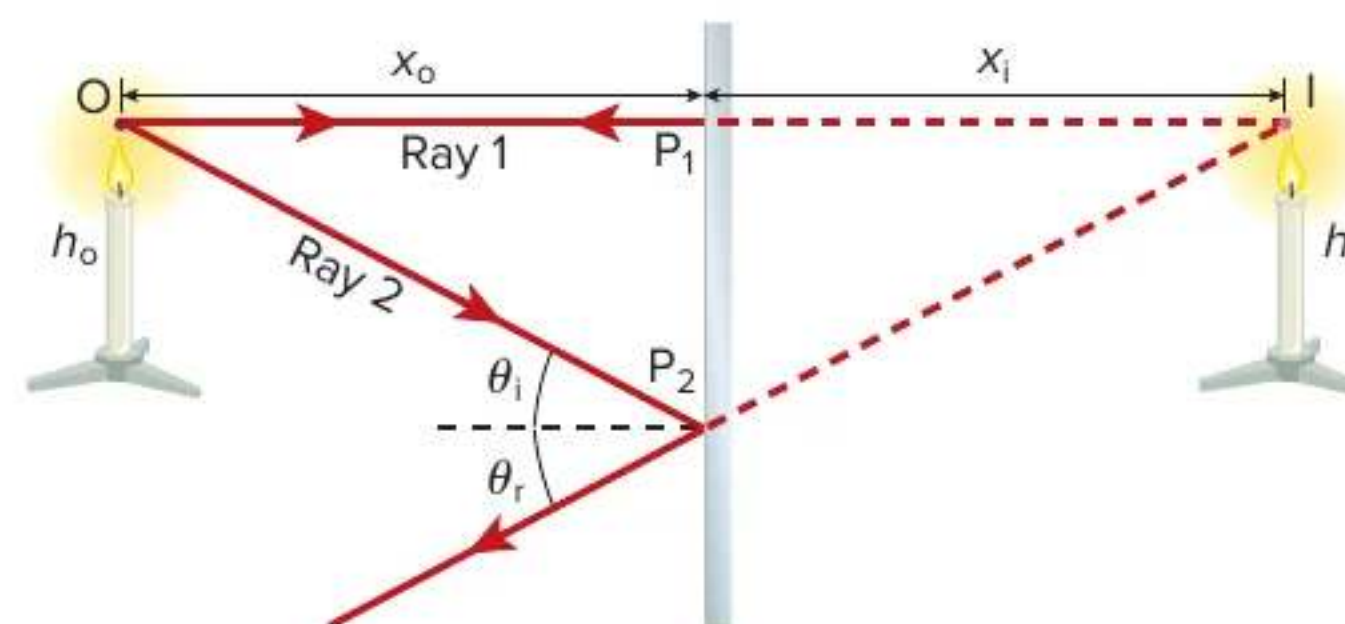


Figure 7 Reflected light rays from the candle (two rays are shown) strike the mirror. Some of those rays reach the viewer's eye. Sight lines (dashed lines) are drawn from where the rays reflect from the mirror to where they converge. The image is located where the sight lines converge.

Explain why $x_i = -x_o$.

Plane-Mirror Image Position

With a plane mirror, the image position is equal to the negative of the object position. The negative sign indicates that the image is behind the mirror (and therefore virtual).

$$x_i = -x_o$$



Get It?

Summarize the properties of a plane-mirror image.

STEM CAREER Connection

Set Designer

Have you ever attended a school play or other production and noticed eerie light effects? You could use your understanding of the properties of light to create sets for movies, theaters, or concerts. Set designers must understand how to use different properties of light, such as reflection, refraction, color, intensity, and brightness, to create mood and special effects.

You can draw more light rays from the object to the mirror to determine the size of the image. The sight lines of two rays originating from the bottom of the candle in **Figure 7** will converge at the bottom of the image. From the law of reflection and congruent-triangle geometry, we find the following is true of the object height (h_o) and image height (h_i) and any other dimension of the object and image.

Plane-Mirror Image Height

With a plane mirror, image height is equal to object height.

$$h_i = h_o$$

Image orientation A plane mirror produces an image with the same orientation as the object. If you

stand upright, a plane mirror produces an image of you standing upright. If you do a headstand, the mirror shows you doing a headstand. However, the mirror gives a front-to-back reversal. You are looking face-to-face at the image of yourself.

Follow the sight lines in **Figure 8**. The image is behind the mirror such that the boy appears to be facing himself. When the boy extends his right hand, it appears as though the left hand of the image is extended, but from the boy's perspective, the hand that is extended is on his right-hand side. This happens because a plane mirror does not really reverse left and right. The mirror in **Figure 8** only reverses the boy's image such that it is facing in the opposite direction as the boy, or, in other words, it produces a front-to-back reversal.

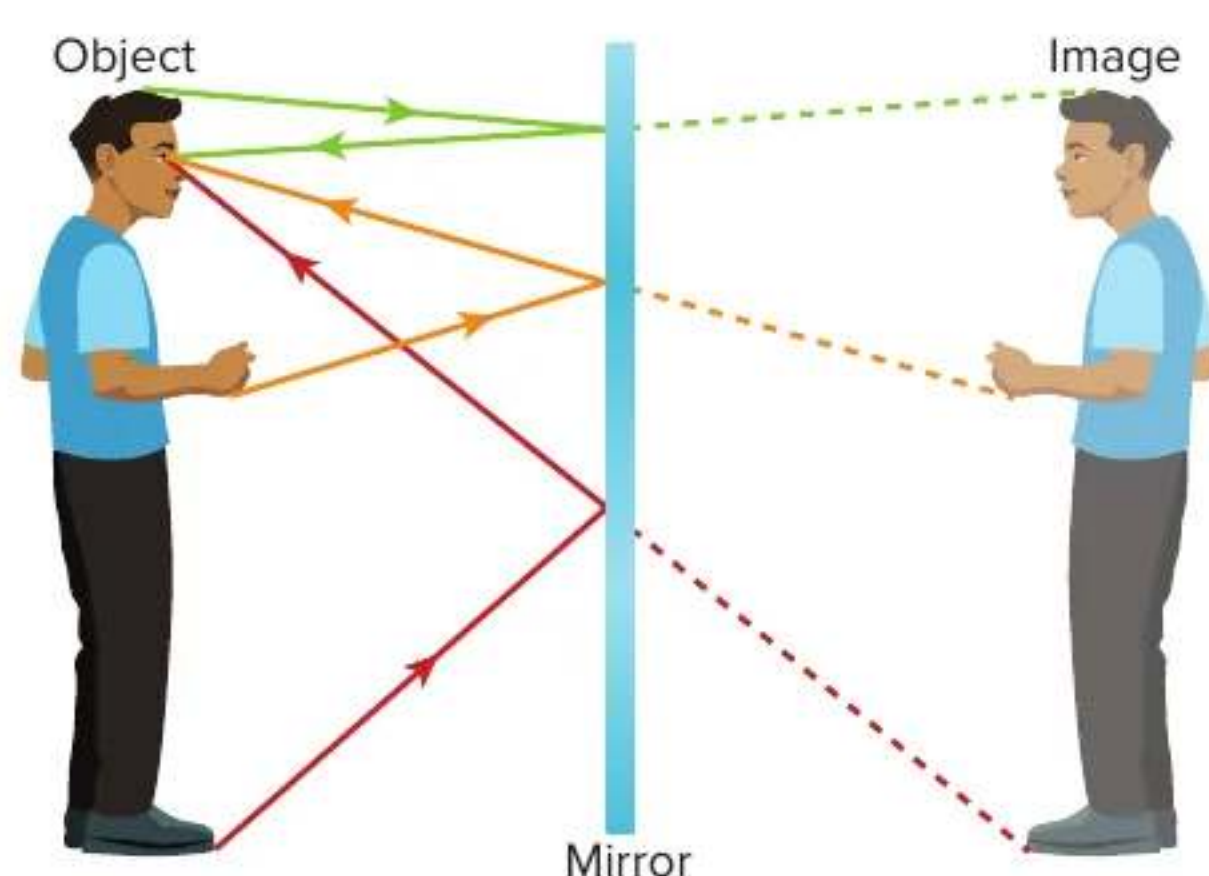


Figure 8 Viewed in a mirror, your height and distance from the mirror appear the same. There is, however, a difference. You are facing the opposite direction.



Check Your Progress

7. **Reflection** A light ray strikes a flat, smooth, reflecting surface at an angle of 80° to the normal. What angle does the reflected ray make with the surface of the mirror?
8. **Image Properties** A dog looks at its image, as shown in **Figure 9**. What are the position, height, and type of image?

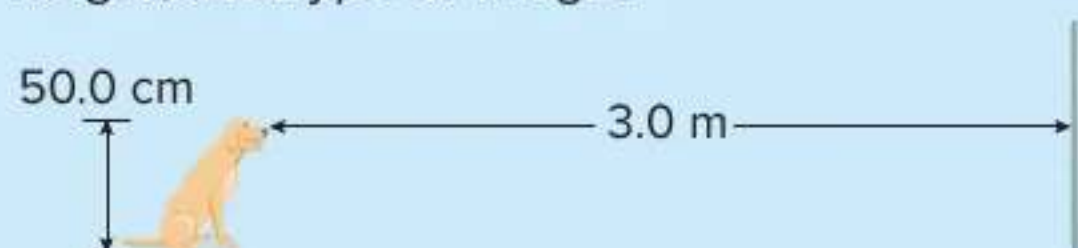


Figure 9

9. **Law of Reflection** Explain how the law of reflection applies to diffuse reflection.
10. **Image Diagram** A car is following another car along a straight road. The first car has a rear window tilted at 45° to the horizontal. Draw a ray diagram showing the position of the Sun that would cause sunlight to reflect into the eyes of the driver of the second car.
11. **Critical Thinking** Explain how diffuse reflection of light off an object enables you to see that object from any angle.

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LESSON 2 CURVED MIRRORS

FOCUS QUESTION

What are some advantages and disadvantages of curved mirrors compared to plane mirrors?

Properties of Curved Spherical Mirrors

Move a metal spoon toward and away from your face. Look at your image on both sides of the spoon. Your image may appear larger or smaller, or it may even be inverted. The properties of curved mirrors, such as a metal spoon, and the images that they form depend on the shape of the mirror and the object's position.

The way that a spherical mirror reflects light depends on its curve. **Figure 10** shows two spherical mirrors and some important points and distances for understanding how they form images. A spherical mirror is shaped as if it were a section of a hollow sphere. The mirror (M) has the same geometric center (C) as a sphere of radius r . The distance r is also called the radius of curvature.

The straight line that includes line segment CM is called the **principal axis**, or line perpendicular to the mirror's surface that divides the mirror in half.

Focal point Light rays from the Sun to Earth are almost parallel because the Sun is so far away. When you point the principal axis of a concave mirror toward the Sun, all the rays reflect through a point. You can locate this point by moving a sheet of paper toward and away from the mirror until the smallest and sharpest spot of sunlight is focused on the paper. This spot is the mirror's **focal point**, which is the point at which incident light rays that are parallel to the principal axis converge after reflecting from the mirror. This is point F.

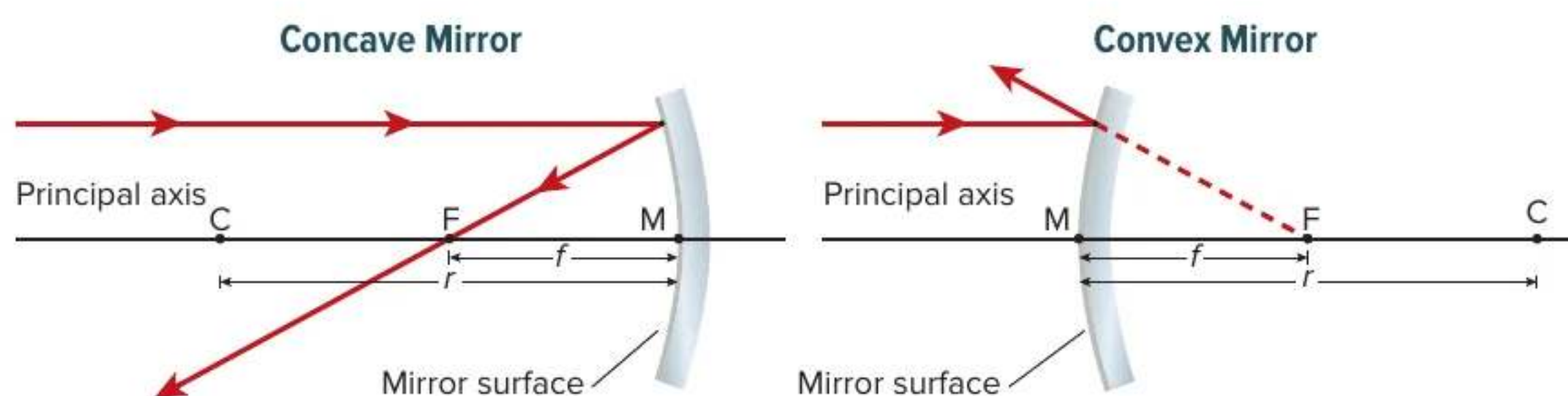


Figure 10 For concave and convex mirrors, the distance from M to F (the focal point) is half the distance from M to C.



3D THINKING

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



Virtual Investigation: Ray Tracing for Mirrors

Use a computer model to investigate how the properties of a curved mirror affect how light is reflected.

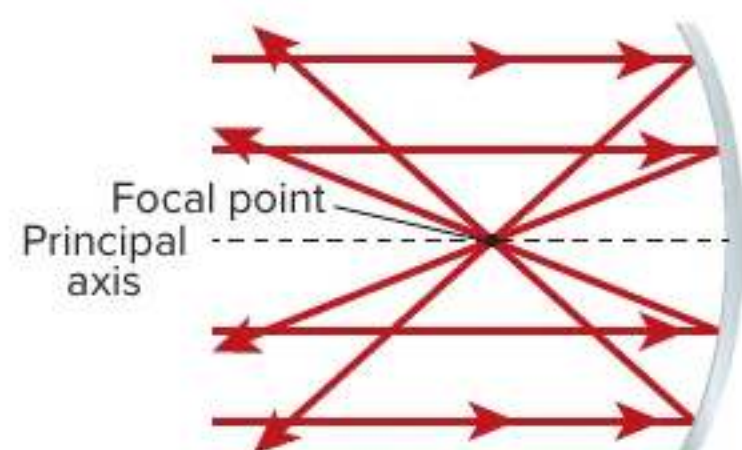


PhysicsLAB: Concave Mirror Images

Construct an explanation for why different mirrors produce different images when reflecting light.

For the mirror on the left in **Figure 10**, a ray parallel to the principal axis is reflected and crosses the principal axis at point F. Note that the distance from M to F is half the distance from M to C. The **focal length** (f) is the distance between the mirror and the focal point and can be expressed as $f = \frac{r}{2}$.

Concave mirrors The inside surface of a shiny metal spoon acts as a concave mirror. A **concave mirror**, such as the one on the left in **Figure 10**, has an inwardly curving reflective surface, the edges of which curve toward the observer. Incident light rays parallel to the principal axis are reflected and intersect the principal axis at point F. The focal length is positive for a concave mirror because it is a point of converging rays. Rays reflected from concave mirrors are parallel, as shown in **Figure 11**. Concave mirrors are placed in car headlights. It is a concave mirror that allows the light from a spotlight to be focused into a spot.



Light rays parallel to the principal axis reflect off a concave mirror and converge at the focal point.



A lightbulb is placed at the focal point of a concave mirror. The parallel reflected rays are evident in this flashlight beam.

Figure 11 Rays traced from the focal point to a concave mirror reflect off the surface as parallel rays.

Convex mirrors A spoon's outer surface acts as a **convex mirror**, an outwardly curving reflective surface with edges that curve away from the observer, as shown on the right side of **Figure 10**. Properties of a spherical convex mirror are also shown in **Figure 10**. The focal point (F) and the geometric center of the mirror (C) are behind the mirror. Store security mirrors and many side-view mirrors on cars are convex mirrors.

Magnification Another property of a spherical mirror is **magnification** (m), which is the ratio of the image height to the object height.

The sign of the magnification tells you whether the image is upright or inverted. For virtual images such as those formed by convex mirrors, x_i is negative. This means m is positive. Note that virtual images are always upright, so the height is always positive.

Magnification

The magnification of an object by a spherical mirror, defined as the image height divided by the object height, is equal to the negative of the image position, divided by the object position.

$$m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$



Get It?

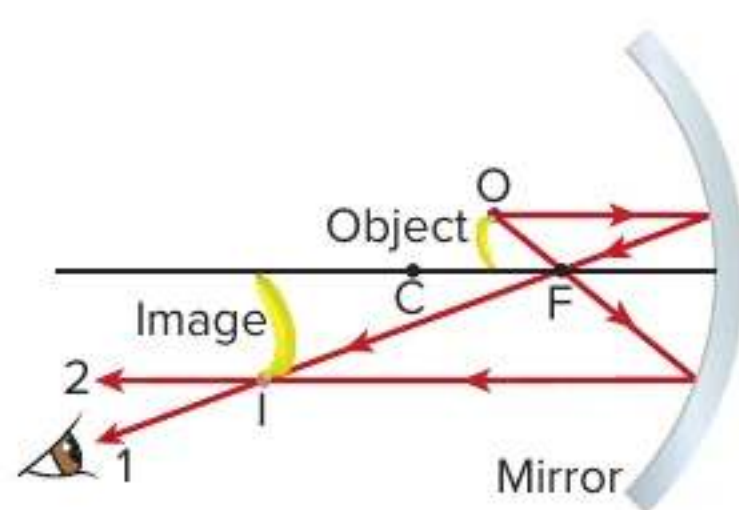
Explain What does it mean if $m < 1$?

Ray Diagrams for Curved Mirrors

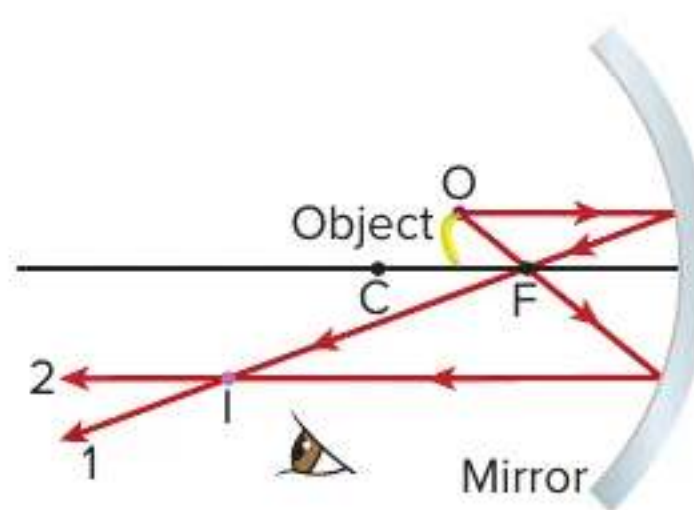
Ray diagrams give you a visual representation of images formed by a curved mirror, depending on where the object is placed in relation to the focal point.

Figure 12 shows rays reflected from a concave mirror converging at the point I, where an image is located. The converging light rays form a **real image** that is inverted and larger than the object. You can see the image floating in space if you place your eye so that the rays that form the image fall on your eye, as in **Figure 12**. You must face a direction that allows the light rays coming from the image to enter your eyes, however. You cannot look at the image from behind. If you were to place a piece of paper or a movie screen at this point, the image would appear on the screen, as shown on the right in **Figure 12**. You cannot project virtual images on a screen, since they are not formed by converging light rays.

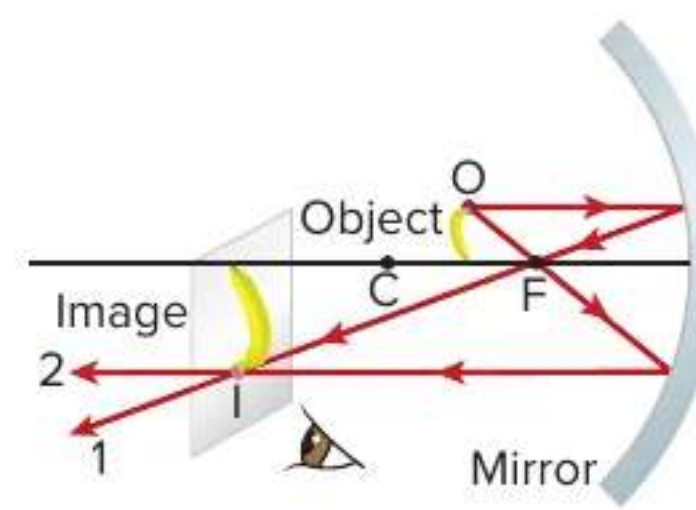
In **Figure 12**, if the object is beyond point C, the absolute value of the magnification for the real image is less than 1. This means that the image is smaller than the object. If the object is placed between point C and point F, the absolute value of the magnification is greater than 1, which means the image is larger than the object.



The eye is positioned so that the rays that form the real image strike the eye, allowing the image to be seen.



Rays from the object do not reach the eye, so the image cannot be seen from this position.



The image can be seen when projected on a white opaque screen.

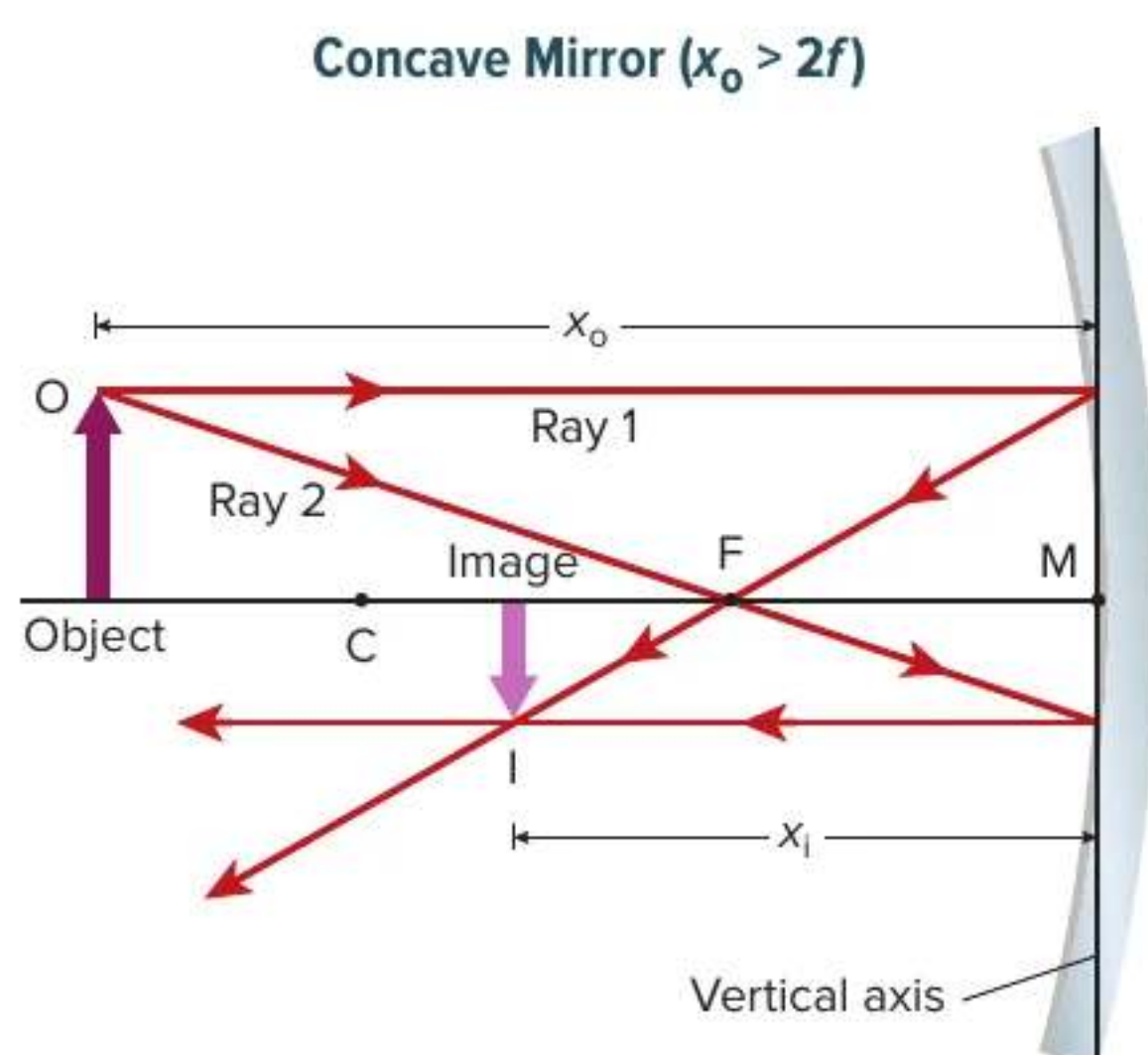
Figure 12 Ray diagrams can be used to locate an image reflected from a curved mirror.

Real Images with Concave Mirrors

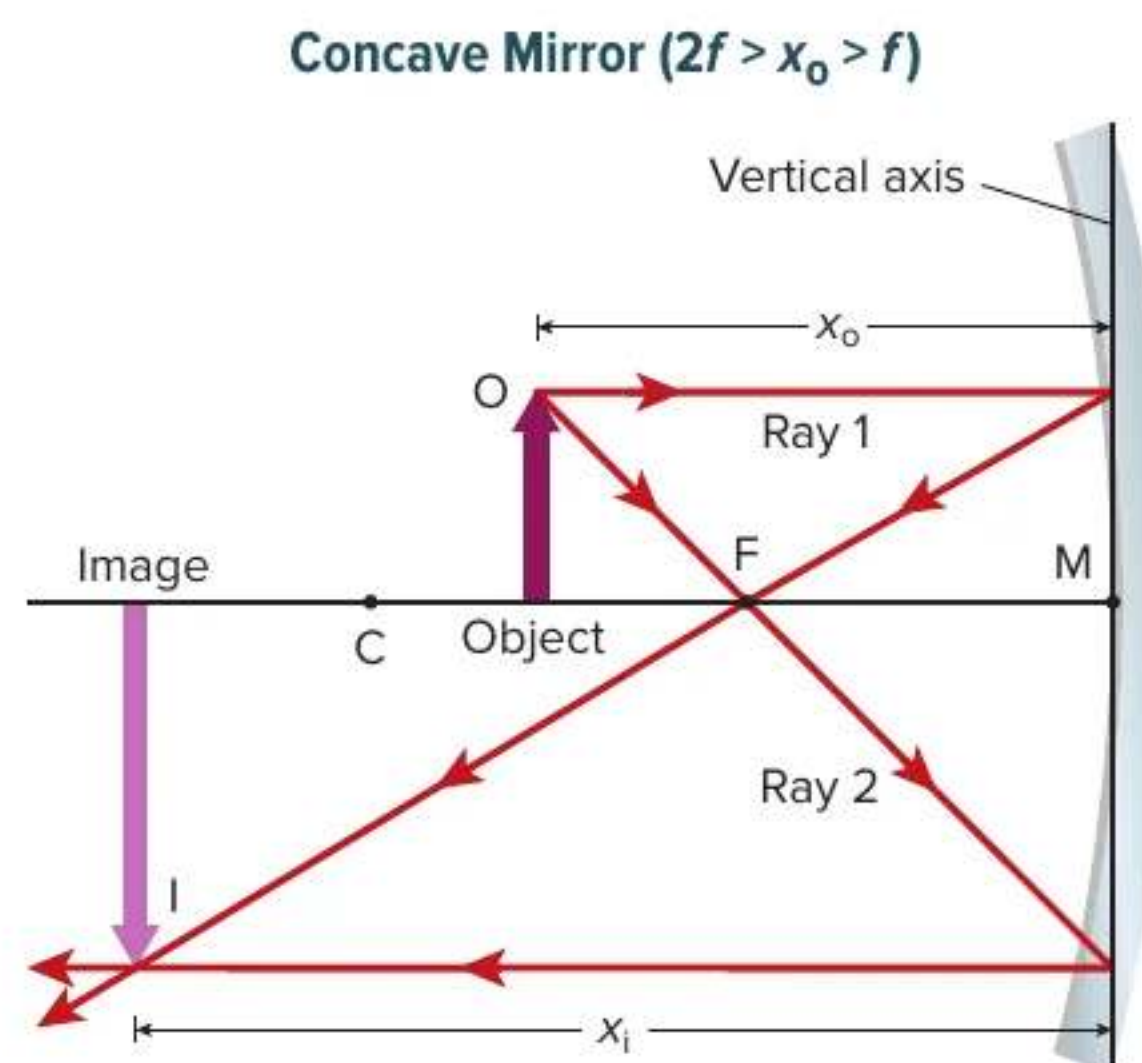
You can often simplify ray diagrams by using simple, one-dimensional objects, such as the arrow shown in **Figure 13** on the next page. A spherical concave mirror produces an inverted real image if the object position (x_o) is greater than twice the focal length (f). The object is then beyond the center of curvature (C). If the object is placed between the center of curvature and the focal point, as shown on the right in **Figure 13**, the image is again real and inverted. The size of the image is now greater than the size of the object.

CCC CROSSCUTTING CONCEPTS

Cause and Effect Systems can be designed to cause a desired effect. You want to design a mirror that can be used for shaving, applying makeup, or both, so you want your mirror to produce a large image. Would you use a plane mirror, a convex mirror, or a concave mirror? What evidence will you use to support your choice? Make a simple sketch of your design.



The image is reduced for an object located a distance greater than $2f$ from the mirror.



The image is enlarged for an object between f and $2f$.

Figure 13 The type of image depends on the object's distance from the mirror. A real, inverted image is formed in both situations shown here. Remember that f , the focal length, is the distance from M to F.

Diagram how other points on the image are formed.

COLOR CONVENTION

Object		magenta
Image		lilac
Light ray		red

PROBLEM-SOLVING STRATEGIES

Using Ray Diagrams to Locate Images Produced by Spherical Mirrors

Use the following strategies for curved spherical-mirror problems. Refer to **Figure 13**, as well as **Figure 14** and **Figure 15** on the next page.

- Using lined or graph paper, draw the principal axis of the mirror as a horizontal line from the left side to the right side of your paper, leaving six blank lines above and six blank lines below.
- Place points and labels on the principal axis for the object, C, and F as follows:
 - If the mirror is concave and the object is beyond C away from the mirror, place the mirror at the right side of the page, the object at the left side of the page, and make C and F to scale.
 - If the mirror is concave and the object is between C and F, place the mirror at the right side of the page, C at the center of the paper, F halfway between the mirror and C, and the object to scale.
 - For any other situation, place the mirror in the center of the page, the object or F (whichever is the greater distance from the mirror) at the left side of the page, and the other to scale.
- To represent the mirror, draw a vertical line at the mirror point that extends for 12 lines. This is the mirror's principal plane.
- Draw the object as an arrow, and label its top O. For concave mirrors, objects inside of C should not be higher than three lines. For all other situations, the objects should be six lines high. The scale for the height of the object will be different from the scale along the principal axis.
- Draw ray 1, the parallel ray. It is parallel to the principal axis, reflects off the principal plane, and passes through F.
- Draw ray 2, the focus ray. It passes through F, reflects off the principal plane, and is reflected parallel to the principal axis.
- If reflected rays 1 and 2 diverge from each other, then extend the sight lines behind the mirror as dashed lines.
- The image is located where rays 1 and 2 or their sight lines cross after reflection. For either case, label the point of intersection I. Draw the image as a perpendicular arrow from the principal axis to I.

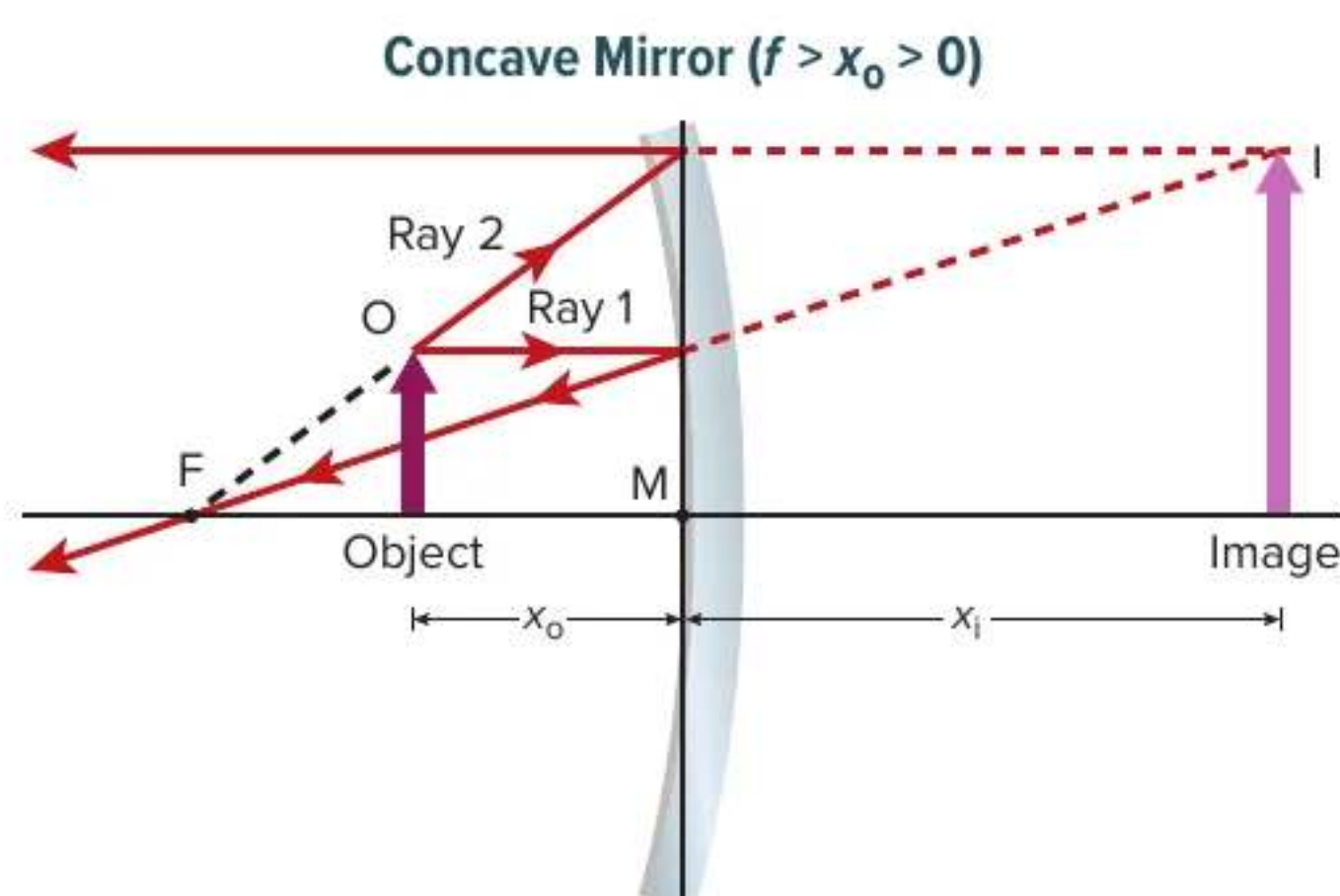


Figure 14 A virtual, upright, enlarged image is formed when an object, such as the block tower on the right, is placed between the focal point and the surface of a concave mirror.

Consider the cause of the appearance of the other images in the mirror.

Virtual Images with Concave Mirrors

You have seen that as an object approaches the focal point (F) of a concave mirror, the image moves farther away from the mirror. If the object is at the focal point, all reflected rays are parallel. They never meet, therefore, and the image is said to be at infinity, so the image could never be seen.

What happens if you move the object even closer to the mirror? The image is upright and behind the mirror. A concave mirror produces a virtual image if the object is located between the mirror and the focal point. This situation is shown in the ray diagram in **Figure 14**.

You draw two rays to locate the image of a point on an object. As before, you draw ray 1 parallel to the principal axis and then reflected through the focal point. Ray 2 is drawn as a line from the point on the object to the mirror, along a line defined by the focal point and the point on the object. At the mirror, ray 2 is reflected parallel to the principal axis. Note in **Figure 14** that ray 1 and ray 2 diverge as they leave the mirror, so there cannot be a real image. However, sight lines extended behind the mirror converge, showing that a virtual image forms behind the mirror. From the position of the point of convergence, you can see that the image is upright and larger than the object.



Get It?

Explain how you can determine whether an image is virtual.

Virtual Images with Convex Mirrors

Rays reflected from a convex mirror always diverge. Thus, convex mirrors form only virtual images. The ray diagram in **Figure 15** shows how a spherical convex mirror forms an image. The figure shows two rays, but remember that there is an infinite number of rays reflecting from the object. Ray 1 approaches the mirror parallel to the principal axis. Ray 1 reflects off the mirror following the line between F and the point where ray 1 strikes the principal plane.

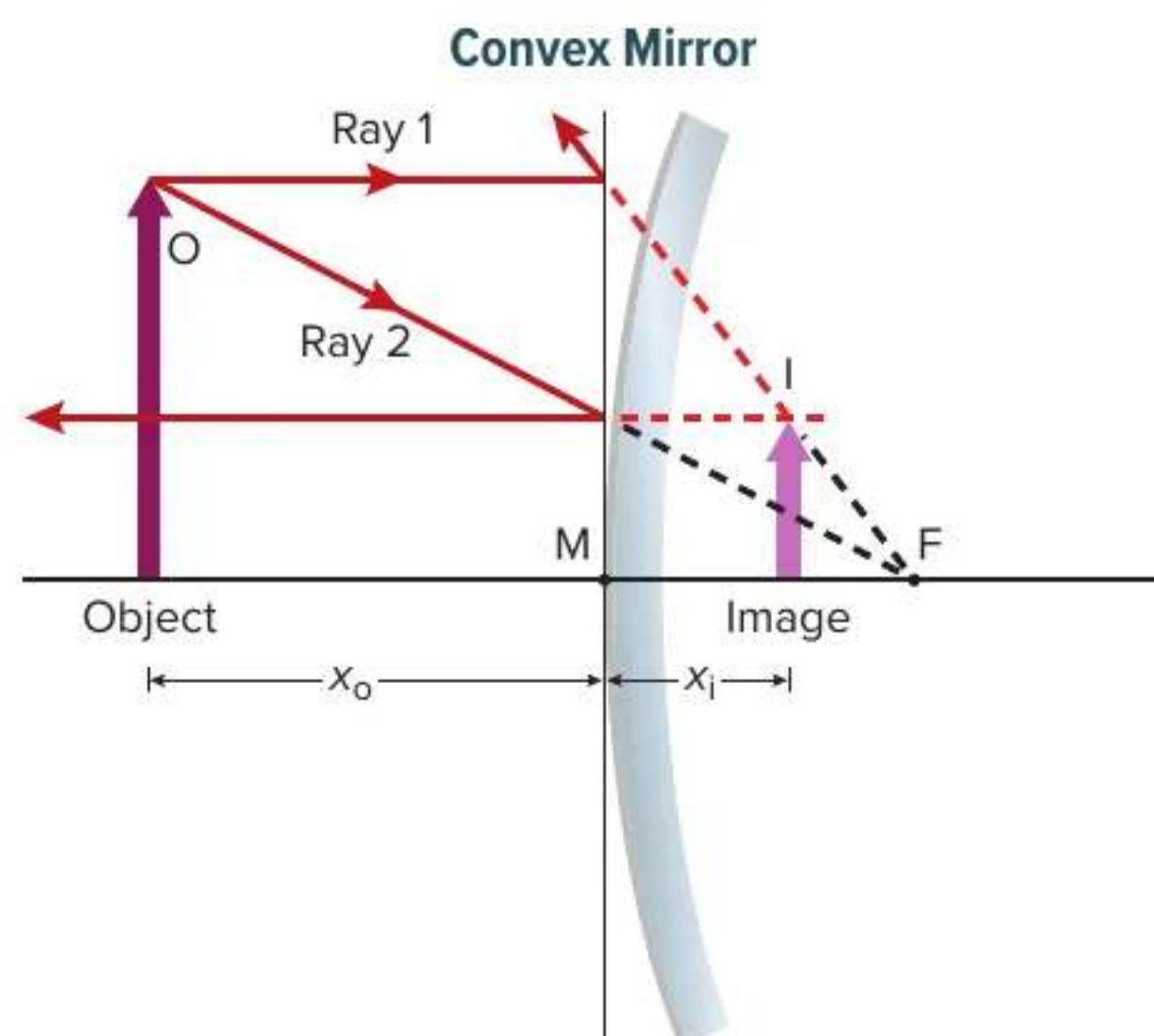


Figure 15 Convex mirrors form images that are virtual, upright, and smaller than the object.

A sight line extends from the point where ray 1 strikes the principle plane through F. Ray 2 approaches the mirror on a path that, if extended behind the mirror, would pass through F. The reflected part of ray 2 and its sight line are parallel to the principal axis. The two reflected rays diverge. However, the sight lines intersect behind the mirror. The point where they intersect is the location of the image. An image produced by a convex mirror is a virtual image that is upright and smaller than the object.

Field of view It might seem that convex mirrors would have little use because the images they form are smaller than the objects. This property of convex mirrors, however, does have practical uses. By forming smaller images, convex mirrors enlarge the area that an observer sees, which is called the field of view. Also, the image is visible from a large range of angles; thus, the field of view is visible from a wide perspective.

Because of the increased field of view, convex mirrors are used in cars as passenger-side rearview mirrors, as shown in **Figure 16**. However, because the reduced image size makes the objects appear farther away than they really are, there is often a warning on passenger-side rearview mirrors of cars stating that objects may be closer than they appear.



Figure 16 Images from convex mirrors are smaller than the object. This increases the field of view and decreases the driver's blind spot.

Explain why a warning stating that objects in a convex side-view mirror are closer than they appear may be useful.

itself, as shown in **Figure 17**. Only parallel rays that are close to the principal axis reflect through the focal point. Other rays converge at points closer to the mirror. This defect, called **spherical aberration**, occurs because the light rays do not all converge at the focal point, which makes an image look fuzzy.

A mirror ground to the shape of a parabola, as in **Figure 17**, suffers no spherical aberration. Large, parabolic glass mirrors are expensive to manufacture; so, many telescopes use spherical mirrors and smaller, specially designed secondary mirrors or lenses to correct for spherical aberration.

Defects in Concave Mirrors

In diagramming rays, you used a vertical line to represent the mirror. In reality, light rays reflect off the mirror

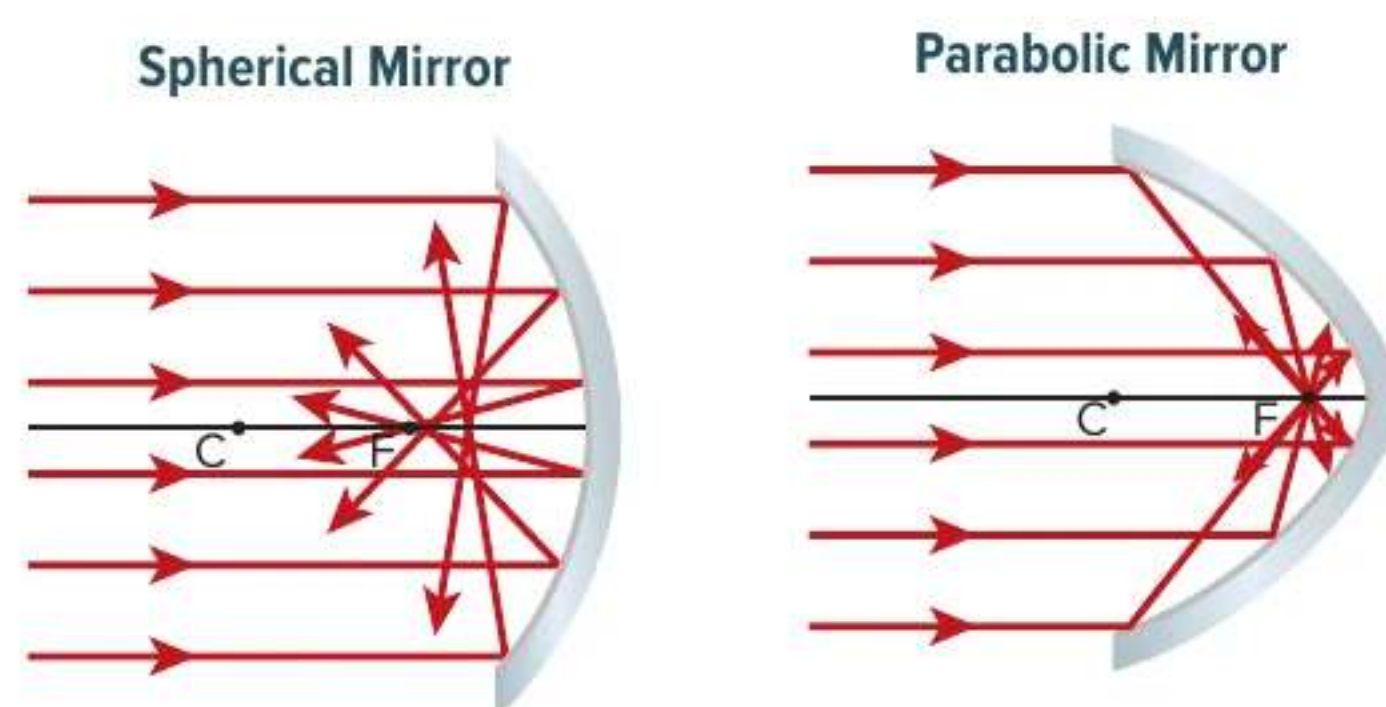


Figure 17 Spherical aberration occurs for spherical mirrors but does not occur for parabolic mirrors.



Get It?

Explain why a spherical mirror produces spherical aberration but a mirror in the shape of a parabola does not produce spherical aberration.

Calculating Image Position

We can use the spherical mirror model to develop a simple equation for spherical mirrors. You must use the paraxial ray approximation, which states that only rays that are close and almost parallel to the principal axis are used to form an image. Using this with the law of reflection leads to the mirror equation, relating the focal length (f), object position (x_o), and image position (x_i) of a spherical mirror.

Mirror Equation

The reciprocal of the focal length of a spherical mirror is equal to the sum of the reciprocals of the image position and the object position.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

Real-World Physics

IN 1990, NASA launched the *Hubble Space Telescope* into orbit around Earth. Scientists expected *Hubble* to provide clear images without atmospheric distortions. Soon after it was deployed, however, scientists found that *Hubble* had a spherical aberration. In 1993, astronauts installed corrective optics called COSTAR on *Hubble*, enabling *Hubble* to produce clear images. For over twenty-five years, *Hubble* provided the world with stunning images and enabled scientists to make important discoveries about the universe.

Negative values When virtual images are formed, the image position (x_i) has a negative value, indicating that it is located behind the mirror. For concave mirrors, a virtual image only forms when the object is between a concave mirror and the focal point. The focal point is in front of the mirror and the focal length has a positive value. For convex mirrors, the focal point is always behind the mirror, and the focal length has a negative value.

Remember that the mirror equation is only approximately correct. In reality, light coming from an object toward a mirror is diverging, so not all of the light is close to or parallel to the axis. When the mirror diameter is large relative to the radius of curvature to minimize spherical aberration, this equation predicts image properties more precisely.

CONNECTING MATH to Physics

Adding and Subtracting Fractions When using the mirror equation, you first use math to move the fraction that contains the quantity you are seeking to the left-hand side of the equation and everything else to the right. Then you combine the two fractions on the right-hand side by using a common denominator that results from multiplying the denominators.

Math	Physics
$\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$	$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$
$\frac{1}{y} = \frac{1}{x} - \frac{1}{z}$	$\frac{1}{x_i} = \frac{1}{f} - \frac{1}{x_o}$
$\frac{1}{y} = \left(\frac{1}{x}\right)\left(\frac{z}{z}\right) - \left(\frac{1}{z}\right)\left(\frac{x}{x}\right)$	$\frac{1}{x_i} = \left(\frac{1}{f}\right)\left(\frac{x_o}{x_o}\right) - \left(\frac{1}{x_o}\right)\left(\frac{f}{f}\right)$
$\frac{1}{y} = \frac{z-x}{xz}$	$\frac{1}{x_i} = \frac{x_o - f}{fx_o}$
$y = \frac{xz}{z-x}$	$x_i = \frac{fx_o}{x_o - f}$

Using this approach, the following relationships can be derived for image position, object position, and focal length:

$$x_i = \frac{fx_o}{x_o - f} \quad x_o = \frac{fx_i}{x_i - f} \quad f = \frac{x_i x_o}{x_i + x_o}$$

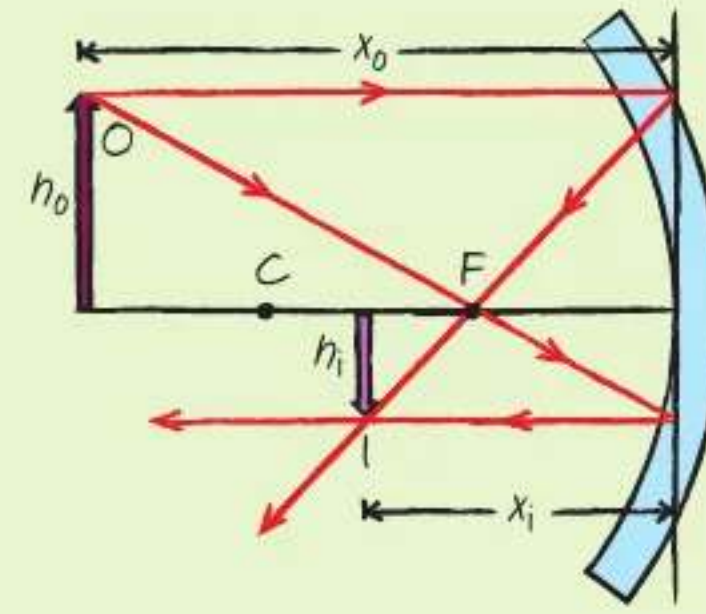
EXAMPLE Problem 2

REAL IMAGE FORMATION BY A CONCAVE MIRROR A concave mirror has a radius of curvature of 20.0 cm. You place a 2.0-cm-tall object 30.0 cm from the mirror. What are the image position and image height?

1 ANALYZE AND SKETCH THE PROBLEM

- Draw a diagram with the object and the mirror.
- Draw two principal rays to locate the image in the diagram.

Known	Unknown
$h_o = 2.0 \text{ cm}$	$x_i = ?$
$x_o = 30.0 \text{ cm}$	$h_i = ?$
$r = 20.0 \text{ cm}$	

**2 SOLVE FOR THE UNKNOWN**

Focal length is half the radius of curvature.

$$\begin{aligned} f &= \frac{r}{2} \\ &= \frac{20.0 \text{ cm}}{2} && \text{Substitute } r = 20.0 \text{ cm.} \\ &= 10.0 \text{ cm} \end{aligned}$$

Use the relationship between focal length and object position to solve for image position.

$$\begin{aligned} \frac{1}{f} &= \frac{1}{x_i} + \frac{1}{x_o} \\ x_i &= \frac{fx_o}{x_o - f} \\ &= \frac{(10.0 \text{ cm})(30.0 \text{ cm})}{30.0 \text{ cm} - 10.0 \text{ cm}} && \text{Substitute } f = 10.0 \text{ cm, } x_o = 30.0 \text{ cm.} \\ &= 15.0 \text{ cm (real image, in front of the mirror)} \end{aligned}$$

Use the relationship between object height and object and image position to solve for image height.

$$\begin{aligned} m &\equiv \frac{h_i}{h_o} = \frac{-x_i}{x_o} \\ h_i &= \frac{-x_i h_o}{x_o} \\ &= -\frac{(15.0 \text{ cm})(2.0 \text{ cm})}{30.0 \text{ cm}} && \text{Substitute } x_i = 15.0 \text{ cm, } h_o = 2.0 \text{ cm, } x_o = 30.0 \text{ cm.} \\ &= -1.0 \text{ cm (inverted, smaller image)} \end{aligned}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** All positions and heights are in centimeters.
- **Do the signs make sense?** Positive position and negative height agree with the drawing.

PRACTICE Problems**ADDITIONAL PRACTICE**

- Use a ray diagram drawn to scale to solve Example Problem 2.
- You place an object 36.0 cm in front of a concave mirror with a 16.0-cm focal length. Determine the image position.
- You place a 3.0-cm-tall object 20.0 cm from a 16.0-cm-radius concave mirror. Determine the image position and image height.
- A concave mirror has a 7.0-cm focal length. You place a 2.4-cm-tall object 16.0 cm from the mirror. Determine the image height.
- CHALLENGE** You place an object near a concave mirror with a 10.0-cm focal length. The image is 3.0 cm tall, inverted, and 16.0 cm from the mirror. What are the object position and object height?

EXAMPLE Problem 3

IMAGE IN A SECURITY MIRROR A convex security mirror in a warehouse has a -0.50-m focal length. A 2.0-m -tall forklift is 5.0 m from the mirror. What are the forklift's image position and image height?

1 ANALYZE AND SKETCH THE PROBLEM

- Draw a diagram with the mirror and the object.
- Draw two principal rays to locate the image in the diagram.

Known

$$h_o = 2.0\text{ m}$$

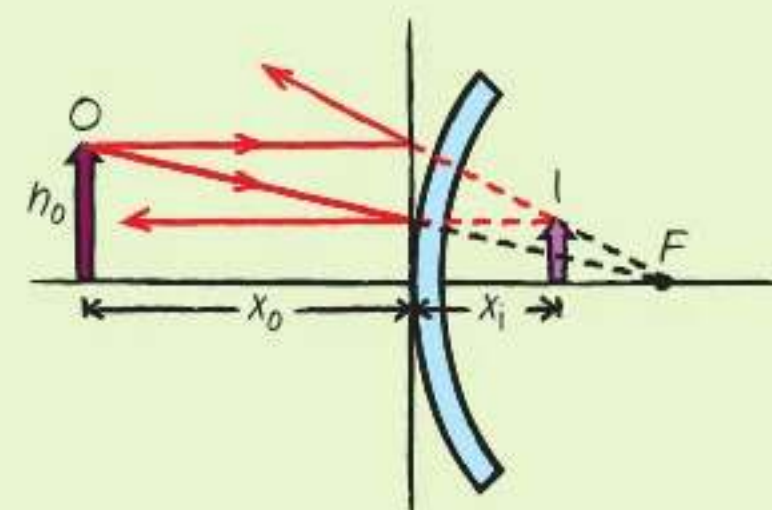
$$x_o = 5.0\text{ m}$$

$$f = -0.50\text{ m}$$

Unknown

$$x_i = ?$$

$$h_i = ?$$

**2 SOLVE FOR THE UNKNOWN**

Use the relationship between focal length and object position to solve for image position.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

$$x_i = \frac{fx_o}{x_o - f}$$

$$= \frac{(-0.50\text{ m})(5.0\text{ m})}{5.0\text{ m} - (-0.50\text{ m})} \quad \text{Substitute } f = -0.50\text{ m}, x_o = 5.0\text{ m.}$$

$$= -0.45\text{ m (virtual image, behind the mirror)}$$

Use the relationship between object height and object and image position to solve for image height.

$$m \equiv \frac{h_i}{h_o} = \frac{-x_i}{x_o}$$

$$h_i = \frac{-x_i h_o}{x_o}$$

$$= \frac{-(-0.45\text{ m})(2.0\text{ m})}{5.0\text{ m}} \quad \text{Substitute } x_i = -0.45\text{ m}, h_o = 2.0\text{ m}, x_o = 5.0\text{ m.}$$

$$= 0.18\text{ m (upright, smaller image)}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** All positions and heights are in meters.
- **Do the signs make sense?** A negative position indicates a virtual image; a positive height indicates an image that is upright. These agree with the diagram.

PRACTICE Problems**ADDITIONAL PRACTICE**

- You place an object 20.0 cm in front of a convex mirror with a -15.0-cm focal length. Find the image position using both a scale diagram and the mirror equation.
- A convex mirror has a focal length of -13.0 cm . You place a 6.0-cm diameter lightbulb 60.0 cm from that mirror. What are the lightbulb's image position and diameter?
- A 7.6-cm -diameter ball is located 22.0 cm from a convex mirror with a radius of curvature of 60.0 cm . What are the ball's image position and diameter?
- A 1.8-m -tall girl stands 2.4 m from a store's security mirror. Her image appears to be 0.36 m tall.
 - What is the image's distance?
 - What is the focal length of the mirror?
- CHALLENGE** A convex mirror is needed to produce an image that is three-fourths the size of an object and located 24 cm behind the mirror.
 - What is the object's distance?
 - What focal length should be specified?

Mirror Comparison

Table 1 summarizes the properties of single-mirror systems with objects that are located on the principal axis of the mirror.

Notice that the single plane mirror and convex mirror produce only virtual images. A concave mirror produces real images when the object is farther than the focal distance. A concave mirror produces virtual images when the object is closer than the focal distance. Plane mirrors give reflections on scale with the objects, and convex mirrors provide reduced images, expanding the field of view. A concave mirror acts as a magnifier when an object is within the focal length of the mirror.

A concave mirror enlarges and inverts the image when the object is between the focal length and the radius of curvature. Beyond the radius of curvature, a concave mirror produces a reduced, inverted image.

Table 1 Single-Mirror System Properties

Mirror Type	f	x_o	x_i	m	Image
Plane	∞	$x_o > 0$	$ x_i = x_o$ (negative)	same size	virtual
Concave	+	$x_o > r$	$r > x_i > f$	reduced, inverted	real
		$r > x_o > f$	$x_i > r$	enlarged, inverted	real
		$f > x_o > 0$	$ x_i > x_o$ (negative)	enlarged	virtual
Convex	—	$x_o > 0$	$ f > x_i > 0$ (negative)	reduced	virtual



Check Your Progress

22. **Image Properties** If you know the focal length of a concave mirror, where should you place an object so that its image is upright and larger compared to the object? Will this produce a real or virtual image?
23. **Magnification** You place an object 20.0 cm in front of a concave mirror with a focal length of 9.0 cm. What is the magnification of the image?
24. **Object Position** The placement of an object in front of a concave mirror with a focal length of 12.0 cm forms a real image that is 22.3 cm from the mirror. What is the object's position?
25. **Image Position and Height** You place a 3.0-cm-tall object 22.0 cm in front of a concave mirror that has a focal length of 12.0 cm. Find the image position and height by drawing a ray diagram to scale. Verify your answer using the mirror and magnification equations.
26. **Ray Diagram** You place a 4.0-cm-tall object 14.0 cm from a convex mirror with a focal length of -12.0 cm. In a scale ray diagram show the image position and height. Verify your answer using the mirror and the magnification equations.
27. **Radius of Curvature** You place a 6.0-cm-tall object 16.4 cm from a convex mirror. If the image of the object is 2.8 cm tall, what is the mirror's radius of curvature?
28. **Focal Length** A convex mirror is used to produce an image that is two-thirds the size of an object and located 12 cm behind the mirror. What is the focal length of the mirror?
29. **Critical Thinking** Would spherical aberration be less for a mirror whose height, compared to its radius of curvature, is small or large? Explain your answer.

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Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

LESSON 3 REFRACTION OF LIGHT

FOCUS QUESTION

What happens to light when it enters a new medium?

Light and Boundaries

Recall that when light crosses a boundary between two mediums, it bends. You might be familiar with this phenomenon already. Have you ever looked down through the water of a swimming pool and thought the water looked shallower than it was? This strange appearance is due to the bending of light at the boundaries between two mediums, or refraction.

Refraction Observe the light rays in **Figure 18**. Identical rays of light start in air and pass into three different mediums: water, glass, and diamond. The light

rays hit the surface of each medium at the same angle and bend, or refract, as they cross the boundaries.

What difference do you notice between the three mediums shown? The light rays bend more when traveling from air to diamond than from air to water or air to glass. The amount of refraction depends on properties of the mediums that the light rays are traveling from and into.

What do you think the relationship is between the angle of the light as it crosses the boundary between mediums and refraction?

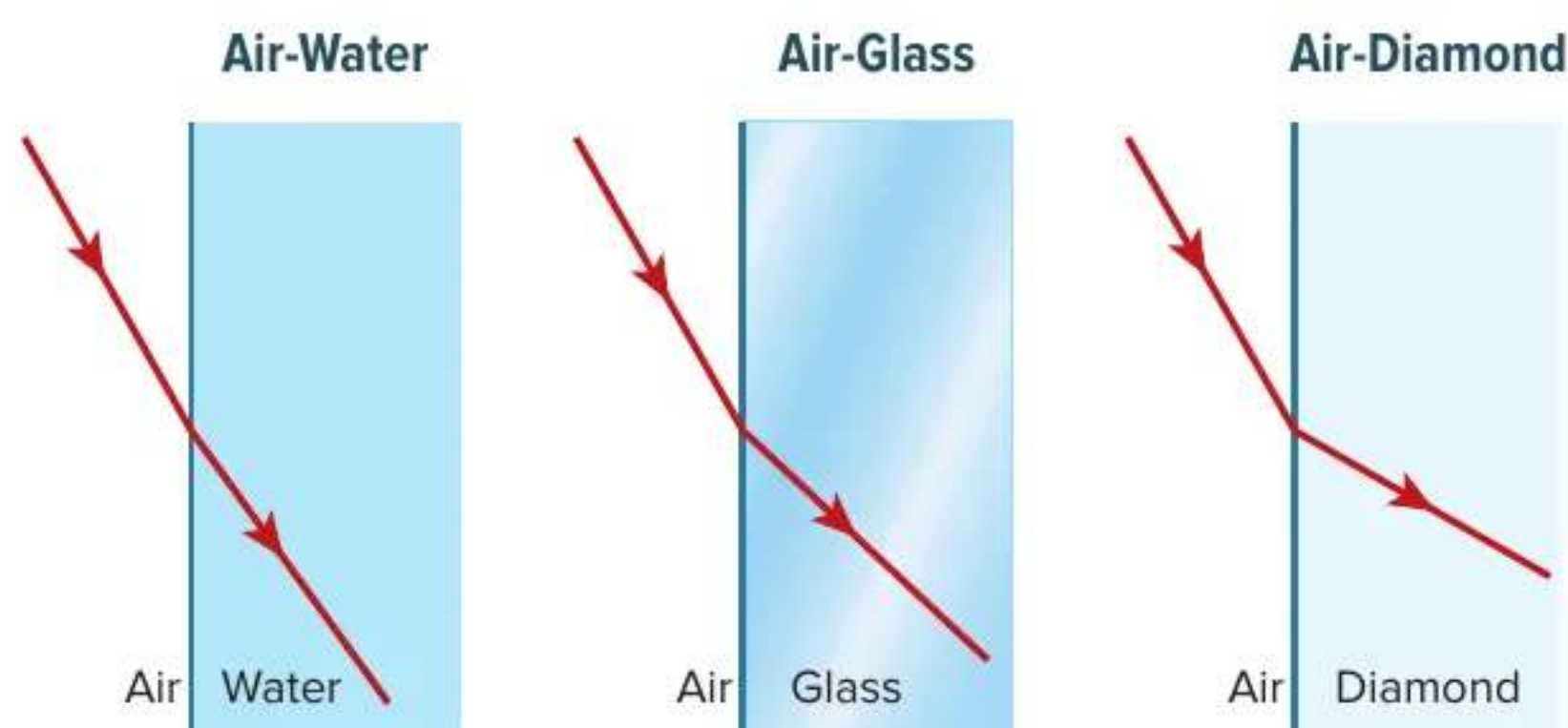


Figure 18 Light refracts as it crosses a boundary. The amount of refraction depends on the properties of the mediums. (Angles are not to scale.)



3D THINKING

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

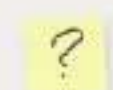
INVESTIGATE

GO ONLINE to find these activities and more resources.



Forensics Lab: A Whole Spectrum of Possibilities

Analyze data about how the index of refraction being dependent on wavelength causes the dispersion of light.



Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

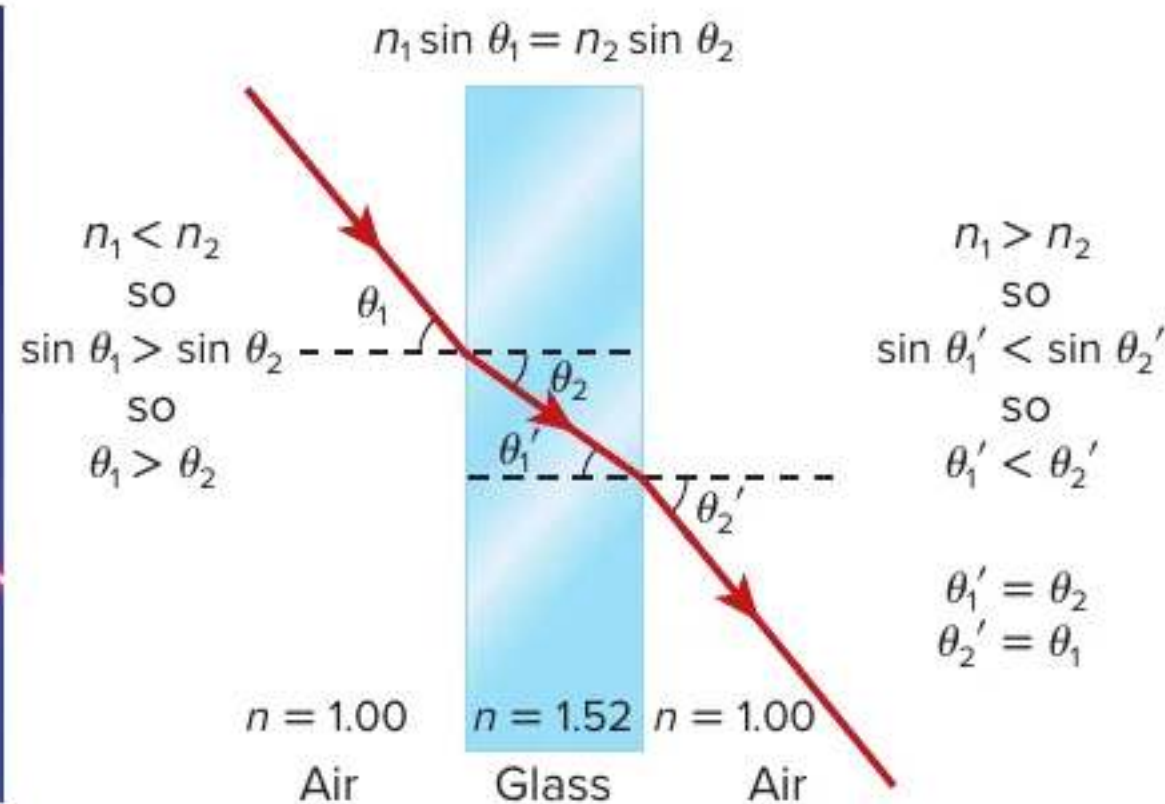
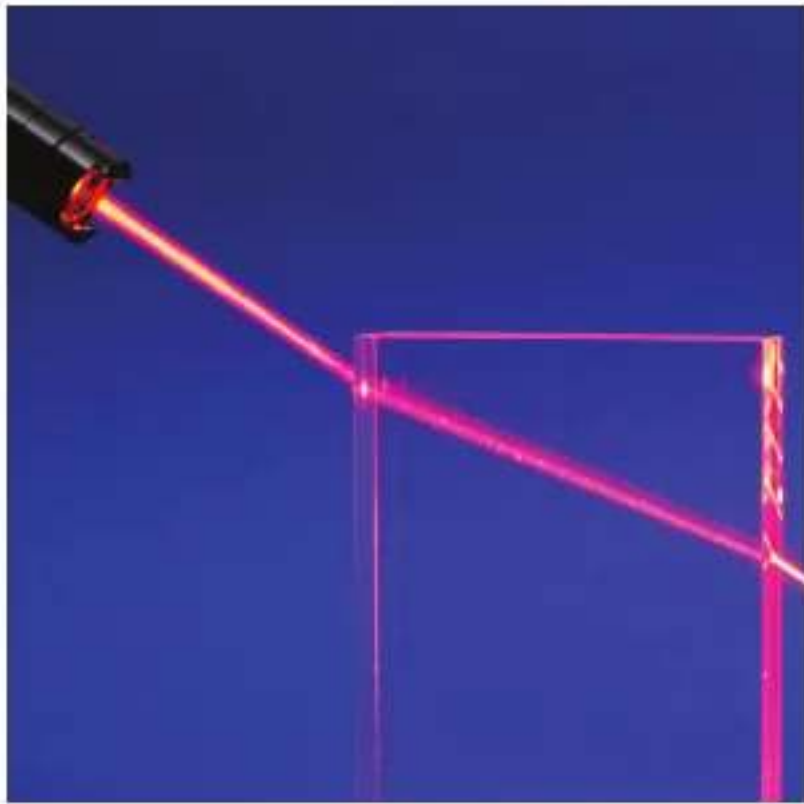


Figure 19 When light travels from air through glass and back to air, it refracts toward and then away from the normal.

Snell's Law of Refraction

René Descartes and Willebrord Snell first studied refraction in the seventeenth century by shining a narrow beam of light onto a transparent medium, such as the glass shown in **Figure 19**. They measured two angles—the angle of incidence and the angle of refraction. The angle of incidence (θ_1) is the angle at which the light ray strikes the surface. The angle of refraction (θ_2) is the angle at which the transmitted light leaves the surface. Both angles are measured with respect to the normal.

Index of refraction Snell found that when light passes from air into a transparent medium, the sines of the angles are related. This **index of refraction** (n) determines the angle of refraction of light as it crosses the boundary between two mediums. Properties of the mediums light is traveling through determine indices of refraction. Values of n for several mediums are listed in **Table 2**.

Table 2 Indices of Refraction for Yellow Light ($\lambda = 589 \text{ nm}$ in a vacuum)

Medium	n
Vacuum	1.00
Air	1.0003*
Water	1.33
Ethanol	1.36
Float glass	1.52
Quartz	1.54
Flint glass	1.62
Diamond	2.42

*The value given for air contains additional significant figures to distinguish it from that for a vacuum. Use a value of $n = 1.00$ for air in problems in this text.

Snell's Law of Refraction

The product of the index of refraction of the first medium and the sine of the angle of incidence is equal to the product of the index of refraction of the second medium and the sine of the angle of refraction.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

We can use **Figure 19** and **Table 2** to show how Snell's law applies when light travels through a piece of glass with parallel surfaces, such as a windowpane. The light is refracted when it enters the glass and again when it leaves the glass. When light goes from air into glass, it travels from a medium with a lower n of 1.00 to a medium with a higher n of 1.52. The light bends toward the normal.

Traveling from glass to air, light moves from a medium with a higher n (1.52) to one with a lower n (1.00). The light is bent away from the normal. The relative values of n determine whether the light will bend toward or away from the normal. Note the direction of the ray when it leaves the glass. It is the same as before it struck the glass, $\theta_1 = \theta_2'$. This is because the boundaries are between the same two mediums.

EXAMPLE Problem 4

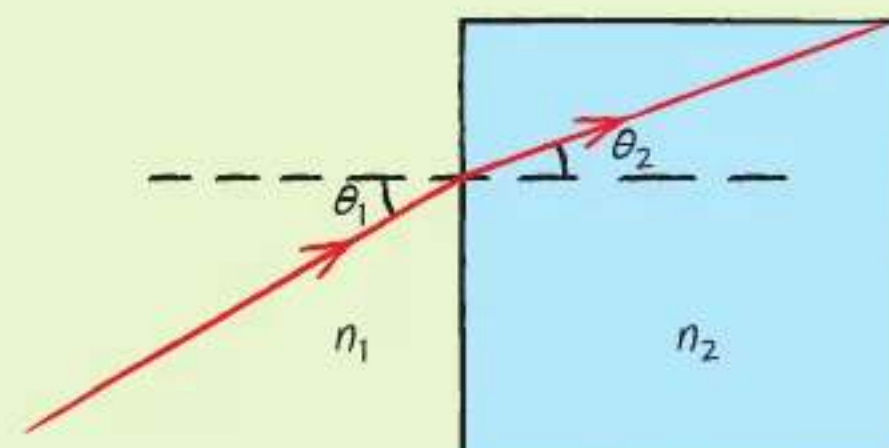
ANGLE OF REFRACTION A beam of light in air hits a sheet of float glass at an angle of 30.0° . What is the angle of refraction of the light ray?

1 ANALYZE AND SKETCH THE PROBLEM

- Make a sketch of the air and float glass boundary.
- Draw the light ray and label θ_1 , θ_2 , n_1 , and n_2 .

Known
 $\theta_1 = 30.0^\circ$
 $n_1 = 1.00$
 $n_2 = 1.52$

Unknown
 $\theta_2 = ?$

**2 SOLVE FOR THE ANGLE OF REFRACTION**

Use Snell's law to solve for the sine of the angle of refraction.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_2 = \left(\frac{n_1}{n_2} \right) \sin \theta_1$$

$$\theta_2 = \sin^{-1} \left(\left(\frac{n_1}{n_2} \right) \sin \theta_1 \right)$$

$$= \sin^{-1} \left(\left(\frac{1.00}{1.52} \right) \sin 30.0^\circ \right) \quad \text{Substitute } n_1 = 1.00, n_2 = 1.52, \theta_1 = 30.0^\circ.$$

$$= 19.2^\circ$$

3 EVALUATE THE ANSWER

- **Does the answer make sense?** The light beam travels into a medium with a higher n . Therefore, θ_2 should be less than θ_1 .

PRACTICE Problems**ADDITIONAL PRACTICE**

- 30.** A laser beam in air enters ethanol at an angle of incidence of 37.0° . What is the angle of refraction?
- 31.** As light travels from air into water, the angle of refraction is 25.0° to the normal. Find the angle of incidence.
- 32.** Light in air enters a diamond facet at 45.0° . What is the angle of refraction?
- 33.** A block of unknown material is submerged in water. Light in the water enters the block at an angle of incidence of 31° . The angle of refraction of the light in the block is 27° . What is the index of refraction of the material of the block?
- 34. CHALLENGE** Light travels from air into another medium. The angle of incidence is 45.0° and the angle of refraction is 27.7° . What is the other medium?

The Meaning of the Index of Refraction

Looking at light as a wave yields an interesting connection between the propagation of light through a medium and its index of refraction. Snell's work was done prior to the development of the wave model of light; after the development of the wave model, it was understood that light interacts with atoms in such a way that it moves more slowly through a medium than it does through a vacuum.

Remember that for light traveling through a vacuum, $\lambda_0 = \frac{c}{f}$. This can be written more generally as $\lambda = \frac{v}{f}$, where v is the speed of light in any medium and λ is the wavelength in that medium. Frequency (f) is the number of oscillations a wave makes per second.

Frequency does not change at the boundary, so the wavelength decreases when light slows down as it crosses a boundary. And because the speed of light in any medium is slower than the speed of light in a vacuum, the wavelength in any medium is shorter than it is in a vacuum.

Wave model Figure 20 shows a beam of light made up of a series of parallel, straight wavefronts. Each wavefront represents the crest of a wave and is perpendicular to the beam's direction. The beam's angle of incidence is θ_1 . Consider $\triangle PQR$. Because the wavefronts are perpendicular to the beam's direction, $\angle PQR$ is a right angle. $\angle QRP$ is equal to θ_1 . Therefore, $\sin \theta_1$ is equal to $\overline{PQ}$ divided by $\overline{PR}$.

$$\sin \theta_1 = \frac{\overline{PQ}}{\overline{PR}}$$

The angle of refraction (θ_2) can be related in a similar way to $\triangle PSR$. In this case,

$$\sin \theta_2 = \frac{\overline{RS}}{\overline{PR}}$$

By taking the ratio of the sines of the two angles, $\overline{PR}$ is canceled, leaving the following equation.

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{\overline{RS}}{\overline{PQ}}$$

In Figure 20, $\overline{PQ}$ is the length of three wavelengths of light in air, which can be written as $\overline{PQ} = 3\lambda_1$. In a similar way, $\overline{RS} = 3\lambda_2$. Substituting these values into the previous equation and canceling the common factor of 3 yields an equation relating the angles of incidence and refraction to the wavelength of light in each medium.

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{3\lambda_2}{3\lambda_1} = \frac{\lambda_2}{\lambda_1}$$

Using $\lambda = \frac{v}{f}$ and canceling the common factor f , because frequency stays constant, the equation can be rewritten:

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$$

Snell's law also can be written as a ratio of the sines of the angles of incidence and refraction.

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{n_1}{n_2}$$

Using the transitive property of equality, the previous two equations lead to the following equation:

$$\frac{n_1}{n_2} = \frac{v_2}{v_1}$$

Wavefronts

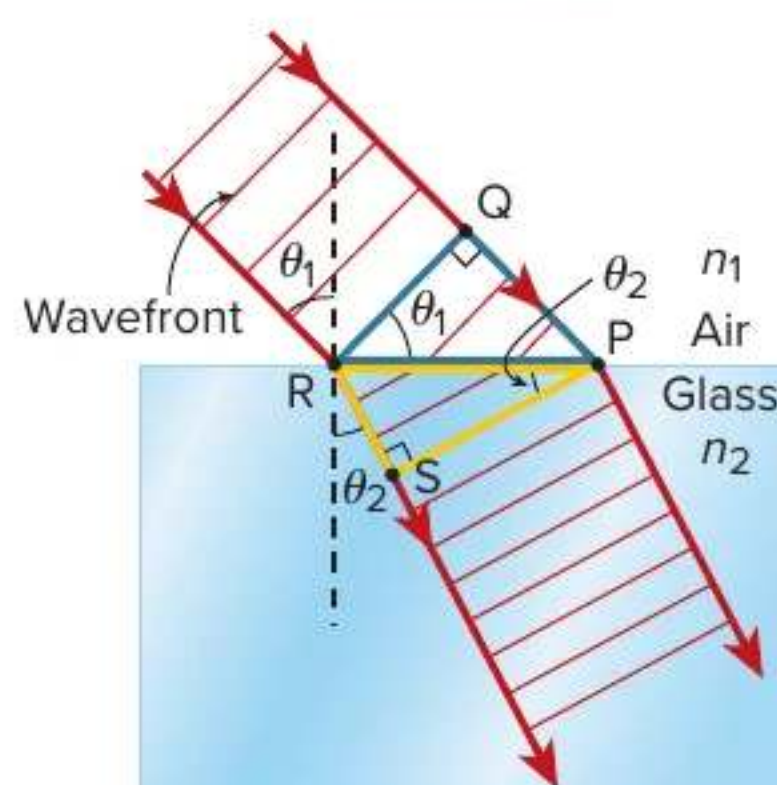


Figure 20 Each wavefront passes the boundary from air to glass at an angle. Part of the wavefront slows, and the ray bends. Since the wave slows and the frequency stays constant, for $\lambda = \frac{v}{f}$ to be true, the wavelength must decrease.

Infer Which medium has a higher index of refraction?

In a vacuum, $n = 1.00$ and $v = c$. If either medium is a vacuum, then the equation is simplified to an equation that relates the index of refraction to the speed of light in a medium.

Index of Refraction

The index of refraction of a medium is equal to the speed of light in a vacuum divided by the speed of light in the medium.

$$n = \frac{c}{v}$$

This definition of the index of refraction can be used to find the wavelength of light in a medium. In a medium with an index of refraction n , the speed of light is given by $v = \frac{c}{n}$. The wavelength of the light in a vacuum is $\lambda_0 = \frac{c}{f}$. Solve for frequency, and substitute $f = \frac{c}{\lambda_0}$ and $v = \frac{c}{n}$ into $\lambda = \frac{v}{f}$.

$$\lambda = \frac{(c/n)}{(c/\lambda_0)} = \frac{\lambda_0}{n}$$

Note that the wavelength of light in a medium is smaller than its wavelength in a vacuum.



Get It?

Describe the relationship between the index of refraction and the speed of light in a medium.

Total Internal Reflection

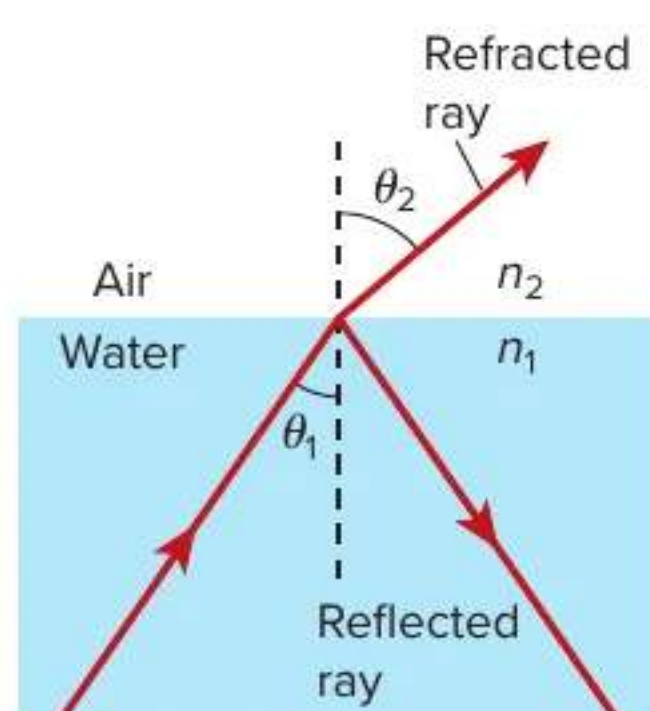
Recall that when light strikes a transparent boundary, some of the light is transmitted and some is reflected. As light travels from a medium of higher index of refraction to a medium of lower index of refraction, the angle of refraction is larger than the angle of incidence, as shown in **Figure 21**.

This leads to an interesting phenomenon. As the angle of incidence increases, the angle of refraction increases. At a certain angle of incidence known as the **critical angle** (θ_c), the refracted light ray lies along the boundary of the two mediums.

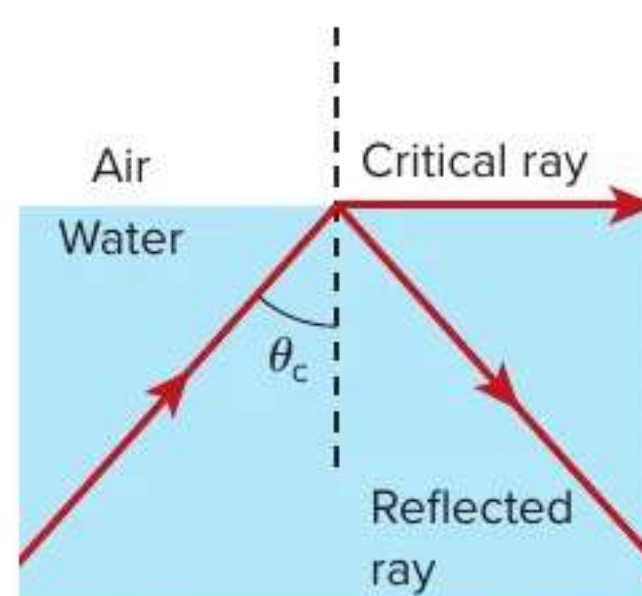
Total internal reflection occurs when light traveling from a region of a higher index of refraction to a region of a lower index of refraction strikes the boundary at an angle greater than θ_c such that all the light reflects back into the region of the higher index of refraction. This is shown in the third diagram of **Figure 21**. Using Snell's law and substituting $\theta_1 = \theta_c$, and $\sin \theta_2 = \sin 90.0^\circ = 1$:

$$\sin \theta_c = \frac{n_2}{n_1}$$

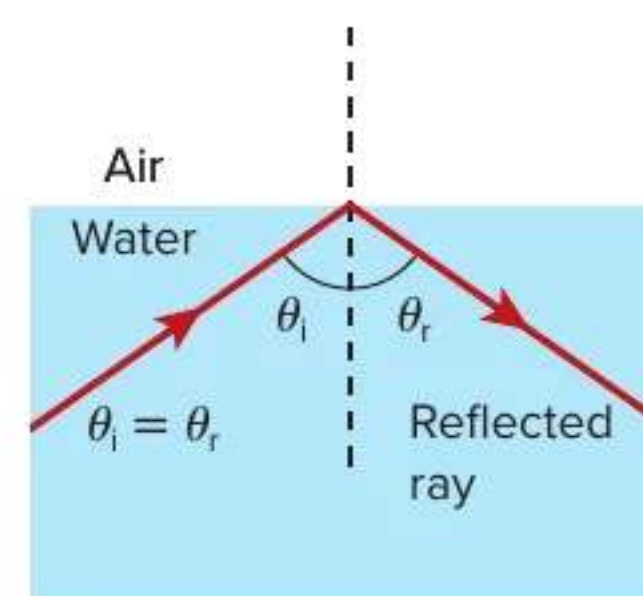
Total internal reflection causes some curious effects. Suppose you are looking up at the surface from under water in a pool. You might see an upside-down reflection of another nearby object that is also under water. You might see a reflection of the bottom of the pool itself. The surface of the water acts like a mirror, reflecting the image back into the water.



At an angle of incidence less than the critical angle, light is partially refracted and partially reflected.



A ray refracted along the boundary of the medium forms the critical angle.



An angle of incidence greater than the critical angle results in total internal reflection, which follows the law of reflection.

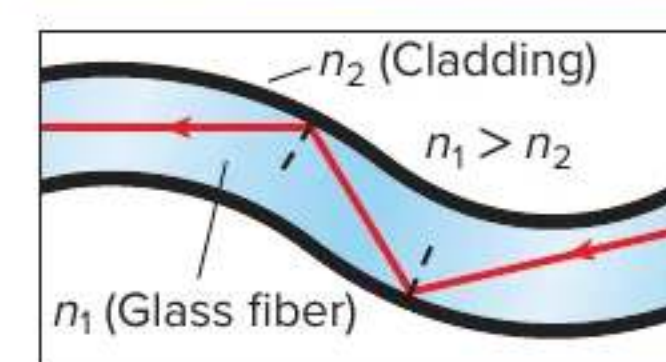
Figure 21 Refraction and reflection of light traveling between mediums depend on the angle of incidence θ_i .



Get It?

Compare and contrast data transmission using the fiber optics cable at the beginning of this module and data transmission by satellite.

Real-World Physics



TOTAL INTERNAL REFLECTION is used in communication via optical fibers. The light traveling through the transparent fiber always hits the internal boundary of the optical fiber at an angle greater than the critical angle. All the light is reflected, and none of the light is transmitted through the boundary. Light pulses in fiber optic cables carry larger amounts of information over longer distances than other forms of communication. The outer material of each fiber is called cladding.

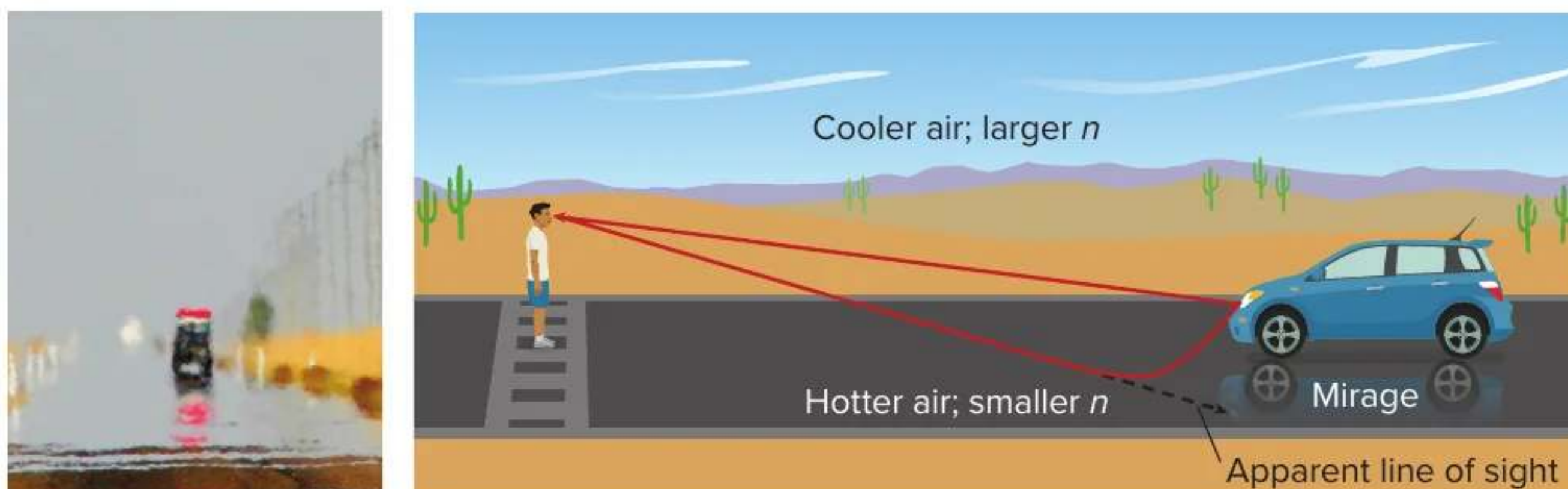


Figure 22 Light waves reflected from the vehicle on the road are refracted when the air near the surface of the road is hotter than the air above it.

Apply Are the light waves traveling faster near the surface of the road or near the top of the vehicle?

Mirages

On a hot summer day, you sometimes can see what appears to be the reflection of an oncoming vehicle in a pool of water as shown on the left in **Figure 22**. The pool, however, disappears as you approach it. This mirage is the result of the Sun heating the road. The speed of light and, therefore, the index of refraction, for a gaseous medium can change slightly with temperature. The hot road heats the air above it and produces a thermal layering of air, causing light traveling toward the road from the car to gradually bend upward. This makes the light appear to be coming from a reflection in a pool.

The diagram in **Figure 22** shows how this occurs. As light from a distant object travels downward toward the road, the index of refraction of the air decreases as the air gets hotter, but the temperature change is gradual. Recall that light wavefronts are comprised of Huygens' wavelets. In the case of a mirage, the Huygens' wavelets closer to the ground travel faster than those higher up, causing the wavefronts to gradually turn upward.

Dispersion of Light

If you look carefully at the spectrum formed by light passing through the prism in **Figure 23**, you will notice that violet light is refracted more than red light. This occurs because the speed of violet light through glass is less than the speed of red light through glass. Violet light has a higher frequency than red light, which causes it to interact differently with the atoms of the

glass. This results in glass having a slightly higher index of refraction for violet light than it has for red light. This separation of white light into a spectrum of colors is called **dispersion**.

The speed of light in a medium is determined by interactions between the light and the atoms that make up the medium. The speed of light and the index of refraction for a solid or liquid medium vary for different wavelengths of light.



Figure 23 Dispersion through a prism occurs because the index of refraction varies with the wavelength of light.

Rainbows A rainbow is a spectrum formed when sunlight is dispersed by water droplets in the atmosphere. Sunlight that falls on a water droplet is refracted. Since each color has a different wavelength, it is refracted at a slightly different angle, as shown in **Figure 24** on the next page, resulting in dispersion. At the back surface of the droplet, some of the light undergoes internal reflection. On the way out of the droplet, the light once again is refracted and dispersed.

Although each droplet produces a complete spectrum, an observer positioned between the Sun and the rain will see only a certain wavelength of light from each droplet. The wavelength observed depends on the relative positions of the Sun, the droplet, and the observer, as shown in **Figure 24**.

Second-order rainbow Sometimes, you can see a faint second-order rainbow like the one shown in **Figure 25**. The second rainbow is outside of the first. It is also fainter and has the order of the colors reversed. Light rays that are reflected twice inside water droplets produce this effect.



Figure 25 Light reflected within raindrops can cause the appearance of a secondary rainbow.

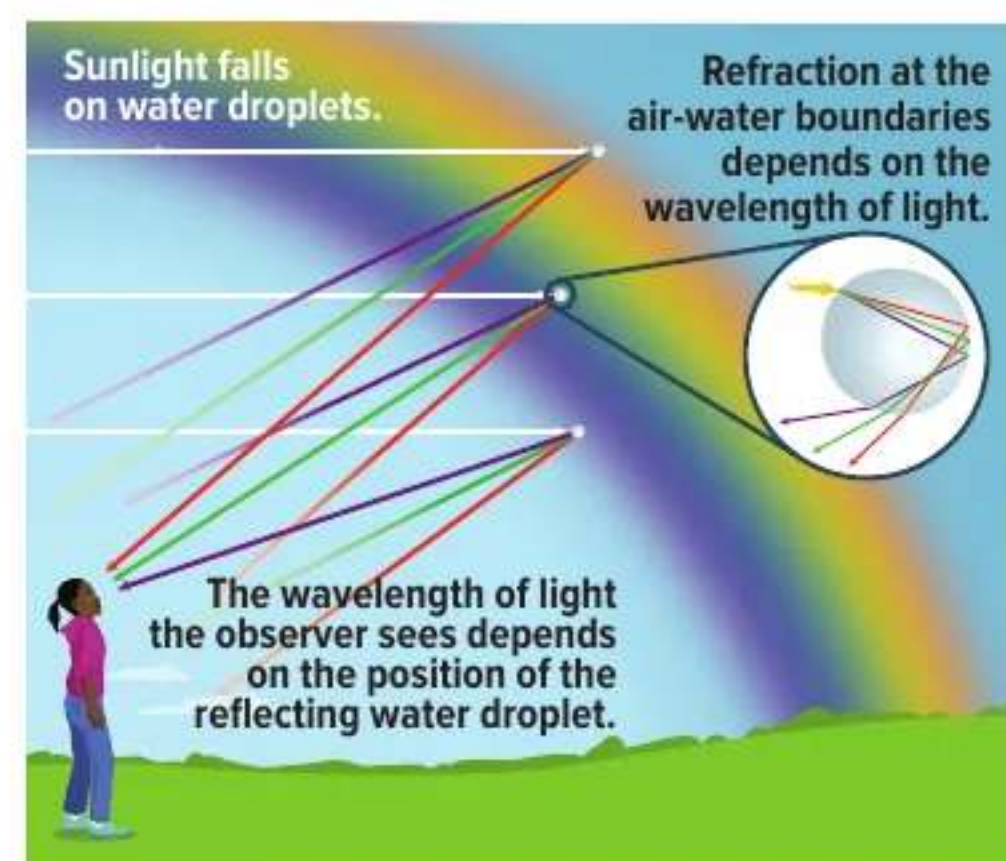


Figure 24 Light from the Sun is dispersed by water droplets to form rainbows. Because there are many droplets at various positions in the sky, a complete spectrum is visible.

Consider whether it is possible for a rainbow to be close enough to touch.

Check Your Progress

35. **Index of Refraction** When a ray of light enters a certain liquid from water, it is bent toward the normal, but when it enters the same liquid from float glass, it is bent away from the normal. What can you conclude about the liquid's index of refraction?
36. **Index of Refraction** A ray of light in air has an angle of incidence of 30.0° on a block of unknown material and an angle of refraction of 20.0° , as shown in **Figure 26**. What is the index of refraction of the material?
37. **Angle of Refraction** A beam of light passes from water into polyethylene ($n = 1.50$). If $\theta_1 = 57.5^\circ$, what is the angle of refraction in the polyethylene?
38. **Speed of Light** What is the speed of light in chloroform ($n = 1.51$)?
39. **Critical Angle** Is there a critical angle for light traveling from glass to water? From water to glass? Explain your answer.
40. **Total Internal Reflection** Would you use quartz or float glass for the cladding of an optical fiber? Explain your answer.
41. **Speed of Light** Could an index of refraction ever be less than 1? What would this imply about the speed of light in that medium?
42. **Critical Thinking** In what direction would you have to be looking to see a rainbow on a late rainy afternoon? Explain.

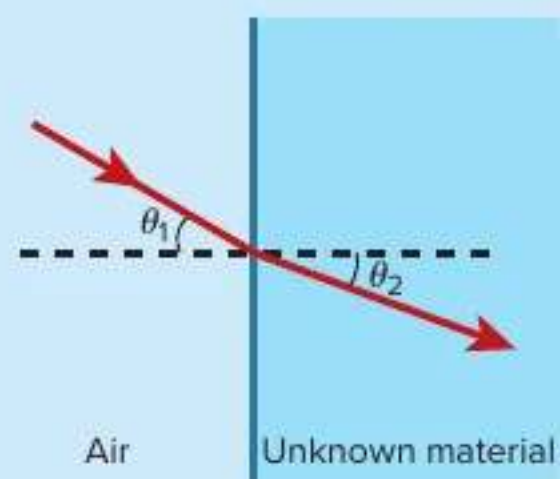


Figure 26

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Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

LESSON 4

CONVEX AND CONCAVE LENSES

FOCUS QUESTION

How are systems of lenses used to make optical devices?

Types of Lenses

Refraction of light can be quite useful. In 1303, French physician Bernard of Gordon wrote of using lenses to correct eyesight. Around 1610, Galileo discovered the four major moons of Jupiter using a telescope made with two lenses. Since that time, lenses have been used in instruments such as microscopes and cameras.

A **lens** is a piece of transparent material that focuses light and forms an image. Each of a lens's two faces might be either curved or flat. A lens that is thicker at the center than at the edges is called a **convex lens**, as shown on the left in **Figure 27**. When it is surrounded by material with a lower index of refraction, such as air, it refracts parallel light rays so that they all pass through a common point, called the focal point, after going

through the lens. For this reason, a convex lens is often called a converging lens.

A lens that is thinner in the middle than at the edges is called a **concave lens**, as shown on the right in **Figure 27**. When surrounded by material with a lower index of refraction, rays passing through it spread out, so it is often called a diverging lens.

You can determine the paths of rays passing through lenses using Snell's law. To simplify such problems, we assume that all refraction occurs at the principal plane, which passes through the center of the lens. This approximation, called the thin lens model, applies to all the lenses discussed in this module.

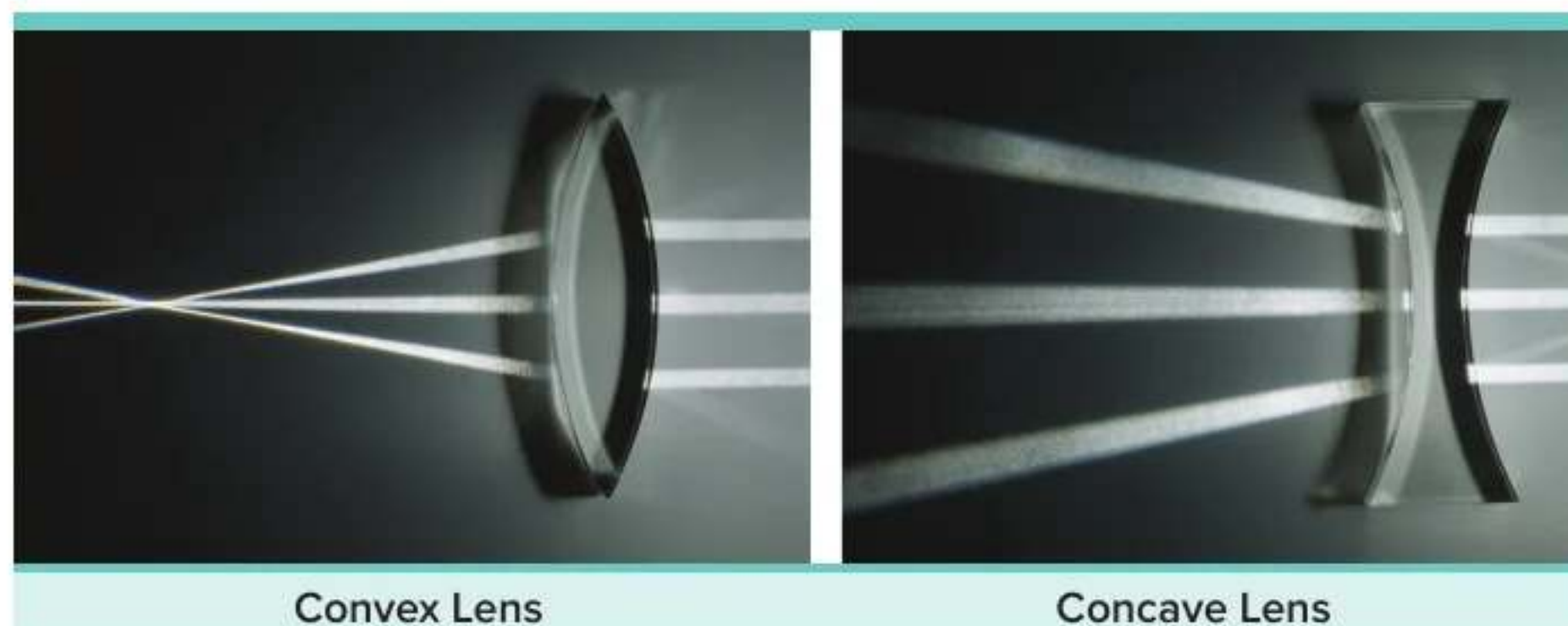


Figure 27 Convex lenses refract light so that the rays converge after passing through. The light passing through a concave lens does not meet at the focal point.



3D THINKING

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



Virtual Investigation: Ray Tracing for Lenses

Use a **computer model** to investigate the properties of a lens **affect** how **light** is transmitted.



Review the News

Obtain **information** from a current news story about technologies that use **light**. **Evaluate** your source and **communicate** your findings to your class.

Convex Lenses

To determine an image's location, we can use ray diagrams. In a ray diagram, as with mirrors you represent a few important rays to find out how a lens affects the light that passes through it. You can use any two rays to locate the image; with experience, you will learn that some rays are easier to draw and use than others.

In ray diagrams for convex lenses, x_o is the distance of the object from the lens and x_i is the distance of the image from the lens. In all ray diagrams in this module, the thin lens model is used. In this model, light refracts at the center of the lens rather than at the boundaries between air and the surface of the lens.

$x_o \geq 2f$ In **Figure 28**, rays are traced from an object located far from the lens. Ray 1 is parallel to the principal axis. It refracts and passes through the focal point (F) on the other side of the lens. Ray 2 passes through F on its way to the lens. After refraction, its path is parallel to the principal axis.

The two rays intersect at a point beyond F and locate the image. Rays selected from other points on the object converge at corresponding points to form the complete image. Note that this is a real image that is inverted and smaller compared to the object.

$x_o = 2f$ If the object is placed at twice the focal length from the lens at the point 2F, the diagram is similar. The image is real and is found at 2F, but it is no longer reduced in size. Because of symmetry, the image and the object have the same size. Thus, you can conclude that if an object is more than twice the focal length from the lens, the image is smaller than the object.

$2f > x_o > f$ You can use **Figure 29** to locate the image of an object that is between F and 2F viewed through a convex lens. This is similar to the ray diagram for the object located at a distance greater than twice the focal length with the image and object interchanged. The direction of the rays would be reversed. In this case, the image is also real and inverted.

For an object placed between F and 2F, the image is enlarged. When an object is placed at the focal point of a convex lens, ray diagrams cannot be drawn. The refracted rays will emerge in a parallel beam and no image will be seen.

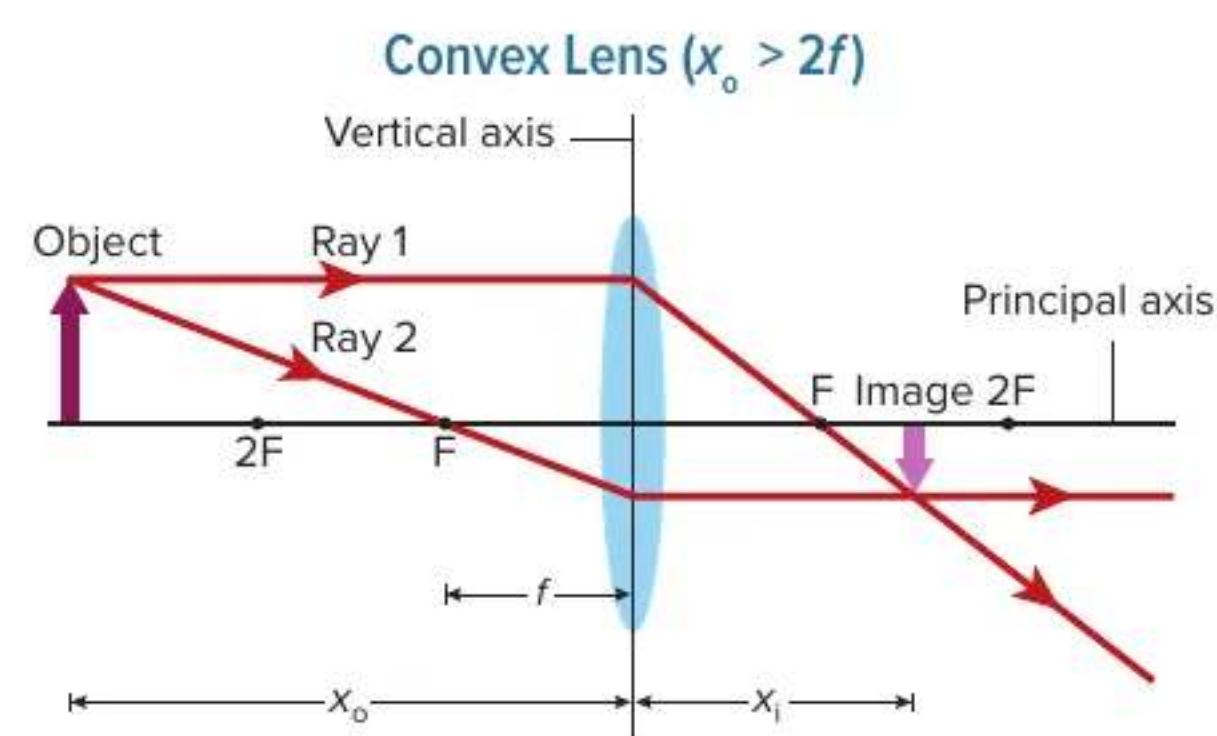


Figure 28 An object placed at a distance greater than twice the focal length from the lens will produce an image that is real, reduced in size, and inverted.

COLOR CONVENTION

Light ray		red
Object		magenta
Image		lilac

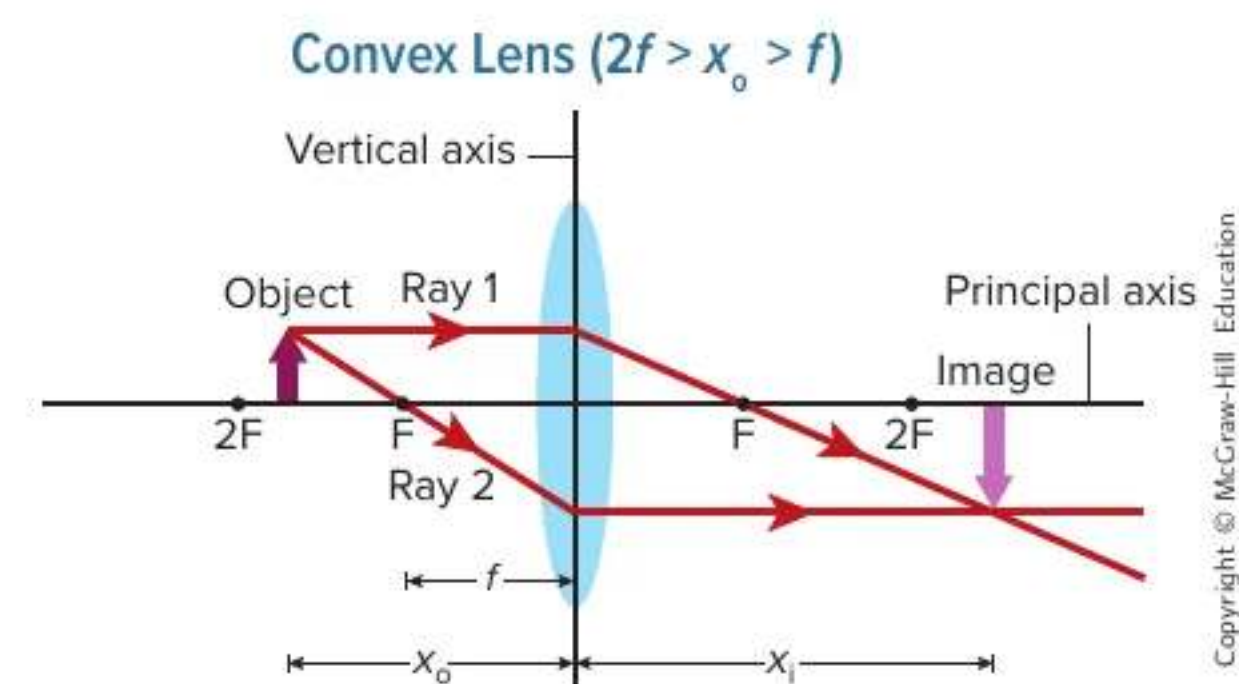


Figure 29 An object placed at a distance less than twice the focal length but greater than one focal length from the lens will produce an image that is real, enlarged, and inverted.

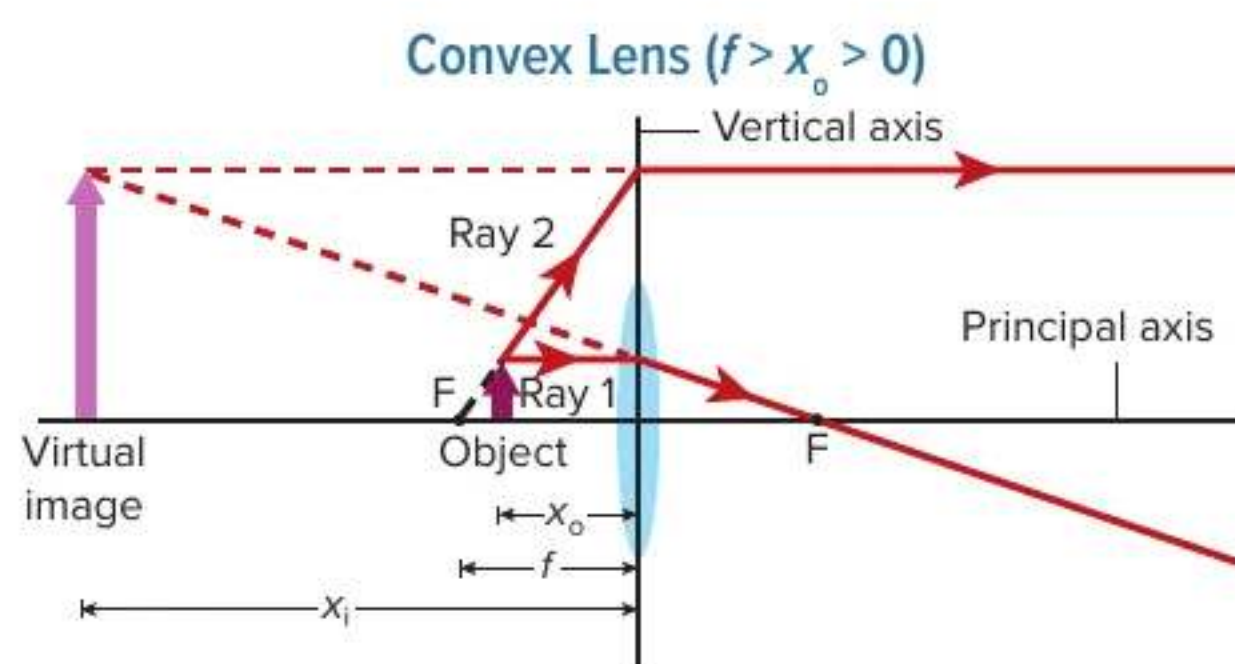


Figure 30 An object placed at a distance less than the focal length from the lens will produce an image that is virtual and enlarged.

$f > x_o > 0$ **Figure 30** shows how a convex lens forms a virtual image. The object in this situation is located between F and the lens. Ray 1, as usual, approaches the lens parallel to the principal axis and is refracted through the focal point, F. Ray 2 travels from the tip of the object in the direction it would have if it had started at F on the object side of the lens. The dashed line from F to the object shows you how to draw ray 2. Ray 2 leaves the lens parallel to the principal axis. Ray 1 and ray 2 diverge as they leave the lens.

The reflection appears to an observer to come from a spot on the same side of the lens as the object. This is a virtual image that is upright and larger compared to the object. No real image is possible. Drawing sight lines for the two rays back to their apparent intersection locates the virtual image. Note that the actual image is formed by light that passes through the lens, but you can still determine the location of the image by drawing rays that do not have to pass through the lens.



Get It?

Classify an image as virtual or real based on the side of the lens it is on.

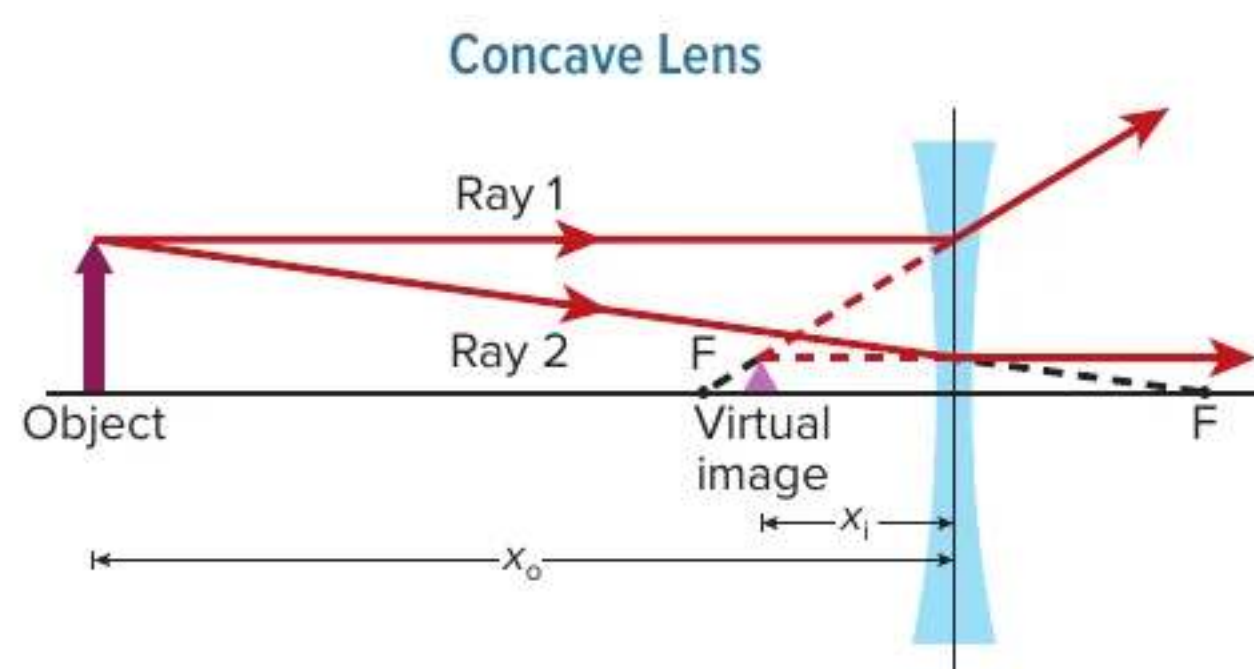


Figure 31 An object placed any distance from a concave lens will always produce an image that is virtual and reduced.

Concave Lenses

A concave lens causes all rays to diverge. **Figure 31** shows how a concave lens forms a virtual image. Ray 1 approaches the lens parallel to the principal axis. It leaves the lens along a line that extends back through the focal point on the object side of the lens. The focal point of a concave lens is found on the same side of the lens as the incoming light. Ray 2 approaches the lens as if it is going to pass through the focal point on the opposite side but leaves the lens parallel to the principal axis.

The sight lines of rays 1 and 2 intersect on the same side of the lens as the object. The image is located at the point from where the two rays appear to intersect, creating a virtual image. The image is upright and smaller than the object. This is true no matter how far from the lens the object is located. The focal length for a diverging lens is negative.



Get It?

Describe why a concave lens always produces a virtual image.

Table 3 Properties of a Single Spherical Lens System

Lens Type	f	x_o	x_i	m	Image
Convex	+	$x_o > 2f$	$2f > x_i > f$	reduced, inverted	real
		$2f > x_o > f$	$x_i > 2f$	enlarged, inverted	real
		$f > x_o > 0$	$ x_i > x_o$ (negative)	enlarged	virtual
Concave	–	$x_o > 0$	$ f > x_i > 0$ (negative)	reduced	virtual

Lens Equations

Lenses can be constructed with a variety of shapes, but you will only consider thin lenses, with faces having spherical curvatures. Based on this model of thin spherical lenses, an equation has been developed that looks exactly like the one for spherical mirrors. For lenses, virtual images are always on the same side of the lens as the object, which means the image position is negative. Note that a concave lens produces only virtual images, whereas a convex lens can produce real images or virtual images.

Thin lens equation The **thin lens equation** relates the focal length of a spherical thin lens, the object position, and the image position.

Thin Lens Equation

The inverse of the focal length of a spherical lens is equal to the sum of the inverses of the image position and the object position.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

When solving problems for concave lenses using the thin lens equation, remember that the sign convention for focal length is different from that of convex lenses. This is because a concave lens is a divergent lens. If the focal point for a concave lens is 24 cm from the lens, you should use the value $f = -24$ cm in the thin lens equation. All images for a concave lens are virtual. Thus, if an image distance is given as 20 cm from the lens, then you should use $x_i = -20$ cm. The object position always will be positive.

Magnification A property of spherical thin lenses that measures how much larger or smaller the image is than the object is magnification. The magnification equation for spherical mirrors also can be used for spherical thin lenses. It is used to determine the height

and the orientation of the image formed by a spherical thin lens.

Magnification

The magnification of an object by a spherical lens, defined as the image height divided by the object height, is equal to the negative of the image position divided by the object position.

$$m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$

Magnification gives information about the size and orientation of the image relative to the object. When the absolute value of a magnification is between zero and one, the image is smaller than the object. Magnifications with absolute values greater than one represent images that are larger than the objects. A negative magnification means the image is inverted compared to the object.

Using the equations for lenses It is important that you use the proper sign conventions when using these equations. **Table 3** shows a comparison of the image position, magnification (m), and type of image formed by single convex and concave lenses when an object is placed at various object positions (x_o) relative to the lens. For convex lenses, the object position relative to the focal point influences the image type.

Notice how this table is similar to **Table 1** in Lesson 1, the table for mirrors. As with mirrors, the distance from the principal plane of a lens to its focal point is the focal length (f). The focal length depends on the shape of the lens and the index of refraction of the lens material. Focal lengths, image positions, and image heights can be negative.

EXAMPLE Problem 5

AN IMAGE FORMED BY A CONVEX LENS An object is placed 32.0 cm from a convex lens that has a focal length of 8.0 cm.

- Where is the image?
- If the object is 3.0 cm high, how tall is the image?
- What is the orientation of the image?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation, locating the object and the lens.
- Draw the two principal rays.

Known

$$x_o = 32.0 \text{ cm}$$

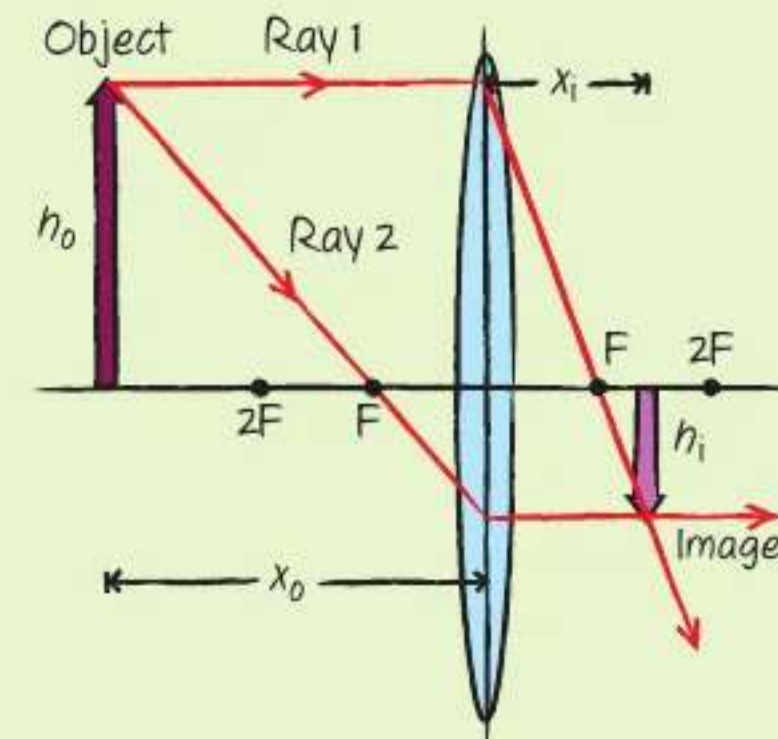
$$h_o = 3.0 \text{ cm}$$

$$f = 8.0 \text{ cm}$$

Unknown

$$x_i = ?$$

$$h_i = ?$$

**2 SOLVE FOR THE IMAGE POSITION AND HEIGHT**

- Use the thin lens equation to determine x_i .

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

$$x_i = \frac{fx_o}{x_o - f}$$

$$= \frac{(8.0 \text{ cm})(32.0 \text{ cm})}{32.0 \text{ cm} - 8.0 \text{ cm}}$$

Substitute $f = 8.0 \text{ cm}$, $x_o = 32.0 \text{ cm}$.

$$= 11 \text{ cm (11 cm away from the lens on the side opposite the object)}$$

- Use the magnification equation to solve for image height.

$$m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$

$$h_i = -\frac{x_i h_o}{x_o}$$

$$= -\frac{(11 \text{ cm})(3.0 \text{ cm})}{32.0 \text{ cm}}$$

Substitute $x_i = 11 \text{ cm}$, $h_o = 3.0 \text{ cm}$, $x_o = 32.0 \text{ cm}$.

$$= -1.0 \text{ cm (1.0 cm tall)}$$

- The negative sign for the height in part **b** means the image is inverted.

3 EVALUATE THE ANSWER

- Are the units correct?** All are in centimeters.
- Do the signs make sense?** Image position is positive (real image), and image height is negative (inverted compared to the object), which make sense for a convex lens.

PRACTICE Problems**ADDITIONAL PRACTICE**

- A 2.25-cm-tall object is 8.5 cm to the left of a convex lens whose focal length is 5.5 cm. Find the image position and height.
- An object near a convex lens produces a 1.8-cm-tall real image that is 10.4 cm from the lens and inverted. If the focal length of the lens is 6.8 cm, what are the object position and height?
- An object is placed to the left of a convex lens with a 25-mm focal length so that its image is the same size as the object. What are the image and object positions?
- Calculate the image position and height of a 2.0-cm-tall object located 25 cm from a convex lens with a focal length of 5.0 cm. What is the orientation of the image?
- Use a scale ray diagram to find the image position of an object that is 30 cm to the left of a convex lens with a 10-cm focal length.
- CHALLENGE** A magnifier with a focal length of 30 cm is used to view a 1-cm-tall object. Use a ray diagram to determine the location and size of the image when the magnifier is positioned 10 cm from the object.

Defects of Spherical Lenses

Throughout this section, you have studied lenses that produce perfect images at specific positions. In reality, spherical lenses, just like spherical mirrors, have intrinsic defects that cause problems with the focus and color of images. Spherical lenses exhibit an aberration associated with their spherical design, just as mirrors do. In addition, the dispersion of light through a spherical lens causes an aberration that mirrors do not exhibit.

Spherical aberration The model you have used for drawing rays through spherical lenses suggests that all parallel rays focus at the same position. However, this is only an approximation. In reality, parallel rays that pass through the edges of a spherical lens focus at positions different from those of parallel rays that pass through the center. This inability of a spherical lens to focus all parallel rays to a single point is called spherical aberration. The effects are shown in **Figure 32**. In reality, most lenses have a slightly different shape to address this, but the spherical approximation works well enough for our purposes. In high-precision instruments, many lenses, often five or more, are used to form sharp, well-defined images.

Chromatic aberration Lenses have a second defect that mirrors do not have. Because the index of refraction of a medium depends on wavelength, different wavelengths of light are refracted at slightly different angles, as you can see on the right in **Figure 33**. Light that passes through a lens is slightly dispersed, especially near the edges, causing an effect called **chromatic aberration**. This is seen as an apparent ring of color around an object viewed through a lens, as shown on the left in **Figure 33**.

Chromatic aberration is always present when a single lens is used. However, this defect can be greatly reduced by an **achromatic lens**, which is a system of two or more lenses, such as a convex lens with a concave lens, that have different indices of refraction. A simple lens and an achromatic lens are shown in **Figure 33**. Both lenses disperse light, but the dispersion caused by the achromatic lens is less than the dispersion caused by the simple lens. The index of refraction of the achromatic lens is chosen so that the light will still converge at the desired location.



Figure 32 Spherical aberration from the camera lens blurs the edges of this image.

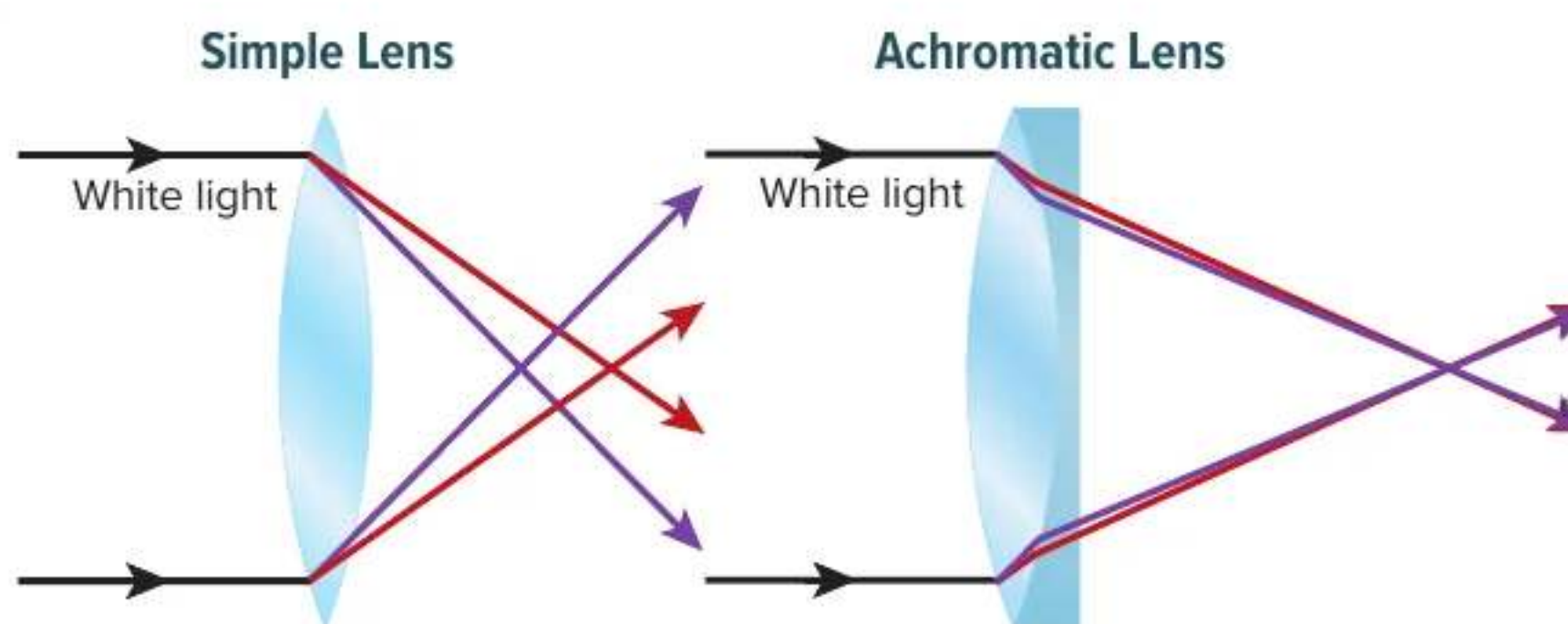


Figure 33 Simple lenses, such as the one shown in the photo on the left, exhibit a rainbow fringing effect called chromatic aberration. Achromatic lenses reduce chromatic aberration.

Explain why the index of refraction is important for achromatic lenses.

Lenses in Eyes

The concepts that you have learned for the refraction of light through lenses apply to almost every optical device, including the eye. The eye is a remarkable optical device. As shown in **Figure 34**, the eye is a fluid-filled, almost spherical vessel. Light that is emitted from or reflected off an object travels into the eye through the cornea and pupil. The light then passes through the lens and focuses onto the retina, which is at the back of the eye. Specialized cells on the retina absorb this light and send information about the image along the optic nerve to the brain.

BIOLOGY Connection Because of its name, you might assume that the lens of an eye is responsible for focusing light onto the retina. In fact, light entering the eye is primarily focused by the cornea, because the air-cornea boundary has the greater difference in indices of refraction. The lens is responsible for the fine focus that allows you to clearly see both distant and nearby objects.



Get It?

Describe the roles of the cornea and the lens in your eye.

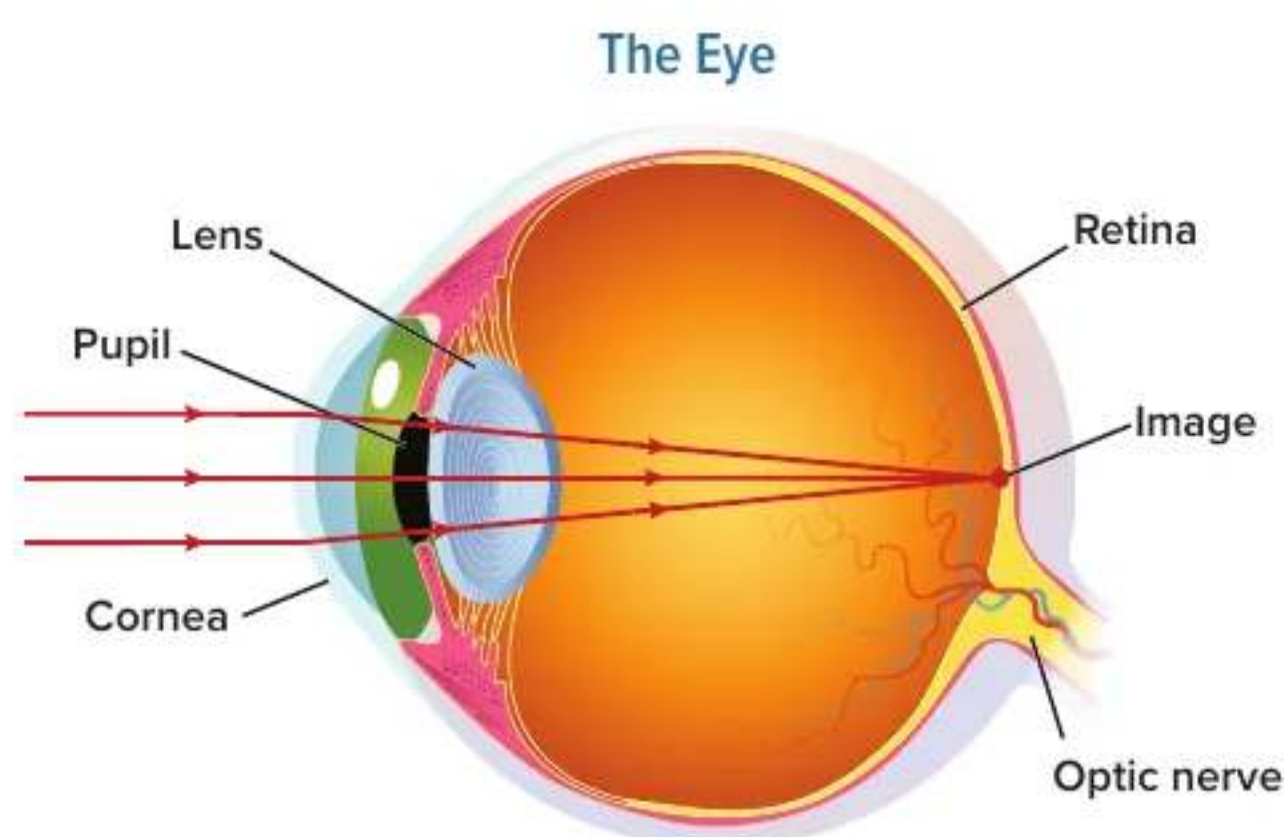


Figure 34 The cornea and the lens of your eye refract light that is reflected off every object you see.

Summarize why most of the refraction occurs at the air-cornea boundary rather than at the lens.

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The eye lens uses a process called accommodation. Muscles surrounding the lens can contract or relax, thereby changing the shape of the lens. This, in turn, changes the focal length of the eye. For a healthy eye, when the muscles are relaxed, the image of distant objects is focused on the retina. When the muscles contract, the focal length is shortened, and this allows images of closer objects to be focused on the retina.

Nearsightedness The eyes of many people focus images either in front of the retina or behind it. External lenses, such as eyeglasses and contact lenses, adjust the focal length and move images to the retina. **Figure 35** on the next page shows the condition of **nearsightedness**, also called myopia.

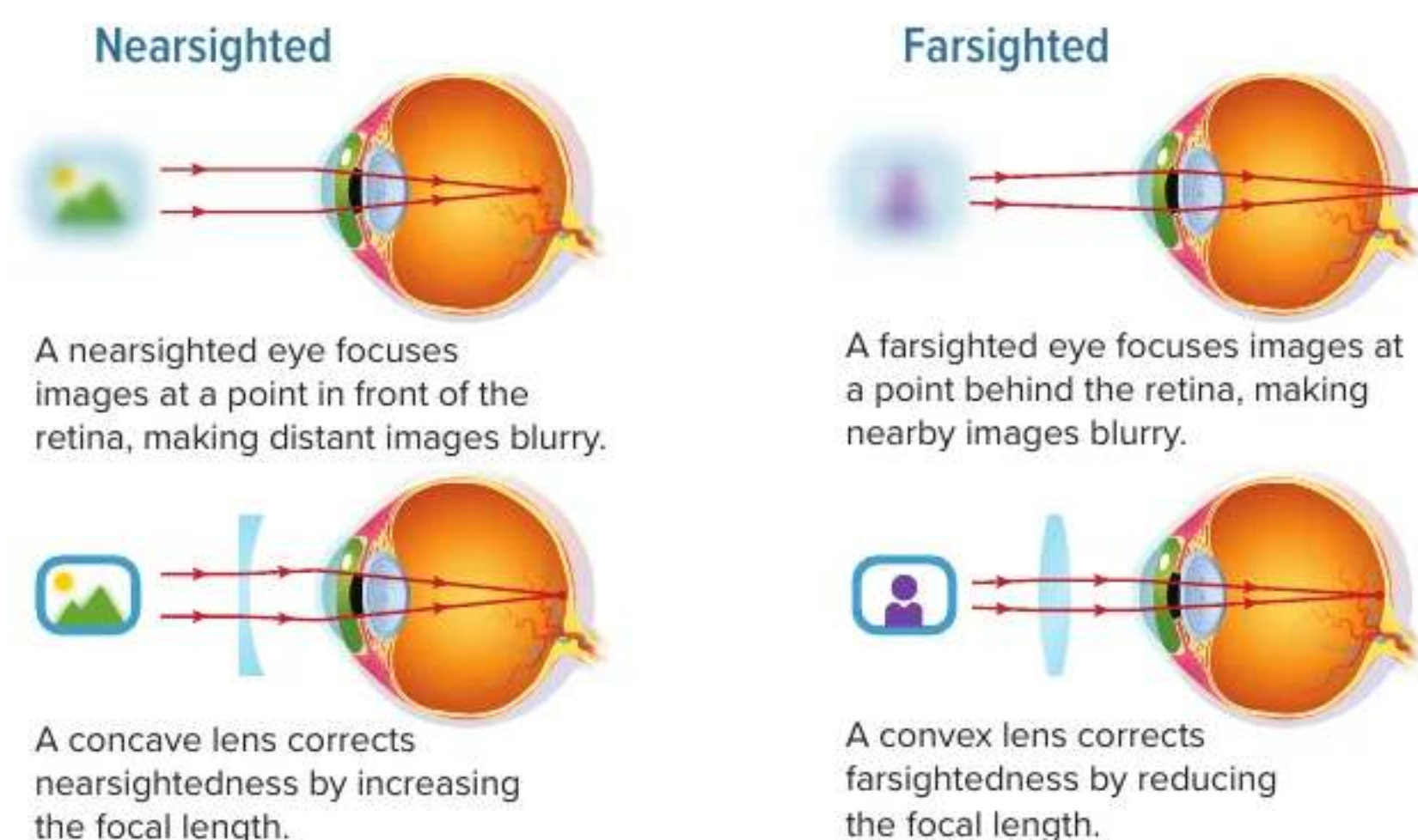


Figure 35 Objects at a distance are blurred for a nearsighted person. For a farsighted person, objects nearby are blurred.

Explain how bifocal lenses might be made.

With myopia, the focal length of the eye is too short to focus light on the retina. Images are formed in front of the retina. As shown in **Figure 35**, concave lenses correct this by diverging light, thereby increasing images' distances from the lens, and forming images on the retina.

Farsightedness You also can see in **Figure 35** that **farsightedness**, also called hyperopia, is the condition in which the focal length of the eye is too long. Images are therefore not formed on the retina.

A similar result is caused by the increasing rigidity of the lenses in the eyes of people who are more than about 45 years old. Their muscles cannot shorten the focal length enough to focus images of close objects on the retina.

For either condition, convex lenses produce virtual images farther from the eye than the associated objects, as shown in **Figure 35**. The images then become the objects for the eye lens and can be focused on the retina, thereby correcting the condition.

Real-World Physics

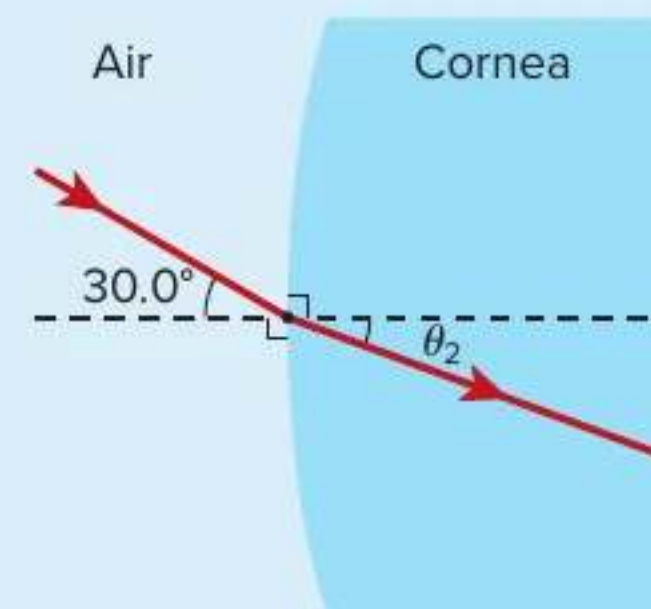


CONTACT LENSES produce the same results as eyeglasses. These small, thin lenses are placed directly on the corneas. A thin layer of tears between the cornea and lens keeps the lens in place. Most of the refraction occurs at the air-lens boundary, where the difference in indices of refraction is greatest.

PHYSICS Challenge

As light enters the eye, it first encounters the air-cornea boundary. Consider a ray of light that strikes the interface between the air and a person's cornea at an angle of 30.0° to the normal. The index of refraction of the cornea is approximately 1.4.

1. Use Snell's law to calculate the angle of refraction.
2. What would the angle of refraction be if the person were swimming underwater with his or her eyes open?
3. Is the refraction greater in air or in water? Does this mean objects underwater seem closer or more distant than they would in air?
4. If you want the angle of refraction for the light ray in water to be the same as it is for air, what should the new angle of incidence be?



Refracting Telescopes

An astronomical refracting telescope uses lenses to magnify distant objects. **Figure 36** shows the optical system for a Keplerian telescope. Light from stars and other astronomical objects comes from so far away that the incoming rays can be considered parallel. These rays enter the objective convex lens, which focuses them as a real image at the focal point of the objective lens. The image is inverted compared to the object. This image then becomes the object for the convex lens of the eyepiece.

Notice that the eyepiece lens is positioned so that the focal point of the objective lens is between the eyepiece lens and its focal point. This means a virtual image is produced that is upright and larger than the first image. However, because the first image was already inverted, the final image is still inverted. For viewing astronomical objects, an image that is inverted is acceptable.

In a telescope, the convex lens of the eyepiece is almost always an achromatic lens. Recall that an achromatic lens is a combination of lenses that functions as one lens. The combination of lenses greatly reduces the peripheral colors that can form on images due to chromatic aberration.



Get It?

Explain how a refracting telescope works.

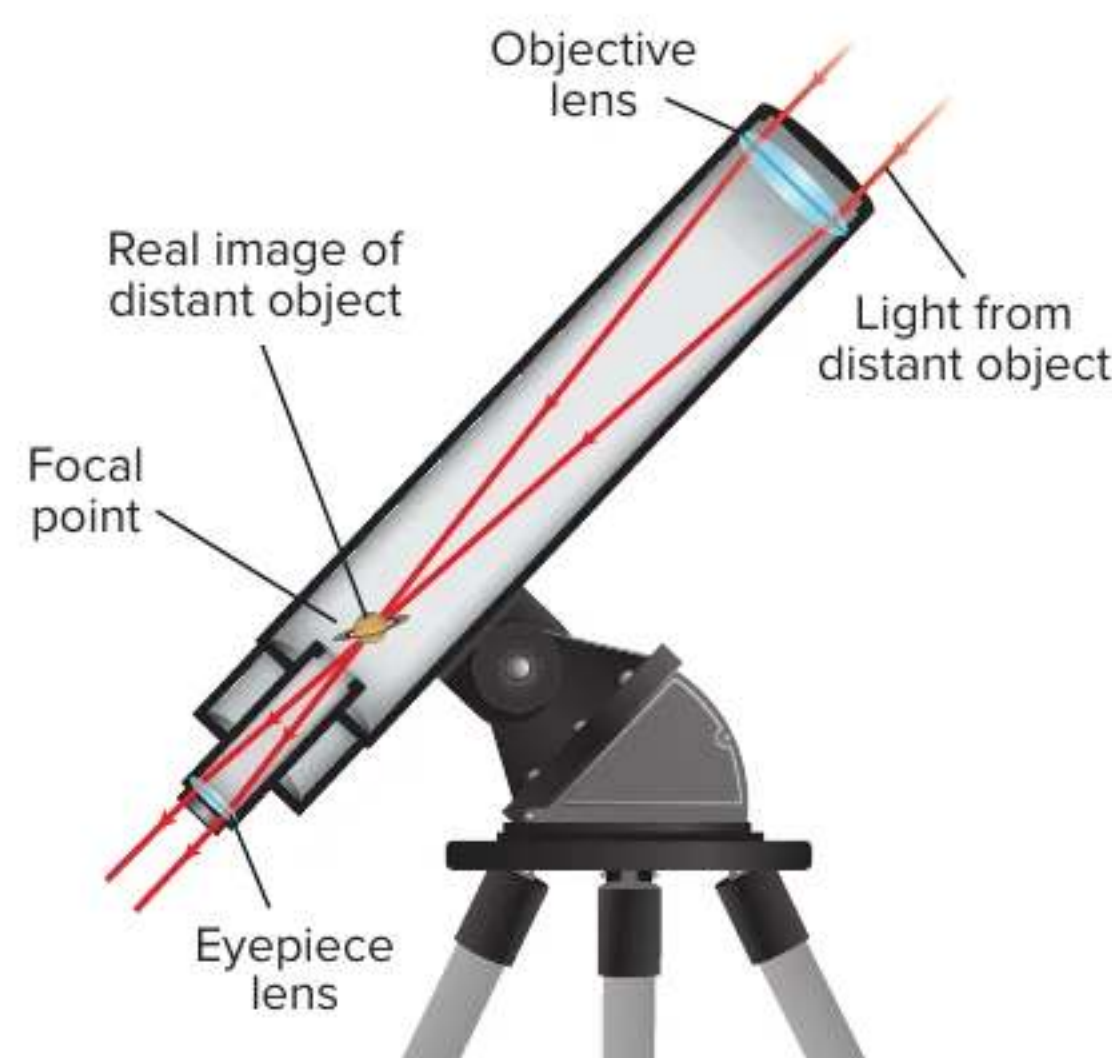


Figure 36 Light from distant objects is gathered by the objective lens and the eyepiece of a refracting telescope.

Evaluate Why is an inverted image acceptable for viewing astronomical objects?

Cameras

Figure 37 shows the optical system used in a single-lens reflex camera. As light enters the camera through the aperture, it passes through an achromatic lens. This lens system refracts the light much like a single convex lens would, forming an image that is inverted on the reflex mirror. The image is reflected upward to a prism that inverts and redirects the light to the viewfinder.

When the person holding the camera takes a photograph, he or she presses the shutter-release button, which briefly raises the mirror, as shown in **Figure 37**. The light, instead of being diverted upward to the prism, then travels along a straight path to form an image on the sensor.

The image sensor, which is often a charge-coupled device (CCD), captures a two-dimensional image that corresponds to the image projected onto it. The image information is collected on the photoactive region of the CCD as electric charge proportional to the light intensity. The CCD then processes the charge and transfers the information to a storage device.

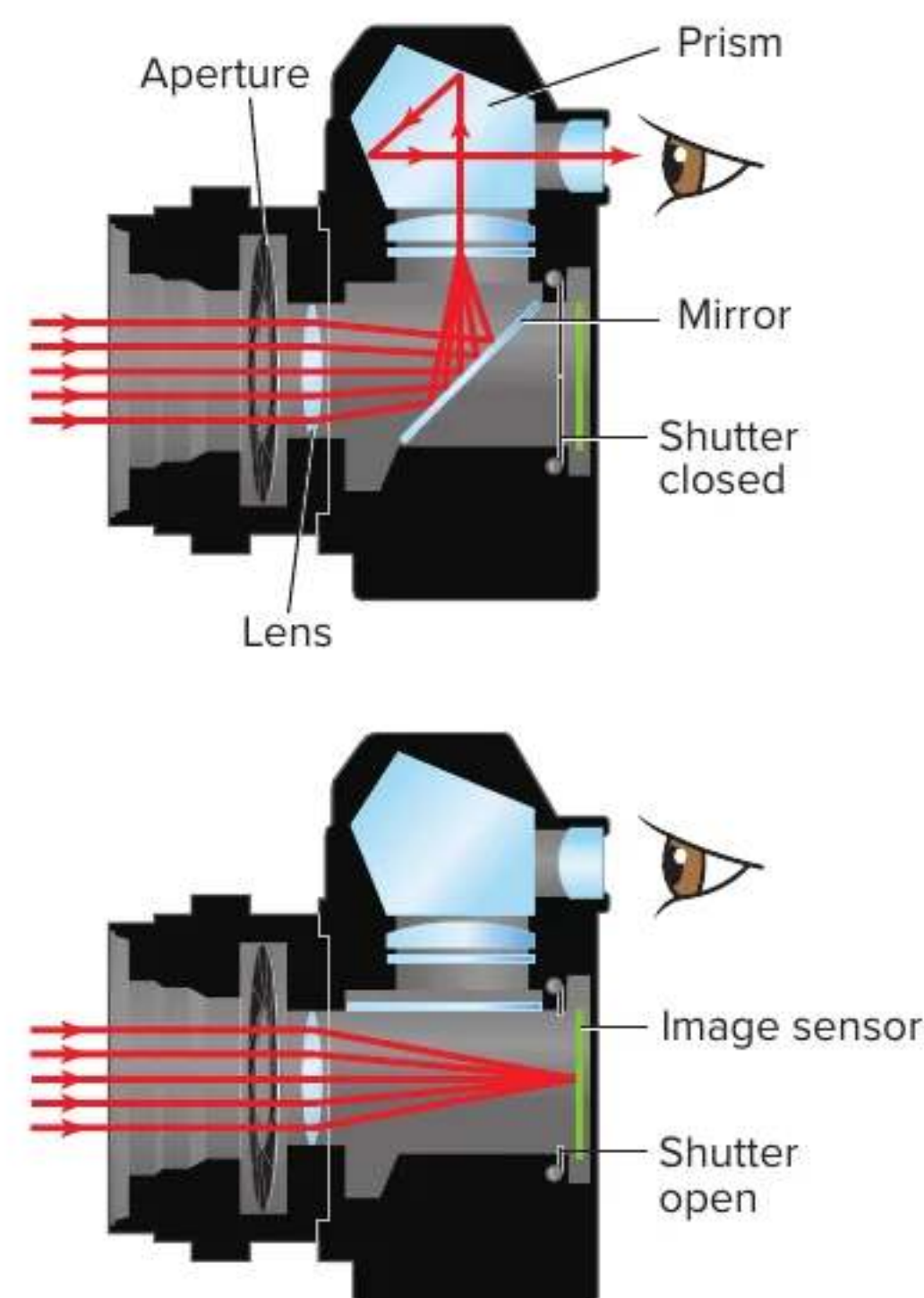


Figure 37 The lens of a camera can be adjusted to focus an image on the image sensor. When the shutter is closed, a mirror diverts the image to the viewer's eye.

Infer Why would you want the shutter to be open longer when taking a photo in dim light?

Check Your Progress

49. **Lenses** Magnifying lenses normally are used to produce images that are larger than the related objects, but they also can produce images that are smaller than the related objects. Explain.

50. **Types of Lenses** The cross sections of four different thin lenses are shown in **Figure 38**.

- Which of these lenses, if any, are convex, or converging, lenses?
- Which of these lenses, if any, are concave, or diverging, lenses?



Figure 38

51. **Image Position and Height** An object is placed 1.5 m from a convex lens with a focal length of 1.0 m. Use the thin lens equation to determine the distance of the image from the lens. If the object height is 2.0 m, what is the image height? Is the image real or virtual? Is the image inverted or upright?

52. **Thin Lens Approximation** What is a thin lens approximation and why is it used?

53. **Chromatic Aberration** All simple lenses have chromatic aberration. Infer why you do not see this effect when you look through a microscope, which has two convex lenses.

54. **Type of Image** Use the ray diagram in **Figure 39** to determine whether the image for object 1 will be reduced or enlarged, inverted or upright, and real or virtual. Do the same for object 2.

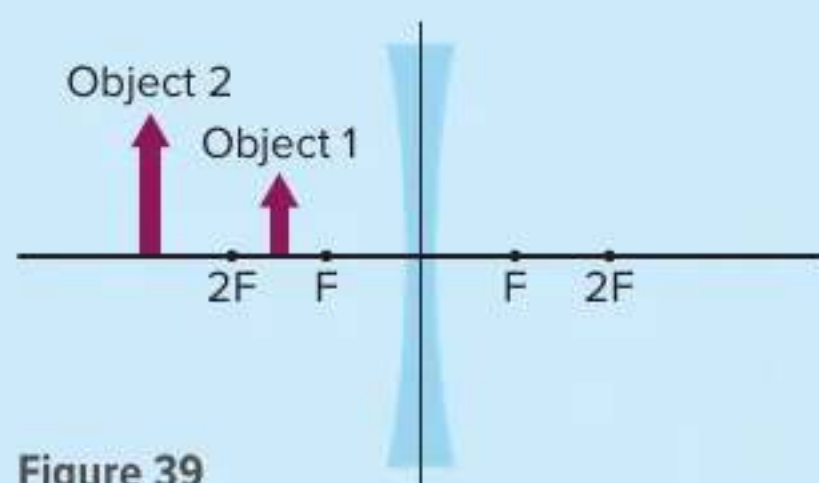


Figure 39

55. **Image Position and Height** A 6.0-cm-tall object is placed 5.0 cm from a convex lens with a focal length of 4.0 cm. Draw a ray diagram to determine the image location and size. Confirm your results using the thin lens equation and the magnification equation.

56. **Diverging Lens** A 6.5-cm-tall salt shaker is viewed through a diverging lens with a focal length of 5.0 cm.

- If the shaker is 6.0 cm from the lens, what is the image distance from the lens? Is the image virtual or real?
- What is the magnification? Is the image smaller or larger than the object?
- The salt shaker is now 4 cm from the lens. What is the distance of the image from the lens? The magnification? Is the image now smaller or larger than the object?

57. **Eyeglass Lenses** Which type of lens, convex or concave, should a nearsighted person use? Which type should a farsighted person use? See **Figure 40**. Explain.



Figure 40

58. **Critical Thinking** An air lens constructed of two watch glasses is placed in a tank of water. Copy **Figure 41** and draw the effect of this lens on parallel light rays incident on the lens.

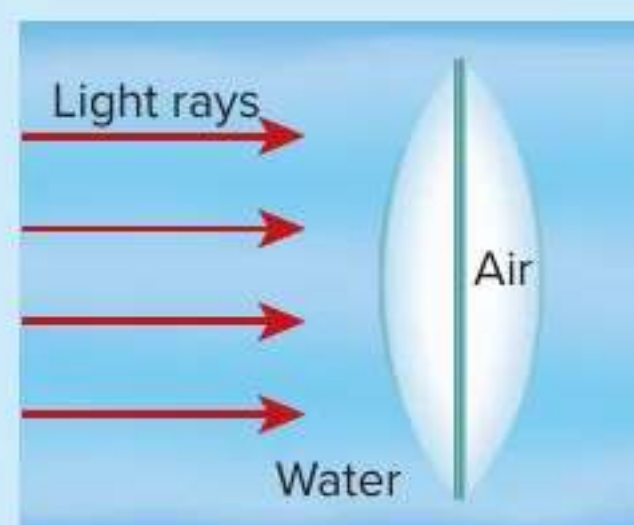


Figure 41

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Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

ENGINEERING & TECHNOLOGY

Scientists Hope the James Webb Space Telescope Will Help Them See the Light

Expected to launch in 2020, the James Webb Space Telescope (JWST) has been designed to produce images of deep space objects. Its ability to do so relies heavily on how its state of the art mirror captures and reflects light.



Building a Better Telescope

The way in which backyard reflecting telescopes produce images is fairly straightforward. A concave primary mirror captures light from an object. This light is then reflected by another mirror to produce an image at a focal point. The larger the primary mirror, the more light it can capture. Many backyard reflecting telescopes used by amateur astronomers have a primary mirror that measures about 25 cm in diameter.

The primary mirror on the JWST measures 6.5 m in diameter. Its diameter not only dwarfs that of the average backyard telescope, but it's also much larger than that of two existing space telescopes.

The diameter of the *Hubble Space Telescope's* primary mirror, which has been in use since 1990, is 2.4 m. The telescope that has orbited Earth in the *Herschel Space Observatory* since 2009 has a primary mirror with a diameter of 3.5 m.

Scientists think that the JWST will be able to see some of the oldest objects in the universe. Some of

The ability of the JWST to produce images of deep space objects is largely due to the construction of its primary mirror.

the universe's oldest galaxies are over 13 billion light years away. At such a distance, the light that reaches Earth is faint. The JWST will be able to capture more of this light. The JWST will collect data in the infrared to see processes and objects not seen in visible light.

Unlike the *Hubble* and *Herschel* telescopes, the JWST will not be in Earth's orbit. It will be set about 1.5 million kilometers from Earth. Its primary mirror is too massive to launch in one piece, so it is constructed out of 18 smaller, six-sided mirrors made out of lightweight gold-plated beryllium. The components will fold up tightly for launch and unfold once in orbit. Each individual mirror can also be controlled to ensure that the pieces work together as a single large mirror and that light is reflected from the mirror accurately to produce an image that's not distorted.

Scientists are hopeful that the images from the JWST will provide them with more clues about the origins of the universe.



EVALUATE DESIGN SOLUTIONS

NASA officials have stated that the JWST has been designed to succeed, but not replace, other existing space telescopes. Compare the JWST to another telescope, giving an example of a space object that can be visualized using each.

STUDY GUIDE

 **GO ONLINE** to study with your Science Notebook.

Lesson 1 REFLECTION OF LIGHT

- The law of reflection says that the angle of incidence equals the angle of reflection, and applies to specular and diffuse reflection.

$$\theta_i = \theta_r$$

- Plane mirrors form virtual images, which are the same size, orientation, and distance from the mirror as the object.

$$x_i = -x_o; \quad h_i = h_o$$

- specular reflection
- diffuse reflection
- plane mirror
- object
- image
- virtual image

Lesson 2 CURVED MIRRORS

- A spherical concave mirror is shaped as if it were a section of a hollow sphere with the same geometric center (C) and radius of curvature (r) as a sphere of radius r . The focal point (F) is the point where rays parallel to the principal axis converge after reflection.
- You can locate the image created by a curved mirror by drawing ray diagrams. The type of image formed by a concave mirror depends on the object's position. A convex mirror always forms a virtual image that is upright and smaller compared to the object.
- The magnification of a mirror image is given by equations relating either the positions or the heights of the image and the object.

$$m \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$

- The mirror equation gives the relationship between image position, object position, and focal length of a spherical mirror.

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

- principal axis
- focal point
- focal length
- concave mirror
- convex mirror
- magnification
- real image
- spherical aberration

Lesson 3 REFRACTION OF LIGHT

- Light refracts when it travels across a boundary from one medium with an index of refraction (n_1) into a medium with a different index of refraction (n_2) according to Snell's law.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

- The ratio of the speed of light in a vacuum (c) to the speed of light in a medium (v) is the index of refraction (n) of the medium.
- When light traveling through a medium hits a boundary with a medium of a smaller index of refraction, if the angle of incidence exceeds the critical angle (θ_c), the light will be reflected back into the original medium by total internal reflection.
- Refraction causes optical effects such as mirages and rainbows.

- index of refraction
- critical angle
- total internal reflection
- dispersion

Lesson 4 CONVEX AND CONCAVE LENSES

- The image formed by a convex lens depends on the position of the object. The image formed by a concave lens is always virtual.
- The thin lens equation provides the relationship between focal length (f), object position (x_o), and image position (x_i).

$$\frac{1}{f} = \frac{1}{x_i} + \frac{1}{x_o}$$

- Concave and convex lenses correct eye conditions such as nearsightedness and farsightedness.

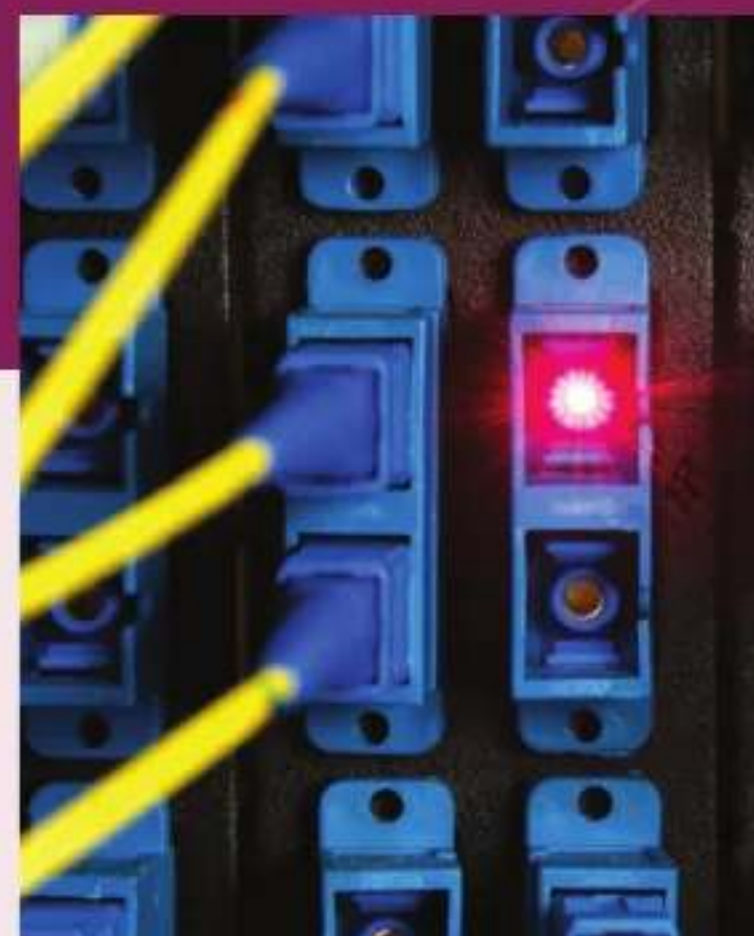
- lens
- convex lens
- concave lens
- thin lens equation
- chromatic aberration
- achromatic lens
- nearsightedness
- farsightedness



THREE-DIMENSIONAL THINKING Module Wrap-Up

REVISIT THE PHENOMENON

How does light transmit information through a communication network?



CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.



STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

Which material should you use for fiber optic cables?

You are planning a colony on a very cold, icy planet. The engineer in charge of communications suggests that instead of hauling glass fiber optic cables from Earth, the expedition should make them from ice once they arrive on the planet.

CER Analyze and Interpret Data

1. Find the critical angle for light traveling from ice ($n = 1.31$) to air.
2. **Claim** Would fiber optic cables made of ice or those made of glass do a better job of keeping light inside the cable?
3. **Evidence and Reasoning** Use your calculations and what you know about refraction to support and explain your claims.

MODULE 8

ROTATIONAL MOTION

NASA Earth Observatory image by Joshua Stevens, using VIIRS data from LANCE/EOSDIS Rapid Response



MODULE 8

ROTATIONAL MOTION

ENCOUNTER THE PHENOMENON

Why do all tropical cyclones in the northern hemisphere rotate the same direction?



 **GO ONLINE** to play a video about the rotation of tropical cyclones.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about why tropical cyclones in the northern hemisphere rotate in the same direction. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

 **GO ONLINE** to access your CER chart and explore resources that can help you collect evidence.



LESSON 2: Explore & Explain:
Torque



LESSON 3: Explore & Explain:
Rotating Frames of Reference



Smithsonian

Additional Resources

LESSON 1

DESCRIBING ROTATIONAL MOTION

FOCUS QUESTION

How would you describe the rotation of a hurricane?

Angular Displacement

You probably have observed a spinning object many times. How would you measure such an object's rotation? Find a circular object, such as a DVD. Mark one point on the edge of the DVD so that you can keep track of its position. Rotate the DVD to the left (counterclockwise), and as you do so, watch the location of the mark. When the mark returns to its original position, the DVD has made one complete revolution.

Measuring revolution How can you measure a fraction of one revolution? It can be measured in several different ways, but the two most used are degrees and radians. A degree is $\frac{1}{360}$ of a revolution and is the usual scale marking on a protractor. In mathematics and physics, the radian is related to the ratio of the circumference of a circle to its radius. In one revolution, a point on the edge of a wheel travels a distance equal to 2π times the radius of the wheel. For this reason, the **radian** is defined as $\frac{1}{(2\pi)}$ of a revolution. One complete revolution is an angle of 2π radians. The radian is abbreviated *rad*.

The Greek letter theta (θ) is used to represent the angle of revolution. **Figure 1** shows the angles in radians for several common fractions of a revolution. Note that counterclockwise rotation is designated as positive, while clockwise is negative. As an object rotates, the change in the angle is called the object's **angular displacement**.

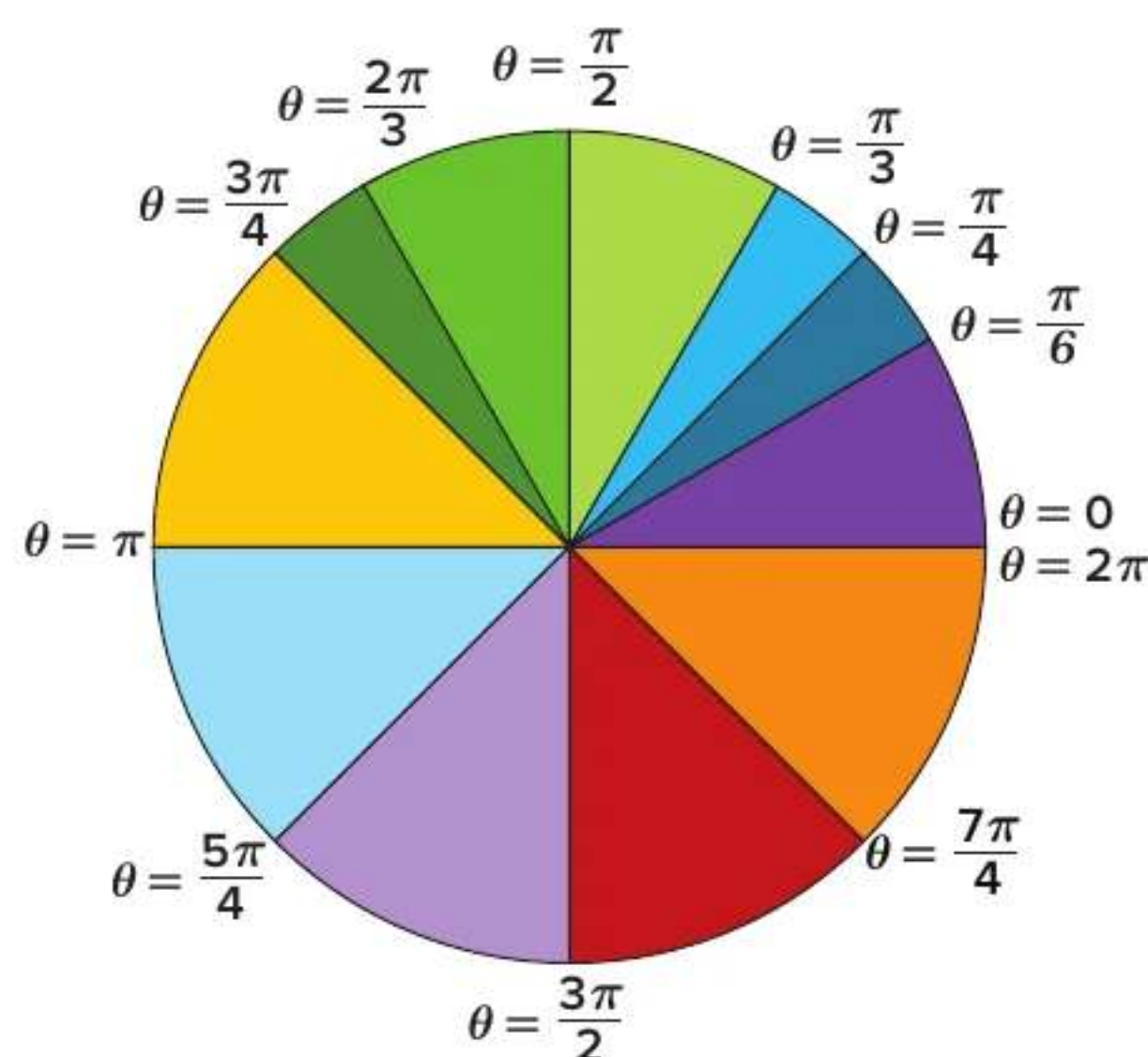


Figure 1 A fraction of a revolution can be measured in degrees or radians. Some common angles are shown below measured in radians. Each angle is measured in the counterclockwise direction from $\theta = 0$.



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Identify Crosscutting Concepts

Create a table of the **crosscutting concepts** and fill in examples you find as you read.



Review the News

Obtain information from a current news story about rotational motion. Evaluate your source and communicate your findings to your class.

Earth's revolution As you know, Earth turns one complete revolution, or 2π rad, in 24 h. In 12 h, it rotates through π rad. Through what angle does Earth rotate in 6 h? Because 6 h is one-fourth of a day, Earth rotates through an angle of $\frac{\pi}{2}$ rad during that period. Earth's rotation as seen from the North Pole is positive. Is it positive or negative when viewed from the South Pole?



Get It?

Identify the angle that Earth rotates in 48 h.

Measuring distance How far does a point on a rotating object move? You already found that a point on the edge of an object moves 2π times the radius in one revolution. In general, for rotation through an angle (θ), a point at a distance r from the center, as shown in **Figure 2**, moves a distance given by $x = r\theta$. If r is measured in meters, you might think that multiplying it by θ rad would result in x being measured in m·rad. However, this is not the case. Radians indicate the dimensionless ratio between x and r . Thus, x is measured in meters.

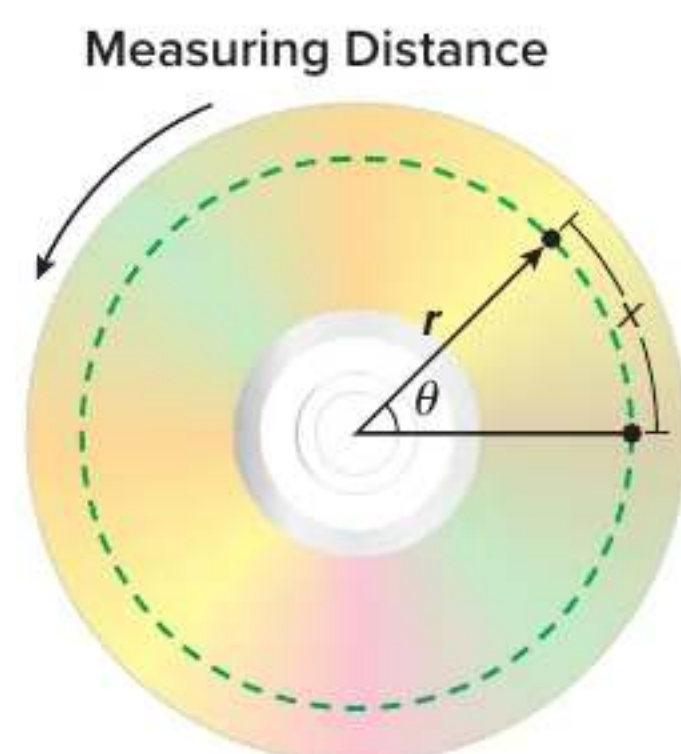


Figure 2 The dashed line shows the path of a point on a DVD as the DVD rotates counterclockwise about its center. The point is located a distance r from the center of the DVD and moves a distance x as it rotates.

Explain what the variables r , x , and θ represent.

Angular Velocity

How fast does a DVD spin? How can you determine its speed of rotation? Recall that velocity is displacement divided by the time taken to make the displacement. Likewise, the **angular velocity** of an object is angular displacement divided by the time taken to make the angular displacement. The angular velocity of an object is given by the following ratio, and is represented by the Greek letter omega (ω).

Average Angular Velocity of an Object

The angular velocity equals the angular displacement divided by the time required to make the rotation.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Recall that if the velocity changes over a time interval, the average velocity is not equal to the instantaneous velocity at any given instant. Similarly, the angular velocity calculated in this way is actually the average angular velocity over a time interval (Δt). Instantaneous angular velocity equals the slope of a graph of angular position versus time.

Earth's angular velocity Angular velocity is measured in rad/s. You can calculate Earth's angular velocity as follows:

$$\omega_E = \frac{(2\pi \text{ rad})}{(24.0 \text{ h})(3600 \text{ s/h})} = 7.27 \times 10^{-5} \text{ (rad/s)}$$

In the same way that counterclockwise rotation produces positive angular displacement, it also results in positive angular velocity.

If an object's angular velocity is ω , then the linear velocity of a point a distance r from the axis of rotation is given by $v = r\omega$. The speed at which an object on Earth's equator moves as a result of Earth's rotation is given by $v = r\omega = (6.38 \times 10^6 \text{ m/rad})(7.27 \times 10^{-5} \text{ rad/s}) = 464 \text{ m/s}$. Earth is an example of a rotating, rigid body, as shown in **Figure 3**. Even though different points at different latitudes on Earth do not move the same distance in each revolution, all points rotate through the same angle. All parts of a rigid body rotate at the same rate. The Sun is not a rigid body. Different parts of the Sun rotate at different angular velocities. Most objects that we will consider in this chapter are rigid bodies.

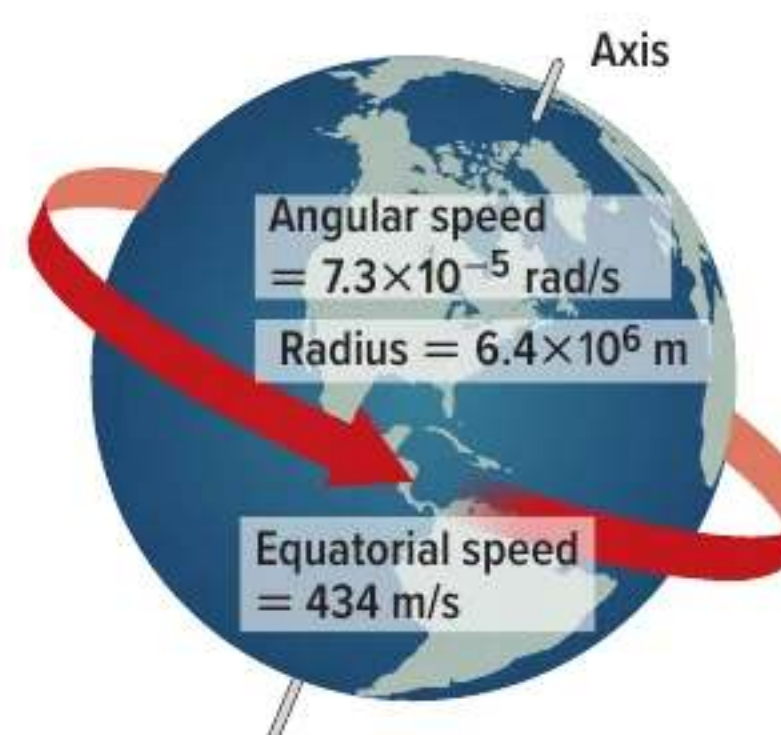


Figure 3 Earth is a rotating, rigid body and all parts rotate at the same rate.

Angular Acceleration

What if angular velocity is changing? For example, a car could accelerate from 0.0 m/s to 25 m/s in 15 s. In the same 15 s, the angular velocity of the car's 0.64 m diameter wheels would change from 0.0 rad/s to 78 rad/s. The wheels would undergo **angular acceleration**, which is the change in angular velocity divided by the time required to make the change. Angular acceleration (α) is represented by the following equation and is measured in rad/s^2 .

Average Angular Acceleration of an Object

Angular acceleration is equal to the change in angular velocity divided by the time required to make that change.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

If the change in angular velocity is positive, then the angular acceleration also is positive. Angular acceleration defined in this way is also the average angular velocity over the time interval Δt . One way to find the instantaneous angular acceleration is to find the slope of a graph of angular velocity as a function of time. The linear acceleration of a point at a distance (r) from the axis of an object with angular acceleration (α) is given by $a = r\alpha$. Table 1 is a summary of linear and angular relationships discussed previously in this lesson.



Get It?

Compare the angular velocity and angular acceleration of a rotating body.

Table 1 Linear and Angular Measures

Quantity	Linear	Angular	Relationship
Displacement	x (m)	θ (rad)	$x = r\theta$
Velocity	v (m/s)	ω (rad/s)	$v = r\omega$
Acceleration	a (m/s ²)	α (rad/s ²)	$a = r\alpha$

PRACTICE Problems

ADDITIONAL PRACTICE

- What is the angular displacement of each of the following hands of a clock in 1.00 h? State your answer in three significant digits.
 - the second hand
 - the minute hand
 - the hour hand
- A rotating toy above a crib makes one complete counterclockwise rotation in 1 min.
 - What is its angular displacement in 3 min?
 - What is the toy's angular velocity in rad/min?
 - If the toy is turned off, does it have positive or negative angular acceleration? Explain.
- If a truck has a linear acceleration of 1.85 m/s^2 and the wheels have an angular acceleration of 5.23 rad/s^2 , what is the diameter of the wheels?
- The truck in the previous problem is towing a trailer with wheels that have a diameter of 48 cm.
 - How does the linear acceleration of the trailer compare with that of the truck?
 - How do the angular accelerations of the wheels of the trailer and the wheels of the truck compare?
- CHALLENGE** You replace the tires on your car with tires of larger diameter. How will the angular velocity and number of revolutions change for trips at the same speed and over the same distance?

Angular frequency The number of complete revolutions made by an object in 1 s is called angular frequency. Angular frequency is defined as $f = \frac{\omega}{2\pi}$. One example of such a rotating object is a computer hard drive. Listen carefully when you start a computer. You often will hear the hard drive spinning. Hard drive frequencies are measured in revolutions per minute (RPM). Inexpensive hard drives rotate at 4800, 5400, and 7200 RPM. More advanced hard drives operate at 10,000 or 15,000 RPM. The faster the hard drive rotates, the quicker it can access or store information.

 Check Your Progress

- Angular Velocity** The Moon ($r = 1.74 \times 10^6 \text{ m}$) rotates once on its axis in 27.3 days.
 - What is its period of rotation in seconds?
 - What is the frequency of its rotation in rad/s?
 - A rock sits on the surface at the Moon's equator. What is its linear speed due to the Moon's rotation?
 - Compare this speed with that of a person at the equator due to Earth's rotation.
- Angular Displacement** A movie lasts 2 h. During that time, what is the angular displacement of each of the following?
 - the hour hand
 - the minute hand
 - the second hand
- Angular Acceleration** In the spin cycle of a clothes washer, the drum turns at 635 rev/min. If the lid is opened, the motor is turned off. If the drum requires 8.0 s to slow to a stop, what is the angular acceleration of the drum?
- Angular Displacement** Do all parts of the minute hand on a watch such as the one shown in **Figure 4** have the same angular displacement? Do they move the same linear distance? Explain.
- Critical Thinking** A CD-ROM has a spiral track that starts 2.7 cm from the center of the disk and ends 5.5 cm from the center. The disk drive must turn the disk so that the linear velocity of the track is a constant 1.4 m/s. Find the following.
 - the angular velocity of the disk (in rad/s and rev/min) for the start of the track
 - the disk's angular velocity at the end of the track
 - the disk's angular acceleration if the disk is played for 76 min



Figure 4

LEARNSMART™

Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

LESSON 2 ROTATIONAL DYNAMICS

FOCUS QUESTION

How does force affect rotation?

Force and Angular Velocity

How do you start the rotation of an object? That is, how do you increase its angular velocity? If you wrap a string around a toy top and pull hard, you can make the top spin rapidly. The force of the string is exerted at the outer edge of the top and at right angles to the line from the center of the top to the point where the string leaves the top's surface. You have learned that a force changes the velocity of a point object. In the case of a toy top, a force that is exerted in a very specific way changes the angular velocity of an extended object. An extended object is an object that has a definite shape and size.

How can you exert a force to open a door easily? To get the most effect from the least force, you exert the force as far from the axis of rotation as possible, as shown in **Figure 5**. In this case, the axis of rotation is an imaginary vertical line through the hinges. The doorknob is near the outer edge of the door. You exert the force on the doorknob at right angles to the door. Thus, the magnitude of the force, the distance from the axis to the point where the force is exerted, and the direction of the force determine the change in angular velocity.

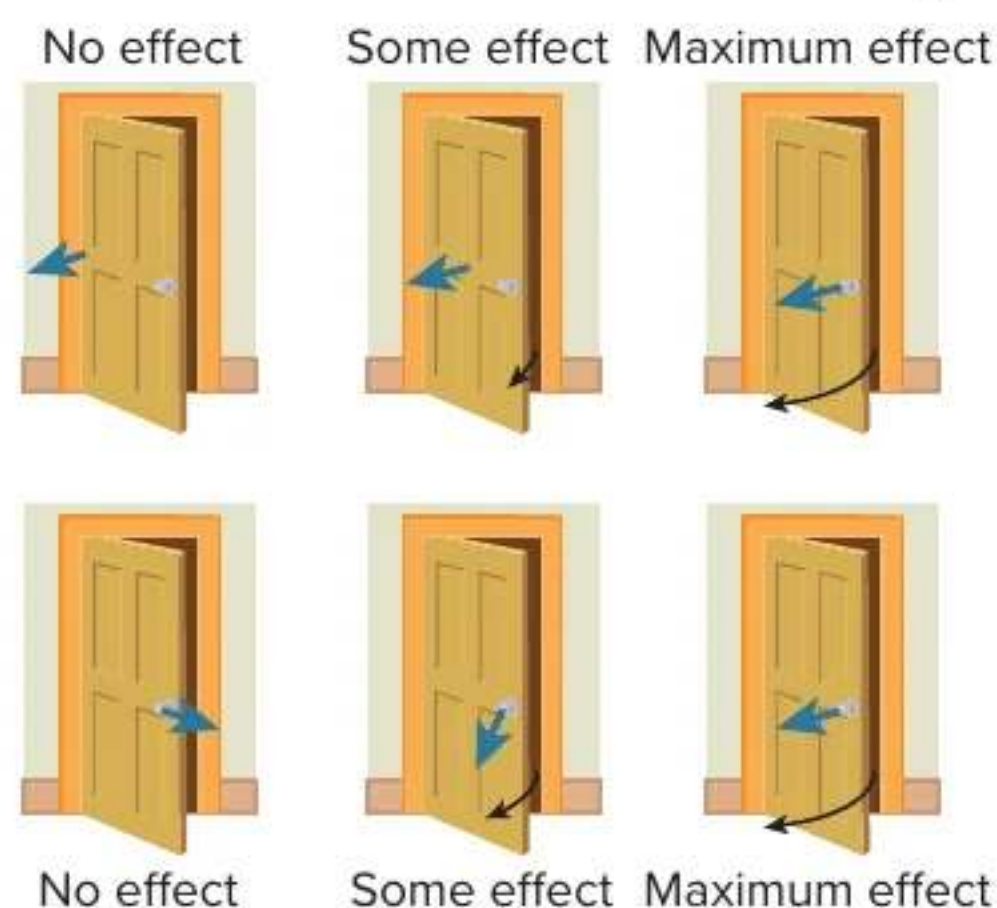


Figure 5 When opening a door that is free to rotate about its hinges, apply the force farthest from the hinges, at an angle perpendicular to the door.



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



Forensics Lab: Leverage

Carry out an investigation to determine how a small force can produce a large torque.



PhysicsLAB: Torques

Calculate the torque on a system when the forces and lever arms are known.

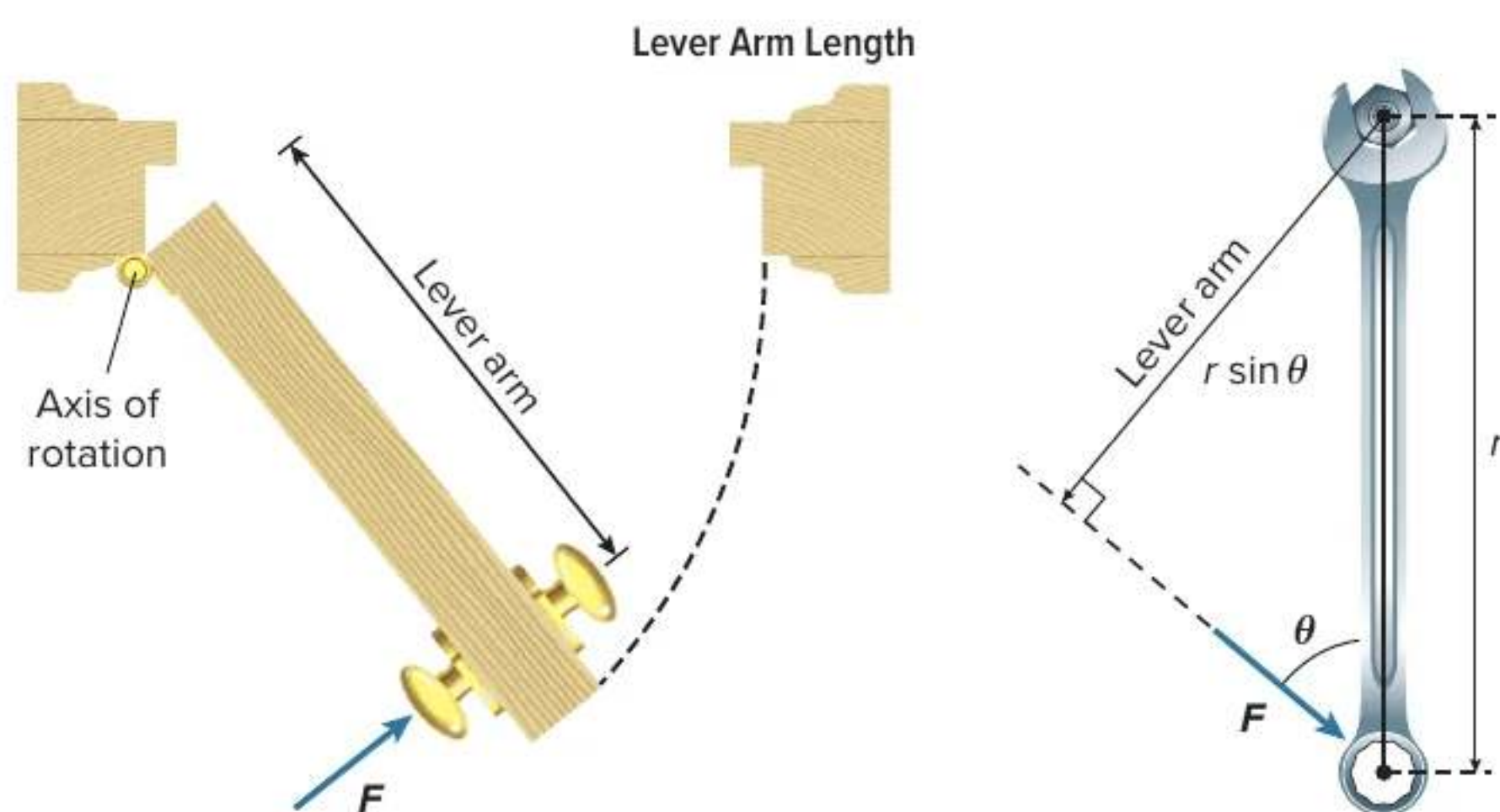


Figure 6 The lever arm is the perpendicular distance from the axis of rotation to the point where the force is exerted. For the door, the lever arm is along the width of the door, from the hinge to the point where the force is exerted. For the wrench, the lever arm is equal to $r \sin \theta$, when the angle (θ) between the force and the radius of rotation is not equal to 90° .

Explain why the formula $r \sin \theta$ is used to find the length of the lever arm.

Lever arm For a given applied force, the change in angular velocity depends on the lever arm, which is the perpendicular distance from the axis of rotation to the point where the force is exerted. If the force is perpendicular to the radius of rotation, as it was with the toy top, then the **lever arm** is the distance from the axis (r). For the door example, the lever arm is the distance from the hinges to the point where you exert the force, as illustrated on the left in **Figure 6**. If a force is not exerted perpendicular to the radius, the length of the lever arm is reduced. You must use mathematics to find the length of the lever arm.

To find the lever arm, you must extend the line of the force until it forms a right angle with a line from the center of rotation, as shown in **Figure 6**. The distance between this intersection and the axis is the lever arm. Using trigonometry, the lever arm (L) can be calculated by the equation $L = r \sin \theta$. In this equation, r is the distance from the axis of rotation to the point where the force is exerted, and θ is the angle between the force and the radius from the axis of rotation to the point where the force is applied.



Get It?

Explain what each of the variables represents in the equation $L = r \sin \theta$.

Torque The term **torque** describes the combination of force and lever arm that can cause an object to rotate. The magnitude of a torque is the product of the force and the perpendicular lever arm. Because force is measured in newtons and distance is measured in meters, torque is measured in newton-meters (N·m). Torque is represented by the Greek letter tau (τ) and is represented by the equation shown below.

Torque

Torque is equal to the force F times the lever arm ($r \sin \theta$).

$$\tau = Fr \sin \theta$$



Get It?

Identify what each of the variables in the torque equation — τ , F , r , and θ — represents.

EXAMPLE Problem 1

LEVER ARM A bolt on a car engine must be tightened with a torque of 35 N·m. You use a 25-cm-long wrench and pull the end of the wrench at an angle of 60.0° to the handle of the wrench. How long is the lever arm, and how much force must you exert?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation. Find the lever arm by extending the force vector backward until a line that is perpendicular to it intersects the axis of rotation.

Known

$$r = 0.25 \text{ m} \quad \tau = 35 \text{ N}\cdot\text{m}$$

$$\theta = 60.0^\circ$$

Unknown

$$L = ?$$

$$F = ?$$

2 SOLVE FOR THE UNKNOWN

Solve for the length of the lever arm.

$$L = r \sin \theta$$

$$= (0.25 \text{ m})(\sin 60.0^\circ)$$

$$= 0.22 \text{ m}$$

Solve for the force.

$$\tau = Fr \sin \theta$$

$$F = \frac{\tau}{r \sin \theta}$$

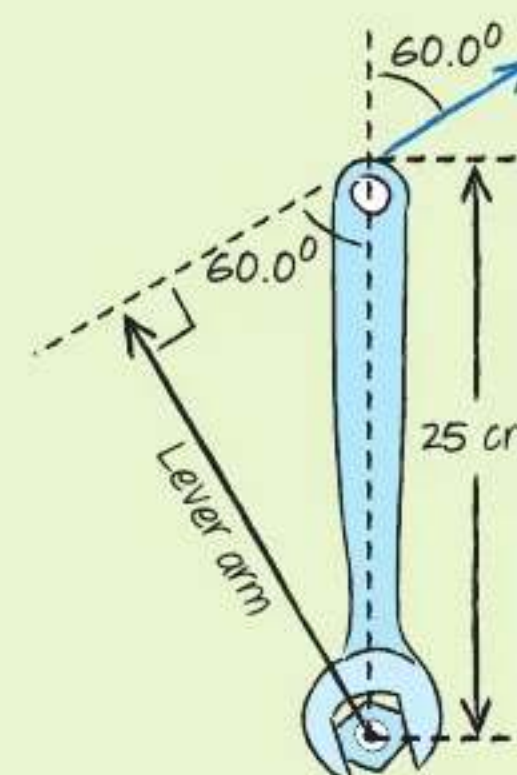
$$= \frac{35 \text{ N}\cdot\text{m}}{(0.25 \text{ m})(\sin 60.0^\circ)}$$

$$= 1.6 \times 10^2 \text{ N}$$

Substitute $r = 0.25 \text{ m}$ and $\theta = 60.0^\circ$ into the equation. Then, solve the equation.

Substitute $\tau = 35 \text{ N}\cdot\text{m}$, $r = 0.25 \text{ m}$, and $\theta = 60.0^\circ$ into the equation.

Then, solve the equation. Remember to use significant digits.

**3 EVALUATE THE ANSWER**

- **Are the units correct?** Force is measured in newtons.
- **Does the sign make sense?** Only the magnitude of the force needed to rotate the wrench clockwise is calculated.

PRACTICE Problems**ADDITIONAL PRACTICE**

- Consider the wrench in Example Problem 1. What force is needed if it is applied to the wrench pointing perpendicular to the wrench?
- If a torque of 55.0 N·m is required to turn a bolt and the largest force you can exert is 135 N, how long a lever arm must you use to turn the bolt?
- You have a 0.234-m-long wrench. A job requires a torque of 32.4 N·m, and you can exert a force of 232 N.
 - What is the smallest angle, with respect to the handle of the wrench, at which you can pull on the wrench and get the job done?
 - A friend can exert 275 N. What is the smallest angle she can use to accomplish the job?

- You stand on a bicycle pedal, as shown in **Figure 7**. Your mass is 65 kg. If the pedal makes an angle of 35° above the horizontal and the pedal is 18 cm from the center of the chain ring, how much torque would you exert?



Figure 7

- CHALLENGE** If the pedal in the previous problem is horizontal, how much torque would you exert? How much torque would you exert when it is vertical?

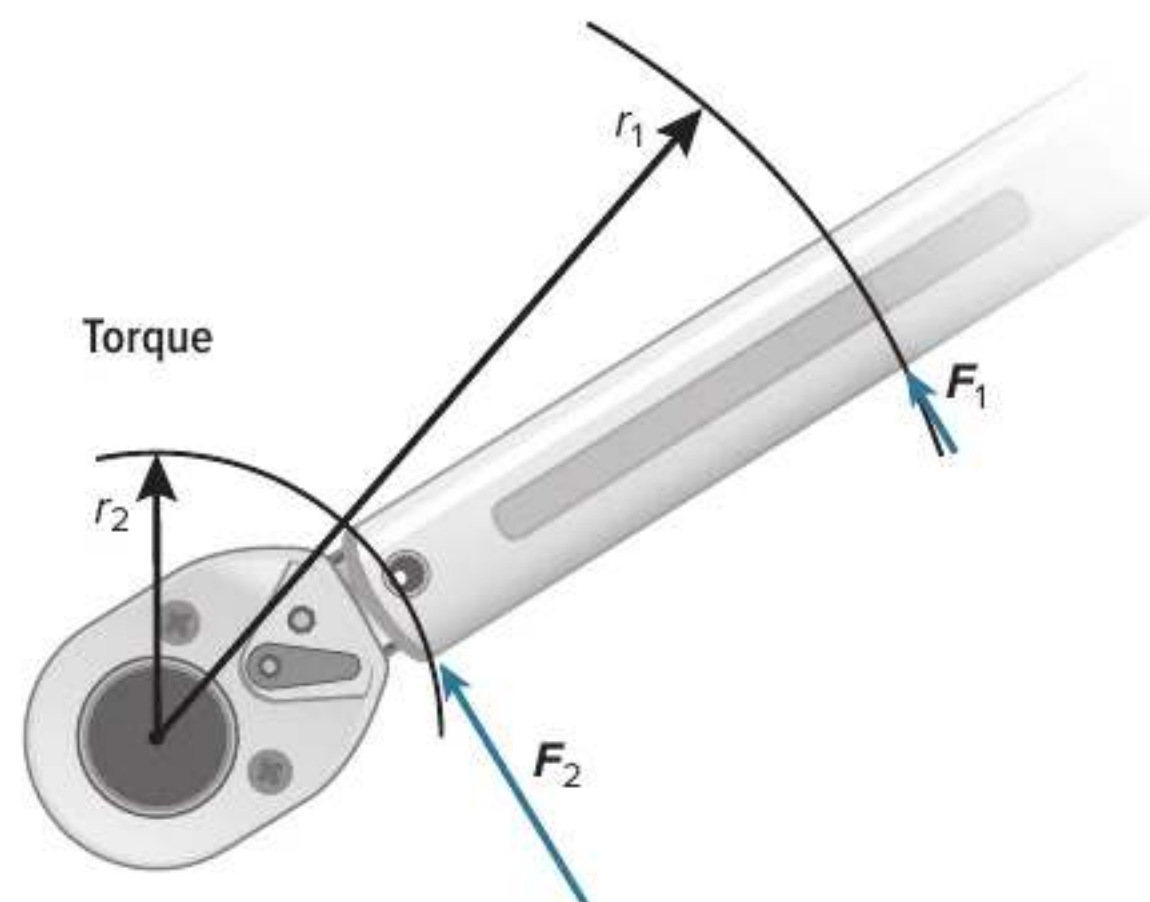


Figure 8 This worker uses a long wrench because it requires him to exert less force to tighten and loosen the nut. The wrench has a long lever arm, and less force is required if the force is applied farther from the axis of rotation (the center of the nut).

Finding Net Torque

Figure 8 shows a practical application of increasing torque to make a task easier. For another example of torque, try the following investigation. Collect two pencils, some coins, and some transparent tape. Tape two identical coins to the ends of the pencil and balance it on the second pencil, as shown in **Figure 9**. Each coin exerts a torque that is equal to the distance from the balance point to the center of the coin (r) times its weight (F_g), as follows:

$$\tau = rF_g$$

But the torques are equal and opposite in direction. Thus, the net torque is zero:

$$\tau_1 - \tau_2 = 0$$

or

$$r_1 F_{g1} - r_2 F_{g2} = 0$$

How can you make the pencil rotate? You could add a second coin on top of one of the two coins, thereby making the two forces different. You also could slide the balance point toward one end or the other of the pencil, thereby making the two lever arms of different length.

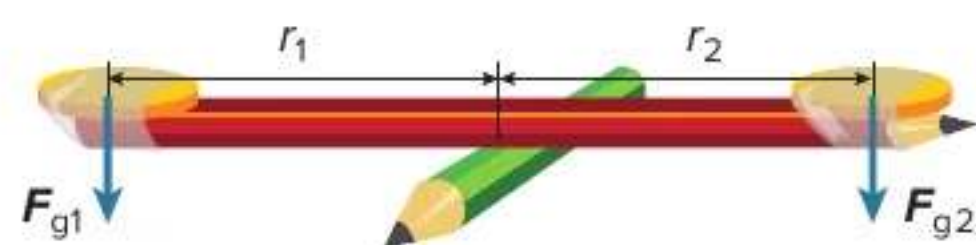


Figure 9 The torque exerted by the first coin ($F_{g1}r_1$) is equal and opposite in direction to the torque exerted by the second coin ($F_{g2}r_2$) when the pencil is balanced.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Sketch models to describe the variations of the system in **Figure 9** described above. Use the models to predict how the different arrangements will affect the system. Test the arrangements and compare your predictions with your evidence.

STEM CAREER Connection

Race Car Mechanic

If you are competitive and love to hear the roar of a high-performance engine, race car mechanic might be the career for you. These specialists use their understanding of torque to help keep their team's car at the front of the pack.

EXAMPLE Problem 2

BALANCING TORQUES Kariann (56 kg) and Aysha (43 kg) want to balance on a 1.75-m-long seesaw. Where should they place the pivot point of the seesaw? Assume that the seesaw is massless.

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Draw and label the vectors.

Known

$$m_K = 56 \text{ kg}$$

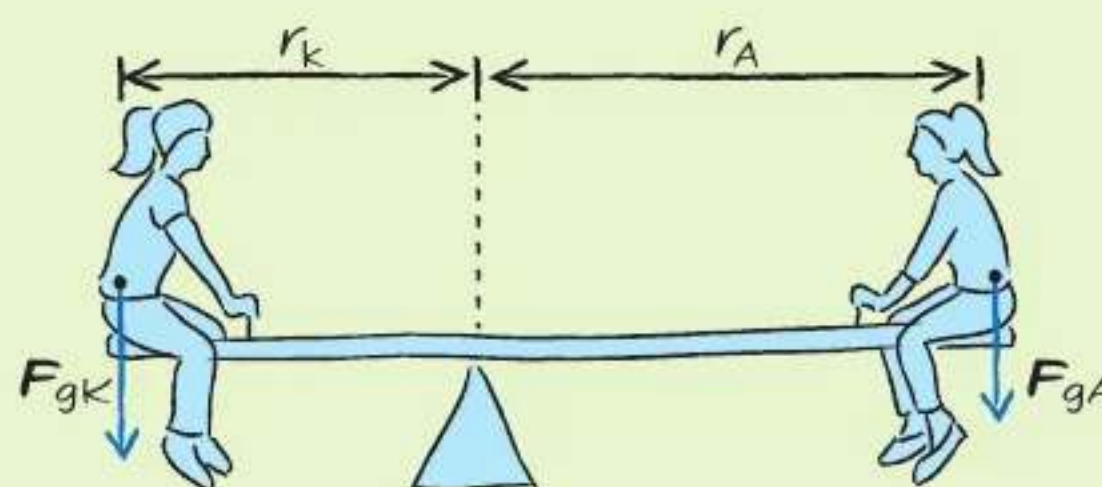
$$m_A = 43 \text{ kg}$$

$$r_K + r_A = 1.75 \text{ m}$$

Unknown

$$r_K = ?$$

$$r_A = ?$$

**2 SOLVE FOR THE UNKNOWN**

Find the two forces.

Kariann:

$$F_{gK} = m_K g$$

$$= (56 \text{ kg})(9.8 \text{ N/kg})$$

$$= 5.5 \times 10^2 \text{ N}$$

Substitute the known values into the equation: $m_K = 56 \text{ kg}$, $g = 9.8 \text{ N/kg}$.

Aysha:

$$F_{gA} = m_A g$$

$$= (43 \text{ kg})(9.8 \text{ N/kg})$$

$$= 4.2 \times 10^2 \text{ N}$$

Substitute the known values into the equation: $m_A = 43 \text{ kg}$, $g = 9.8 \text{ N/kg}$.

Define Kariann's distance in terms of the length of the seesaw and Aysha's distance.

$$r_K = 1.75 \text{ m} - r_A$$

When there is no rotation, the sum of the torques is zero.

$$F_{gK} r_K = F_{gA} r_A$$

$$F_{gK} r_K - F_{gA} r_A = 0.0 \text{ N}\cdot\text{m}$$

$$F_{gK}(1.75 \text{ m} - r_A) - F_{gA} r_A = 0.0 \text{ N}\cdot\text{m}$$

Substitute the relationship between Kariann's distance in terms of Aysha's distance in to the equation: $r_K = 1.75 \text{ m} - r_A$.

Solve for r_A .

$$F_{gK}(1.75 \text{ m}) - F_{gK} r_A - F_{gA} r_A = 0.0 \text{ N}\cdot\text{m}$$

$$F_{gK} r_A + F_{gA} r_A = F_{gK}(1.75 \text{ m})$$

$$(F_{gK} + F_{gA}) r_A = F_{gK}(1.75 \text{ m})$$

$$r_A = \frac{F_{gK}(1.75 \text{ m})}{(F_{gK} + F_{gA})}$$

$$= \frac{(5.5 \times 10^2 \text{ N})(1.75 \text{ m})}{(5.5 \times 10^2 \text{ N} + 4.2 \times 10^2 \text{ N})}$$

$$= 0.99 \text{ m}$$

Substitute $F_{gK} = 5.5 \times 10^2 \text{ N}$ and $F_{gA} = 4.2 \times 10^2 \text{ N}$.

3 EVALUATE THE ANSWER

- **Are the units correct?** Distance is measured in meters.
- **Do the signs make sense?** Distances are positive.
- **Is the magnitude realistic?** Aysha is about 1 m from the center, so Kariann is about 0.75 m away from it. Because Kariann's weight is greater than Aysha's weight, the lever arm on Kariann's side should be shorter. Aysha is farther from the pivot, as expected.

PRACTICE Problems

ADDITIONAL PRACTICE

16. Ashok, whose mass is 43 kg, sits 1.8 m from a pivot at the center of a seesaw. Steve, whose mass is 52 kg, wants to seesaw with Ashok. How far from the center of the seesaw should Steve sit?
17. A bicycle-chain wheel has a radius of 7.70 cm. If the chain exerts a 35.0-N force on the wheel in the clockwise direction, what torque is needed to keep the wheel from turning?
18. Two stationary baskets of fruit hang from strings on pulleys of different diameters, as shown in **Figure 10**. What is the mass of basket A?

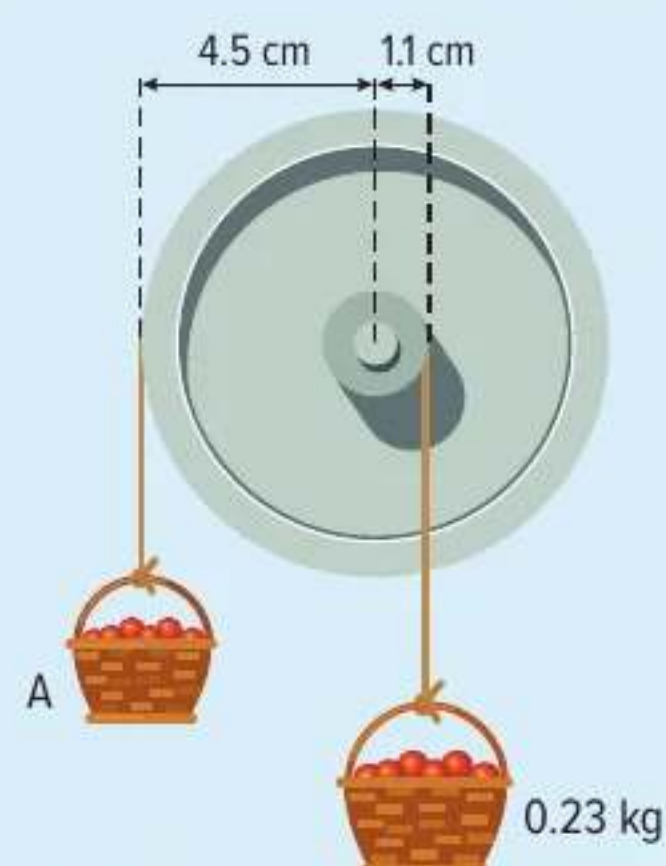


Figure 10

19. Suppose the radius of the larger pulley in problem 18 was increased to 6.0 cm. What is the mass of basket A now?
20. **CHALLENGE** A bicyclist, of mass 65.0 kg, stands on the pedal of a bicycle. The crank, which is 0.170 m long, makes a 45.0° angle with the vertical, as shown in **Figure 11**. The crank is attached to the chain wheel, which has a radius of 9.70 cm. What force must the chain exert to keep the wheel from turning?

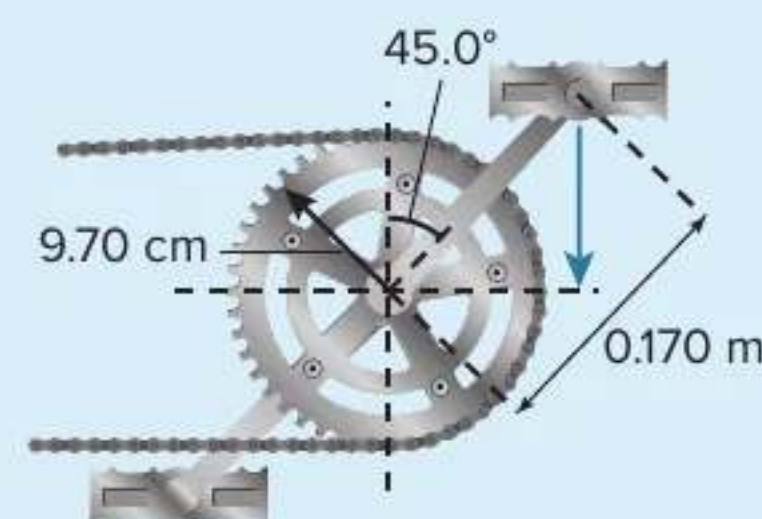


Figure 11

The Moment of Inertia

If you exert a force on a point mass, its acceleration will be inversely proportional to its mass. How does an extended object rotate when a torque is exerted on it? To observe firsthand, recover the pencil, the coins, and the transparent tape that you used earlier in this chapter. First, tape the coins at the ends of the pencil. Hold the pencil between your thumb and forefinger, and wiggle it back and forth. Take note of the forces that your thumb and forefinger exert. These forces create torques that change the angular velocity of the pencil and coins.

Now move the coins so that they are only 1 or 2 cm apart. Wiggle the pencil as before. Did you notice that the pencil is now easier to rotate? The torque that was required was much less this time. The mass of an object is not the only factor that determines how much torque is needed to change its angular velocity; the distribution or location of the mass also is important.

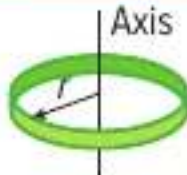
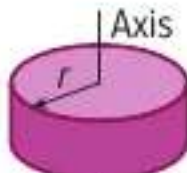
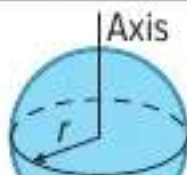
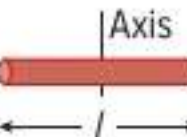
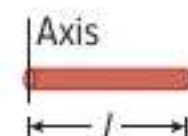
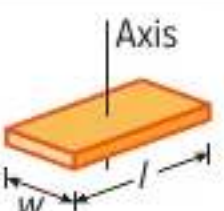
The resistance to rotation is called the **moment of inertia**, which is represented by the symbol I and has units of mass times the square of the distance. For a point object located at a distance (r) from the axis of rotation, the moment of inertia is given by the following equation.

Moment of Inertia of a Point Mass

The moment of inertia of a point mass is equal to the mass of the object times the square of the object's distance from the axis of rotation.

$$I = mr^2$$

Table 2 Moments of Inertia for Various Objects

Object	Location of Axis	Diagram	Moment of Inertia
Thin hoop of radius r	through central diameter		mr^2
Solid, uniform cylinder of radius r	through center		$\left(\frac{1}{2}\right)mr^2$
Uniform sphere of radius r	through center		$\left(\frac{2}{5}\right)mr^2$
Long, uniform rod of length l	through center		$\left(\frac{1}{12}\right)ml^2$
Long, uniform rod of length l	through end		$\left(\frac{1}{3}\right)ml^2$
Thin, rectangular plate of length l and width w	through center		$\left(\frac{1}{12}\right)m(l^2 + w^2)$

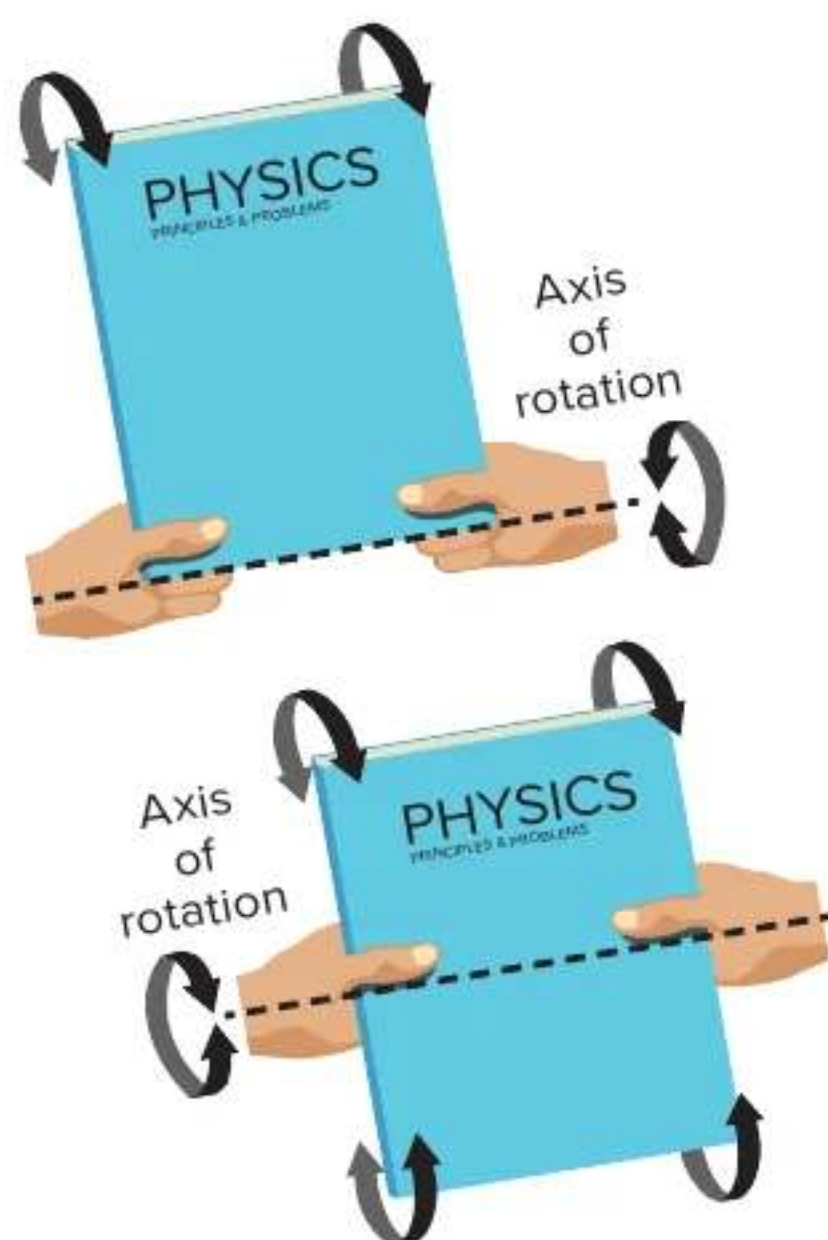


Figure 12 The moment of inertia of a book depends on the axis of rotation. The moment of inertia of the book on the top is larger than the moment of inertia of the book on the bottom because the average distance of the book's mass from the rotational axis is larger.

Identify which book requires more torque to rotate it and why.

Moment of inertia and mass As you have seen, the moments of inertia for extended objects, such as the pencil and coins, depend on how far the masses are from the axis of rotation. A bicycle wheel, for example, has almost all of its mass in the rim and tire. Its moment of inertia about its axle is almost exactly equal to mr^2 , where r is the radius of the wheel. For most objects, however, the mass is distributed closer to the axis so the moment of inertia is less than mr^2 . For example, as shown in **Table 2**, for a solid cylinder of radius r , $I = \left(\frac{1}{2}\right)mr^2$, while for a solid sphere, $I = \left(\frac{2}{5}\right)mr^2$.



Get It?

Write the equation for the moment of inertia of a hoop.

Moment of inertia and rotational axis The moment of inertia also depends on the location and direction of the rotational axis, as illustrated in **Figure 12**. To observe this effect, hold a book in the upright position by placing your hands at the bottom of the book. Feel the torque needed to rock the book toward you and then away from you. Now put your hands in the middle of the book and feel the torque needed to rock the book. Note that much less torque is needed when your hands are placed in the middle of the book because the average distance of the book's mass from the rotational axis is much less.

EXAMPLE Problem 3

MOMENT OF INERTIA A simplified model of a twirling baton is a thin rod with two round objects at each end. The length of the baton is 0.66 m, and the mass of each object is 0.30 kg. Find the moment of inertia of the baton as it is rotated about an axis at the midpoint between the round objects and perpendicular to the rod. What is the moment of inertia of the baton if the axis is moved to one end of the rod? Which is greater? Assume the mass of the rod is negligible.

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation. Show the baton with the two different axes of rotation and the distances from the axes of rotation to the masses.

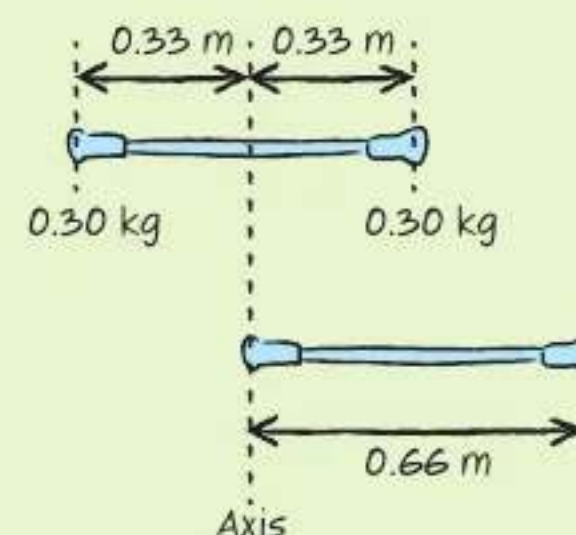
Known

$$m = 0.30 \text{ kg}$$

$$l = 0.66 \text{ m}$$

Unknown

$$I = ?$$

**2 SOLVE FOR THE UNKNOWN**

Calculate the moment of inertia of each mass separately.
Rotating about the center of the rod:

$$\begin{aligned} r &= \left(\frac{1}{2}\right)l \\ &= \left(\frac{1}{2}\right)(0.66 \text{ m}) \\ &= 0.33 \text{ m} \end{aligned}$$

Substitute the known value, $l = 0.66 \text{ m}$, into the equation.

$$\begin{aligned} I_{\text{single mass}} &= mr^2 \\ &= (0.30 \text{ kg})(0.33 \text{ m})^2 \\ &= 0.033 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Substitute $m = 0.30 \text{ kg}$ and $r = 0.33 \text{ m}$ into the equation.

Find the moment of inertia of the baton.

$$\begin{aligned} I &= 2I_{\text{single mass}} \\ &= 2(0.033 \text{ kg}\cdot\text{m}^2) \\ &= 0.066 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Substitute $I_{\text{single mass}} = 0.033 \text{ kg}\cdot\text{m}^2$ into the equation.

Rotating about one end of the rod:

$$\begin{aligned} I_{\text{single mass}} &= mr^2 \\ &= (0.30 \text{ kg})(0.66 \text{ m})^2 \\ &= 0.13 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Substitute $m = 0.30 \text{ kg}$ and $r = 0.66 \text{ m}$ into the equation.

$$\begin{aligned} I_{\text{single mass}} &= mr^2 \\ &= (0.30 \text{ kg})(0.0 \text{ m})^2 \\ &= 0 \text{ for the other mass.} \end{aligned}$$

Find the moment of inertia of the baton.

$$\begin{aligned} I &= I_{\text{single mass}} + 0 \\ &= 0.13 \text{ kg}\cdot\text{m}^2 \end{aligned}$$

The moment of inertia is greater when the baton is swung around one end.

3 EVALUATE THE ANSWER

- **Are the units correct?** Moment of inertia is measured in $\text{kg}\cdot\text{m}^2$.
- **Is the magnitude realistic?** Masses and distances are small, and so are the moments of inertia. Doubling the distance increases the moment of inertia by a factor of 4. Thus, doubling the distance increases the moment of inertia more than having only one mass decreases the moment of inertia.

PRACTICE Problems

- 21.** Two children of equal masses sit 0.3 m from the center of a seesaw. Assuming that their masses are much greater than that of the seesaw, by how much is the moment of inertia increased when they sit 0.6 m from the center? Ignore the moment of inertia for the seesaw.
- 22.** Suppose there are two balls with equal diameters and masses. One is solid, and the other is hollow, with all its mass distributed at its surface. Are the moments of inertia of the balls equal? If not, which is greater?
- 23.** Calculate the moments of inertia for each object below using the formulas in **Table 2**. Each object has a radius of 2.0 m and a mass of 1.0 kg.
- a thin hoop
 - a solid, uniform cylinder
 - a solid, uniform sphere

ADDITIONAL PRACTICE

24. CHALLENGE Figure 13 shows three equal-mass spheres on a rod of very small mass. Consider the moment of inertia of the system, first when it is rotated about sphere A and then when it is rotated about sphere C.

- Are the moments of inertia the same or different? Explain. If the moments of inertia are different, in which case is the moment of inertia greater?
- Each sphere has a mass of 0.10 kg. The distance between spheres A and C is 0.20 m. Find the moment of inertia in the following instances: rotation about sphere A, rotation about sphere C.

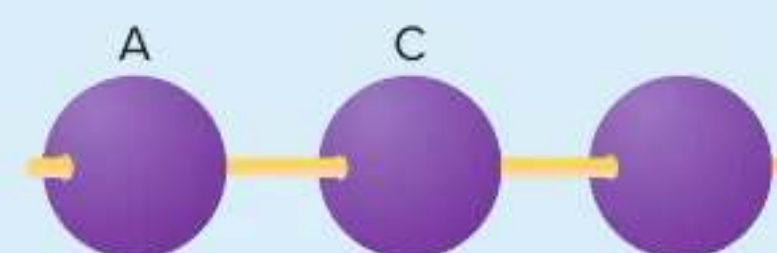


Figure 13

Newton's Second Law for Rotational Motion

Newton's second law for linear motion is expressed as $a = \frac{F_{\text{net}}}{m}$. If you rewrite this equation to represent rotational motion, acceleration is replaced by angular acceleration (α), force is replaced by net torque (τ_{net}), and mass is replaced by moment of inertia (I). Angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia as stated in **Newton's second law for rotational motion**. This law is expressed by the following equation.

Newton's Second Law for Rotational Motion

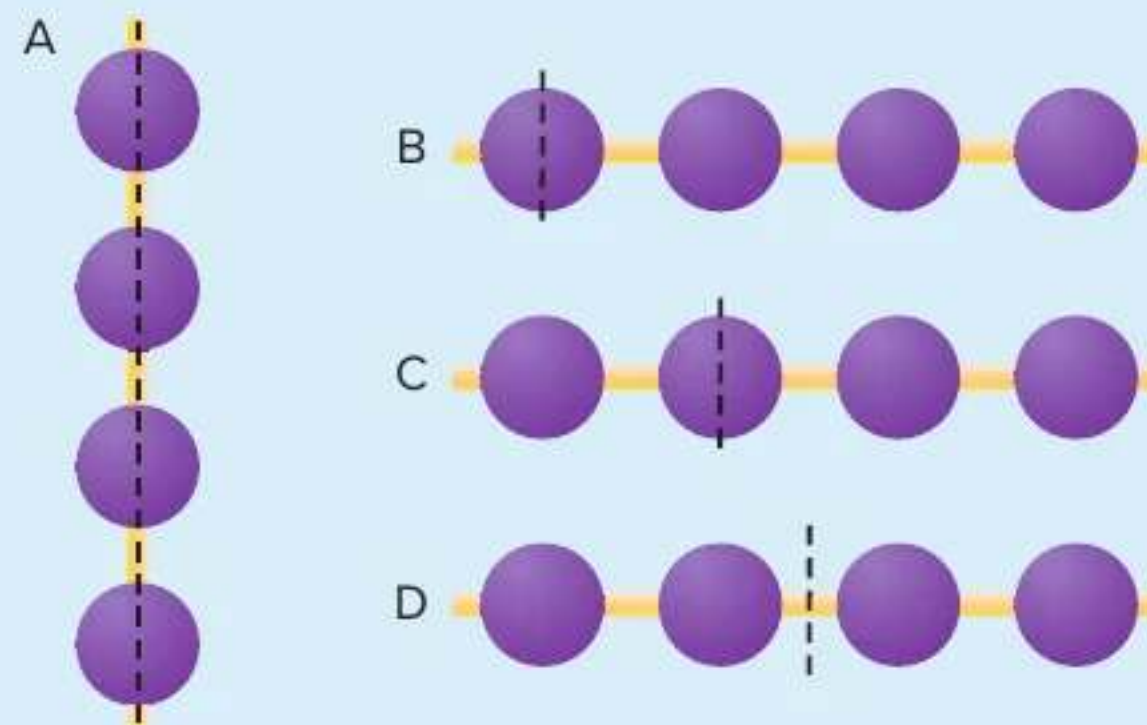
The angular acceleration of an object about a particular axis equals the net torque on the object divided by the moment of inertia.

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

If the torque on an object and the angular velocity of that object are in the same direction, then the angular velocity of the object increases. If the torque and angular velocity are in different directions, then the angular velocity decreases.

PHYSICS Challenge

Moments of Inertia Rank the objects shown in the diagram from least to greatest according to their moments of inertia about the indicated axes. All spheres have equal masses and all separations are the same. Assume that the rod's mass is negligible.



EXAMPLE Problem 4

TORQUE A solid steel wheel is free to rotate about a motionless central axis. It has a mass of 15 kg and a diameter of 0.44 m and starts at rest. You want to increase this wheel's rotation about its central axis to 8.0 rev/s in 15 s.

- What torque must be applied to the wheel?
- If you apply the torque by wrapping a strap around the outside of the wheel, how much force should you exert on the strap?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation. The torque must be applied in a counterclockwise direction; force must be exerted as shown.

Known

$$m = 15 \text{ kg}$$

$$r = \left(\frac{1}{2}\right)(0.44 \text{ m}) = 0.22 \text{ m}$$

$$\omega_i = 0.0 \text{ rad/s}$$

$$\omega_f = 2\pi(8.0 \text{ rev/s})$$

$$t = 15 \text{ s}$$

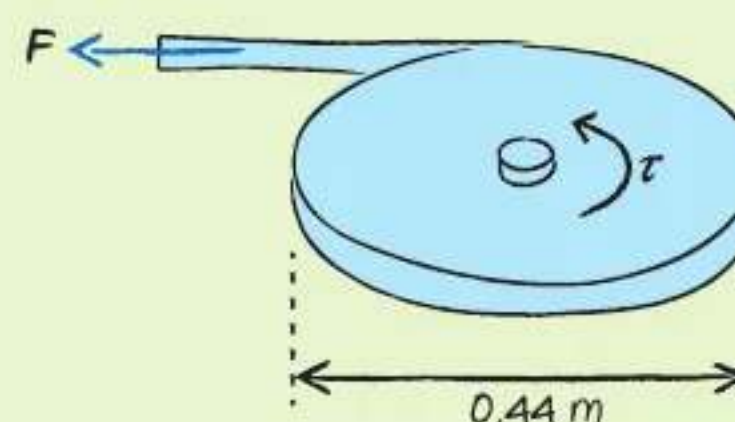
Unknown

$$\alpha = ?$$

$$I = ?$$

$$\tau = ?$$

$$F = ?$$

**2 SOLVE FOR THE UNKNOWN**

- Solve for angular acceleration.

$$\begin{aligned}\alpha &= \frac{\Delta\omega}{\Delta t} \\ &= \frac{(16\pi \text{ rad/s} - (0.0 \text{ rad/s}))}{15 \text{ s}} \\ &= 3.4 \text{ rad/s}^2\end{aligned}$$

Substitute $\omega_f = 16\pi \text{ rad/s}$ and $\omega_i = 0.0 \text{ rad/s}$ into the equation.

Solve for the moment of inertia.

$$\begin{aligned}I &= \left(\frac{1}{2}\right)mr^2 \\ &= \left(\frac{1}{2}\right)(15 \text{ kg})(0.22 \text{ m})^2 \\ &= 0.36 \text{ kg}\cdot\text{m}^2\end{aligned}$$

Substitute $m = 15 \text{ kg}$ and $r = 0.22 \text{ m}$ into the equation.

Solve for torque.

$$\begin{aligned}\tau &= I\alpha \\ &= (0.36 \text{ kg}\cdot\text{m}^2)(3.4 \text{ rad/s}^2) \\ &= 1.2 \text{ kg}\cdot\text{m}^2/\text{s}^2 \\ &= 1.2 \text{ N}\cdot\text{m}\end{aligned}$$

Substitute $I = 0.36 \text{ kg}\cdot\text{m}^2$ and $\alpha = 3.4 \text{ rad/s}^2$ into the equation.

- Solve for force.

$$\begin{aligned}\tau &= Fr \\ F &= \frac{\tau}{r} \\ &= \frac{(1.2 \text{ N}\cdot\text{m})}{(0.22 \text{ m})} \\ &= 5.5 \text{ N}\end{aligned}$$

Substitute $\tau = 1.2 \text{ N}\cdot\text{m}$ and $r = 0.22 \text{ m}$ into the equation.

3 EVALUATE THE ANSWER

- Are the units correct?** Torque is measured in $\text{N}\cdot\text{m}$ and force is measured in N .
- Is the magnitude realistic?** The size of the wheel makes it relatively easy to spin despite its mass.

PRACTICE Problems

25. Consider the wheel in Example Problem 4. If the force on the strap were twice as great, what would be the angular frequency of the wheel after 15 s?
26. A solid wheel accelerates at 3.25 rad/s^2 when a force of 4.5 N exerts a torque on it. If the wheel is replaced by a wheel which has all of its mass on the rim, the moment of inertia is given by $I = mr^2$. What force should be exerted on the strap to give the same angular velocity?
27. A bicycle wheel on a repair bench can be accelerated either by pulling on the chain that is on the gear or by pulling on a string wrapped around the tire. The tire's radius is 0.38 m, while the radius of the gear is 0.14 m. What force would you need to pull on the string to produce the same acceleration you obtained with a force of 15 N on the chain?
28. The same bicycle wheel in the previous problem is used with a smaller gear whose radius is 0.11 m. If you obtained the needed acceleration with a force of 15 N on the chain, what force would you need to exert on the string?

ADDITIONAL PRACTICE

29. A chain is wrapped around a pulley and pulled with a force of 16.0 N. The pulley has a radius of 0.20 m. The pulley's rotational speed increases from 0.0 to 17.0 rev/min in 5.00 s. What is the moment of inertia of the pulley?
30. **CHALLENGE** A disk with a moment of inertia of $0.26 \text{ kg}\cdot\text{m}^2$ is attached to a smaller disk mounted on the same axle. The smaller disk has a diameter of 0.180 m and a mass of 2.5 kg. A strap is wrapped around the smaller disk, as shown in **Figure 14**. Find the force needed to give this system an angular acceleration of 2.57 rad/s^2 .



Figure 14

 Check Your Progress

31. **Torque** Vijesh enters a revolving door that is not moving. Explain where and how Vijesh should push to produce a torque with the least amount of force.
32. **Lever Arm** You open a door by pushing at a right angle to the door. Your friend pushes at the same place, but at an angle of 55° from the perpendicular. If both you and your friend exert the same torque on the door, how do the forces you and your friend applied compare?
33. **Torque and Force** The solid wheel shown in **Figure 15** has a mass of 5.2 kg and a diameter of 0.55 m. It is at rest, and you need it to rotate at 12 rev/s in 35 s.
- What torque must be applied?
 - If a nylon strap is wrapped around the outside of the wheel, how much force do you need to exert on the strap?
34. **Net Torque** Two people pull on ropes wrapped around the edge of a large wheel. The wheel has a mass of 12 kg and a diameter of 2.4 m. One person pulls in a clockwise direction with a 43-N force, and the other pulls counterclockwise with a 67-N force. What is the net torque?
35. **Newton's Second Law for Rotational Motion** A rope is wrapped around a pulley and pulled with a force of 13.0 N. The pulley's radius is 0.150 m. The pulley's rotational speed increases from 0.0 to 14.0 rev/min in 4.50 s. What is the moment of inertia of the pulley?
36. **Critical Thinking** A ball on a low-friction, tilted surface will slide downhill without rotating. On a rough surface, the ball will roll. Explain why, using a free-body diagram.

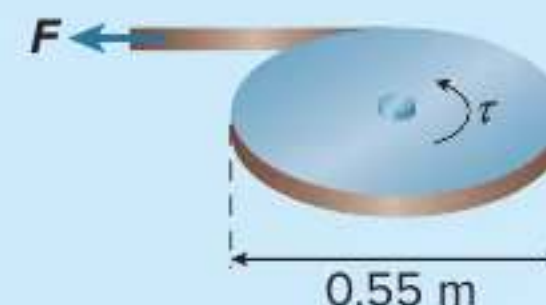


Figure 15

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LESSON 3 EQUILIBRIUM

FOCUS QUESTION

Why are some vehicles more likely to roll over than others?

The Center of Mass

How does a freely moving object rotate around its center of mass? A wrench may spin about its handle or end-over-end. Does any single point on the wrench follow a straight path? **Figure 16** shows the path of the wrench. You can see that there is a single point whose path traces a straight line, as if the wrench could be replaced by a point particle at that location. The white dot in the photo represents this point. The point on the object that moves in the same way that a point particle would move is the **center of mass** of an object.

Locating the center of mass How can you locate the center of mass of an object? First, suspend the object from any point. When the object stops swinging, the center of mass is somewhere along the vertical line drawn from the suspension point. Draw the line. Then, suspend the object from another point. Again, the center of mass must be directly below this point. Draw a second vertical line. The center of mass is at the point at which these two lines cross. The wrench and all other objects that are freely moving through space rotate about an axis that goes through their center of mass. Where would you think the center of mass of a human body is located?

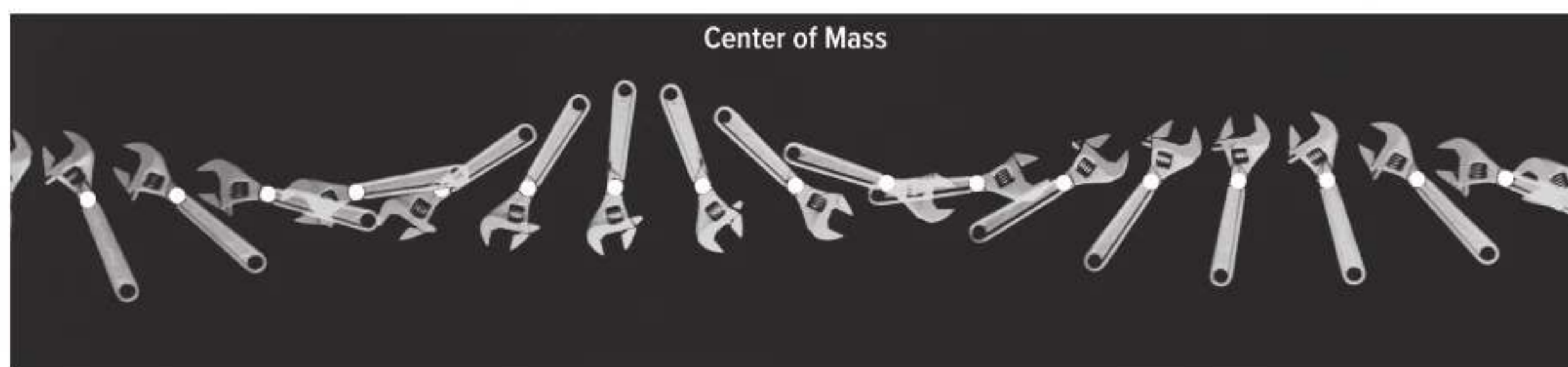


Figure 16 The path of the center of mass of a wrench is a straight line.

Berenice Abbott/Science Source



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



PhysicsLAB: Equilibrium

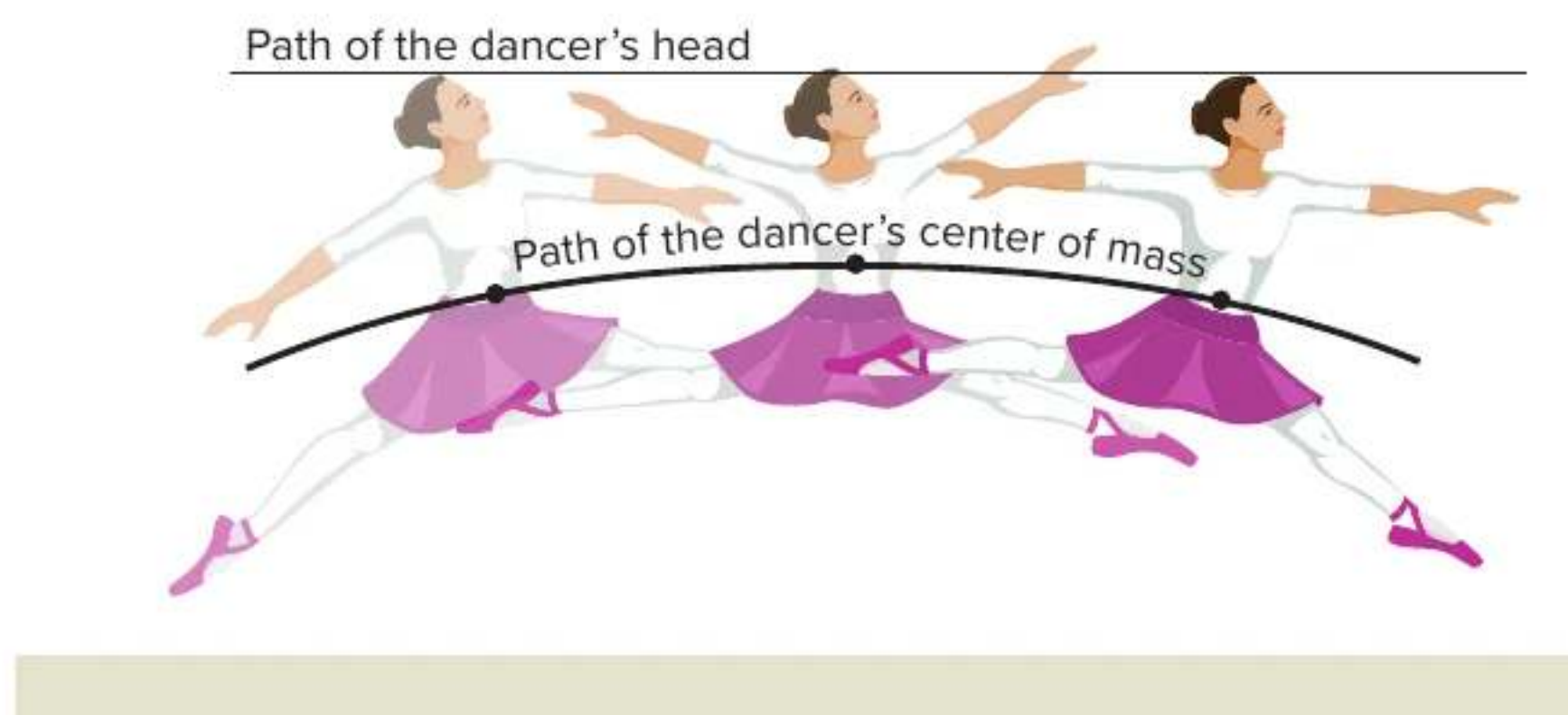
Use **mathematics** in the form of **Newton's second law** for both linear and rotational motion to determine if an object is in **static equilibrium**.



Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Figure 17 The upward motion of the ballet dancer's head is less than the upward motion of the center of mass. Thus, the head and torso move in a nearly horizontal path. This creates an illusion of floating.



The human body's center of mass For a person who is standing with her arms hanging straight down, the center of mass is a few centimeters below the navel, midway between the front and back of the person's body. The center of mass is farther below the navel for women than men, which often results in better balance for women than men. Because the human body is flexible, however, its center of mass is not fixed. If you raise your hands above your head, your center of mass rises 6 to 10 cm. A ballet dancer, for example, can appear to be floating on air by changing her center of mass in a leap. By raising her arms and legs while in the air, as shown in **Figure 17**, the dancer moves her center of mass up. The path of the center of mass is a parabola, but the dancer's head stays at almost the same height for a surprisingly long time.

Real-World Physics

THE FOSBURY FLOP In high jumping, a technique called the Fosbury flop allows a high jumper to clear the bar when it is placed at the highest position. This is possible because the athlete's center of mass passes below the bar as he or she somersaults over the bar, with his or her back toward it.

Center of Mass and Stability

What factors determine whether a vehicle is stable or prone to roll over in an accident? To understand the problem, think about tipping over a box. A tall, narrow box standing on end tips more easily than a low, broad box. Why? To tip a box, as shown in **Figure 18**, you must rotate it about a corner. You pull at the top with a force (F) applying a torque (τ_F). The weight of the box, acting on the center of mass (F_g) applies an opposing torque (τ_w). When the center of mass is directly above the point of support, τ_w is zero. The only torque is the one applied by you. As the box rotates farther, its center of mass is no longer above its base of support, and both torques act in the same direction. At this point, the box tips over rapidly.

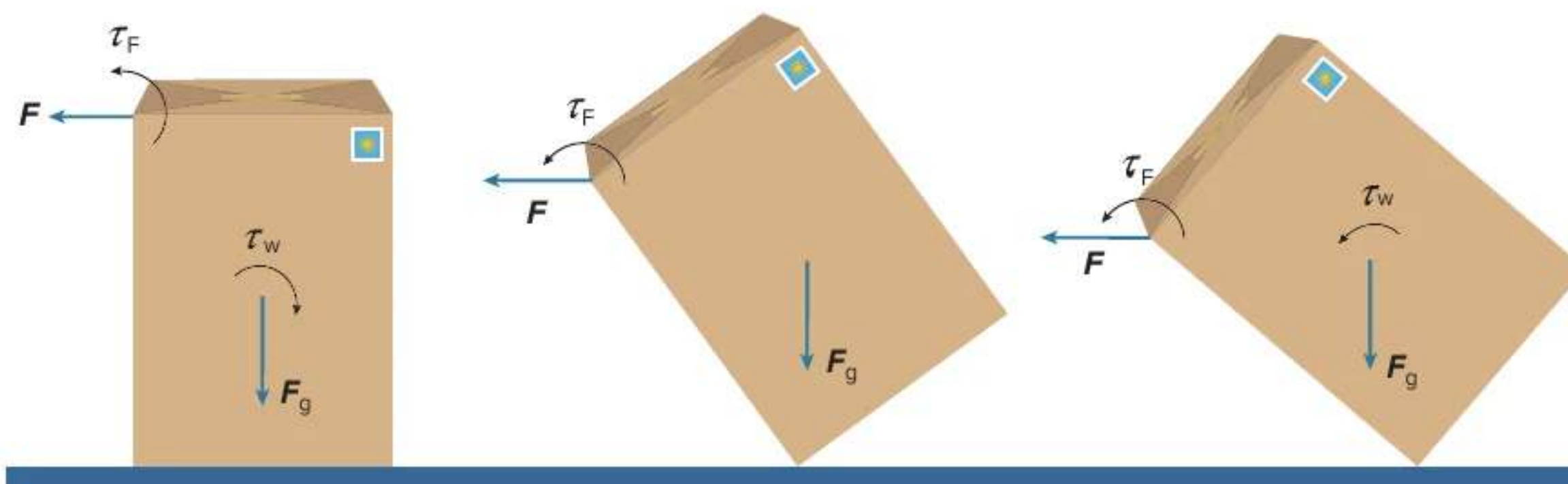


Figure 18 The curved arrows show the direction of the torque produced by the force exerted to tip over a box.

Stability An object is said to be stable if a large external force is required to tip it. The box in **Figure 18** is stable as long as the direction of the torque due to its weight (τ_w) tends to keep it upright. This occurs as long as the box's center of mass lies above its base. To tip the box over, you must rotate its center of mass around the axis of rotation until it is no longer above the base of the box. To rotate the box, you must lift its center of mass. The broader the base, the more stable the object is. Passengers on a city bus, for example, often stand with their feet spread apart to avoid toppling over as the bus starts and stops and weaves through traffic.

Why do vehicles roll over? **Figure 19** shows two vehicles about to roll over. Note that the one with the higher center of mass does not have to be tilted very far for its center of mass to be outside its base—its center of mass does not have to be raised as much as the other vehicle's center of mass. As demonstrated by the vehicles, the lower the location of an object's center of mass, the greater its stability.

You are stable when you stand flat on your feet. When you stand on tiptoe, however, your center of mass moves forward directly above the balls of your feet, and you have very little stability. In judo, aikido, and other martial arts, the fighter uses torque to rotate the opponent into an unstable position, where the opponent's center of mass does not lie above his or her feet. A small person can use torque, rather than force, to defend himself or herself against a stronger person.

In summary, if the center of mass is not located above the base of an object, it is unstable and will roll over without additional torque. If the center of mass is above the base of the object, it is stable. If the base of the object is very narrow and the center of mass is high, then the object might be stable, but the slightest force will cause it to tip over.



Get It?

Describe when an object is the most stable.

Conditions for Equilibrium

If your pen is at rest, what is needed to keep it at rest? You could either hold it up or place it on a desk or some other surface. An upward force must be exerted on the pen to balance the downward force of gravity. You must also hold the pen so that it will not rotate. An object is said to be in static equilibrium if both its velocity and angular velocity are zero or constant. Thus, for an object to be in static equilibrium, it must meet two conditions. First, it must be in translational equilibrium; that is, the net force exerted on the object must be zero. Second, it must be in rotational equilibrium; that is, the net torque exerted on the object must be zero.



Figure 19 Larger vehicles have a higher center of mass than smaller ones. The higher the center of mass, the smaller the tilt needed to cause the vehicle's center of mass to move outside its base and cause the vehicle to roll over.

EXAMPLE Problem 5

STATIC EQUILIBRIUM A 5.8-kg ladder, 1.80 m long, rests on two sawhorses. Sawhorse A is 0.60 m from one end of the ladder, and sawhorse B is 0.15 m from the other end of the ladder. What force does each sawhorse exert on the ladder?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Choose the axis of rotation at the point where F_A acts on the ladder. Thus, the torque due to F_A is zero.

Known

$$m = 5.8 \text{ kg}$$

$$l = 1.80 \text{ m}$$

$$l_A = 0.60 \text{ m}$$

$$l_B = 0.15 \text{ m}$$

Unknown

$$F_A = ?$$

$$F_B = ?$$

2 SOLVE FOR THE UNKNOWN

For a ladder that has a constant density, the center of mass is at the center rung.

The net force is the sum of all forces on the ladder.

The ladder is in translational equilibrium, so the net force exerted on it is zero.

$$F_{\text{net}} = F_A + F_B + (-F_g)$$

$$0.0 \text{ N} = F_A + F_B + (-F_g)$$

The ladder is in translational equilibrium, so the net force exerted on it is zero.

Solve for F_A .

$$F_A = F_g - F_B$$

Find the torques due to F_g and F_B .

$$\tau_g = -F_g r_g$$

τ_g is in the clockwise direction.

$$\tau_B = +F_B r_B$$

τ_B is in the counterclockwise direction.

The net torque is the sum of all torques on the object.

$$\tau_{\text{net}} = \tau_B + \tau_g$$

$$0.0 \text{ N}\cdot\text{m} = \tau_B + \tau_g$$

The ladder is in rotational equilibrium, so $\tau_{\text{net}} = 0.0 \text{ N}\cdot\text{m}$.

$$\tau_B = -\tau_g$$

$$F_B r_B = F_g r_g$$

Substitute $\tau_B = F_B r_B$ and $\tau_g = -F_g r_g$ into the equation.

Solve for F_B .

$$F_B = \frac{F_g r_g}{r_B}$$

$$= \frac{r_g mg}{r_B}$$

Substitute $F_g = mg$

Using the expression $F_A = F_g - F_B$, substitute in the expressions for F_B and F_g .

$$F_A = F_g - F_B$$

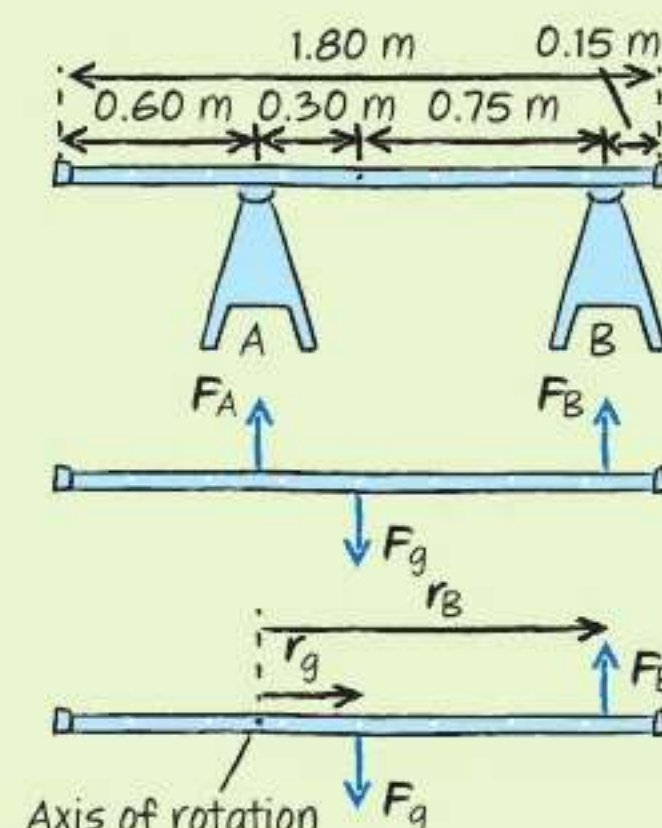
$$= F_g - \frac{r_g mg}{r_B}$$

Substitute $F_B = \frac{r_g mg}{r_B}$ into the equation.

$$= m_g - \frac{r_g mg}{r_B}$$

Substitute $F_g = mg$ into the equation.

$$= mg \left(1 - \frac{r_g}{r_B} \right)$$



EXAMPLE Problem 5 (continued)Solve for r_g .

$$\begin{aligned}
 r_g &= \left(\frac{1}{2}\right)l - l_A \\
 &= 0.90 \text{ m} - 0.60 \text{ m} \\
 &= 0.30 \text{ m}
 \end{aligned}$$

Solve for r_B .

$$\begin{aligned}
 r_B &= (0.90 \text{ m} - l_B) + (0.90 \text{ m} - l_A) \\
 &= (0.90 \text{ m} - 0.15 \text{ m}) + (0.90 \text{ m} - 0.60 \text{ m}) \\
 &= 0.75 \text{ m} + 0.30 \text{ m} \\
 &= 1.05 \text{ m}
 \end{aligned}$$

Calculate F_B .

$$\begin{aligned}
 F_B &= \frac{r_g mg}{r_B} \\
 &= \frac{(0.30 \text{ m})(5.8 \text{ kg})(9.8 \text{ N/kg})}{(1.05 \text{ m})} \\
 &= 16 \text{ N}
 \end{aligned}$$

Calculate F_A .

$$\begin{aligned}
 F_A &= mg \left(1 - \frac{r_g}{r_B}\right) \\
 &= (5.8 \text{ kg})(9.8 \text{ N/kg}) \left(1 - \frac{(0.30 \text{ m})}{(1.05 \text{ m})}\right) \\
 &= 41 \text{ N}
 \end{aligned}$$

For a ladder, which has a constant density, the center of mass is at the center rung.

Substitute $\frac{l}{2} = 0.90 \text{ m}$ and $l_A = 0.60 \text{ m}$ into the equation.

Substitute $l_B = 0.15 \text{ m}$ and $l_A = 0.60 \text{ m}$ into the equation.

Substitute $r_g = 0.30 \text{ m}$, $m = 5.8 \text{ kg}$, $g = 9.8 \text{ N/kg}$, and $r_B = 1.05 \text{ m}$ into the equation.

Substitute $r_g = 0.30 \text{ m}$, $m = 5.8 \text{ kg}$, $g = 9.8 \text{ N/kg}$, and $r_B = 1.05 \text{ m}$ into the equation.

3 EVALUATE THE ANSWER

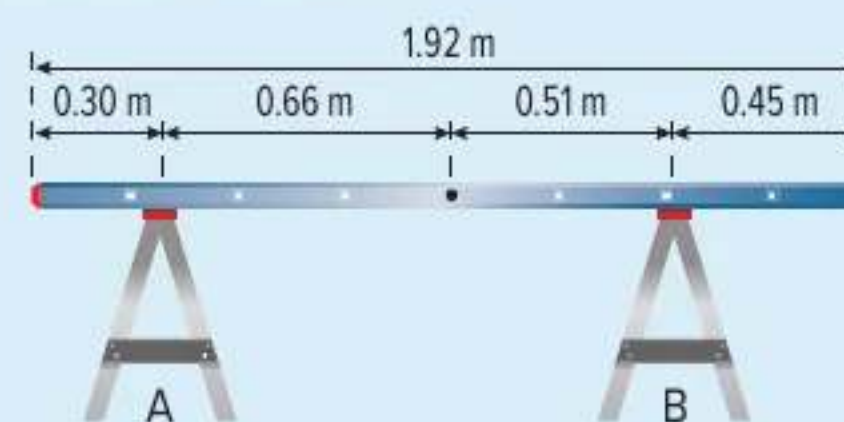
- Are the units correct? Forces are measured in newtons.
- Do the signs make sense? Both forces are upward.
- Is the magnitude realistic? The forces add up to the weight of the ladder, and the force exerted by the sawhorse closer to the center of mass is greater, which makes sense.

PRACTICE Problems**ADDITIONAL PRACTICE**

- 37.** What would be the forces exerted by the two sawhorses if the ladder in Example Problem 5 had a mass of 11.4 kg?
- 38.** A 7.3-kg ladder, 1.92 m long, rests on two sawhorses, as shown in **Figure 20**. Sawhorse A, on the left, is located 0.30 m from the end, and sawhorse B, on the right, is located 0.45 m from the other end. Choose the axis of rotation to be the center of mass of the ladder.
- What are the torques acting on the ladder?
 - Write the equation for rotational equilibrium.
 - Solve the equation for F_A in terms of F_g .
 - How would the forces exerted by the two sawhorses change if A were moved very close to, but not directly under, the center of mass?

- 39.** A 4.5-m-long wooden plank with a 24-kg mass has two supports. One is directly under the center of the board, and the other is at one end. What forces are exerted by the two supports?

- 40. CHALLENGE** A 85-kg diver walks to the end of a diving board. The board, which is 3.5 m long with a mass of 14 kg, is supported at the center of mass of the board and at one end. What are the forces on the two supports?

Figure 20

Rotating Frames of Reference

When you sit on a spinning amusement-park ride, it feels as if a strong force is pushing you to the outside. A pebble on the floor of the ride would accelerate outward without a horizontal force being exerted on it in the same direction. The pebble would not move in a straight line relative to the floor of the ride. In other words, Newton's laws of motion would not seem to apply. This is because in the ride your point of view, called your frame of reference, is rotating. Newton's laws are valid only in non-rotating or nonaccelerated frames of reference. Motion in a rotating reference frame is important to us because Earth rotates. The effects of the rotation of Earth are too small to be noticed in the classroom or lab, but they are significant influences on the motion of the atmosphere and, therefore, on climate and weather.

Centrifugal “Force” Suppose you fasten one end of a spring to the center of a rotating platform. An object lies on the platform and is attached to the other end of the spring. As the platform rotates, an observer on the platform sees the object stretch the spring. The observer might think that some force toward the outside of the platform is pulling on the object. This apparent force seems to pull on a moving object but does not exert a physical outward push on it. This apparent force, which seems to push an object outward, is observed only in rotating frames of reference and is called the **centrifugal “force.”** It is not a real force because there is no physical outward push on the object.

As the platform rotates, an observer on the ground would see things differently. This observer sees the object moving in a circle, and it accelerates toward the center because of the force of the spring. As you know, this centripetal acceleration is given by $a_c = \frac{v^2}{r}$. It also can be written in terms of angular velocity, as $a_c = \omega^2 r$. Centripetal acceleration is proportional to the distance from the axis of rotation and depends on the square of the angular velocity. Thus, if you double the rotational frequency, the angular acceleration increases by a factor of four.

The Coriolis “Force” A second effect of rotation is shown in **Figure 21**. Suppose a person standing at the center of a rotating disk throws a ball toward the edge of the disk. Consider the horizontal motion of the ball as seen by two observers, and ignore the vertical motion of the ball as it falls. An observer standing outside the disk sees the ball travel in a straight line at a constant speed toward the edge of the disk. However, the other observer, who is stationed on the disk and rotating with it, sees the ball follow a curved path at a constant speed. The apparent force that seems to deflect a moving object from its path and is observed only in rotating frames of reference is called the **Coriolis “force.”** Like the centrifugal “force,” the Coriolis “force” is not a real force. It seems to exist because we observe a deflection in horizontal motion when we are in a rotating frame of reference.

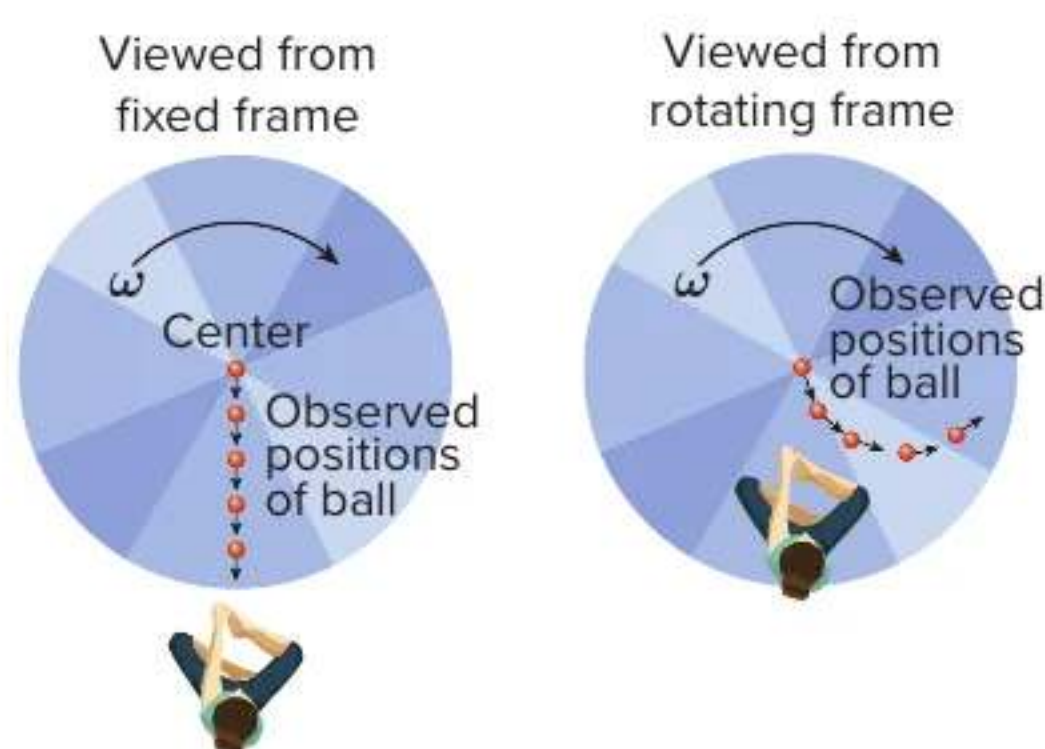


Figure 21 The Coriolis “force” is not a real force. It exists only in rotating reference frames.

Coriolis “force” due to Earth’s rotation Suppose a cannon is fired from a point on the equator toward a target due north of it. If the projectile were fired directly northward, it would also have an eastward velocity component because of the rotation of Earth. This eastward speed is greater at the equator than at any other latitude. Thus, as the projectile moves northward, it also moves eastward faster than points on Earth below it do. The result is that the projectile lands east of the target as shown in **Figure 22**. While an observer in space would see Earth’s rotation, an observer on Earth could claim that the projectile missed the target because of the Coriolis “force” on the rocket. Note that for objects moving toward the equator, the direction of the apparent force is westward. A projectile will land west of the target when fired due south.

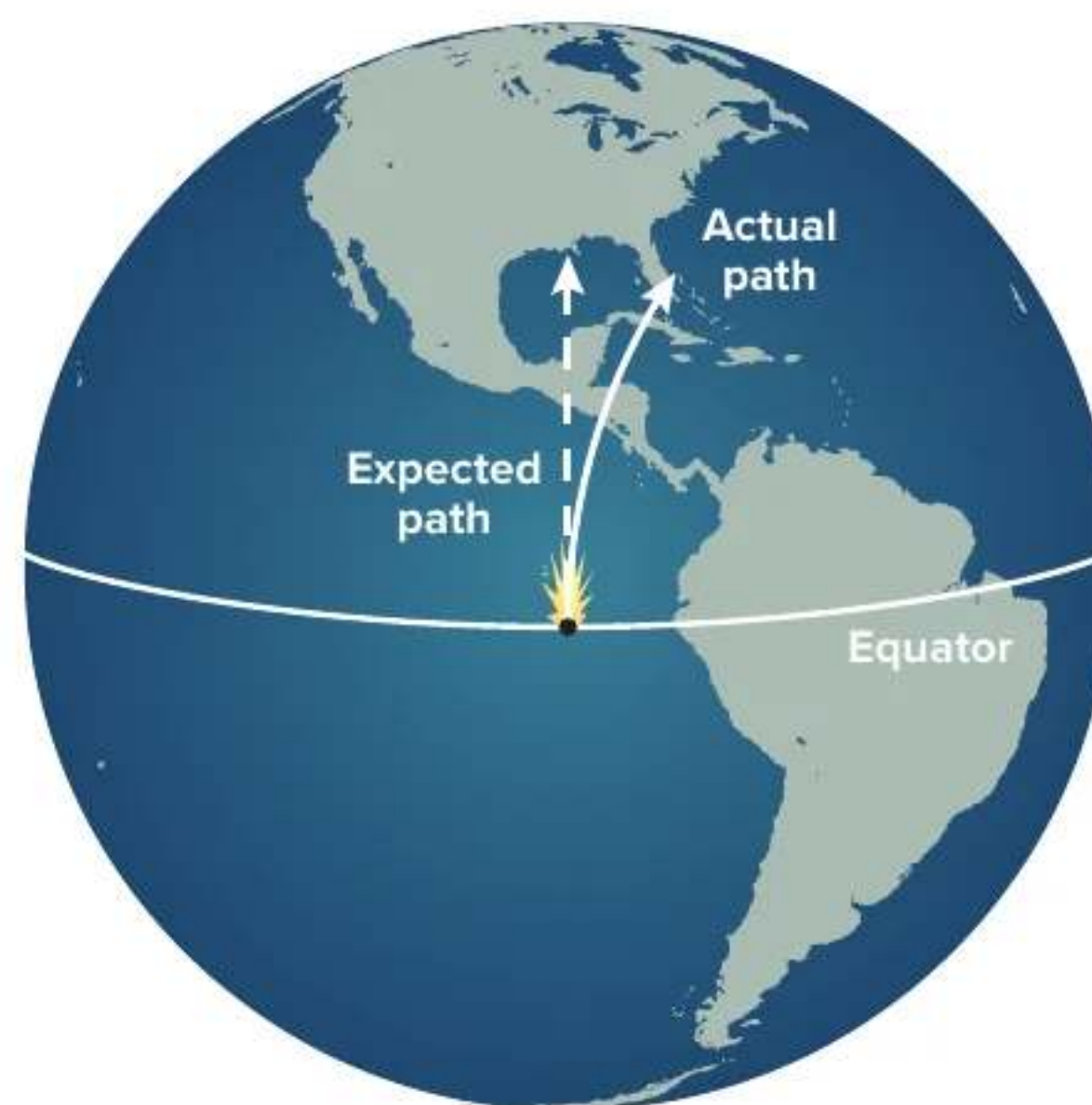


Figure 22 An observer on Earth sees the Coriolis “force” cause a projectile fired due north to deflect to the right of the intended target.

EARTH SCIENCE Connection The direction of winds around high- and low-pressure areas results from the Coriolis “force.” Winds flow from areas of high to low pressure.

Because of the Coriolis “force” in the northern hemisphere, winds from the south go to the east of low-pressure areas.

Winds from the north end up west of low-pressure areas.

Therefore, winds rotate counterclockwise around low-pressure areas in the northern hemisphere. In the southern hemisphere, however, winds rotate clockwise around low-pressure areas. This is why tropical cyclones, also called hurricanes, rotate clockwise in the southern hemisphere and counterclockwise in the northern hemisphere.



Get It?

Explain why hurricanes Katia, Irma, and Jose, mentioned at the beginning of this module, rotate counterclockwise.



Check Your Progress

41. **Equilibrium** Give an example of an object for each of the following conditions.
 - a. rotational but not translational equilibrium
 - b. translational but not rotational equilibrium
42. **Location of Center of Mass** Can the center of mass of an object be located in an area where the object has no mass? Explain.
43. **Stability of an Object** Why is a modified vehicle with its body raised high on risers less stable than a similar vehicle with its body at normal height?
44. **Center of Mass** Where is the center of mass on a roll of masking tape?
45. **Center of Mass** Describe how you would find the center of mass of this textbook.
46. **Rotating Frames of Reference** A penny is placed on a rotating old-fashioned record turntable. At the highest speed, the penny starts sliding outward. What are the forces acting on the penny?
47. **Critical Thinking** Predict the direction of the flow of surface ocean currents in the northern and southern hemispheres.

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SCIENCE & SOCIETY

Quantum Jump

The sport of figure-skating is physics in motion. Some of the fundamental concepts of physics—force, motion, velocity, torque—are the key to success in this beautiful display of athleticism. The quadruple jump, one of most difficult in figure skating, is a stellar example of basic mechanics.

The “Quad” Revolution

Skaters complete a quadruple, “quad,” jump when they leap into the air, rotate four times, and then land back on the ice. In 2017, 17-year-old American skater Nathan Chen became the first skater in history to land five quad jumps in a single program during competition.

The Physics of a Jump

When skaters set up for a jump, they first increase their velocity, and thus their linear momentum. Then, they push downward on the ice with the ball of one foot to launch themselves into the air. This torque generates angular momentum and puts them into a spin.

During a jump, skaters’ legs are close together. This position increases their rotational speed. To spin even faster, they pull their arms in tight against their body. This action reduces their moment of inertia by changing the distribution of their weight, so their bodies rotate faster due to conservation of angular momentum. Skaters



In this photo, Japanese figure-skater Yuzuru Hanyu completes a quadruple jump during the men’s figure-skating short program of the 2014 Winter Olympics.

attempting a quadruple jump contract their bodies as much as possible in order to complete the four required rotations.

Skaters finish a jump by bringing their arms away from their bodies and extending one of their legs as they land on the other. Skaters coming down from a quad jump land with an enormous impact force—somewhere between eight and ten times their body weight. They do all this while keeping their balance and continuing with the rest of their routine.

Not every quadruple jump is the same. Depending on the type of jump, skaters launch themselves with their toe pick or the inside of their foot, and they launch from and land on different feet. The skater even switches direction while completing a quad jump called the Lutz. But no matter the type they choose, the quadruple jump is a marvel of motion.



ASK QUESTIONS TO CLARIFY

Brainstorm several questions about the physics of figure skating. Use print or online sources to find answers to your questions. Make a poster that lists your questions and answers. Display the poster in your classroom.

MODULE 8

STUDY GUIDE

 **GO ONLINE** to study with your Science Notebook.

Lesson 1 DESCRIBING ROTATIONAL MOTION

- Angular displacement is the change in the angle (θ) as an object rotates. It is usually measured in degrees or radians.
- Average angular velocity is the object's angular displacement divided by the time taken to make the angular displacement. Average angular velocity is represented by the Greek letter omega (ω) and is determined by the following equation:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Average angular acceleration is the change in angular velocity divided by the time required to make the change.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

- radian
- angular displacement
- angular velocity
- angular acceleration

Lesson 2 ROTATIONAL DYNAMICS

- Torque describes the combination of a force and a lever arm that can cause an object to rotate. Torque is represented by the Greek letter tau (τ) and is determined by the following equation:

$$\tau = Fr \sin \theta$$

- The moment of inertia is a point object's resistance to changes in angular velocity. The moment of inertia is represented by the letter I and for a point mass, it is represented by the following equation:

$$I = mr^2$$

- Newton's second law for rotational motion states that angular acceleration is directly proportional to the net torque and inversely proportional to the moment of inertia.

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

- lever arm
- torque
- moment of inertia
- Newton's second law for rotational motion

Lesson 3 EQUILIBRIUM

- The center of mass of an object is the point on the object that moves in the same way that a point particle would move.
- An object is stable against rollover if its center of mass is above its base.
- An object in equilibrium has no net force exerted on it and there is no net torque acting on it.
- Centrifugal "force" and the Coriolis "force" are two apparent, but nonexistent, forces that seem to exist when an object is analyzed from a rotating frame of reference.

- center of mass
- centrifugal "force"
- Coriolis "force"



THREE-DIMENSIONAL THINKING Module Wrap-Up



REVISIT THE PHENOMENON

Why do all tropical cyclones in the northern hemisphere rotate the same direction?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.



STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will apply your evidence from this module and complete your project.

GO FURTHER

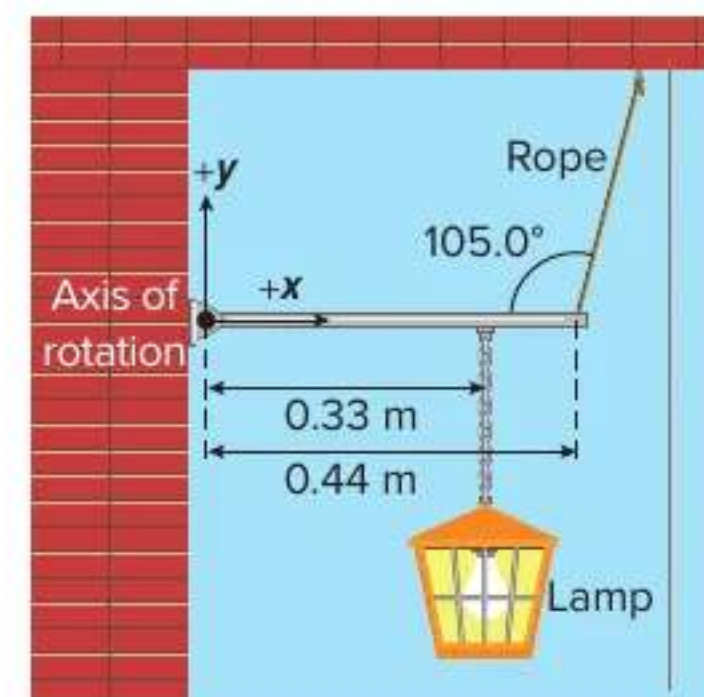
SEP Data Analysis Lab

Is the system stable?

Madhu is installing a pivoted pole lamp in front of her house, as shown in the diagram. The pole weighs 27 N, the lamp weighs 64 N, and the rope can support a tension of 60 N.

CER Analyze and Interpret Data

1. Determine the torque for each caused by the pole and by the lamp.
2. Determine the torque the rope must produce to keep the set-up in static equilibrium.
3. **Claim** Will Madhu's set up work or will the rope break?
4. **Evidence and Reasoning** Justify your claim.





MODULE 9

MOMENTUM AND ITS CONSERVATION

ENCOUNTER THE PHENOMENON

How do rockets accelerate once they reach space?



 **GO ONLINE** to play a video about early work on rockets.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about how rockets accelerate in space. Explain your reasoning.

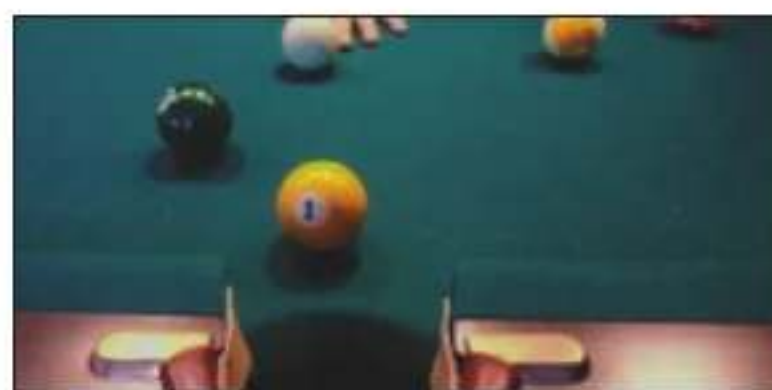
Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

 **GO ONLINE** to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain:
Using the Impulse-Momentum
Theorem to Save Lives



LESSON 2: Explore & Explain:
Two Particle Collisions



Additional Resources

LESSON 1

IMPULSE AND MOMENTUM

FOCUS QUESTION

How do air bags work to save lives?

Impulse-Momentum Theorem

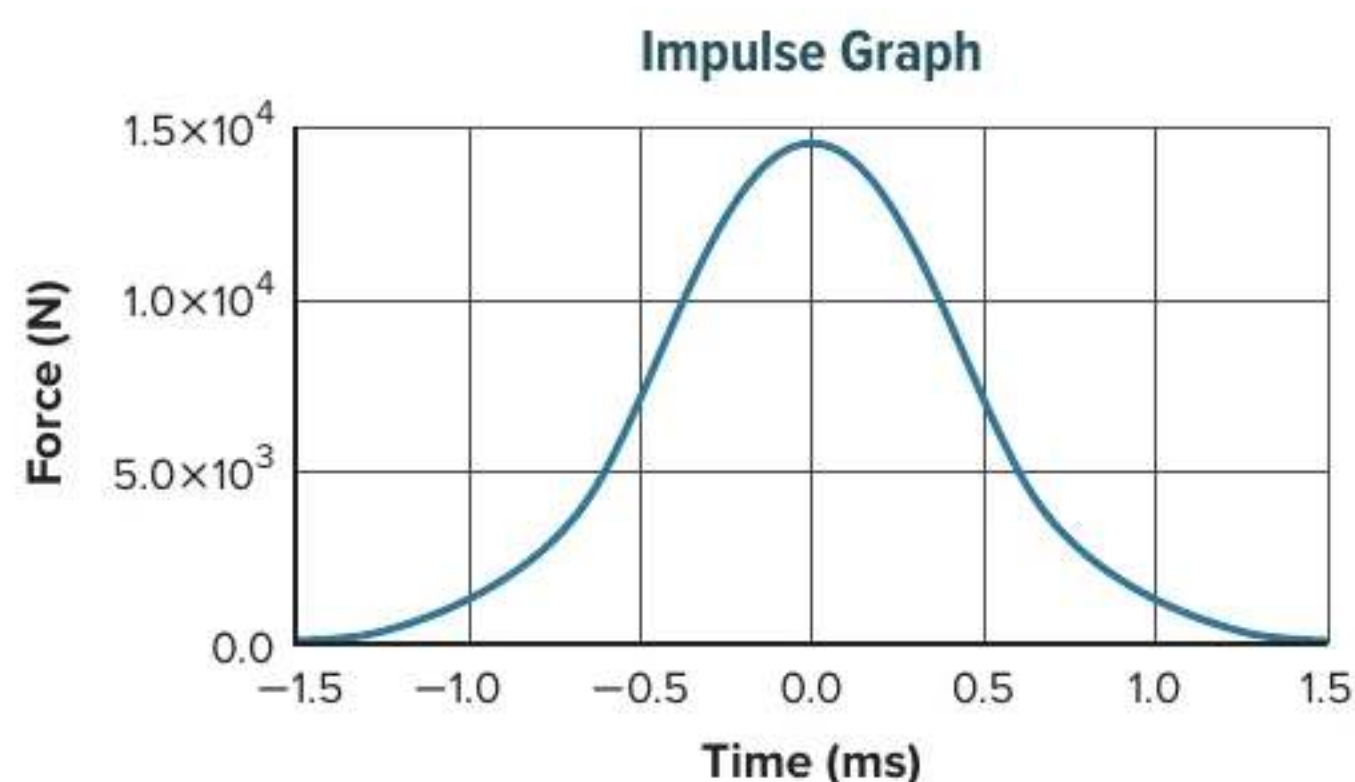
What happens when a baseball player hits a home run? According to Newton's second law of motion, the force from the bat changes the ball's velocity. This force changes over time, as shown in **Figure 1**. Just after contact, the ball is squeezed, and the force increases to a maximum more than 10,000 times the weight of the ball. The ball then recovers its shape and rebounds from the bat. The force on the ball rapidly returns to zero. This whole event takes place within about 3.0 ms. How can you calculate the change in velocity of the baseball?

Impulse Newton's second law of motion ($F = ma$) can be rewritten in terms of the change in velocity divided by the time for that change.

$$F = ma = m\left(\frac{\Delta v}{\Delta t}\right)$$

Multiplying both sides by the time interval (Δt) results in this equation.

$$F\Delta t = m\Delta v = m(v_f - v_i)$$



The left side of the equation ($F\Delta t$) is defined as the **impulse**. The impulse on an object is the product of the average force on an object and the time interval over which it acts. Impulse is measured in newton-seconds. If the force F varies with time, the magnitude of the impulse equals the area under the curve of a force-time graph, as in **Figure 1**.

Figure 1 The force on a baseball increases then decreases during a collision.



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

**Applying Practices: Egg Heads**

HS-PS2-3. Apply science and engineering ideas to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision.

**PhysicsLAB: Sticky Collisions**

Analyze data from a collision to determine how momentum is transferred within a system.

Momentum The right side of the equation ($m\Delta v$) involves the change in velocity, where $m\Delta v = mv_f - mv_i$. The product of the object's mass (m) and the object's velocity (v) is defined as the **momentum** of the object. Momentum is measured in $\text{kg}\cdot\text{m/s}$. It is always defined for a particular frame of reference. From a stationary frame of reference, a moving object such as a car has momentum. To the driver of the car, the car has zero momentum because the car has zero velocity in its own frame of reference. When finding momentum, assume the frame of reference is stationary unless otherwise indicated.

Momentum

The momentum of an object equals the mass of the object times the object's velocity.

$$p = mv$$

Now, you can rewrite the impulse as $F\Delta t = m\Delta v = mv_f - mv_i = p_f - p_i$. Thus, the impulse on an object is equal to the change which is called the **impulse-momentum theorem**.

Impulse-Momentum Theorem

An impulse acting on an object is equal to the object's final momentum minus the object's initial momentum.

$$F\Delta t = p_f - p_i$$

If the force on an object is constant, the impulse is the product of the force multiplied by the time interval over which it acts. If force is not constant, you can find the impulse using an average force multiplied by the time interval over which it acts, or by finding the area under a force-time graph. Momentum and impulse are vectors because velocity and force are vectors.

Using the impulse-momentum theorem The impulse from a bat changes a baseball's momentum. In **Figure 1**, the area under the curve is about $15 \text{ N}\cdot\text{s}$. The direction of the impulse is in the direction of the force. Therefore, the change in momentum of the ball also is $15 \text{ N}\cdot\text{s}$, or $15 \text{ kg}\cdot\text{m/s}$, in the direction of the force. Suppose a batter hits a fastball. Assume the positive direction is toward the pitcher. Before the collision, the ball, with a mass of 0.145 kg , has a velocity of -47 m/s . Therefore, the baseball's momentum is $p_i = (0.145 \text{ kg})(-47 \text{ m/s}) = -6.8 \text{ kg}\cdot\text{m/s}$. The momentum of the ball after the bat hits it is found by solving $p_f = p_i + F\Delta t$.

$$p_f = p_i + F\Delta t = -6.8 \text{ kg}\cdot\text{m/s} + 15 \text{ kg}\cdot\text{m/s} = +8.2 \text{ kg}\cdot\text{m/s}$$

Because $p_f = mv_f$, solving for v_f yields the ball's final velocity:

$$v_f = \frac{p_f}{m} = \frac{+8.2 \text{ kg}\cdot\text{m/s}}{+0.145 \text{ kg}} = +57 \text{ m/s}$$

A speed of 52 m/s is fast enough to clear most outfields if the ball is hit in the right direction.

SCIENCE USAGE v. COMMON USAGE

Impulse

Science usage: the product of average net force and the time interval over which the force acts

The impulse on the rocket from its exhaust lifted the astronauts into orbit.

Common usage: a sudden tendency toward some action
After seeing the commercial, Amad had an impulse to eat a candy bar.

Momentum

Science usage: the product of an object's mass and its velocity

A moving truck usually has much more momentum than a moving bicycle.

Common usage: strength gained through the development of events
The team hoped to continue the momentum of the regular season into the play-offs.

EXAMPLE Problem 1

AVERAGE FORCE A 2200-kg vehicle traveling at 94 km/h (26 m/s) can be stopped in 21 s by gently applying the brakes. It can be stopped in 3.8 s if the driver slams on the brakes, or in 0.22 s if it hits a concrete wall. What is the impulse exerted on the vehicle? What average force is exerted on the vehicle in each of these stops?

1 ANALYZE AND SKETCH THE PROBLEM

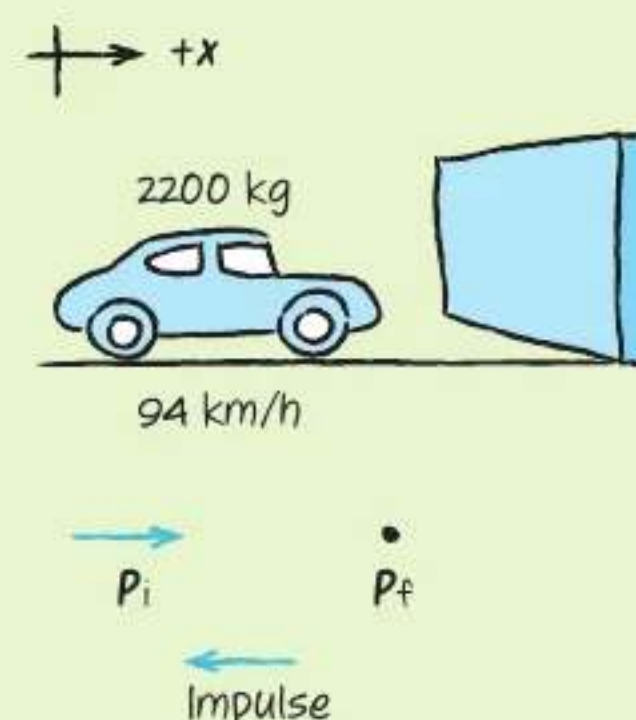
- Sketch the system.
- Include a coordinate axis. Select the car's direction as positive.
- Draw a vector diagram for momentum and impulse.

Known

$$\begin{aligned} m &= 2200 \text{ kg} & \Delta t_{\text{gentle braking}} &= 21 \text{ s} \\ v_i &= +26 \text{ m/s} & \Delta t_{\text{hard braking}} &= 3.8 \text{ s} \\ v_f &= 0.0 \text{ m/s} & \Delta t_{\text{hitting a wall}} &= 0.22 \text{ s} \end{aligned}$$

Unknown

$$\begin{aligned} F\Delta t &= ? \\ F_{\text{gentle braking}} &= ? \\ F_{\text{hard braking}} &= ? \\ F_{\text{hitting a wall}} &= ? \end{aligned}$$

**2 SOLVE FOR THE UNKNOWN**

Determine the initial momentum (p_i).

$$\begin{aligned} p_i &= mv_i \\ &= (2200 \text{ kg})(+26 \text{ m/s}) \\ &= +5.7 \times 10^4 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Substitute $m = 2200 \text{ kg}$, $v_i = +26 \text{ m/s}$.

Determine the final momentum (p_f).

$$\begin{aligned} p_f &= mv_f \\ &= (2200 \text{ kg})(0.0 \text{ m/s}) \\ &= 0.0 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Substitute $m = 2200 \text{ kg}$, $v_f = 0.0 \text{ m/s}$.

Apply the impulse-momentum theorem to determine the impulse and the force needed to stop the vehicle.

$$F\Delta t = p_f - p_i$$

$$\begin{aligned} F\Delta t &= (0.0 \text{ kg}\cdot\text{m/s}) - (5.7 \times 10^4 \text{ kg}\cdot\text{m/s}) \\ &= -5.7 \times 10^4 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Substitute $p_f = 0.0 \text{ kg}\cdot\text{m/s}$, $p_i = 5.7 \times 10^4 \text{ kg}\cdot\text{m/s}$.

$$F = \frac{-5.7 \times 10^4 \text{ kg}\cdot\text{m/s}}{\Delta t}$$

$$\begin{aligned} F_{\text{gentle braking}} &= \frac{-5.7 \times 10^4 \text{ kg}\cdot\text{m/s}}{21 \text{ s}} \\ &= -2.7 \times 10^3 \text{ N} \end{aligned}$$

Substitute $\Delta t_{\text{gentle braking}} = 21 \text{ s}$.

$$\begin{aligned} F_{\text{hard braking}} &= \frac{-5.7 \times 10^4 \text{ kg}\cdot\text{m/s}}{3.8 \text{ s}} \\ &= -1.5 \times 10^4 \text{ N} \end{aligned}$$

Substitute $\Delta t_{\text{hard braking}} = 3.8 \text{ s}$.

$$\begin{aligned} F_{\text{hitting a wall}} &= \frac{-5.7 \times 10^4 \text{ kg}\cdot\text{m}}{0.22 \text{ s}} \\ &= -2.6 \times 10^5 \text{ N} \end{aligned}$$

Substitute $\Delta t_{\text{hitting a wall}} = 0.22 \text{ s}$.

3 EVALUATE THE ANSWER

Are the units correct? Impulse is in $\text{kg}\cdot\text{m/s}$. Force is measured in newtons.

Does the direction make sense? Force is exerted in the direction opposite to the velocity of the car, which means the force is negative.

Is the magnitude realistic? It is reasonable that the force needed to stop a car is thousands or hundreds of thousands of newtons. The impulse is the same for all stops. Thus, as the stopping time decreases by about a factor of 10, the force increases by about a factor of 10.

PRACTICE Problems

ADDITIONAL PRACTICE

1. A compact car, with mass 725 kg, is moving at 115 km/h toward the east. Sketch the moving car.
 - a. Find the magnitude and direction of its momentum. Draw an arrow on your sketch showing the momentum.
 - b. A second car, with a mass of 2175 kg, has the same momentum. What is its velocity?
2. The driver of the compact car in the previous problem suddenly applies the brakes hard for 2.0 s. As a result, an average force of 5.0×10^3 N is exerted on the car to slow it down.
 - a. What is the change in momentum, or equivalently, what is the magnitude and direction of the impulse on the car?
 - b. Complete the “before” and “after” sketches, and determine the momentum and the velocity of the car now.
3. A 7.0-kg object, moving at 2.0 m/s, receives two impulses (one after the other) along the direction of its motion. Both of these impulses are illustrated in **Figure 2**. Find the resulting speed and direction of motion of the object after each impulse.
4. The driver accelerates a 240.0-kg snowmobile, which results in a force being exerted that speeds up the snowmobile from 6.00 m/s to 28.0 m/s over a time interval of 60.0 s.
 - a. Sketch the event, showing the initial and final situations.
 - b. What is the snowmobile’s change in momentum? What is the impulse on the snowmobile?
 - c. What is the magnitude of the average force that is exerted on the snowmobile?
5. **CHALLENGE** Suppose a 60.0-kg person was in the vehicle that hit the concrete wall in Example Problem 1. The velocity of the person equals that of the car both before and after the crash, and the velocity changes in 0.20 s. Sketch the problem.
 - a. What is the average force exerted on the person?
 - b. Some people think they can stop their bodies from lurching forward in a vehicle that is suddenly braking by putting their hands on the dashboard. Find the mass of an object that has a weight equal to the force you just calculated. Could you lift such a mass? Are you strong enough to stop your body with your arms?

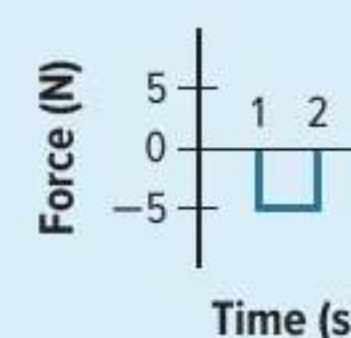
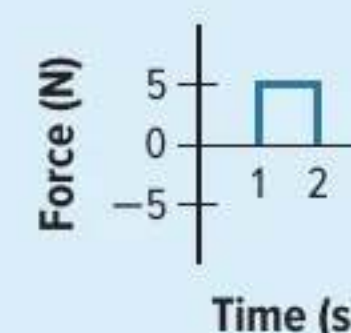


Figure 2

Using the impulse-momentum theorem to save lives

The impulse-momentum theorem shows that a large impulse causes a large change in momentum. This large impulse could result either from a large force acting over a short period of time or from a smaller force acting over a longer period of time. A passenger in a car accident might experience a large impulse, but it is the force on the passenger that causes injuries.

What happens to the driver when a crash suddenly stops a car? An impulse, either from the dashboard or from an air bag, brings the driver’s momentum to zero. According to the impulse-momentum equation, $F\Delta t = p_f - p_i$. The final momentum (p_f) is zero. The initial momentum (p_i) is the same with or without an air bag. Thus, the impulse ($F\Delta t$) also is the same. An air bag, such as the one shown in **Figure 3**, increases the time interval during which the force acts on the passenger. Therefore the required force is less. The air bag also spreads the force over a larger area of the person’s body, thereby reducing the likelihood of injuries.

Caspar Benson/Stop/Getty Images



Figure 3 An air bag reduces injuries by making the force on a passenger less and by spreading that force over a larger area.



Figure 4 The ball's linear momentum is the product of its mass and its velocity. The ball's angular momentum as it rolls down the lane is the product of its moment of inertia and its angular velocity.

Angular Momentum

The impulse-momentum theorem is useful if the momentum of an object is linear, but how can you describe momentum that is angular, as it is for the rotating bowling ball in **Figure 4**? Recall Newton's law for rotational motion, $\tau = \frac{I\Delta\omega}{\Delta t}$. You can rearrange this relationship to produce $\tau\Delta t = I\Delta\omega$. The left side of this equation ($\tau\Delta t$) is the rotating object's angular impulse. You can rewrite the right side as $I\Delta\omega = I\omega_f - I\omega_i$. The product of a rotating object's moment of inertia and angular velocity is called **angular momentum**, which is represented by the symbol L . The following relationship describes an object's angular momentum.

Angular Momentum

The angular momentum of an object is defined as the product of the object's moment of inertia and the object's angular velocity.

$$L = I\omega$$

Angular momentum is measured in $\text{kg}\cdot\text{m}^2/\text{s}$. Just as an object's linear momentum changes when an impulse acts on it, the object's angular momentum changes when an angular impulse acts on it. The object's angular impulse is equal to the change in the object's angular momentum, as stated by the **angular impulse-angular momentum theorem**. This theorem can be represented by the following relationship.

Angular Impulse-Angular Momentum Theorem

The angular impulse on an object is equal to the object's final angular momentum minus the object's initial angular momentum.

$$\tau\Delta t = L_f - L_i$$

If the net force on an object is zero, its linear momentum is constant. If the net torque acting on an object is zero, its angular momentum is also constant, but the two situations are slightly different. Because an object's mass cannot change, if its momentum is constant, then its velocity is also constant. In the case of angular momentum, however, the object's angular velocity can change if its shape changes. This is because the moment of inertia depends on the object's mass and how it is distributed about the axis of rotation. Thus, an object's angular velocity can change even if no torques act on it.

ASTRONOMY Connection Astronomers have discovered many examples of two stars that orbit each other. Such stars are called a binary star system. The torque on the binary system is zero because the gravitational force acts only directly between the stars. Therefore, the binary system's angular momentum is constant. But sometimes the stars are so close that the strong gravitational force rips material from one star and deposits it on the other. This movement of matter changes the moment of inertia of the system. As a result, the angular velocity of the system changes, even though the angular momentum is unchanged.

Real-World Physics



PADDLE SPORTS A table-tennis player controls the flight of the ball by putting spin on the ball. The player hits the ball at an angle so that the paddle exerts both an impulse to change the momentum and an angular impulse to make the ball spin. The interaction with the air will cause a forward-spinning ball to drop rapidly, making it more difficult for the opponent to return.

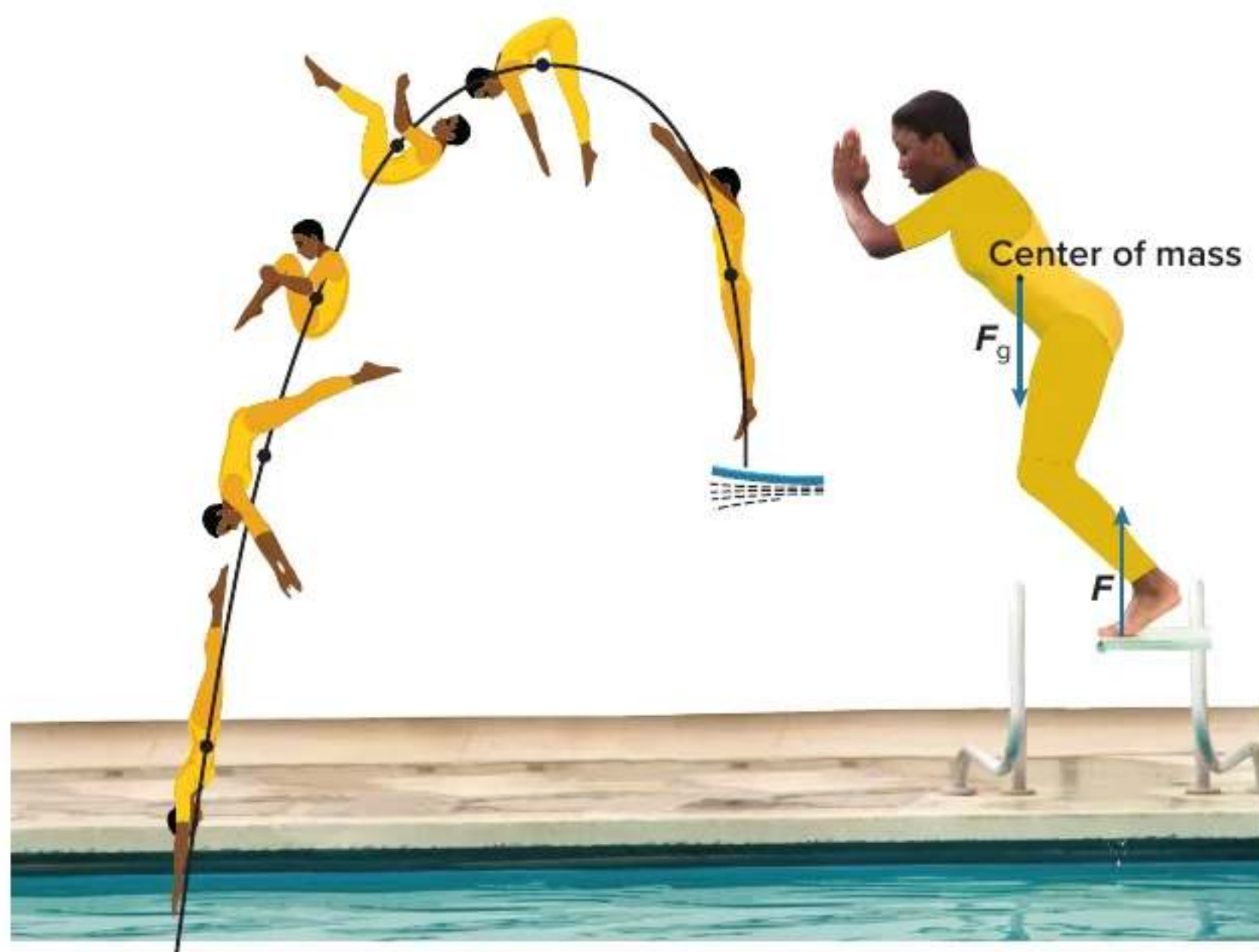


Figure 5 Before she dives, the girl's center of mass is in front of her feet. During her dive, the path of her center of mass forms a parabola. She changes her moment of inertia by moving her arms and legs.

Analyze How does extending her arms affect the diver's motion?

Diving into a pool Consider the diver in **Figure 5**. How does she start rotating her body? She uses the diving board to apply an external force to her body. Then, she moves her center of mass in front of her feet and uses the board to give a final upward push to her feet. This provides a torque that acts over time (Δt) and increases the angular momentum of the diver. Before the diver reaches the water, she can change her angular velocity by changing her moment of inertia. She may go into a tuck position, grabbing her knees with her hands. By moving her mass closer to the axis of rotation, the diver decreases her moment of inertia and increases her angular velocity. When she nears the water, she stretches her body straight, thereby increasing the moment of inertia and reducing the angular velocity. As a result, she goes straight into the water.

Ice-skating An ice-skater uses a similar method to spin. To begin rotating on one foot, the ice-skater causes the ice to exert a force on her body by pushing a portion of one skate into the ice. If the skater in **Figure 6** pushes on the ice in one direction, the ice will exert a force on her in the opposite direction. The force results in a torque if the force is exerted some distance away from the pivot point and in a direction that is not toward it. The greatest torque for a given force will result if the push is perpendicular to the lever arm. The ice-skater then can control her angular velocity by changing her moment of inertia. Both arms and one leg can be extended from the body to slow the rotation or can be pulled in close to the axis of rotation to speed it up. To stop spinning, another torque must be exerted by using the free skate to create a way for the ice to exert the needed force.



Figure 6 When the skater pushes off the ice, the ice exerts a torque on the skater.

CCC CROSSCUTTING CONCEPTS

Cause and Effect Summarize the cause-and-effect relationships that ice-skaters use to spin. Describe how you could gather evidence to test the various ways the skater in **Figure 6** can control her angular velocity.

EXAMPLE Problem 2

ANGULAR IMPULSE AND CHANGE IN ANGULAR MOMENTUM At the conclusion of his competition routine, an ice-skater spins in one spot at a rate of 2.5 rotations per second. If you assume that his moment of inertia is $0.50 \text{ kg}\cdot\text{m}^2$, what is his angular momentum? At the conclusion of the spin, the skater slowly and dramatically slows to a stop over 3.0 s by dragging the toe on one skate. What was the value and sign of the angular impulse on the skater? What average torque did the ice exert on the skater?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the system.
- Draw the axis of rotation and the direction of the angular impulse.

Known

$$\omega_i = 2.5 \text{ rotations/s}$$

$$\omega_f = 0 \text{ rad/s}$$

$$I = 0.50 \text{ kg}\cdot\text{m}^2$$

$$\Delta t = 3.0 \text{ s}$$

$$L_f = 0.0 \text{ kg}\cdot\text{m}^2/\text{s}$$

Unknown

$$L_i = ?$$

$$\text{angular impulse} = ?$$

$$\tau_{\text{avg}} = ?$$

**2 SOLVE FOR THE UNKNOWN**

Determine the initial angular momentum.

$$\begin{aligned} L_i &= I\omega_i \\ &= (0.50 \text{ kg}\cdot\text{m}^2)(2.5 \text{ rotations/s})(2\pi \text{ rad/rotation}) \\ &= 7.9 \text{ kg}\cdot\text{m}^2/\text{s} \end{aligned}$$

Substitute $I = 0.50 \text{ kg}\cdot\text{m}^2$ and $\omega_i = 2.5 \text{ rotations/s}$. Convert from rotations to radians. Radians is a dimensionless unit and is removed.

Determine the angular impulse that the ice exerted on the skater.

$$\begin{aligned} \text{angular impulse} &= \tau_{\text{avg}} \Delta t \\ &= L_f - L_i \\ &= 0.0 \text{ kg}\cdot\text{m}^2/\text{s} - 7.9 \text{ kg}\cdot\text{m}^2/\text{s} \\ &= -7.9 \text{ kg}\cdot\text{m}^2/\text{s} \end{aligned}$$

The angular impulse equals the change in angular momentum.

Substitute $L_f = 0.0 \text{ kg}\cdot\text{m}^2/\text{s}$, $L_i = 7.9 \text{ kg}\cdot\text{m}^2/\text{s}$.

Determine the average torque that the ice exerted on the skater.

From the angular impulse, you can rearrange the quantities to find the average torque.

$$\begin{aligned} \tau_{\text{avg}} &= \frac{L_f - L_i}{\Delta t} \\ &= \frac{0.0 \text{ kg}\cdot\text{m}^2/\text{s} - 7.9 \text{ kg}\cdot\text{m}^2/\text{s}}{3.0 \text{ s}} \\ &= -2.6 \text{ N}\cdot\text{m} \end{aligned}$$

Solve for the average torque.

Substitute $L_f = 0.0 \text{ kg}\cdot\text{m}^2/\text{s}$, $L_i = 7.9 \text{ kg}\cdot\text{m}^2/\text{s}$, and $\Delta t = 3.0 \text{ s}$.

The unit $\text{kg}\cdot\text{m}^2/\text{s}^2$ is equivalent to $\text{N}\cdot\text{m}$.

3 EVALUATE THE ANSWER

- **Are the units correct?** Yes, a $\text{kg}\cdot\text{m}^2/\text{s}^2$ is the equivalent of a $\text{N}\cdot\text{m}$.
- **Do the signs make sense?** The angular impulse and the torque are each negative, indicating that the angular momentum decreases.
- **Are the magnitudes realistic?** Yes, the value of the torque is small, but the product of the torque and time equals the value of the angular momentum.

PRACTICE Problems

ADDITIONAL PRACTICE

6. A 0.25-m-diameter circular saw blade rotates at 5.0×10^3 rpm, as shown in **Figure 7**. The saw is turned off and takes several seconds for to slow to a complete stop. The moment of inertia of the blade is $8.0 \times 10^{-3} \text{ kg}\cdot\text{m}^2$. Friction in the axle provides an average torque of $2.3 \times 10^{-1} \text{ N}\cdot\text{m}$ to slow the blade. How many seconds does it take for the blade to stop?
7. A baseball pitcher throws a 132 km/h (82 mph) curve ball that rotates about 6.0×10^2 rpm. What is the angular velocity of the ball? The pitcher's motion lasts about 0.15 s, and the moment of inertia of the ball is $8.0 \times 10^{-5} \text{ kg}\cdot\text{m}^2$. What average torque did the pitcher exert?
8. A bicycle is clamped upside down on a workbench for the bicycle repair woman to repair a front wheel axle. She gives the front wheel a spin with her hand, and the wheel rotates at 5.0 rev/s. What is the angular velocity of the wheel? If the moment of inertia of the wheel is $0.060 \text{ kg}\cdot\text{m}^2$, what angular impulse did the repair woman give the wheel?

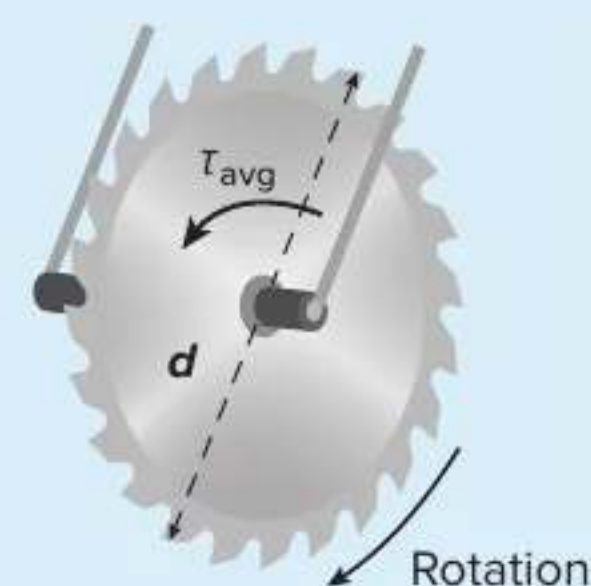


Figure 7

 Check Your Progress

9. **Momentum** Which has more momentum, a truck that is parked or a falling raindrop? Explain.
10. **Momentum** Is the momentum of a car traveling south different from that of the same car when it travels north at the same speed? Draw the momentum vectors to support your answer.
11. **Impulse and Momentum** When you jump from a height to the ground, you bend at the knees as your feet hit the floor. Explain why you do this in terms of impulse and momentum.
12. **Impulse and Momentum** A 0.174-kg softball is pitched horizontally at 26.0 m/s. The ball moves in the opposite direction at 38.0 m/s after it is hit by the bat.
 - a. Draw arrows showing the ball's momentum before and after the bat hits it.
 - b. What is the change in momentum of the ball?
 - c. What is the impulse delivered by the bat?
 - d. If the bat and ball are in contact for 0.80 ms, what is the average force the bat exerts?
13. **Momentum** The speed of a basketball is the same just before and after the ball hits the floor. Are the impulse on and the change in momentum of the basketball zero when the ball hits the floor? If not, in which direction is the change in momentum? Draw the ball's momentum vectors before and after impact.
14. **Frame of Reference** Why is momentum defined for a particular frame of reference? Explain.
15. **Angular Momentum** The ice-skater shown in **Figure 8** spins with his arms outstretched. When he pulls his arms in and raises them above his head, he spins much faster than before. Did a torque act on the ice-skater? What caused his angular velocity to increase?

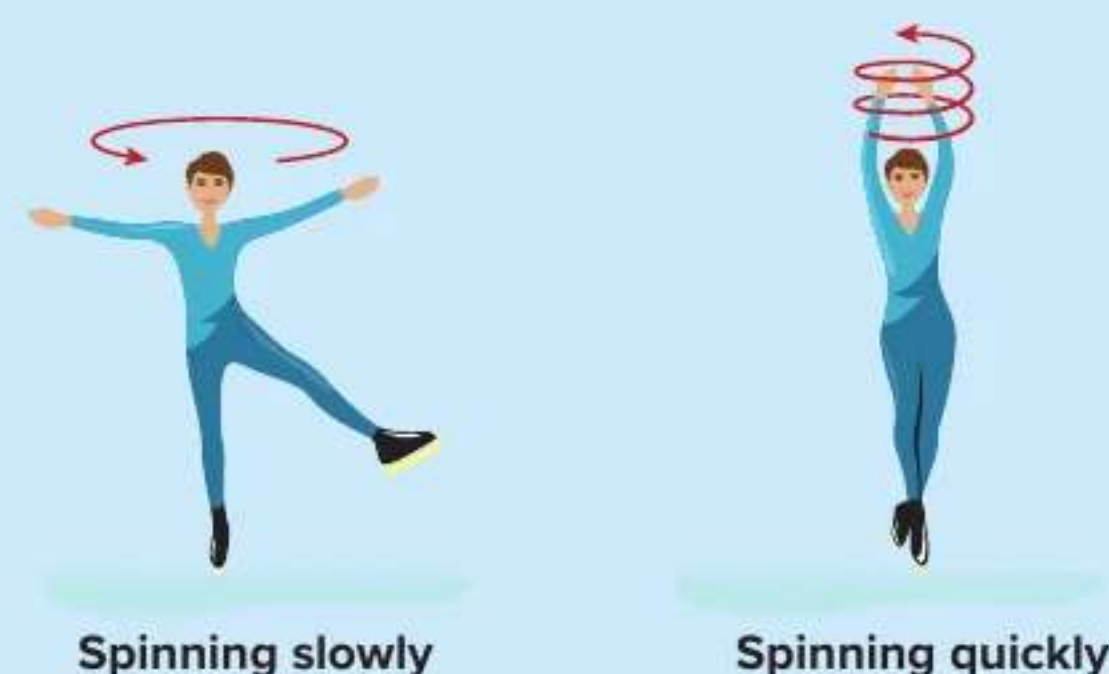


Figure 8

16. **Critical Thinking** An archer shoots arrows at a target. Some of the arrows stick in the target, while others bounce off. Assuming that the masses of the arrows and the velocities of the arrows are the same, which arrows produce a bigger impulse on the target? *Hint: Draw a diagram to show the momentum of the arrows before and after hitting the target for the two instances.*

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LESSON 2

CONSERVATION OF MOMENTUM

FOCUS QUESTION

What is recoil and how is it useful?

Two-Particle Collisions

What will happen when the two balls shown in **Figure 9** collide? During the collision, each ball briefly exerts a force on the other. Despite the differences in sizes and velocities of the balls, the forces they exert on each other are equal and opposite, according to Newton's third law of motion. These forces are represented by the equation $F_{\text{red on blue}} = -F_{\text{blue on red}}$.

How do the impulses imparted by both balls compare? Because the time intervals over which the forces are exerted are the same, the impulses the balls exert on each other must also be equal in magnitude but opposite in direction: $(F\Delta t)_{\text{red on blue}} = -(F\Delta t)_{\text{blue on red}}$. Notice that no mention has been made of the masses of the balls. Even though the balls have different sizes and approach each other with different velocities, and even though they may have different masses, the forces and the impulses they exert on each other have equal strength but opposite directions.

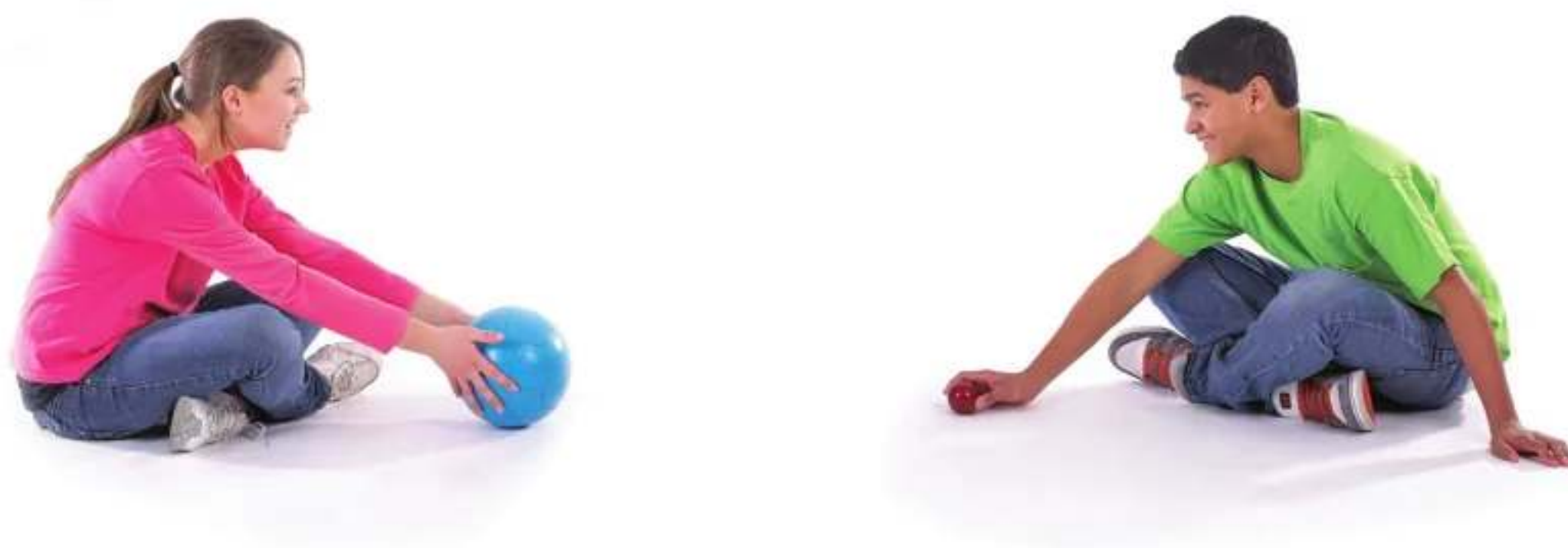


Figure 9 The balls have different sizes, masses, and velocities, but if they interact the forces and impulses they exert on each other are equal and opposite.

Richard Hutchings/Digital Light Source



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Applying Practices: Conservation of Momentum
HS-PS2-2. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.



Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Momentum How did the momenta of the two balls the girl and the boy rolled toward each other in **Figure 9** change as a result of the collision? Compare the changes shown in **Figure 10**.

$$\text{For ball C: } P_{Cf} - P_{Ci} = F_{D \text{ on } C} \Delta t$$

$$\text{For ball D: } P_{Df} - P_{Di} = F_{C \text{ on } D} \Delta t$$

According to the impulse-momentum theorem, the change in momentum is equal to the impulse. Because the impulses are equal in magnitude but opposite in direction, we know that:

$$P_{Cf} - P_{Ci} = -(P_{Df} - P_{Di})$$

$$P_{Cf} + P_{Df} = P_{Ci} + P_{Di}$$

This equation states that the sum of the momenta of the balls in **Figure 10** is the same before and after the collision. That is, the momentum gained by ball D is equal to the momentum lost by ball C. If the system is defined as the two balls, the sum of the gain and loss in momentum is zero, and therefore, momentum is conserved for the system.

Momentum in a Closed, Isolated System

Under what conditions is the momentum of the system of two balls conserved? The first and most obvious condition is that no balls are lost and no balls are gained. Such a system, which does not gain or lose mass, is said to be a **closed system**.

The second condition required to conserve the momentum of a system is that the only forces that are involved are internal forces; that is, there are no unbalanced forces acting on the system from objects outside of it. When the net external force exerted on a closed system is zero, the resulting system is described as an **isolated system**. No system on Earth is absolutely isolated, however, because there will always be some interactions between a system and its surroundings. Often, these interactions are small enough to be ignored when solving physics problems.

Systems can contain any number of objects, and the objects that make up a system can stick together or come apart during a collision. But even under these conditions, momentum is conserved. The **law of conservation of momentum** states that the momentum of any closed, isolated system does not change. This law enables you to make a connection between conditions before and after an interaction without knowing any details of the interaction. If a system is not closed and isolated, and interacts with objects outside itself, the total momentum of the system can change. However, any such change to the system is balanced by changes in the momentum of objects outside the system.

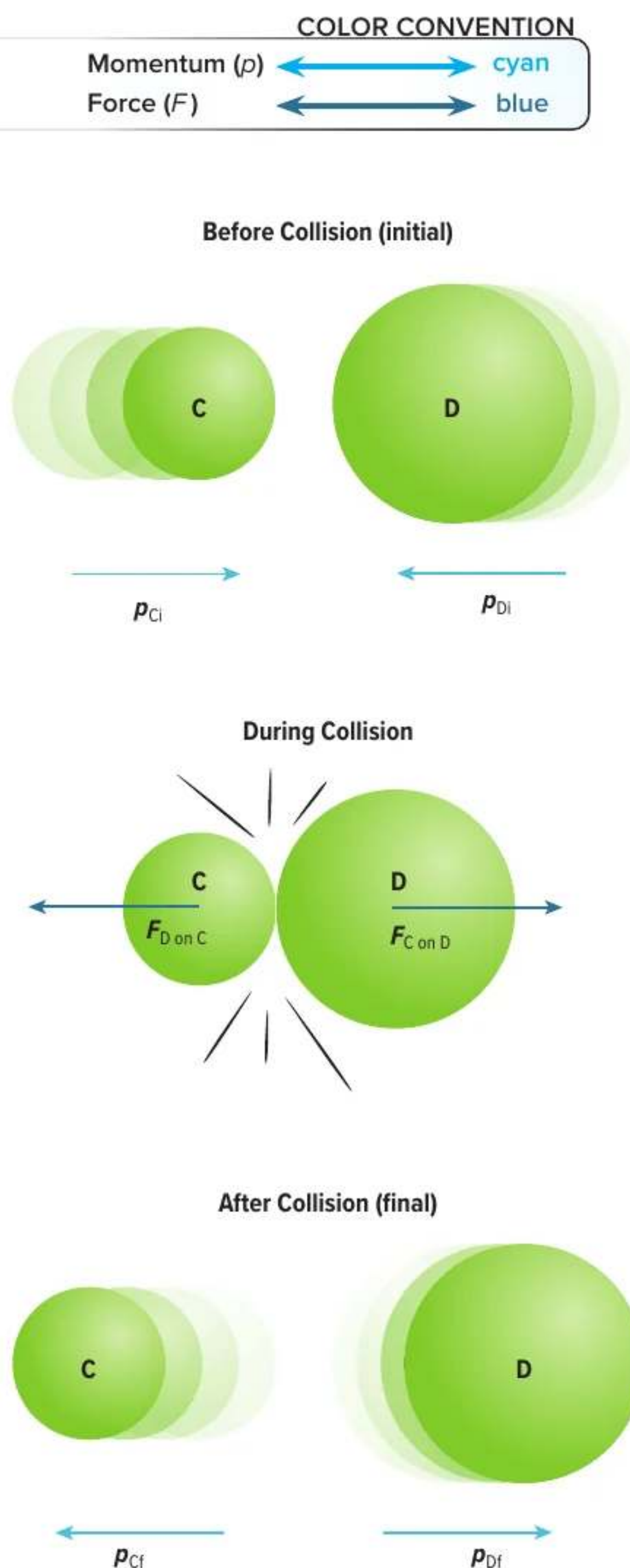


Figure 10 When two balls of different mass and velocity collide, the momentum of each ball is changed. If the system is isolated, however, the sum of their momenta before the collision equals the sum of their momenta after the collision.

Analyze How can you tell from the diagram that the impulses exerted by the balls have the same magnitude?

EXAMPLE Problem 3

SPEED A 1875-kg car going 23 m/s rear-ends a 1025-kg compact car going 17 m/s on ice in the same direction. The two cars stick together. How fast do the two attached cars move immediately after the collision?

1 ANALYZE AND SKETCH THE PROBLEM

- Define the system.
- Establish a coordinate system.
- Sketch the situation showing the before and after states.
- Draw a vector diagram for the momentum

Known

$$m_c = 1875 \text{ kg} \quad m_d = 1025 \text{ kg}$$

$$v_{ci} = +23 \text{ m/s} \quad v_{di} = +17 \text{ m/s}$$

Unknown

$$v_f = ?$$

2 SOLVE FOR THE UNKNOWN

The system is the two cars. Momentum is conserved because the ice makes the total external force on the cars nearly zero.

$$p_i = p_f$$

$$p_{ci} + p_{di} = p_{cf} + p_{df}$$

$$m_c v_{ci} + m_d v_{di} = m_c v_{cf} + m_d v_{df}$$

Because the two cars stick together, their velocities after the collision, denoted as v_f , are equal.

$$v_{cf} = v_{df} = v_f$$

$$m_c v_{ci} + m_d v_{di} = (m_c + m_d) v_f$$

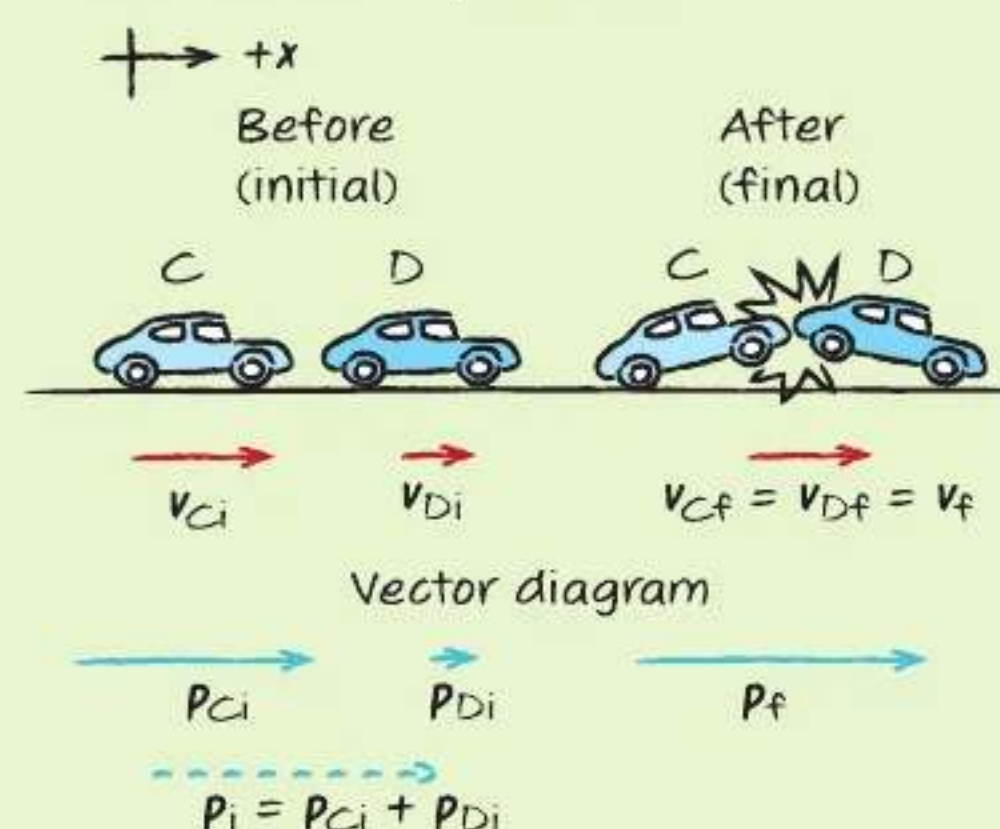
Solve for v_f .

$$v_f = \frac{m_c v_{ci} + m_d v_{di}}{m_c + m_d}$$

$$= \frac{(1875 \text{ kg})(+23 \text{ m/s}) + (1025 \text{ kg})(+17 \text{ m/s})}{1875 \text{ kg} + 1025 \text{ kg}}$$

$$= +21 \text{ m/s}$$

Substitute $m_c = 1875 \text{ kg}$, $v_{ci} = +23 \text{ m/s}$, $m_d = 1025 \text{ kg}$, $v_{di} = +17 \text{ m/s}$.

**3 EVALUATE THE ANSWER**

- **Are the units correct?** Velocity is measured in meters per second.
- **Does the direction make sense?** v_{ci} and v_{di} are in the positive direction; therefore, v_f should be positive.
- **Is the magnitude realistic?** The magnitude of v_f is between the initial speeds of the two cars, but closer to the speed of the more massive one, so it is reasonable.

PRACTICE Problems**ADDITIONAL PRACTICE**

- 17.** Two freight cars, each with a mass of $3.0 \times 10^5 \text{ kg}$, collide and stick together. One was initially moving at 2.2 m/s, and the other was at rest. What is their final speed? Define the system as the two cars.
- 18.** A 0.105-kg hockey puck moving at 24 m/s is caught and held by a 75-kg goalie at rest. With what speed does the goalie slide on the ice after catching the puck? Define the puck and the goalie as a system.

- 19.** A 35.0-g bullet, traveling at 475 m/s, strikes a 2.5-kg steel ball that is at rest. After the collision, the bullet bounces backward at 5.0 m/s. How fast is the ball moving when the bullet bounces backward?
- 20. CHALLENGE** A 0.50-kg ball traveling at 6.0 m/s collides head-on with a 1.00-kg ball moving in the opposite direction at 12.0 m/s. After colliding, the 0.50-kg ball bounces backward at 14 m/s. Find the other ball's speed and direction after the collision.



Figure 11 The boy exerts a force, causing the girl to move to the right. The boy's position relative to the ball shows that the equal but opposite force from the girl caused him to recoil to the left.

Infer How can you tell from the motion that the boy's mass is greater than the girl's mass?

Recoil

It is important to define a system carefully. To see why, consider the in-line skaters in **Figure 11**. Can you find their final velocities? Assume they are skating on a smooth surface with no external forces. They both start at rest, one behind the other. Skater C, the boy, gives skater D, the girl, a push. Now, both are moving in opposite directions. Because the push was an internal force, you can use the law of conservation of momentum to find the skaters' relative velocities. The total momentum of the system was zero before the push. Therefore, it must be zero after the push.

Before	After
$p_{Ci} + p_{Di}$	$= p_{Cf} + p_{Df}$
0	$= p_{Cf} + p_{Df}$
p_{Cf}	$= -p_{Df}$
$m_C v_{Cf}$	$= -m_D v_{Df}$

The coordinate system can be chosen so that the positive direction is to the right. The momenta of the skaters after the push are equal in magnitude but opposite in direction. The backward motion of skater C is an example of recoil. Are the skaters' velocities equal and opposite? The last equation can be written to solve for the velocity of skater C.

$$v_{Cf} = \left(\frac{-m_D}{m_C} \right) v_{Df}$$

The velocities depend on the skaters' relative masses. If skater C has a mass of 68.0 kg and skater D's mass is 45.4 kg, then the ratio of their velocities will be 68.0 : 45.4, or 1.50. The less massive skater moves at the greater velocity. Without more information about how hard skater C pushed skater D, however, you cannot find the velocity of each skater.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Examine **Figure 11**. How does the red ball in the photographs provide evidence for analyzing the system, and why is it necessary to help define the system? Suggest another way this could have been achieved, and make a model of the system to illustrate.

STEM CAREER Connection

Computer Graphics Animator

To create realistic-looking computer graphics for movies and video games, animators must understand how objects move in the real world. Applying the laws of motion to animated interactions results in convincing imagery, be it of spaceship crashes or medieval battles.

Propulsion in Space

How does a rocket in space change velocity? The rocket carries both fuel and oxidizer. When the fuel and oxidizer combine in the rocket motor, the resulting hot gases leave the exhaust nozzle at high speed. The rocket and the chemicals form a closed system. The forces that expel the gases are internal forces, so the system is also an isolated system. Thus, objects in space can accelerate by using the law of conservation of momentum and Newton's third law of motion. A newer type of space propulsion uses the recoil that results from the force of ions. An ion thruster accelerates ions in an electric or magnetic field. As the ions are expelled in one direction, the conservation of momentum results in the movement of the spaceship in the opposite direction.



Get It?

Explain how the rocket shown at the beginning of the module can accelerate in space.

EXAMPLE Problem 4

SPEED An astronaut at rest in space fires a thruster pistol that expels a quick burst of 35 g of hot gas at 875 m/s. The combined mass of the astronaut and pistol is 84 kg. How fast and in what direction is the astronaut moving after firing the pistol?

1 ANALYZE AND SKETCH THE PROBLEM

- Define the system and establish a coordinate axis.
- Sketch before and after conditions. Draw a vector diagram.

Known

$$m_c = 84 \text{ kg} \quad v_{ci} = v_{di} = 0.0 \text{ m/s}$$

$$m_D = 0.035 \text{ kg} \quad v_{Df} = -875 \text{ m/s}$$

Unknown

$$v_{cf} = ?$$

2 SOLVE FOR THE UNKNOWN

The system is the astronaut, the gun, and the gas.

$$p_i = p_{ci} + p_{Di} = 0.0 \text{ kg}\cdot\text{m/s}$$

Use the law of conservation of momentum to find p_{cf} .

$$p_i = p_f$$

$$0.0 \text{ kg}\cdot\text{m/s} = p_{cf} + p_{Df}$$

$$p_{cf} = -p_{Df}$$

Solve for the final velocity of the astronaut (v_{cf}).

$$m_c v_{cf} = -m_D v_{Df}$$

$$v_{cf} = \frac{-m_D v_{Df}}{m_c}$$

$$= \left(\frac{-(0.035 \text{ kg})(-875 \text{ m/s})}{84 \text{ kg}} \right)$$

$$= +0.36 \text{ m/s}$$

Before the pistol is fired, the system is at rest; initial momentum is zero.

The momentum of the astronaut is equal in magnitude, but opposite in direction, to the momentum of the gas leaving the pistol.

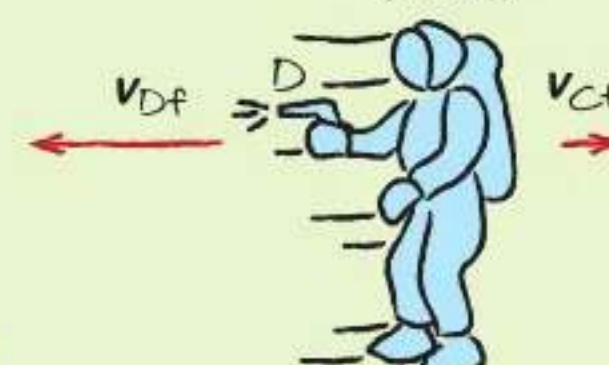
Substitute $m_D = 0.035 \text{ kg}$, $v_{Df} = -875 \text{ m/s}$, $m_c = 84 \text{ kg}$.

Before
(initial)



$$v_i = 0.0 \text{ m/s}$$

After
(final)



Vector diagram

p_i p_{Df} p_{cf}

$p_f = p_{cf} + p_{Df}$

3 EVALUATE THE ANSWER

- Are the units correct?** The velocity is measured in meters per second.
- Does the direction make sense?** The astronaut's direction is opposite that of the gas.
- Is the magnitude realistic?** The astronaut's mass is much greater than that of the gas, so the velocity of the astronaut is much less than that of the expelled gas.

PRACTICE Problems

ADDITIONAL PRACTICE

- 21.** A 4.00-kg model rocket is launched, expelling burned fuel with a mass of 50.0 g at a speed of 625 m/s. What is the velocity of the rocket after the fuel has burned? *Hint: Ignore the external forces of gravity and air resistance.*
- 22.** A thread connects a 1.5-kg cart and a 4.5-kg cart. After the thread is burned, a compressed spring pushes the carts apart, giving the 1.5-kg cart a velocity of 27 cm/s to the left. What is the velocity of the 4.5-kg cart?

- 23. CHALLENGE** Carmen and Judi row their canoe alongside a dock. They stop the canoe, but they do not secure it. The canoe can still move freely. Carmen, who has a mass of 80.0 kg, then steps out of the canoe onto the dock. As she leaves the canoe, Carmen moves forward at a speed of 4.0 m/s, causing the canoe, with Judi still in it, to move also. At what speed and in what direction do the canoe and Judi move if their combined mass is 115 kg?

Two-Dimensional Collisions

Until now you have considered momentum in only one dimension. The law of conservation of momentum holds for all closed systems with no external forces. It is valid regardless of the directions of the particles before or after they interact. But what happens in two or three dimensions? **Figure 12** shows a collision of two billiard balls. Consider the billiard balls to be the system. The original momentum of the moving ball is $\mathbf{p}_{Ci}$ and the momentum of the stationary ball is zero. Therefore, the momentum of the system before the collision is $\mathbf{p}_{Ci}$.

After the collision, both balls are moving and have momenta. Ignoring friction with the tabletop, the system is closed and isolated. Thus, the law of conservation of momentum can be used.

The initial momentum equals the sum of the final momenta, so $\mathbf{p}_{Ci} = \mathbf{p}_{Cf} + \mathbf{p}_{Df}$. The sum of the components of the vectors before and after the collision must also be equal. If the x -axis is defined in the direction of the initial momentum, then the y -component of the initial momentum is zero. The sum of the final y -components also must be zero.

$$p_{Ci,y} + p_{Df,y} = 0$$

The y -components are equal in magnitude but are in the opposite direction and, thus, have opposite signs. The sums of the horizontal components before and after the collision also are equal.

$$p_{Ci} = p_{Cf,x} + p_{Df,x}$$

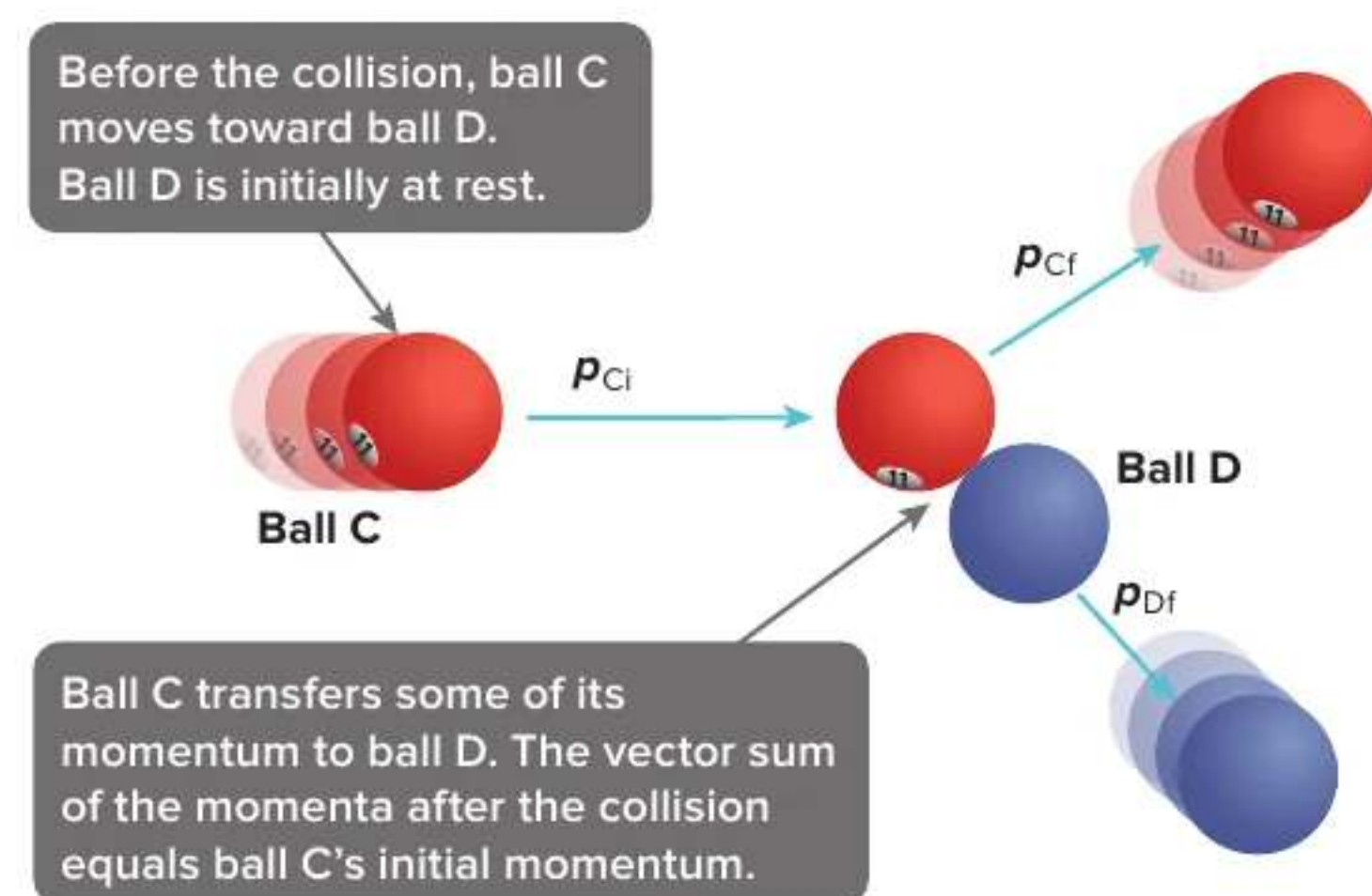


Figure 12 The momentum is conserved in a closed, isolated system, regardless of the velocities of the objects before a collision.

Analyze The momentum of the system is not conserved if the balls are rolling on a high-friction carpet. Explain why, describing how the change in momentum of the system consisting only of the three balls is balanced by changes in momentum outside the system.

EXAMPLE Problem 5

SPEED A 1325-kg car, C, moving north at 27.0 m/s, collides with a 2165-kg car, D, moving east at 11.0 m/s. The collision causes the two cars to stick together. In what direction and with what speed do they move after the collision?

1 ANALYZE AND SKETCH THE PROBLEM

- Define the system.
- Sketch the before and after states.
- Establish the coordinate axis with the y-axis north and the x-axis east.
- Draw a momentum-vector diagram.

Known

$$m_C = 1325 \text{ kg}$$

$$m_D = 2165 \text{ kg}$$

$$v_{C,y} = 27.0 \text{ m/s}$$

$$v_{D,x} = 11.0 \text{ m/s}$$

Unknown

$$v_f = ?$$

$$\theta = ?$$

2 SOLVE FOR THE UNKNOWN

Define the system as the two cars. Determine the magnitudes of the initial momenta of the cars and the momentum of the system.

$$\begin{aligned} p_{Ci} &= m_C v_{Ci,y} \\ &= (1325 \text{ kg})(27.0 \text{ m/s}) \\ &= 3.58 \times 10^4 \text{ kg}\cdot\text{m/s (north)} \end{aligned}$$

$$\begin{aligned} p_{Di} &= m_D v_{Di,x} \\ &= (2165 \text{ kg})(11.0 \text{ m/s}) \\ &= 2.38 \times 10^4 \text{ kg}\cdot\text{m/s (east)} \end{aligned}$$

Use the law of conservation of momentum to find p_f .

$$p_{f,x} = p_{i,x} = 2.38 \times 10^4 \text{ kg}\cdot\text{m/s}$$

$$p_{f,y} = p_{i,y} = 3.58 \times 10^4 \text{ kg}\cdot\text{m/s}$$

Use the diagram to set up equations for $p_{f,x}$ and $p_{f,y}$.

$$\begin{aligned} p_f &= \sqrt{(p_{f,x})^2 + (p_{f,y})^2} \\ &= \sqrt{(2.38 \times 10^4 \text{ kg}\cdot\text{m/s})^2 + (3.58 \times 10^4 \text{ kg}\cdot\text{m/s})^2} \\ &= 4.30 \times 10^4 \text{ kg}\cdot\text{m/s} \end{aligned}$$

Solve for θ .

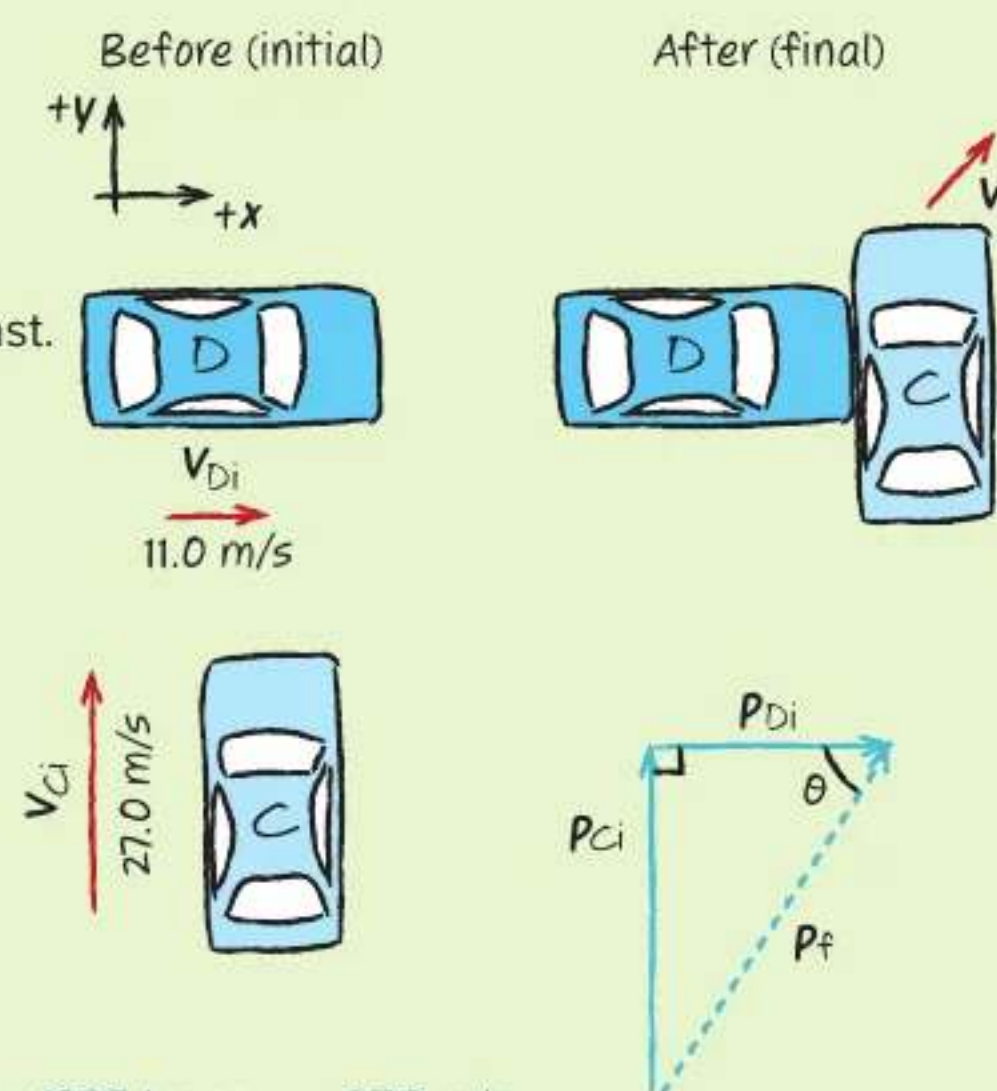
$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{p_{f,y}}{p_{f,x}}\right) \\ &= \tan^{-1}\left(\frac{3.58 \times 10^4 \text{ kg}\cdot\text{m/s}}{2.38 \times 10^4 \text{ kg}\cdot\text{m/s}}\right) \\ &= 56.4^\circ \end{aligned}$$

Determine the final speed.

$$\begin{aligned} V_f &= \frac{p_f}{m_C + m_D} \\ &= \frac{4.30 \times 10^4 \text{ kg}\cdot\text{m/s}}{1325 \text{ kg} + 2165 \text{ kg}} \\ &= 12.3 \text{ m/s} \end{aligned}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** The correct unit for speed is meters per second.
- **Does the direction make sense?** Answers are both positive and at the appropriate angles.
- **Is the magnitude realistic?** The cars stick together, so v_f must be smaller than v_{Ci} .



Substitute $m_C = 1325 \text{ kg}$, $v_{Ci,y} = 27.0 \text{ m/s}$.

Substitute $m_D = 2165 \text{ kg}$, $v_{Di,x} = 11.0 \text{ m/s}$.

Substitute $p_{i,x} = p_{Di} = 2.38 \times 10^4 \text{ kg}\cdot\text{m/s}$.

Substitute $p_{i,y} = p_{Ci} = 3.58 \times 10^4 \text{ kg}\cdot\text{m/s}$.

Substitute $p_{f,x} = 2.38 \times 10^4 \text{ kg}\cdot\text{m/s}$, $p_{f,y} = 3.58 \times 10^4 \text{ kg}\cdot\text{m/s}$.

Substitute $p_{f,y} = 3.58 \times 10^4 \text{ kg}\cdot\text{m/s}$, $p_{f,x} = 2.38 \times 10^4 \text{ kg}\cdot\text{m/s}$.

Substitute $p_f = 4.30 \times 10^4 \text{ kg}\cdot\text{m/s}$, $m_C = 1325 \text{ kg}$, $m_D = 2165 \text{ kg}$.

PRACTICE Problems

ADDITIONAL PRACTICE

- 24.** A 925-kg car moving north at 20.1 m/s collides with a 1865-kg car moving west at 13.4 m/s. After the collision, the two cars are stuck together. In what direction and at what speed do they move after the collision? Define the system as the two cars.
- 25.** A 1383-kg car moving south at 11.2 m/s is struck by a 1732-kg car moving east at 31.3 m/s. After the collision, the cars are stuck together. How fast and in what direction do they move immediately after the collision? Define the system as the two cars.
- 26.** A 1345-kg car moving east at 15.7 m/s is struck by a 1923-kg car moving north. They stick together and move with a velocity of 14.5 m/s at $\theta = 63.5^\circ$. Was the north-moving car exceeding the 20.1 m/s speed limit?
- 27. CHALLENGE** A stationary billiard ball with mass 0.17 kg is struck by an identical ball moving 4.0 m/s. Afterwards, the second ball moves 60.0° to the left of its original direction. The stationary ball moves 30.0° to the right of the moving ball's original direction. What is the velocity of each ball after the collision?

Conservation of Angular Momentum

Like linear momentum, angular momentum can be conserved. The **law of conservation of angular momentum** states that if no net external torque acts on a closed system, then its angular momentum does not change, as represented by the following equation.

Law of Conservation of Angular Momentum

An isolated system's initial angular momentum is equal to its final angular momentum.

$$L_i = L_f$$

The spinning ice-skater in **Figure 13** demonstrates conservation of angular momentum. When he pulls in his arms, he spins faster. Without an external torque, his angular momentum does not change and $L = I\omega$ is constant. The increased angular velocity must be accompanied by a decreased moment of inertia. By pulling in his arms, the skater brings more mass closer to the axis of rotation, decreasing the radius of rotation and decreasing his moment of inertia. You can calculate changes in angular velocity using the law of conservation of angular momentum.

$$L_i = L_f$$

$$\text{thus, } I_i \omega_i = I_f \omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$$



Figure 13 When an ice-skater tucks his arms, his moment of inertia decreases. Because angular momentum is conserved, his angular velocity increases.

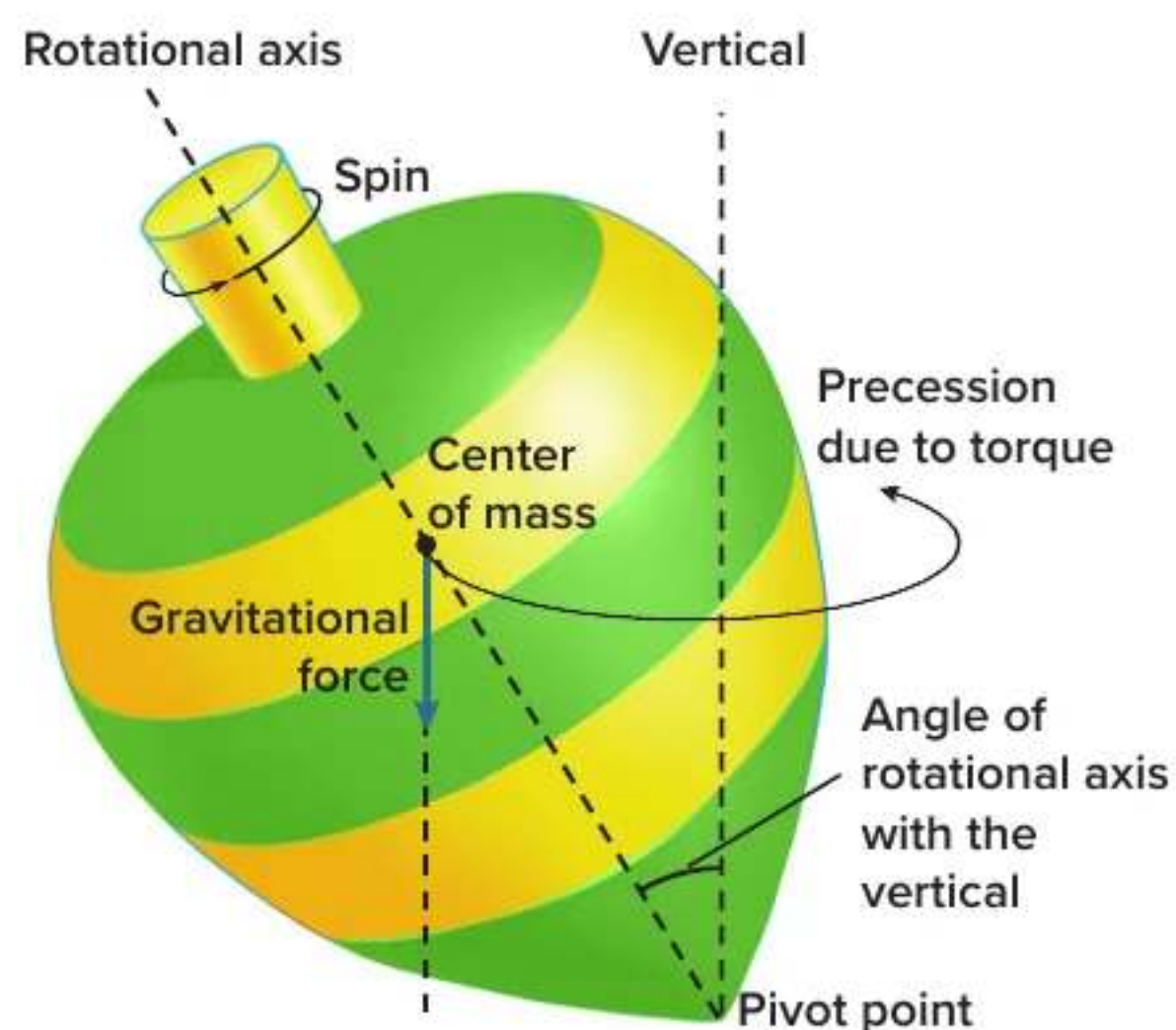


Figure 14 If you spin a top at a tilt, Earth's gravity exerts a torque on it, causing the upper end to precess.

Decide Why does the top precess only when it is tilted?

Tops and Gyroscopes

Because of the conservation of angular momentum, the direction of rotation of a spinning object can be changed only by applying a torque. If you played with a top as a child, you may have spun it by twisting its axle between your fingers or by pulling the string wrapped around its axle. When a top is vertical, the force of gravity on the center of mass of the top points through the pivot point. As a result, there is no torque on the top, and the axis of its rotation does not change. If the top is tipped, as shown in **Figure 14**, a torque tends to rotate it downward. Rather than tipping over, however, the axis of the top slowly precesses, which means it revolves slightly away from the vertical line.

Earth's rotation is another example of precession of a spinning object. Earth is not a perfect sphere. Because it has an equatorial bulge, the gravitational pull of the Sun exerts a torque on it, causing it to precess. It takes about 26,000 years for Earth's rotational axis to go through one cycle of precession.



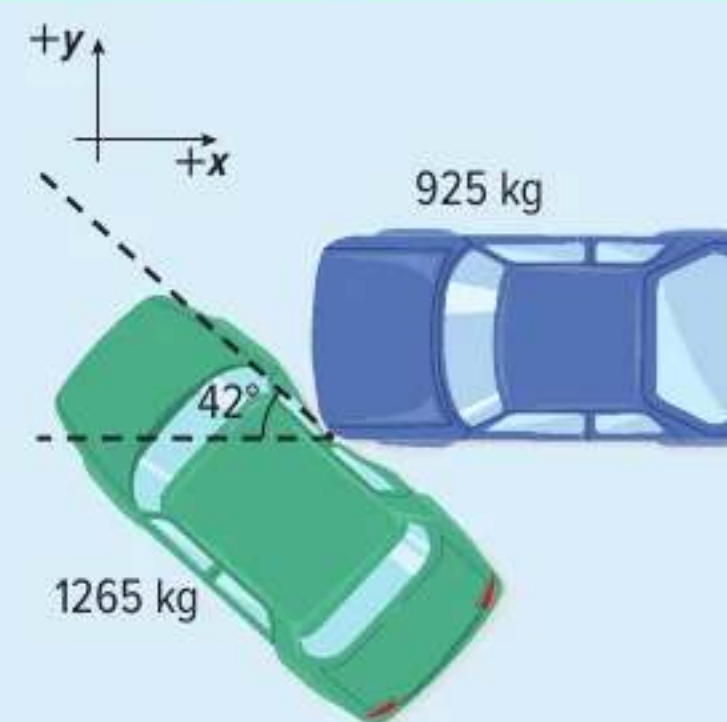
Get It?

Analyze Why does Earth precess as it rotates?

PHYSICS Challenge

Colliding Cars Your friend was driving her 1265-kg car north on Oak Street when she was hit by a 925-kg compact car going west on Maple Street. The cars stuck together and slid 23.1 m at 42° north of west. The speed limit on both streets is 22 m/s (50 mph). Define the two cars as a system. Assume that momentum was conserved during the collision and that acceleration was constant during the skid. The coefficient of kinetic friction between the tires and the pavement is 0.65.

1. Your friend claims she was not speeding but the driver of the other car was. How fast was your friend driving before the crash?
2. How fast was the other car moving before the crash? Can you support your friend's case in court?



A gyroscope, such as the one shown in **Figure 15**, is a wheel or disk that spins rapidly around one axis while being free to rotate around one or two other axes. The direction of its large angular momentum can be changed only by applying an appropriate torque. Without such a torque, the direction of the axis of rotation does not change.

Gyroscopes are used in airplanes, submarines, and spacecraft to keep an unchanging reference direction. Gyroscopic compasses, unlike magnetic compasses, maintain direction even when they are not on a level surface.

A football quarterback uses the gyroscope effect to make an accurate forward pass. A spiral is a pass in which the football spins around its longer axis. If the quarterback throws the ball in the direction of its spin axis of rotation, the ball keeps its pointed end forward, thereby reducing air resistance. Thus, the ball can be thrown far and accurately. If its spin direction is slightly off, the ball wobbles. If the ball is not spun, it tumbles end over end. Spin also stabilizes the flight of a flying disk. A well-spun plastic disk can fly many meters through the air without wobbling. Some people are able to perform tricks with a yo-yo because the yo-yo's fast rotational speed keeps it rotating in one plane.

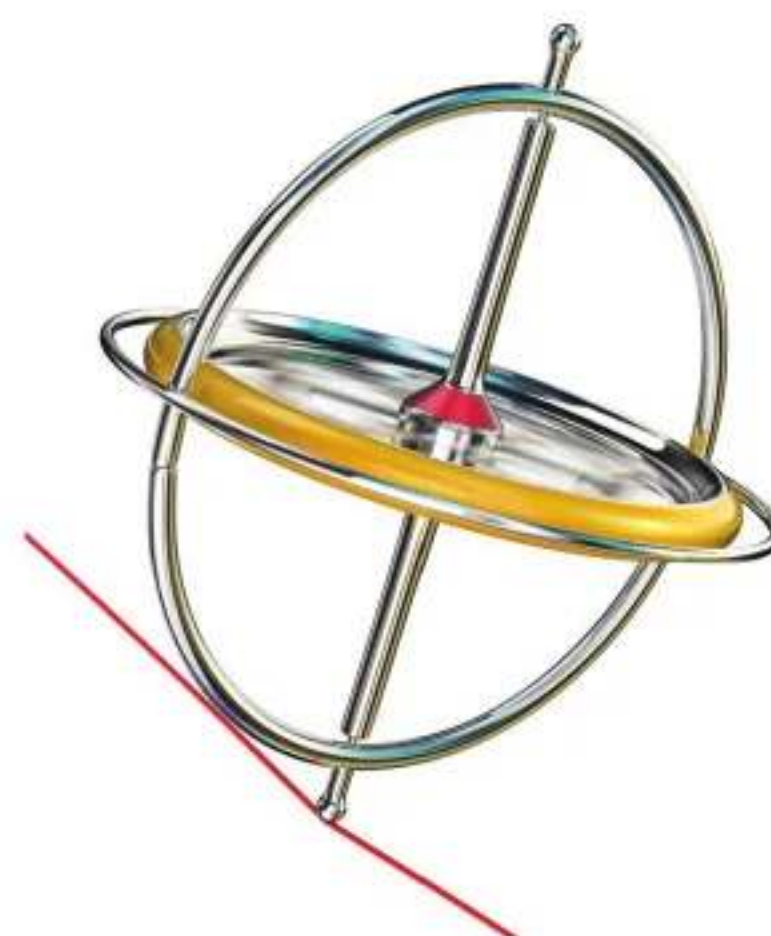


Figure 15 A gyroscope is useful for maintaining a fixed direction because its spin axis does not change orientation, even when the gyroscope is moved.

Check Your Progress

28. **Spin** The outer rim of a plastic disk is thick and heavy. Besides making it easier to catch, how does this affect the rotational properties of the plastic disk?
29. **Speed** A cart, weighing 24.5 N, is released from rest on a 1.00-m ramp, inclined at an angle of 30.0° as shown in **Figure 16**. The cart rolls down the incline and strikes a second cart weighing 36.8 N.
 - a. Define the two carts as the system. Calculate the speed of the first cart at the bottom of the incline.
 - b. If the two carts stick together, with what initial speed will they move along?
30. **Conservation of Momentum** During a tennis serve, the racket of a tennis player continues forward after it hits the ball. Is momentum conserved in the collision between the tennis racket and the ball? Explain, making sure you define the system.
31. **Momentum** A pole-vaulter runs toward the launch point with horizontal momentum. Where does the vertical momentum come from as the athlete vaults over the crossbar?
32. **Initial Momentum** During a soccer game, two players come from opposite directions and collide when trying to head the ball. The players come to rest in midair and fall to the ground. Describe their initial momenta.
33. **Critical Thinking** You catch a heavy ball while you are standing on a skateboard, and then you roll backward. If you were standing on the ground, however, you would be able to avoid moving while catching the ball.
 - a. Identify the system you use in each case.
 - b. Explain both situations using the law of conservation of momentum.

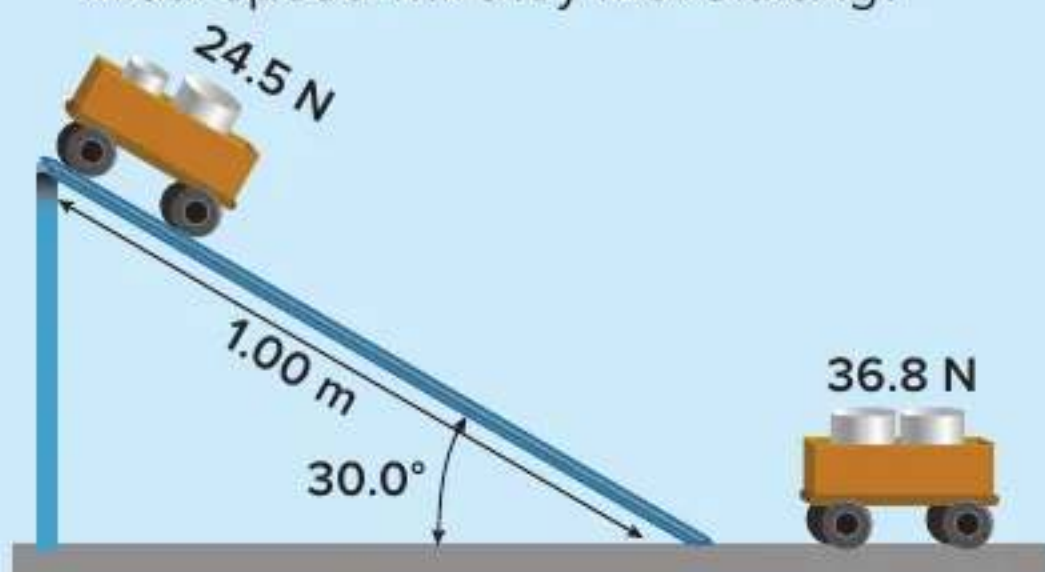


Figure 16

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Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

ENGINEERING & TECHNOLOGY

Pushing Beyond Our Solar System

Propulsion systems push or accelerate objects forward by producing thrust. Space propulsion systems push or accelerate spacecraft and satellites.

Limitations of Rocket Science

Scientists are looking for new ways to move spacecraft and artificial satellites through space. And they are developing ideas for technology that might one day allow us to send spacecraft beyond our solar system.

Most of today's spacecraft are propelled using liquid-fueled rockets that work through chemical combustion. The technology was first developed 90 years ago. It is limited by the energy of the chemical bonds in the fuel. Scientists don't think this technology can be further developed and improved enough to thrust spacecraft to the outer reaches of our solar system and beyond.

Electrical energy has also been used for space propulsion. Thrust is created when the electrical energy produces super-heated plasma that is then fired through a nozzle at a speed faster than sound. The thrust, however, is low and the technology is only useful for changing the position of orbiting satellites.

Potential Future Technologies

One idea is to create thrust by catching photons using a "solar sail." Photons have momentum, and if



Most of today's spacecraft are propelled using liquid-fueled rockets.

enough are caught by a solar sail, there could be enough thrust to move spacecraft at very high velocities, without the need for fuel.

Plasma propulsion engines accelerate ions in plasma and generate thrust using electrical potential and magnetic currents. The idea has yet to be tested in space, but a company developing the technology has said it could send a spacecraft to Mars in 39 days.

Scientists are studying several other ideas, some less probable or feasible than others. Indeed, some of the ideas are seen only as not theoretically impossible. However, with our continuing development of knowledge and technology, one of these ideas might finally lead to a method of propulsion that will take us beyond our solar system.



DEVELOP A MODEL TO ILLUSTRATE

Choose a propulsion technology. Research how thrust is created. Develop a model to illustrate how the technology works.

MODULE 9

STUDY GUIDE

 **GO ONLINE** to study with your Science Notebook.

Lesson 1 IMPULSE AND MOMENTUM

- The impulse on an object is the average net force exerted on the object multiplied by the time interval over which the force acts.

$$\text{impulse} = F\Delta t$$

- The momentum of an object is the product of its mass and velocity and is a vector quantity.

$$p = mv$$

When solving a momentum problem, first define the objects in the system and examine their momentum before and after the event. The impulse on an object is equal to the change in momentum of the object.

$$F\Delta t = p_f - p_i$$

- The angular momentum of a rotating object is the product of its moment of inertia and its angular velocity.

$$L = I\omega$$

- The angular impulse-angular momentum theorem states that the angular impulse on an object is equal to the change in the object's angular momentum.

$$\tau\Delta t = L_f - L_i$$

- impulse
- momentum
- impulse-momentum theorem
- angular momentum
- angular impulse-angular momentum theorem

Lesson 2 CONSERVATION OF MOMENTUM

- According to Newton's third law of motion and the law of conservation of momentum, the forces exerted by colliding objects on each other are equal in magnitude and opposite in direction.
- Momentum is conserved in a closed, isolated system.

$$p_f = p_i$$

- The law of conservation of momentum relates the momenta of objects before and after a collision. Use vector analysis to solve momentum-conservation problems in two dimensions. The law of conservation of angular momentum states that if there are no external torques acting on a system, then the angular momentum is conserved.

$$L_f = L_i$$

Because angular momentum is conserved, the direction of rotation of a spinning object can be changed only by applying a torque.

- closed system
- isolated system
- law of conservation of momentum
- law of conservation of angular momentum



THREE-DIMENSIONAL THINKING Module Wrap-Up

REVISIT THE PHENOMENON

How do rockets accelerate once they reach space?



CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.



STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

How does velocity change in an inelastic collision?

A 92-kg fullback, running at a speed of 5.0 m/s, attempts to dive directly across the goal line for a touchdown. Just as he reaches the line, he is met head-on in midair by two 75-kg linebackers, both moving in the direction opposite the fullback. One is moving at 2.0 m/s, and the other at 4.0 m/s. They all become entangled as one mass.

CER Analyze and Interpret Data

1. Sketch the before and after situations.
2. What is the players' velocity after the collision?
3. **Claim** Does the fullback score a touchdown?
4. **Evidence and Reasoning** Justify your claim.



MODULE 11

THERMAL ENERGY

ENCOUNTER THE PHENOMENON

Why do different parts of the pan appear to be different colors in this thermal image?



 **GO ONLINE** to play a video about how different materials heat at different rates.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

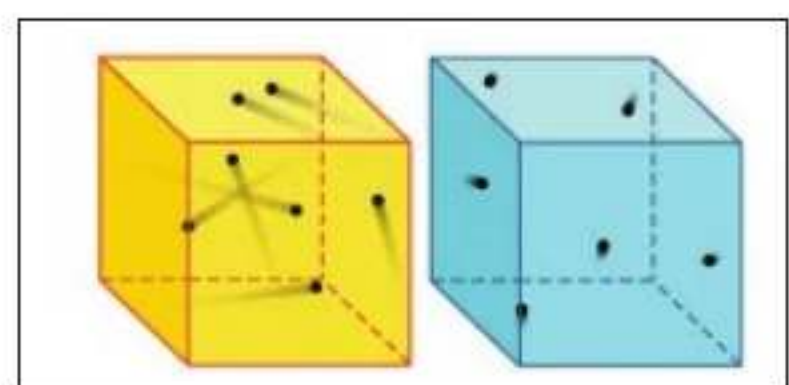
CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about why different parts of the pan appear to be different colors in the thermal image. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

 **GO ONLINE** to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain:
Equilibrium and Thermometers



LESSON 2: Explore & Explain:
The First Law of Thermodynamics



Additional Resources

LESSON 1

TEMPERATURE, HEAT, AND THERMAL ENERGY

FOCUS QUESTION

How is thermal energy transferred when you cook something?

Thermal Energy

You have studied how objects collide and trade kinetic energies. Every material is made of microscopic particles. The many particles present in a gas have linear and rotational kinetic energies. The particles also might have potential energy due to their internal bonds and interactions with each other. As gas particles collide with each other and with the walls of the container, as shown in **Figure 1**, they transfer energy. There are numerous molecules that make up the gas, resulting in many collisions. The energies of the particles become randomly distributed.

Thus, it is convenient to discuss the total energy of the particles that compose the gas and the average energy per particle in the gas. Recall that the sum of the particles' energies is the object's thermal energy. The average kinetic energy per particle is related to the temperature of the gas. The relationship between the particles' random motions and the bulk property of the material is described by kinetic theory.

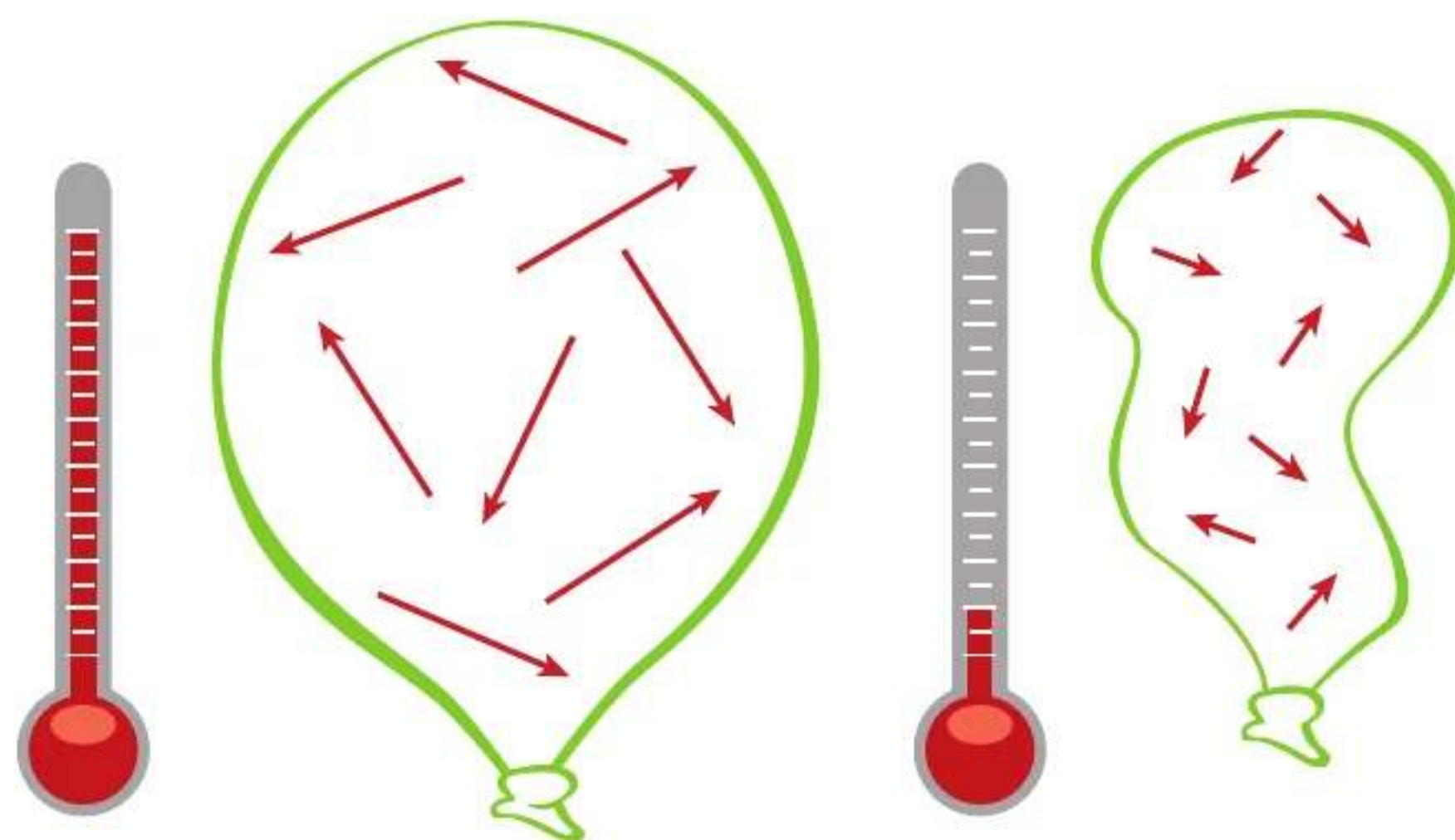


Figure 1 Gas particles collide with each other and the walls of the container. The temperature of an object is related to the average kinetic energy of its particles. The average kinetic energy of the particles that make up a hot object is greater than the average kinetic energy of the particles that make up a cold object.

Explain what the arrows represent and why the arrows are longer in the balloon on the left than in the balloon on the right.



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



Applying Practices: Coffee Cup Calorimetry

HS-PS3-4. Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).

Hot objects and cold objects What makes an object hot? Consider a helium-filled balloon. The balloon is kept inflated by the repeated pounding from helium atoms on the balloon wall. Each of the approximately 10^{22} helium atoms in the balloon collides with the balloon wall, bounces back, and hits the balloon wall again somewhere else. The size and the temperature of the balloon are affected by the average kinetic energy of the helium atoms, as shown in **Figure 1**.

If you put a balloon in sunlight, energy absorbed from the sunlight makes each of the helium atoms move faster in random directions and bounce off the rubber walls of the balloon more often. Each atomic collision with the balloon wall puts a greater force on the balloon and stretches the rubber. Thus, the warmed balloon expands. On the other hand, if you refrigerate a balloon, you will find that it shrinks. It must do so because the particles are moving more slowly. The refrigeration has removed some of the atoms' thermal energy.

Thermal energy in solids The atoms or molecules in solids also have kinetic energy, but they are unable to move everywhere as gas atoms do. One way to illustrate the structure of a solid is to picture a number of atoms that are held in place next to each other by atomic forces that act like springs. The atoms cannot move freely, but they do bounce back and forth, with some bouncing more than others. Each atom has some kinetic energy and some potential energy. If a solid has N atoms, then the total thermal energy in the solid is equal to the average kinetic energy plus potential energy per atom times N .

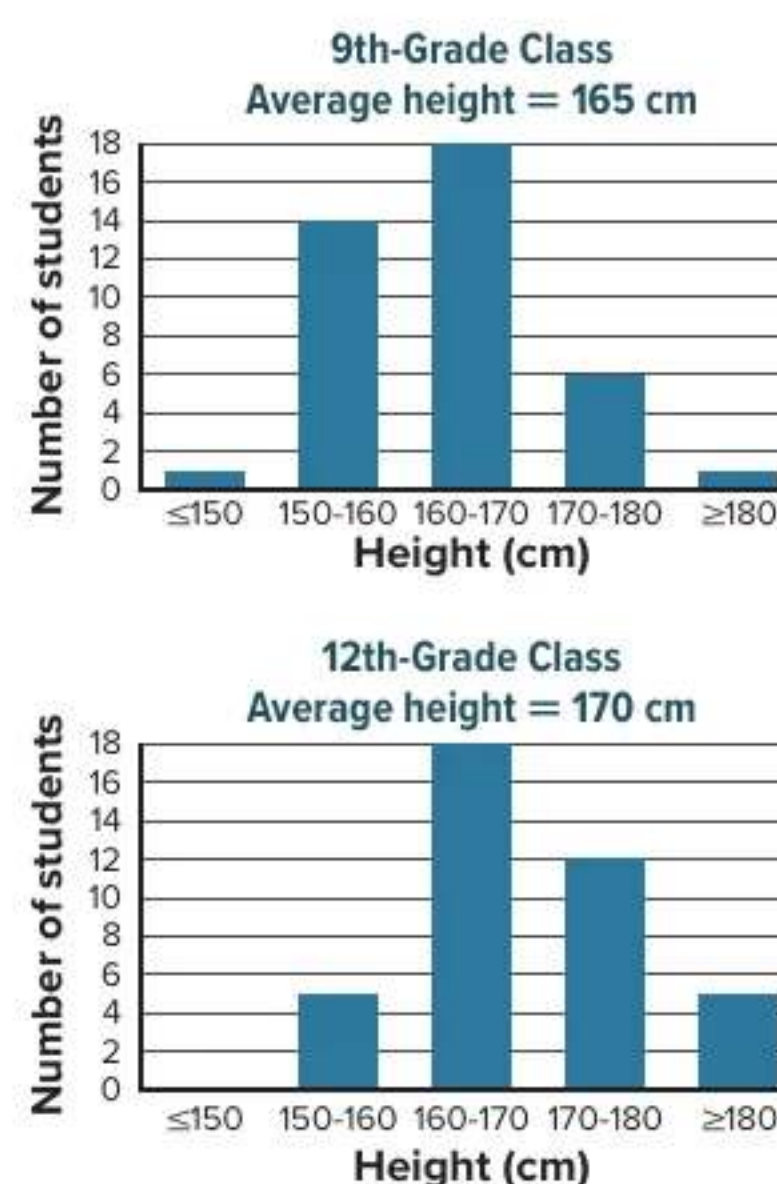


Figure 2 The average height in a 9th-grade class is less than the average height in a 12th-grade class. Similarly, the average kinetic energy of a hot object's particles is greater than the average kinetic energy of a cold object's particles.

Thermal Energy and Temperature

You have just seen that, on average, a particle in a hot object has more kinetic energy than a particle in a cold object. This does not mean that each of the particles that compose an object has the same amount of energy; they have a wide range of energies. The average kinetic energy of the particles that compose a hot object, however, is greater than the average kinetic energy of the particles that compose a cold object.

To understand this, consider the heights of students in a 9th-grade class and the heights of students in a 12th-grade class. The students' heights vary, as shown in **Figure 2**, but you can calculate the average height for each class. The average height of the 12th-grade class is greater than the average height of the 9th-grade class, even though some 9th-grade students might be taller than some 12th-grade students.

STEM CAREER Connection

HVAC Mechanic/Installer

If you like working with your hands and working with mechanical systems, a career as a heating, ventilation, air conditioning (HVAC) mechanic/installer might be for you. HVAC installers install heating and air conditioning systems in schools, homes, and businesses to improve the comfort and health of the occupants.



Figure 3 Two muffins at the same temperature can have different thermal energies.

Temperature depends only on the average kinetic energy of the particles in the object. It does not depend on the number of particles that compose the object. For example, consider the two muffins shown in **Figure 3**. They are at the same temperature, but the large muffin has ten times as many particles as the small muffin. Thus, the large muffin has ten times the thermal energy of the small muffin. The thermal energy of an object depends on both its temperature and the number of particles that make up that object.

Equilibrium and Thermometers

To measure your body temperature, you might place a thermometer in your mouth and wait for a moment before checking the reading. Measuring your temperature involves random collisions and energy transfers between the particles of the thermometer and the particles of your body. Your body is hotter than the thermometer. That is, the average kinetic energy of the particles that compose your body is greater than the average kinetic energy of the thermometer's particles. When the cool thermometer touches your skin, the particles of your skin collide with the particles of the thermometer. On average, the more energetic skin particles will transfer energy to the less energetic particles of the thermometer. **Thermal conduction** is the transfer of thermal energy that occurs when particles collide. As a result of these collisions, the thermal energy of the thermometer's particles increases and the thermal energy of your skin's particles decreases.

Thermal equilibrium The thermometer's particles also transfer energy to your body's particles. As the thermometer's particles gain more energy, the amount of energy they give back to the skin increases. At some point, the rate of energy transfer from the thermometer to your body is equal to the rate of transfer in the other direction. At this point, your body and the thermometer have reached thermal equilibrium. **Thermal equilibrium** is the state in which the rates of thermal energy transfer between two objects are equal and the objects are at the same temperature. In other words, uncontrolled systems always evolve toward more stable states—that is, toward more uniform energy distribution (objects hotter than their surrounding environment cool down until thermal equilibrium is reached by the object and its surroundings). **Figure 4** on the next page shows two blocks reaching equilibrium.



Get It?

Describe an example in your everyday life in which an object and its surroundings move toward equilibrium, or more uniform energy distribution.

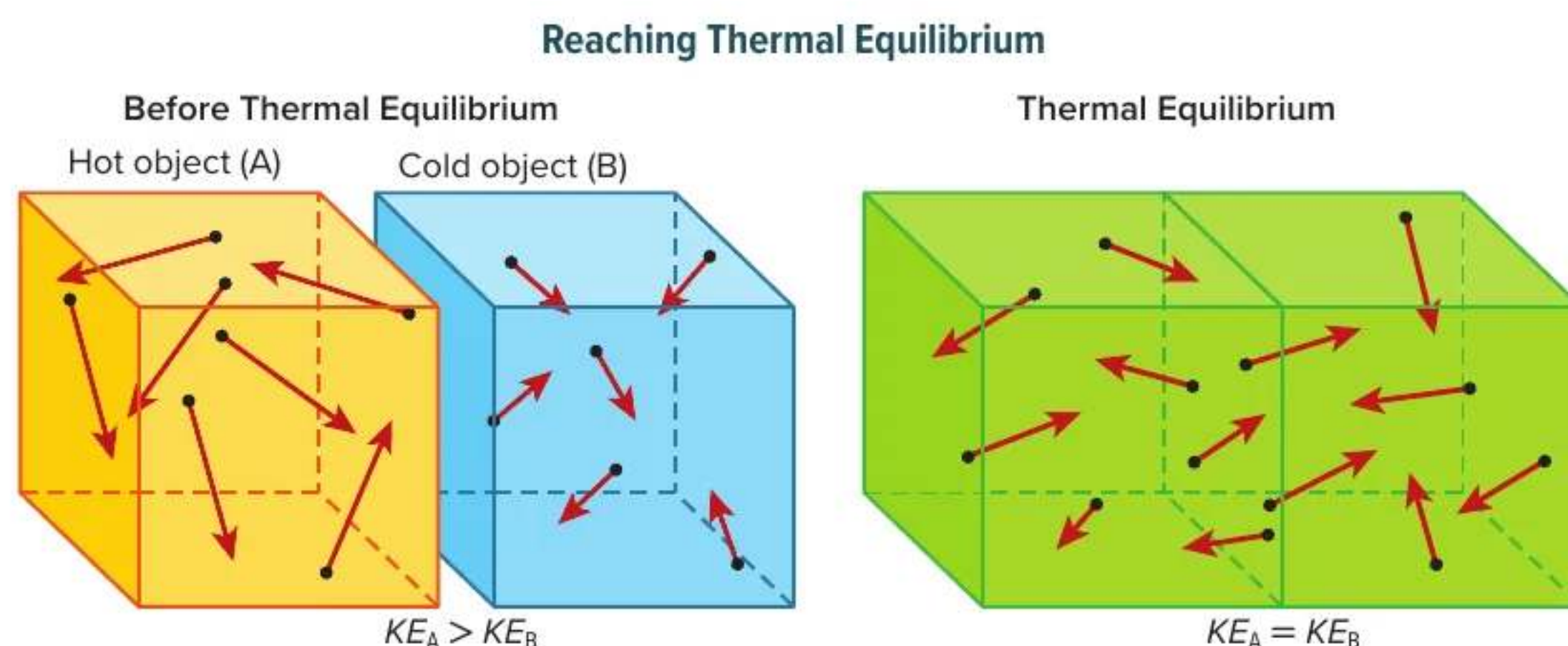


Figure 4 When a hot object and a cold object are in contact, there is a net transfer of thermal energy from the hot object to the cold object. When the two objects reach thermal equilibrium, the transfer of energy between the objects is equal, and the objects are at the same temperature.

Thermometers Every thermometer has some useful property that changes with temperature. Household thermometers often contain colored alcohol that expands when heated. The hotter the thermometer, the more the alcohol expands and the higher it rises in the tube. The liquid crystal thermometer in **Figure 5** uses a variety of long molecules that rearrange and cause a color change at specific temperatures. Medical thermometers and the thermometers that monitor automobile engines use very small, temperature-sensitive electronic circuits to take rapid temperature measurements.



Get It?

Describe in your own words how an alcohol thermometer works.

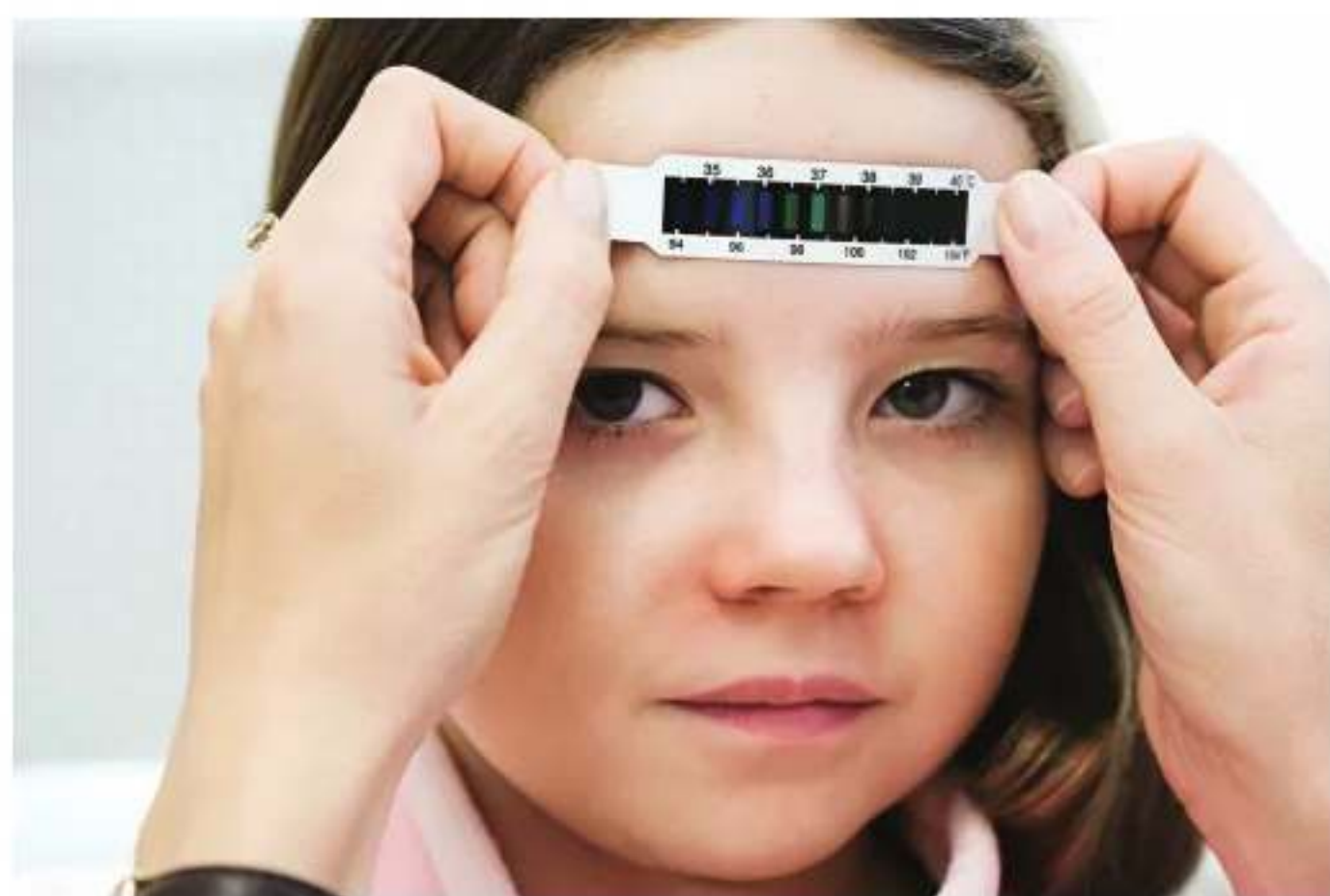


Figure 5 Liquid crystal thermometers change color with temperature.

Summarize the process that occurs when this thermometer is placed on a person's forehead.

Temperature limits You might say that a fire is hot and a freezer is cold. But the temperatures of everyday objects are only a small part of the wide range of temperatures present in the universe, as shown in **Figure 6**. Temperatures do not appear to have an upper limit. The interior of the Sun is at least 1.5×10^7 °C. Supernova cores are even hotter. On the other hand, liquefied gases can be very cold. For example, helium liquefies at -269 °C. Even colder temperatures can be reached by making use of special properties of solids, helium isotopes, and atoms and lasers.

Temperatures do, however, have a lower limit. Generally, materials contract as they cool. If an ideal atomic gas in a balloon were cooled to -273.15 °C, it would contract in such a way that it occupied a volume that is only the size of the atoms, and the atoms would become motionless. At this temperature, all the thermal energy that could be removed has been removed from the gas, and the temperature cannot be reduced any further. Therefore, there can be no temperature less than -273.15 °C, which is called absolute zero.



Get It?

Explain why the term *absolute zero* is appropriate for the coldest temperature possible.

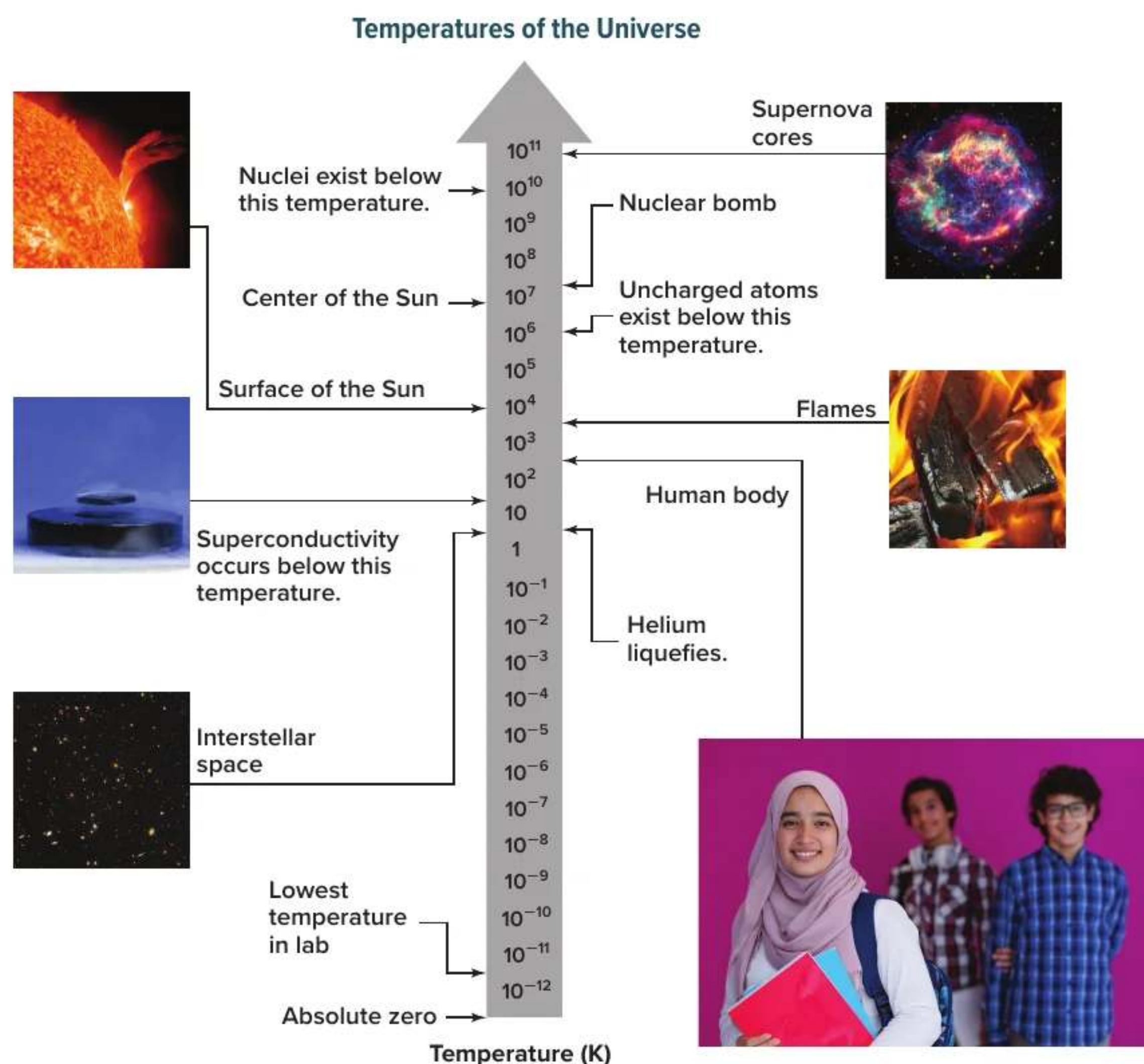


Figure 6 Temperatures in the universe range from just above absolute zero to more than 10^{10} K.

Temperature scales In the United States, weather agencies report temperatures in degrees Fahrenheit. Scientists, however, use the Celsius and Kelvin scales. The Celsius scale is based on the properties of water and was devised in 1741 by Swedish physicist Anders Celsius. On the Celsius scale, the freezing point of pure water at sea level is defined to be 0°C . The boiling point of pure water at sea level is defined to be 100°C . The Celsius scale is useful for day-to-day measurements of temperature.

The Kelvin scale On the Celsius scale, temperatures can be negative. Negative temperatures suggest a particle could have negative kinetic energy. Because temperature represents average kinetic energy of the object's particles, it makes more sense to use a temperature scale whose zero temperature is where the particles' kinetic energy is also zero. Therefore the zero point of the Kelvin scale is defined to be absolute zero. On the Kelvin scale, the freezing point of water (0°C) is about 273 K and the boiling point of water is about 373 K. Each interval on this scale, called a kelvin, is equal to 1°C . Thus, $T_{\text{C}} + 273 = T_{\text{K}}$. **Figure 7** compares the Fahrenheit, Celsius, and Kelvin scales.

Comparing Temperature Scales

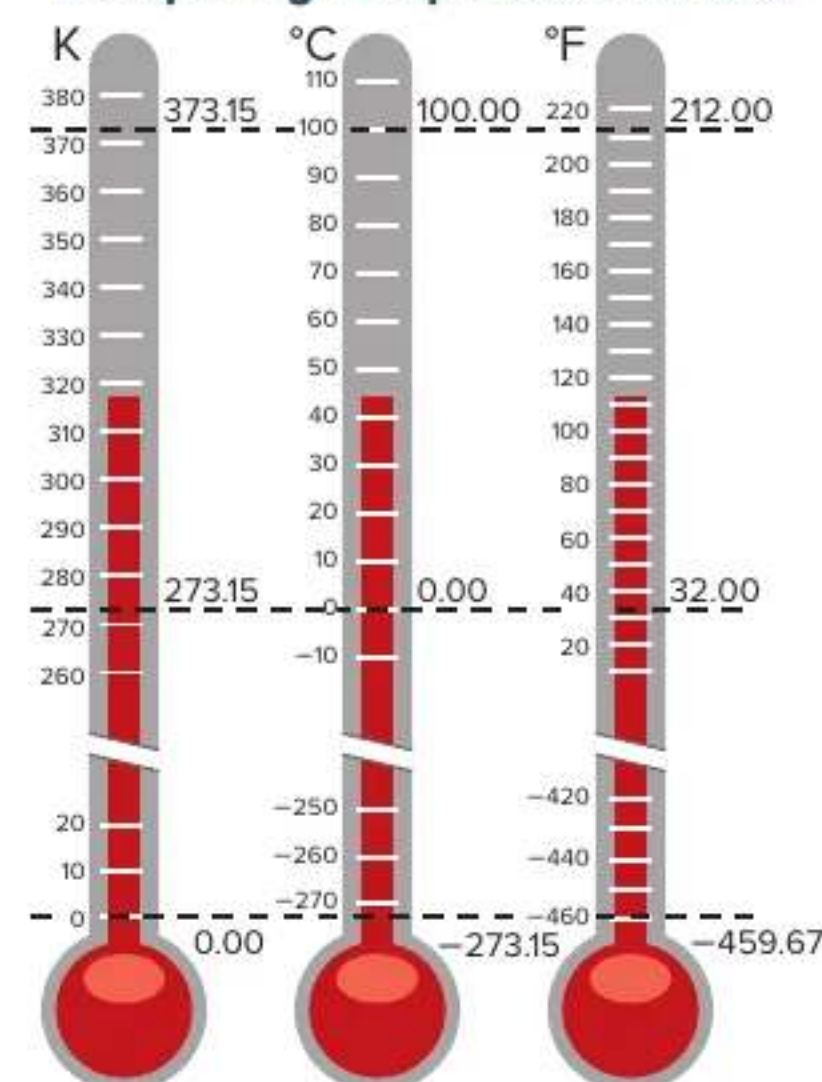


Figure 7 The Kelvin and Celsius scales are used by scientists. In the United States, the Fahrenheit scale is often used for weather reports and cooking.

Heat and Thermal Energy Transfer

When two objects come in contact with each other, they redistribute their thermal energies. **Heat** (Q) is the transfer of thermal energy, which occurs spontaneously from a hotter object to a cooler object. Thermal energy cannot be transferred from a colder object to a hotter object without work being done. Like work and energy, heat is measured in joules. In the thermometer example, thermal energy was transferred from the warm skin to the cold thermometer because of the collisions of particles. If thermal energy has been absorbed by an object, Q is positive. If thermal energy is transferred from an object, Q is negative.

Conduction, convection, and radiation **Figure 8** shows the three types of heat—conduction, convection, and radiation. If you place one end of a metal rod in a flame, the hot gas conducts heat to the rod. The other end of the rod also becomes warm because the particles that make up the rod conduct thermal energy to their neighbors.



Figure 8 Thermal energy can be transferred by conduction, convection, and radiation.

Identify other common occurrences of conduction, convection, and radiation.

Have you ever noticed the motion on the surface of a pot of water just about to boil? The water at the bottom of the pot is heated by conduction, expands, and floats to the top, while the colder, denser water at the top sinks to the bottom. The motion of the hot water rapidly carries heat from the bottom of the pot to the top surface of the water. Heating caused by the motion of fluid in a liquid or gas due to temperature differences is called **convection**. Atmospheric turbulence is caused by convection of gases in the atmosphere. Thunderstorms and hurricanes are excellent examples of large-scale atmospheric convection. Convection also contributes to ocean currents that move water and materials over large distances.

The third method of thermal transfer, unlike the first two, does not depend on the presence of matter. The Sun warms Earth from over 150 million km away via **radiation**, which is the transfer of energy by electromagnetic waves. These waves carry the energy from the hot Sun through the vacuum of space to the much cooler Earth.

Specific Heat

Some objects are easier to warm up than others. On a bright summer day, the Sun radiates thermal energy to the sand on a beach and to the ocean water. The sand on the beach becomes quite hot, while the ocean water stays relatively cool. When an object is heated, its thermal energy increases and its temperature can increase. The amount of the increase in temperature depends on the size of the object and its composition.

The **specific heat** of a material is the amount of energy that must be added to a unit mass of the material to raise its temperature by one temperature unit. In SI units, specific heat (C) is measured in $\text{J}/(\text{kg}\cdot\text{K})$.

Table 1 provides values of specific heat for some common substances.

For example, 897 J must be added to 1 kg of aluminum to raise its temperature by 1 K. The specific heat of aluminum is therefore 897 $\text{J}/(\text{kg}\cdot\text{K})$. Materials with different specific heats are used for different purposes. Metals, such as those used to make the pans in **Figure 9**, have low specific heats and are good thermal conductors. Notice that liquid water has a high specific heat compared to other substances. Ice and water vapor also have relatively high specific heats. These high specific heats have had significant effects on our climate and our bodies.



Figure 9 These pans are made of stainless steel and have copper bottoms and plastic handles.

Explain how the selection of these materials is related to their specific heats.

Table 1 Specific Heat of Common Substances

Material	Specific Heat ($\text{J}/(\text{kg}\cdot\text{K})$)	Material	Specific Heat ($\text{J}/(\text{kg}\cdot\text{K})$)
Aluminum	897	Lead	130
Brass	376	Methanol	2450
Carbon	710	Silver	235
Copper	385	Water Vapor	2020
Glass	840	Water	4180
Ice	2060	Zinc	388
Iron	450		

Measuring heat When a substance is heated, the substance's temperature can change. The change of temperature (ΔT) depends on heat (Q), the mass of the substance (m), and the specific heat of the substance (C). By using the following equation, you can calculate the heat (Q) required to change the temperature of an object.

Heat

Heat is equal to the mass of an object times the specific heat of the object times the difference between the final and initial temperatures.

$$Q = mC\Delta T = mC(T_f - T_i)$$

For example, when the temperature of 10.0 kg of water is increased from 80 K to 85 K, the heat is $Q = (10.0 \text{ kg})(4180 \text{ J/(kg}\cdot\text{K)})(85 \text{ K} - 80 \text{ K}) = 2.1 \times 10^5 \text{ J}$. Remember that the temperature interval for the Kelvin scale is the same as that for the Celsius scale. For this reason, you can calculate ΔT on the Kelvin scale or on the Celsius scale.

EXAMPLE Problem 1

HEAT TRANSFER A 5.10-kg cast-iron skillet is heated on the stove from 295 K to 373 K. How much thermal energy had to be transferred to the iron?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the thermal energy transfer into the skillet from the stove top.

Known

$$m = 5.10 \text{ kg} \quad C = 450 \text{ J/(kg}\cdot\text{K)}$$

$$T_i = 295 \text{ K} \quad T_f = 373 \text{ K}$$

Unknown

$$Q = ?$$

2 SOLVE FOR THE UNKNOWN

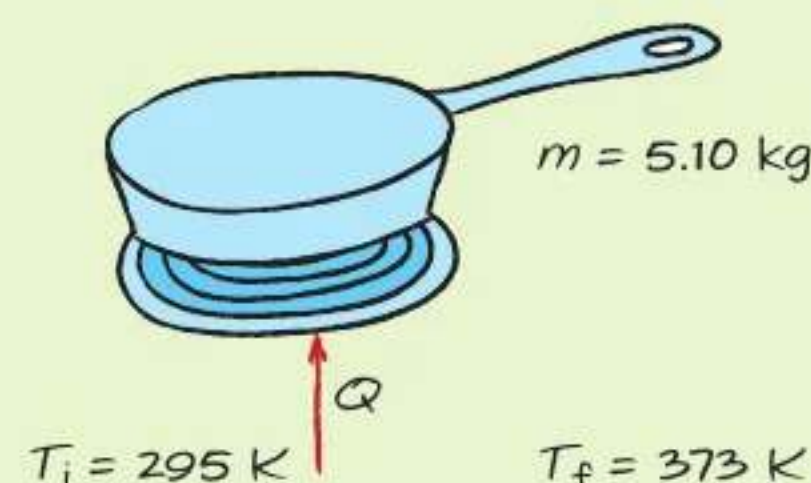
$$Q = mC(T_f - T_i)$$

$$= (5.10 \text{ kg})(450 \text{ J/(kg}\cdot\text{K)})(373 \text{ K} - 295 \text{ K}) \quad \text{Substitute } m = 5.10 \text{ kg, } C = 450 \text{ J/(kg}\cdot\text{K), } T_f = 373 \text{ K, } T_i = 295 \text{ K.}$$

$$= 1.8 \times 10^5 \text{ J}$$

3 EVALUATE THE ANSWER

- Are the units correct? Heat is measured in joules.
- Do the signs make sense? Temperature increased, so Q is positive.



PRACTICE Problems

ADDITIONAL PRACTICE

- When you turn on the hot water to wash dishes, the water pipes heat up. How much thermal energy is absorbed by a copper water pipe with a mass of 2.3 kg when its temperature is raised from 20.0°C to 80.0°C?
- Electrical power companies sell electrical energy by the kilowatt-hour, where $1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$. Suppose that it costs \$0.15 per kWh to run your electric water heater. How much does it cost to heat 75 kg of water from 15°C to 43°C to fill a bathtub?
- CHALLENGE** A car engine's cooling system contains 20.0 L of water (1 L of water has a mass of 1 kg).
 - What is the change in the temperature of the water if 836.0 kJ of thermal energy is added?
 - Suppose that it is winter, and the car's cooling system is filled with methanol. The density of methanol is 0.80 g/cm³. What would be the increase in temperature of the methanol if it absorbed 836.0 kJ of thermal energy?
 - Which coolant, water or methanol, would better remove thermal energy from a car's engine? Explain.

Measuring Specific Heat

A calorimeter, such as the simple one shown in **Figure 10**, is a device that measures changes in thermal energy. A calorimeter is carefully insulated so that thermal energy transfer to the external world is kept to a minimum. A measured mass of a substance that has been heated to a high temperature (T_A) is placed in the calorimeter. The calorimeter also contains a known mass of cold water at a measured temperature (T_B). Thermal energy is transferred from the warmer substance to the cooler water until they come to an equilibrium temperature (T_f). By measuring these three temperatures, the specific heat of the unknown substance can be calculated.

Energy conservation The operation of a calorimeter depends on the conservation of energy in an isolated, closed system composed of the water and the substance being measured. Energy can neither enter nor leave this system but can be transferred from one part of the system to another. Therefore, if the thermal energy of the test substance changes by an amount (ΔE_A) then the change in thermal energy of the water (ΔE_B) must be related by the equation $\Delta E_A + \Delta E_B = 0$. This can be rearranged to form the equation:

$$\Delta E_A = -\Delta E_B$$

The change in energy of the cold water is positive, and the change in energy of the hot test substance is negative. A positive change in energy indicates a rise in temperature, and a negative change in energy indicates a fall in temperature.

In an isolated, closed system, no work is done, so the change in thermal energy for each substance is equal to the heat and can be expressed by the following equation:

$$\Delta E = Q = mC\Delta T = mC(T_f - T_i)$$

Combining this equation with $\Delta E_A = -\Delta E_B$ gives:

$$m_A C_A (T_f - T_A) = -m_B C_B (T_f - T_B)$$

The final temperatures of the two substances are equal because they are in thermal equilibrium. Solving for the unknown specific heat (C_A) gives the equation:

$$C_A = \frac{-m_B C_B \Delta T_B}{m_A \Delta T_A}$$

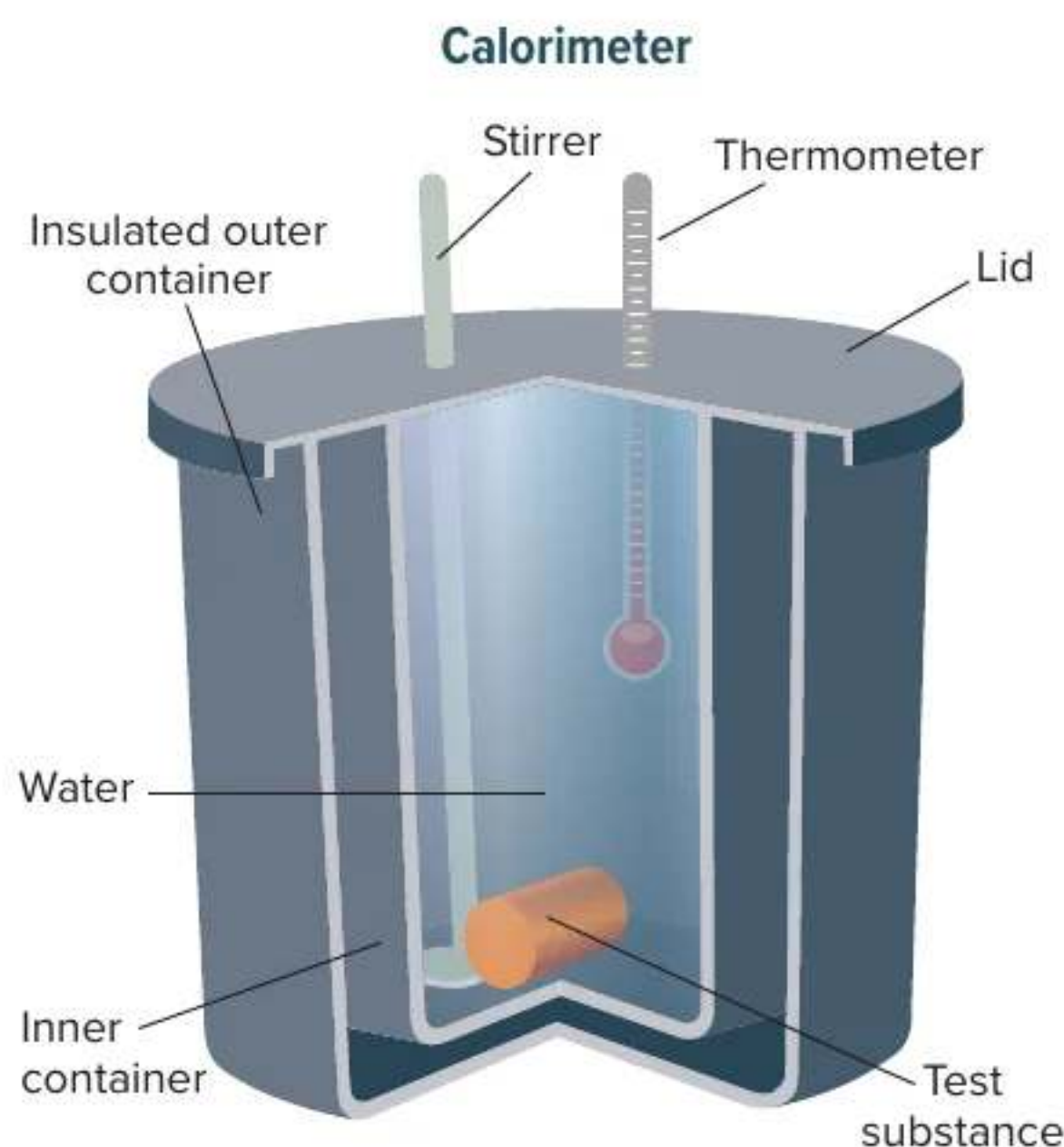


Figure 10 In a simple calorimeter, a hot test substance and a known volume of cold water are placed in an isolated system and allowed to come to thermal equilibrium. The ideal calorimeter has perfect insulation and does not transfer thermal energy to or from the outside. More sophisticated types of calorimeters are used to measure chemical reactions and the energy content of various foods.



Get It?

Evaluate the cup of liquid on the first page of this module. How is energy transferred and transformed?

EXAMPLE Problem 2

TRANSFERRING HEAT IN A CALORIMETER A calorimeter contains 0.50 kg of water at 15°C. A 0.10-kg block of an unknown substance at 62°C is placed in the water. The final temperature of the system is 16°C. What is the substance?

1 ANALYZE AND SKETCH THE PROBLEM

- Let the unknown be sample A and water be sample B.
- Sketch the transfer of thermal energy from the hotter unknown sample to the cooler water.

Known

$$m_A = 0.10 \text{ kg}$$

$$T_A = 62^\circ\text{C}$$

$$m_B = 0.50 \text{ kg}$$

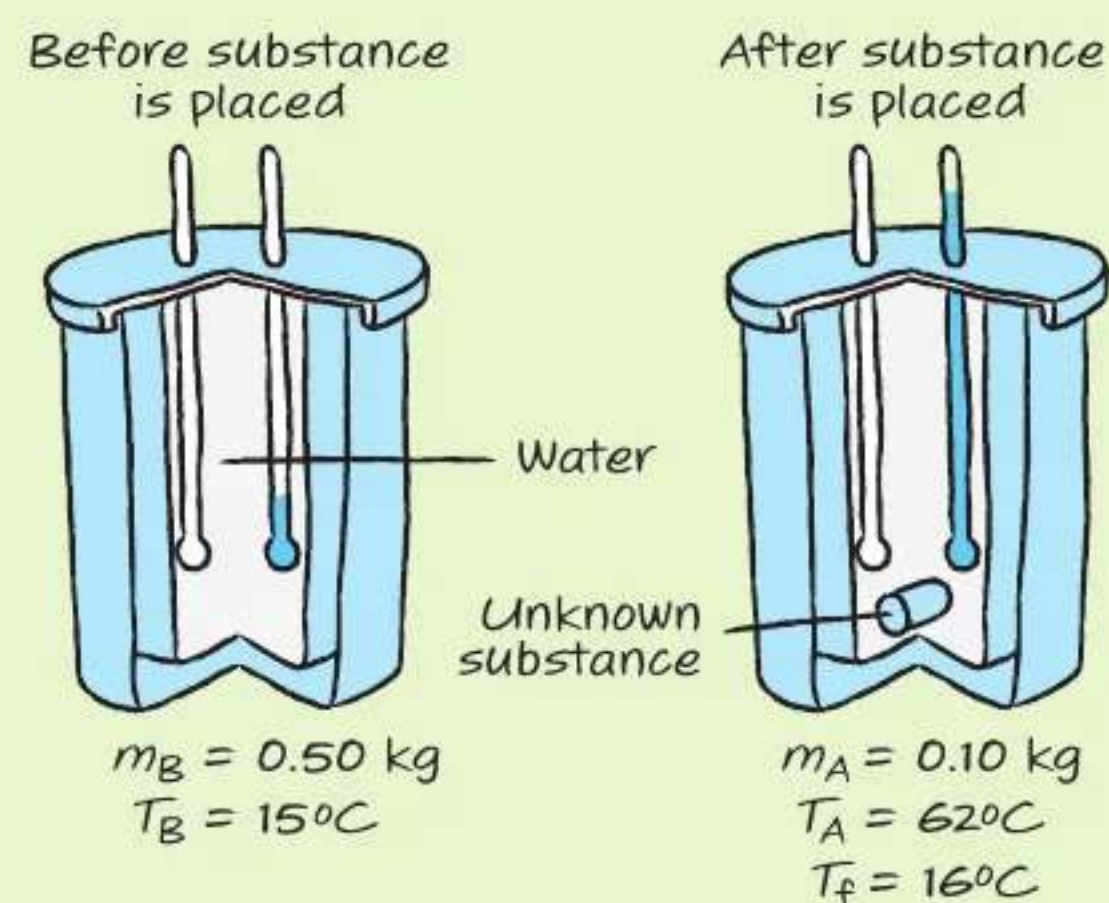
$$C_B = 4180 \text{ J/(kg}\cdot\text{K)}$$

$$T_B = 15^\circ\text{C}$$

$$T_f = 16^\circ\text{C}$$

Unknown

$$C_A = ?$$

**2 SOLVE FOR THE UNKNOWN**

Determine the final temperature using the following equation. Beware of the minus signs.

$$\begin{aligned} C_A &= \frac{-m_B C_B \Delta T_B}{m_A \Delta T_A} \\ &= \frac{-(0.50 \text{ kg})(4180 \text{ J/(kg}\cdot\text{K)})(16^\circ\text{C} - 15^\circ\text{C})}{(0.10 \text{ kg})(16^\circ\text{C} - 62^\circ\text{C})} \\ &= 450 \text{ J/(kg}\cdot\text{K)} \end{aligned}$$

Substitute $m_A = 0.10 \text{ kg}$, $T_A = 62^\circ\text{C}$, $m_B = 0.50 \text{ kg}$, $C_B = 4180 \text{ J/(kg}\cdot\text{K)}$, $T_B = 15^\circ\text{C}$, $T_f = 16^\circ\text{C}$.

According to **Table 1**, the specific heat of the unknown substance equals that of iron.

3 EVALUATE THE ANSWER

- **Are the magnitudes realistic?** Specific heat is measured in J/(kg·K).
- **Is the magnitude realistic?** The answer is of the same magnitude as most metals listed in **Table 1**.

PRACTICE Problems**ADDITIONAL PRACTICE**

- A 1.00×10^2 -g aluminum block at 100.0°C is placed in 1.00×10^2 g of water at 10.0°C . The final temperature of the mixture is 26.0°C . What is the specific heat of the aluminum?
- Three metal fishing weights, each with a mass of 1.00×10^2 g and at a temperature of 100.0°C , are placed in 1.00×10^2 g of water at 35.0°C . The final temperature of the mixture is 45.0°C . What is the specific heat of the metal in the weights?
- A 2.00×10^2 -g sample of water at 80.0°C is mixed with 2.00×10^2 g of water at 10.0°C in a calorimeter. What is the final temperature of the mixture?
- A 1.50×10^2 -g piece of glass at a temperature of 70.0°C is placed in a container with 1.00×10^2 g of water initially at a temperature of 16.0°C . What is the equilibrium temperature of the water?
- CHALLENGE** A 4.00×10^2 -g sample of water at 15.0°C is mixed with 4.00×10^2 g of water at 85.0°C . After the system reaches thermal equilibrium, 4.00×10^2 g of methanol at 15°C is added. Assume there is no thermal energy lost to the surroundings. What is the final temperature of the mixture?

Animals and Thermal Energy

BIOLOGY Connection Animals can be divided into two groups based on how they control their body temperatures. Most, such as the spider in **Figure 11**, are cold-blooded animals. Their body temperatures depend on the environment. A cold-blooded animal regulates the transfer of thermal energy by its behavior, such as hiding under a rock to keep cool or sunning itself to keep warm.

The others are warm-blooded animals whose body temperatures are controlled internally. That is, a warm-blooded animal's body temperature remains stable regardless of the temperature of the environment. For example, humans are warm-blooded and have body temperatures close to 37°C . To regulate its body temperature, a warm-blooded animal relies on bodily responses initiated by the brain, such as shivering and sweating, to counteract a rise or fall in body temperature.

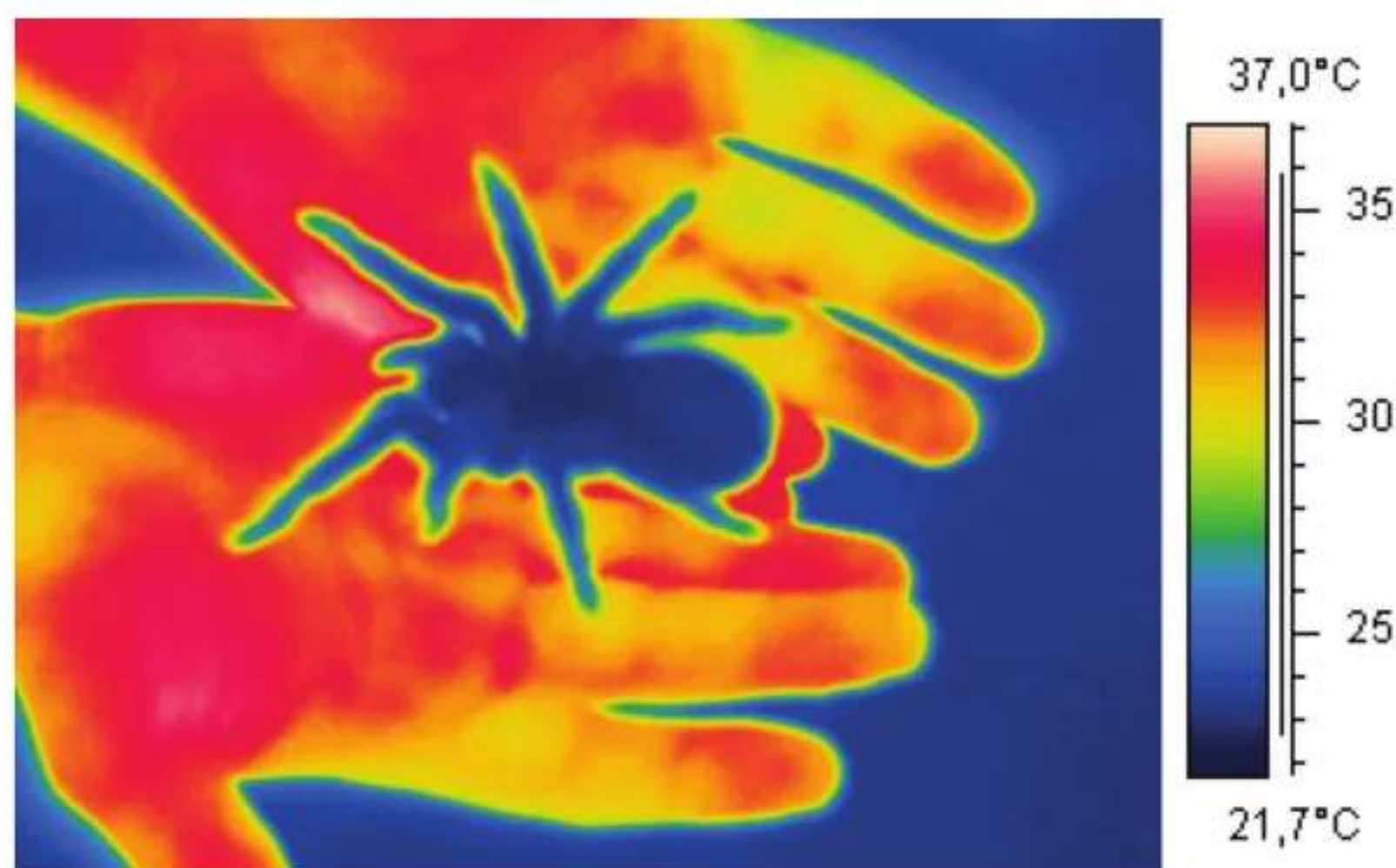


Figure 11 Cold-blooded animals depend on external sources of thermal energy to maintain their body temperatures. In contrast, warm-blooded animals maintain their body temperatures internally. In this thermal image, the spider is at the same temperature as the surrounding air, but the person's hands are significantly warmer than the surrounding air.

Check Your Progress

9. **Conduction** The hard tile floor of a bathroom always feels cold to bare feet even though the rest of the room is warm. Is the floor colder than the rest of the room? Explain.
10. **Temperature** Make the following conversions:
 - a. 5°C to kelvin
 - b. 34 K to degrees Celsius
 - c. 212°C to kelvin
 - d. 316 K to degrees Celsius
11. **Units** Are the units the same for heat (Q) and specific heat (C)? Explain.
12. **Types of Energy** Describe the mechanical energy and the thermal energy of a bouncing basketball.
13. **Thermal Energy** Could the thermal energy of a bowl of hot water equal that of a bowl of cold water? Explain your answer.
14. **Cooling** On a dinner plate, a baked potato always stays hot longer than any other food. Why?
15. **Heat and Food** It takes much longer to bake a whole potato than potatoes that have been cut into pieces. Why?
16. **Cooking** Stovetop pans are made from metals such as copper, iron, and aluminum. Why are these materials used?
17. **Specific Heat** If you take a plastic spoon out of a cup of hot cocoa and put it in your mouth, you are not likely to burn your tongue. However, you could very easily burn your tongue if you put the hot cocoa in your mouth. Why?
18. **Critical Thinking** As water heats in a pot on a stove, it might produce some mist above its surface right before the water begins to roll. What is happening?

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LESSON 2

CHANGES OF STATE AND THERMODYNAMICS

FOCUS QUESTION

Why doesn't a bowl of hot soup spontaneously freeze?

Change of State

The three most common states of matter on Earth are solid, liquid, and gas. As the temperature of a solid rises, that solid usually changes to a liquid. At even higher temperatures, it becomes a gas. If the gas cools, it returns to the liquid state. If the cooling continues, the liquid will return to the solid state. How can these changes be explained? Recall that when the thermal energy of a material changes, the motion of its particles also changes, as does the temperature.

Figure 12 diagrams the changes of state as thermal energy is added to 1.0 kg of water starting at 243 K (ice) and continuing until that water reaches 473 K (water vapor). Between points A and B, the ice is warmed to 273 K. At this point, the added thermal energy gives the water molecules enough energy to partially overcome the forces holding them together. The particles are still touching each other, but they have more freedom of movement. Eventually, the molecules become free enough to slide past each other.

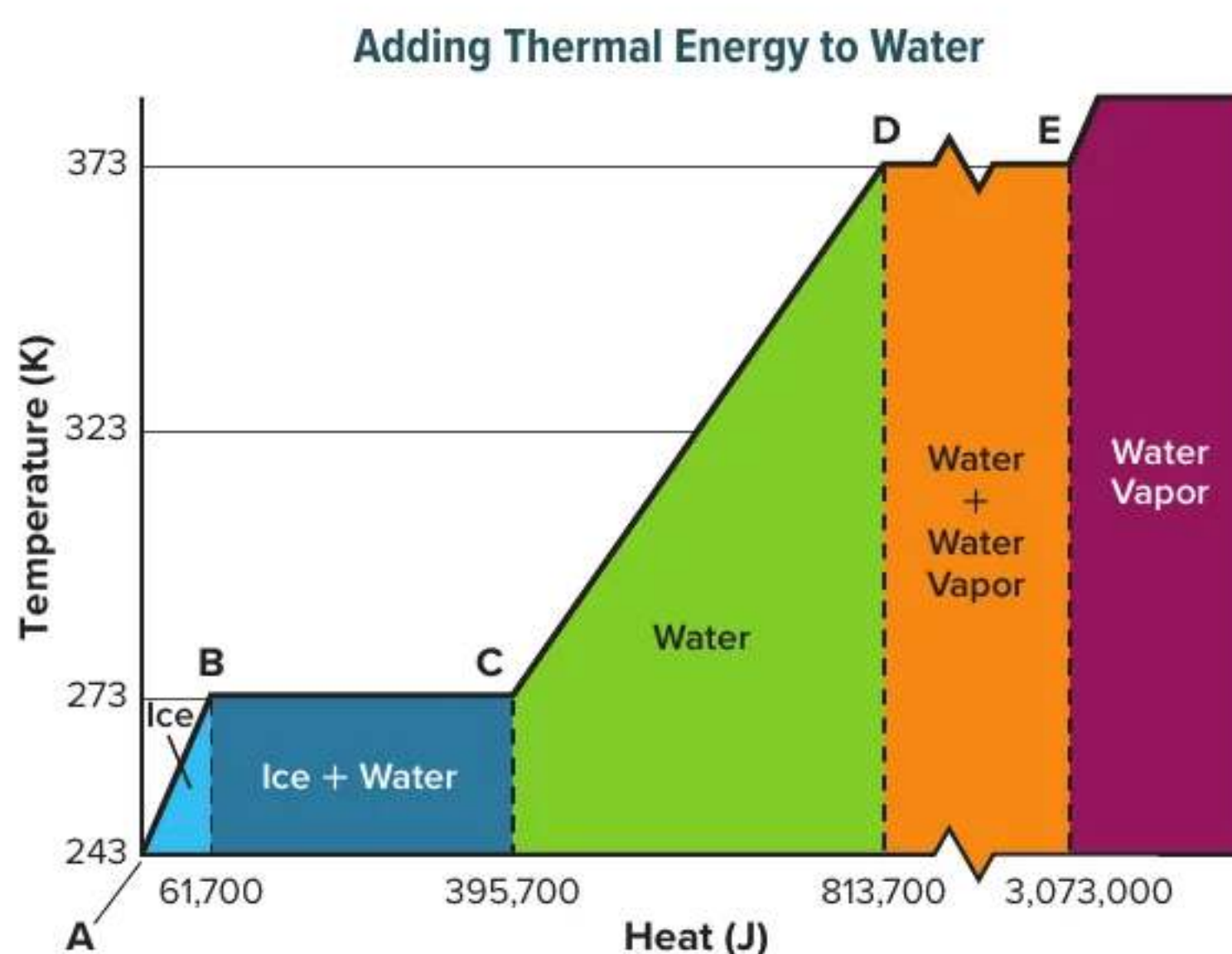


Figure 12 Thermal energy added to a substance can raise the temperature or cause a change in state. Note that the scale is broken between points D and E.



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



Applying Practices: Modeling Energy at Different Scales

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as a combination of energy associated with the motion of particles (objects) and energy associated with the relative positions of particles (objects).

Melting point At this point, water changes from a solid to a liquid, just as the snowman in **Figure 13** does. The temperature at which this change occurs is the melting point of a substance. When a substance melts, the addition of thermal energy allows the particles to move, rotate, and vibrate in ways not available in the solid. Each of these new motions might add new modes of kinetic or potential energy. The added thermal energy does not change the temperature of the material. This can be observed between points B and C in **Figure 12**, where the added thermal energy melts all the ice at a constant 273 K.



Figure 13 Thermal energy is transferred from the warmer air to the snowman, causing the snowman to melt.

Boiling point Once the ice is completely melted, adding more thermal energy increases the motion of the water molecules again. The temperature rises as shown between points C and D in **Figure 12**. As the temperature increases further, some of the particles that make up the liquid acquire enough energy to break free from the other particles.

At a specific temperature, known as the boiling point, adding more energy to a substance causes it to undergo another change of state. All the added thermal energy converts it from the liquid state to the gas state. As in melting, the temperature does not rise while a liquid boils, as shown between points D and E in **Figure 12**. After the water is entirely converted to gas, any added thermal energy again increases the motion of the molecules, and the temperature rises. After point E, the water vapor is heated to temperatures greater than 373 K.

Heat of fusion and heat of vaporization The amount of thermal energy needed to melt 1 kg of a substance is called the substance's **heat of fusion** (H_f). For ice, the heat of fusion is 3.34×10^5 J/kg. If 1 kg of ice at its melting point, 273 K, absorbs 3.34×10^5 J, the ice becomes 1 kg of water at the same temperature. The added energy causes a change in state but not in temperature. The horizontal distance from point B to point C in **Figure 12** represents the heat of fusion.

The thermal energy needed to vaporize 1 kg of a liquid is called the **heat of vaporization** (H_v). Water's heat of vaporization is 2.26×10^6 J/kg. The horizontal distance from point D to point E in **Figure 12** represents the heat of vaporization. Every material has a characteristic heat of fusion (H_f) and heat of vaporization (H_v). Some values are shown in **Table 2**.

Table 2 Heats of Fusion and Vaporization of Common Substances

Material	Heat of Fusion H_f (J/kg)	Heat of Vaporization H_v (J/kg)
Copper	2.05×10^5	5.07×10^6
Mercury	1.15×10^4	2.72×10^5
Gold	6.30×10^4	1.64×10^6
Methanol	1.09×10^5	8.78×10^5
Iron	2.66×10^5	6.29×10^6
Silver	1.04×10^5	2.36×10^6
Lead	2.04×10^4	8.64×10^5
Water (ice)	3.34×10^5	2.26×10^6

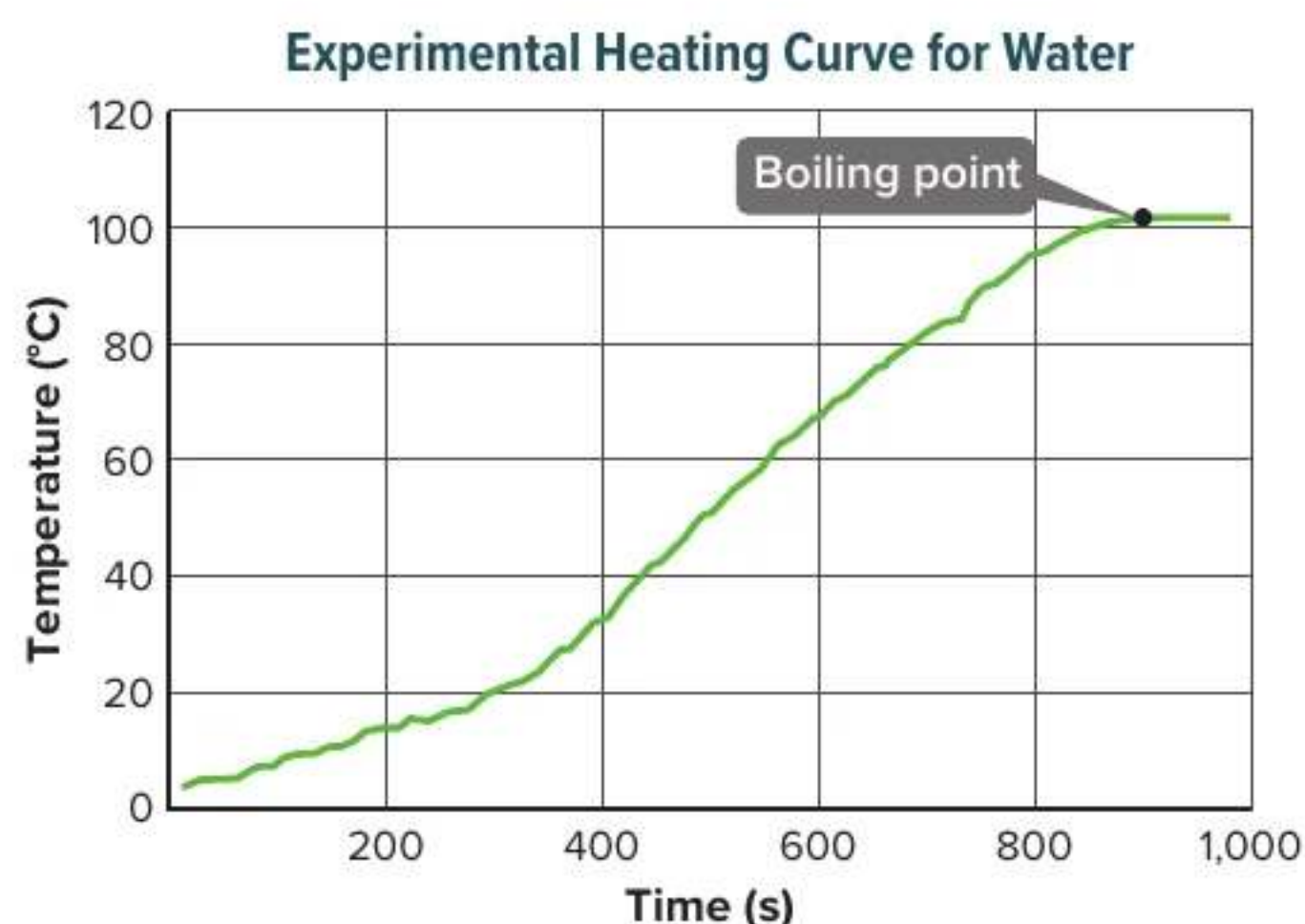


Figure 14 One way to measure the absorption of energy by a material is to add energy from a constant source of thermal energy and measure the change in temperature over time. The plot of temperature v. time is called a heating curve. For this figure, a beaker of cold water was placed on a hot plate. The resulting heating curve is graphed.

Explain why thermal energy must be added at a constant rate to calculate the specific heat of water from this graph.

Energy and changes of state There is a definite slope to the graph in **Figure 14** between about 300 s and 800 s. Heat is added at a constant rate, so this slope is proportional to the reciprocal of the specific heat of water. The slope between points A and B in **Figure 12** is proportional to the reciprocal of the specific heat of ice, and the slope above point E is proportional to the reciprocal of the specific heat of water vapor. The slope for water is less than those of both ice and water vapor. This is because water has a greater specific heat than does ice or water vapor. The heat (Q) required to melt a solid of mass (m) is given by the following equation.

Heat Required to Melt a Solid

The heat required to melt a solid is equal to the mass of the solid times the heat of fusion of the solid.

$$Q = mH_f$$

Similarly, the heat (Q) required to vaporize a mass (m) of liquid is given by the following equation.

Heat Required to Vaporize a Liquid

The heat required to vaporize a liquid is equal to the mass of the liquid times the heat of vaporization of the liquid.

$$Q = mH_v$$

When a liquid freezes, an amount of thermal energy ($Q = -mH_f$) must be removed from the liquid to turn it into a solid. The negative sign indicates that the thermal energy is transferred from the sample to the external world. In the same way, when a vapor condenses to a liquid, an amount of thermal energy ($Q = -mH_v$) must be removed from the vapor.

Water absorbs significant amounts of thermal energy when it melts or evaporates. Every day you use the large heats of fusion and vaporization of water. Each gram of sweat that evaporates from your skin carries off about 2.3 kJ of thermal energy. This is one cooling process that many warm-blooded animals use to regulate their body temperatures. Similarly the melting of a 24-g cube of ice absorbs enough thermal energy, 8 kJ, to cool a glass of water by about 30°C.

EXAMPLE Problem 3

HEAT Suppose that you are camping in the mountains. You need to melt 1.50 kg of snow at 0.0°C and heat it to 70.0°C to make hot cocoa. How much heat will you need?

1 ANALYZE AND SKETCH THE PROBLEM

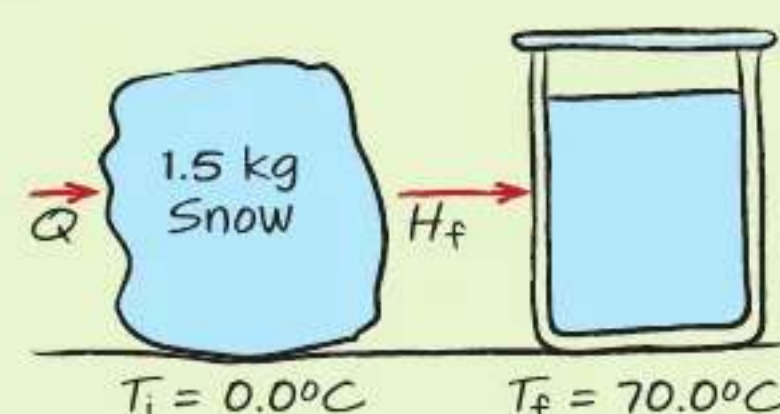
- Sketch the relationship between heat and water in its solid and liquid states.
- Sketch the transfer of heat as the temperature of the water increases.

Known

$$\begin{aligned} m &= 1.50 \text{ kg} & H_f &= 3.34 \times 10^5 \text{ J/kg} \\ T_i &= 0.0^\circ\text{C} & T_f &= 70.0^\circ\text{C} \\ C &= 4180 \text{ J/(kg}\cdot\text{K)} \end{aligned}$$

Unknown

$$\begin{aligned} Q_{\text{melt ice}} &= ? \\ Q_{\text{heat liquid}} &= ? \\ Q_{\text{total}} &= ? \end{aligned}$$

**2 SOLVE FOR THE UNKNOWN**

Calculate the heat needed to melt ice.

$$\begin{aligned} Q_{\text{melt ice}} &= mH_f \\ &= (1.50 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) \\ &= 5.01 \times 10^5 \text{ J} = 5.01 \times 10^2 \text{ kJ} \end{aligned}$$

Substitute $m = 1.50 \text{ kg}$, $H_f = 3.34 \times 10^5 \text{ J/kg}$.

Calculate the temperature change.

$$\begin{aligned} \Delta T &= T_f - T_i \\ &= 70.0^\circ\text{C} - 0.0^\circ\text{C} = 70.0^\circ\text{C} = 70.0\text{K} \end{aligned}$$

Substitute $T_f = 70.0^\circ\text{C}$, $T_i = 0.0^\circ\text{C}$. $^\circ\text{C} = \Delta\text{K}$

Calculate the heat needed to raise the water temperature.

$$\begin{aligned} Q_{\text{heat liquid}} &= mC\Delta T \\ &= (1.50 \text{ kg})(4180 \text{ J/(kg}\cdot\text{K)})(70.0\text{K}) \\ &= 4.39 \times 10^5 \text{ J} = 4.39 \times 10^2 \text{ kJ} \end{aligned}$$

Substitute $m = 1.50 \text{ kg}$, $C = 4180 \text{ J/(kg}\cdot\text{K)}$, $\Delta T = 70.0\text{K}$.

Calculate the total amount of heat needed.

$$\begin{aligned} Q_{\text{total}} &= Q_{\text{melt ice}} + Q_{\text{heat liquid}} \\ &= 5.01 \times 10^2 \text{ kJ} + 4.39 \times 10^2 \text{ kJ} \\ &= 9.40 \times 10^2 \text{ kJ} \end{aligned}$$

Substitute $Q_{\text{melt ice}} = 5.01 \times 10^2 \text{ kJ}$, $Q_{\text{heat liquid}} = 4.39 \times 10^2 \text{ kJ}$.

3 EVALUATE THE ANSWER

- **Are the units correct?** Energy units are in joules.
- **Does the sign make sense?** Q is positive when thermal energy is absorbed.
- **Is the magnitude realistic?** To check the magnitude, perform a quick estimation:
 $Q = (1.5 \text{ kg})(300,000 \text{ J/kg}) + (1.5 \text{ kg})(4000 \text{ J/(kg}\cdot\text{K)})(70 \text{ K}) = 9 \times 10^2 \text{ kJ}$.

PRACTICE Problems**ADDITIONAL PRACTICE**

- How much thermal energy is absorbed by $1.00 \times 10^2 \text{ g}$ of ice at -20.0°C to become water at 0.0°C ?
- A $2.00 \times 10^2 \text{-g}$ sample of water at 60.0°C is heated to water vapor at 140.0°C . How much thermal energy is absorbed?
- Use **Figure 15** to calculate the heat of fusion and heat of vaporization of water in J/kg.
- A steel plant worker will change 100.0 kg of 25°C iron into molten iron (melting point = 1538°C). How much thermal energy must be added?
- CHALLENGE** How much thermal energy is needed to change $3.00 \times 10^2 \text{ g}$ of ice at -30.0°C to water vapor at 130.0°C ?

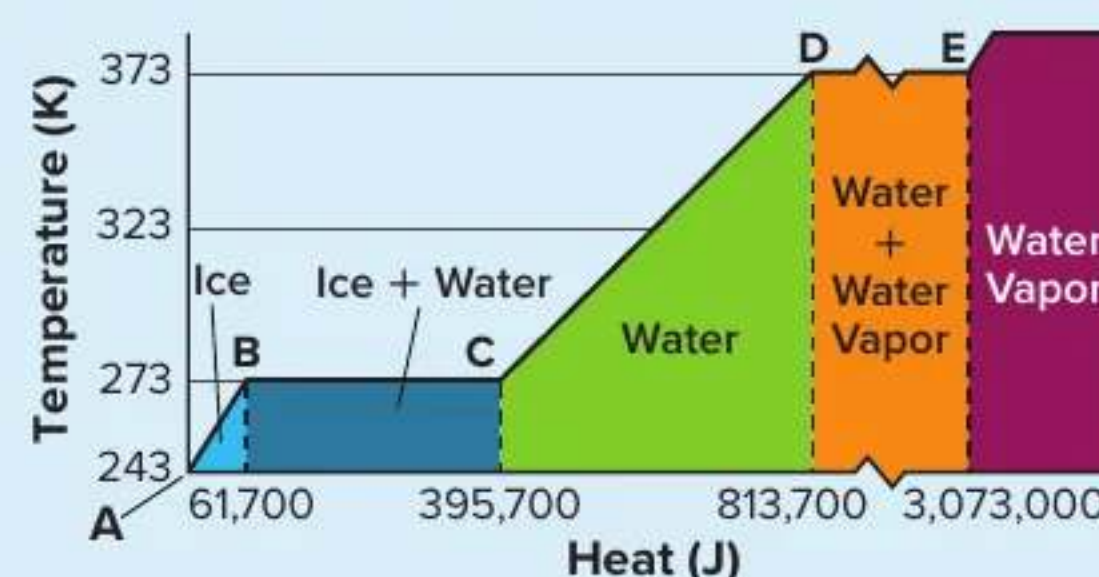


Figure 15

The First Law of Thermodynamics

The first steam engines were built in the eighteenth century and used to power trains and factories. Steam engines change thermal energy into mechanical energy. The invention of the steam engine contributed greatly to the Industrial Revolution and to the study of how heat is related to work. The study of how thermal energy is transformed into other forms of energy is called thermodynamics.

Before 1900, scientists did not realize that the concepts of thermodynamics were linked to the motions of particles in an object, and they considered thermodynamics to be a separate topic from mechanics. Today, engineers routinely apply the concepts of thermodynamics to develop higher performance refrigerators, automobile engines, aircraft engines, and numerous other kinds of machines.

The first law developed for thermodynamics was a statement about what thermal energy is and where it can go. As you know, you can raise the temperature of a glass of cold water by placing it on a hot plate or by stirring it. That is, you can increase the water's thermal energy by heating or by doing work on it. If we consider the system to be the water, the work the system does on you is equal to the negative of the work you do on the system. The **first law of thermodynamics** states that the change in thermal energy (ΔU) of an object is equal to the heat (Q) that is added to the object minus the work (W) done by the object. Note that ΔU , Q , and W are all measured in joules, the unit of energy.

The First Law of Thermodynamics

The change in thermal energy of an object is equal to the heat added to the object minus the work done by the object.

$$\Delta U = Q - W$$

The first law of thermodynamics is merely a restatement of the law of conservation of energy, which states that energy is neither created nor destroyed but can be transported from one place to another or transformed into another kind of energy. An example of transformed thermal energy in a system is a hand pump used to inflate a bicycle tire. As a person pumps, the air and the hand pump become warm. The mechanical energy in the moving piston is transformed into thermal energy of the gas. Similarly, other forms of energy, such as light, sound, and electrical energy, can be transformed into thermal energy. For example, a toaster transforms electrical energy into radiant energy when it toasts bread.

PRACTICE Problems



ADDITIONAL PRACTICE

24. A gas balloon absorbs 75 J of thermal energy. The balloon expands but stays at the same temperature. How much work did the balloon do in expanding?
25. A drill bores a small hole in a 0.40-kg block of aluminum and heats the aluminum by 5.0°C. How much work did the drill do in boring the hole?
26. How many times would you have to drop a 0.50-kg bag of lead shot from a height of 1.5 m to heat the shot by 1.0°C?
27. When you stir a cup of tea, you do about 0.050 J of work each time you circle the spoon in the cup. How many times would you have to stir the spoon to heat a 0.15-kg cup of tea by 2.0°C?
28. **CHALLENGE** An expansion valve does work on 100 g of water. The system is isolated, and all of the work is used to convert the 90°C water into water vapor at 110°C. How much work does the expansion valve do on the water?

Heat engines A **heat engine** is a device that is able to continuously convert thermal energy to mechanical energy. A heat engine requires a high-temperature source, a low-temperature receptacle, called a sink, and a way to convert the thermal energy into work, as shown in **Figure 16**.

Internal combustion engines An automobile's internal combustion engine, shown in **Figure 17**, is one example of a heat engine. In the engine, input heat (Q_H) is transferred from a high-temperature flame to a mixture of air and gas vapor in the cylinder. The hot air expands and pushes on a piston, thereby changing thermal energy into mechanical energy. The heated air is expelled, the piston returns to the top of the cylinder, and the cycle repeats. Car engines repeat this cycle many times each minute. The thermal energy from the flame is converted into mechanical energy, which propels the car.

Waste heat Not all thermal energy from the flame is converted into mechanical energy. When the engine is running, the gases and the engine parts become hot. The exhaust comes in contact with outside air and heats the air. In addition, thermal energy from the engine is transferred to a radiator. Outside air passes through the radiator and becomes warmer. All of this energy (Q_C) transferred out of the automobile engine is called waste heat. When the engine is working continuously, its internal energy does not change. That is, $\Delta U = 0 = Q - W$. The net heat going into the engine is $Q = Q_H - Q_C$. Thus, the work done by the engine is $W = Q_H - Q_C$. All heat engines generate waste heat, and no engine can convert all the energy into useful kinetic energy.

Energy Diagram of a Heat Engine

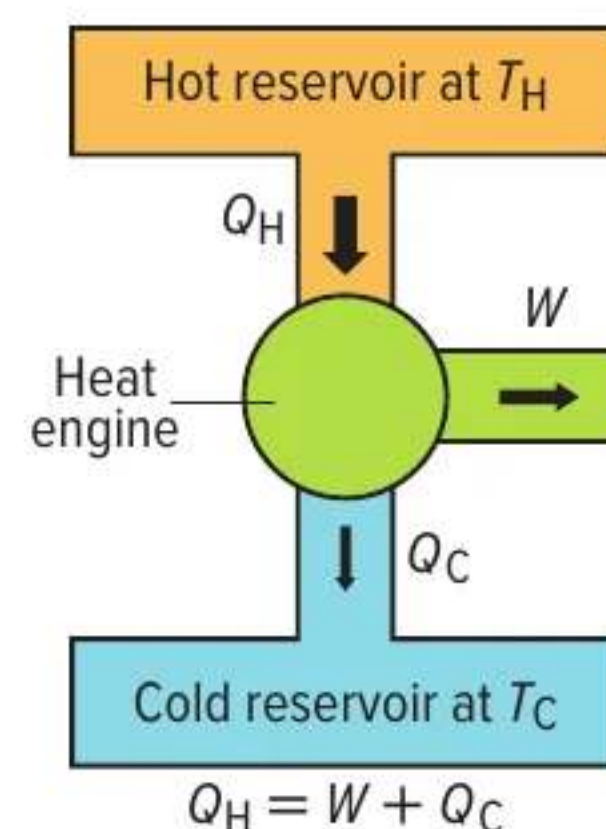
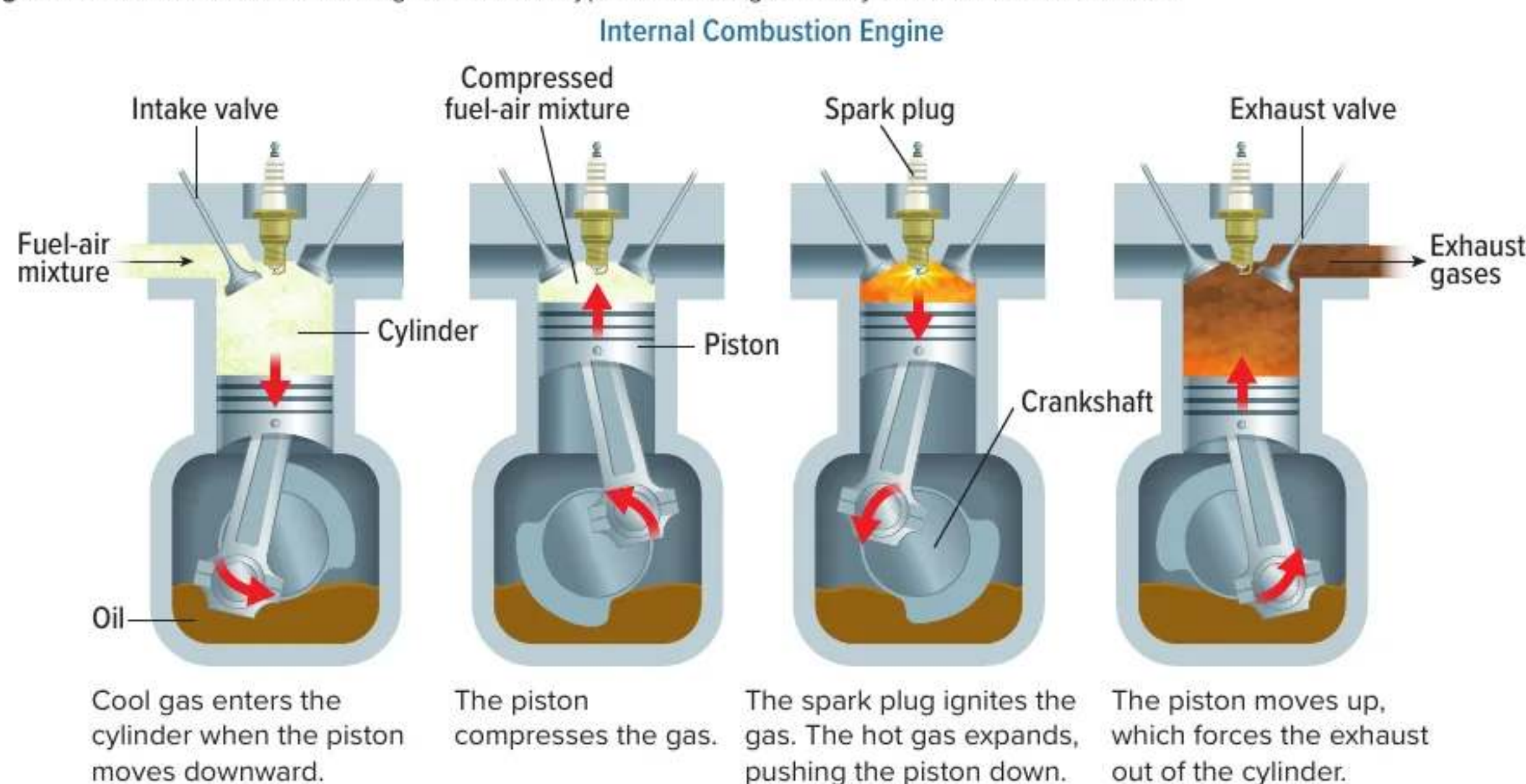


Figure 16 Heat engines transform thermal energy into mechanical energy and waste heat. This schematic shows the energy transfers and transformations.

Figure 17 Internal combustion engines are one type of heat engine. They are used in automobiles.



Efficiency Engineers and car salespeople often discuss the fuel efficiencies of automobile engines. They are referring to the amount of the input heat (Q_H) that is turned into useful work (W). The actual efficiency of an engine is given by the ratio W/Q_H . If all the input heat could be turned into useful work by the engine, the engine would have an efficiency of 100 percent. Because there is always waste heat (Q_C), even the most efficient engines fall short of 100 percent efficiency.

In fact, most heat engines are significantly less than 100 percent efficient. For example, even the most efficient automobile gasoline engines have an efficiency of less than 40 percent. A typical gasoline engine in an automobile has an efficiency that is closer to 20 percent. Energy is not destroyed, but a considerable amount of thermal energy transfers from a hot automobile engine to the surrounding environment.



Get It?

Describe an example of another instance where energy is converted to a less useful form.

Refrigerators Thermal energy is transferred from a warm object to a cold object spontaneously. But it is also possible to remove thermal energy from a colder object and add it to a warmer object if work is done. A refrigerator, such as the one in **Figure 18**, is a common example of a device that accomplishes this transfer. Electrical energy runs a motor that does work on a gas and compresses it.

The gas draws thermal energy from the interior of the refrigerator. It passes from the compressor through the condenser coils on the outside of the refrigerator. The gas cools into a liquid. Thermal energy is transferred into the air in the room. The liquid reenters the interior, vaporizes, and absorbs thermal energy from its surroundings. The gas returns to the compressor, and the process is repeated. The overall change in the thermal energy of the gas is zero. Thus, according to the first law of thermodynamics, the sum of the thermal energy removed from the refrigerator's contents and the work done by the motor is equal to the thermal energy expelled. These energy transfers and transformations are shown in **Figure 19**.

Heat pumps A heat pump is a refrigerator that can be run in two directions. In summer, the pump removes thermal energy from a house and cools the house. In winter, thermal energy is removed from the cold outside air and transferred into the warmer house. In both cases, mechanical energy is required to transfer thermal energy from a colder object to a warmer one.

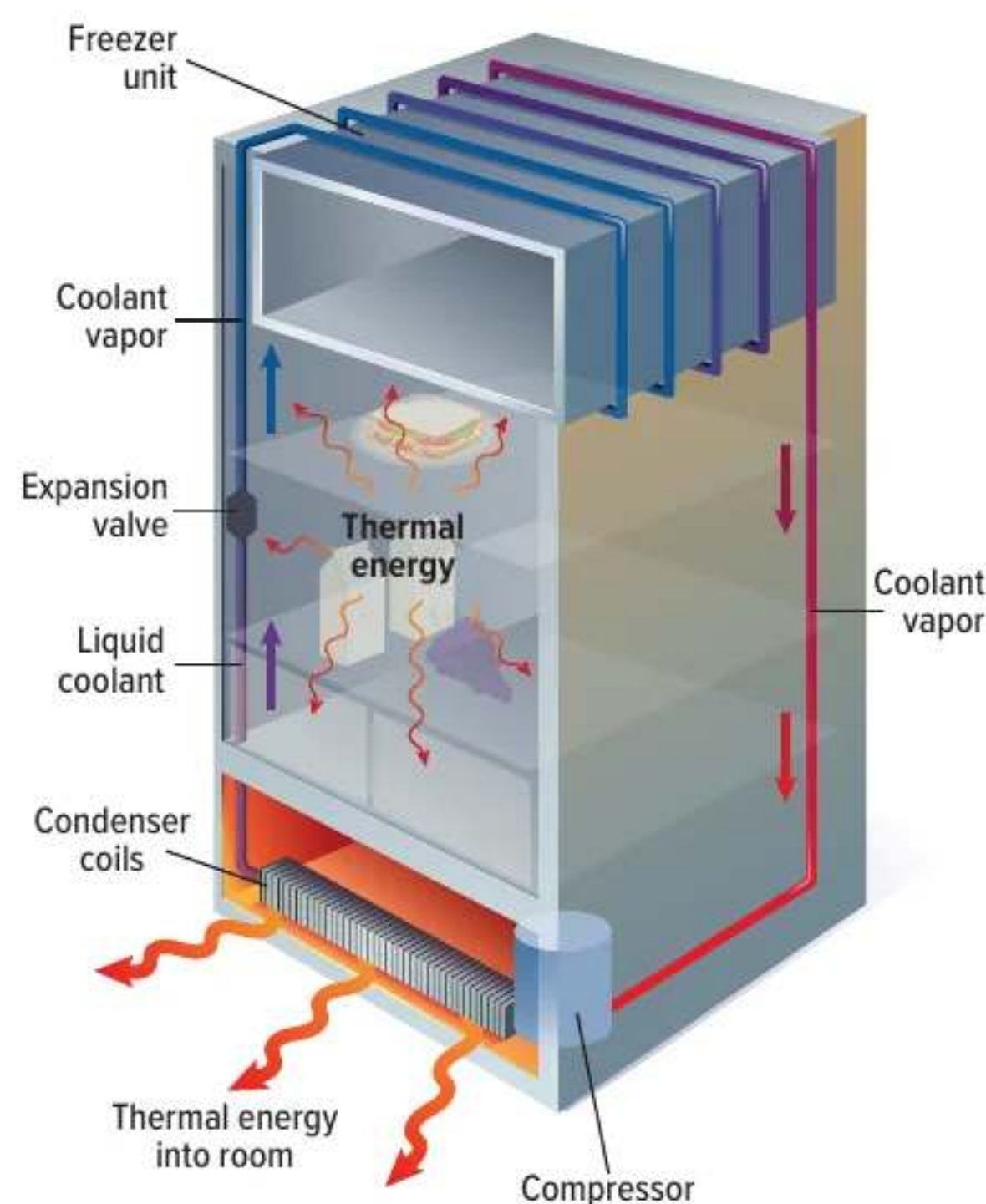


Figure 18 Liquid coolant is pumped into an expansion valve, where it absorbs energy from its surroundings and becomes a gas. The gas then heats up as it absorbs thermal energy from inside the refrigerator. A compressor does work on the gas to cool it to a liquid, and the cycle begins again.

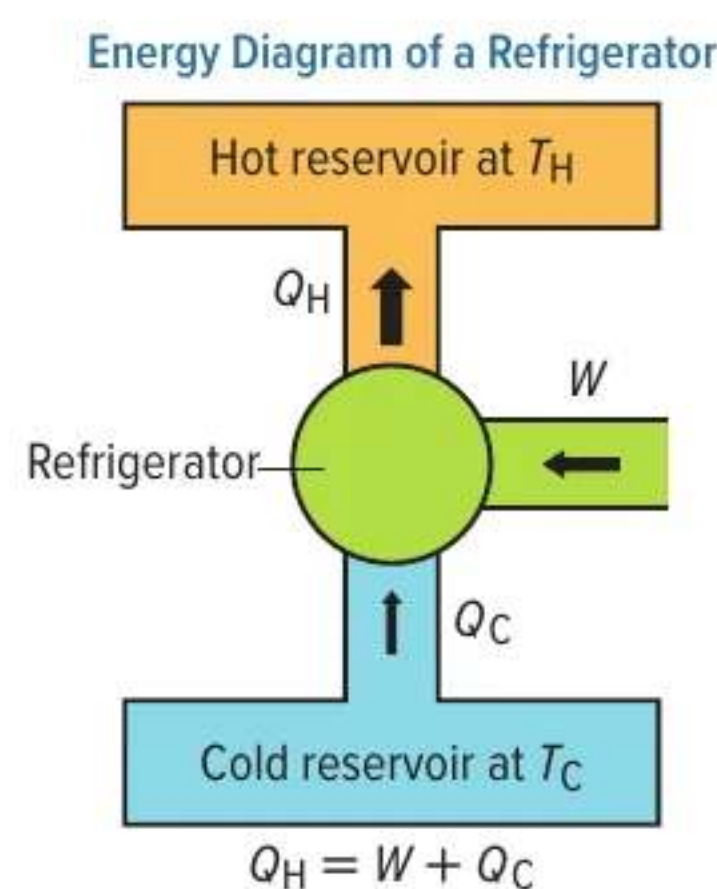


Figure 19 When work is done on the refrigerator, thermal energy is transferred from the cold reservoir to the hot reservoir.

Second Law of Thermodynamics

You are not surprised when a metal spoon, heated at one end, soon becomes uniformly hot. Consider your reaction, however, if a spoon lying on a table suddenly, on its own, became red hot at one end and icy cold at the other. This reverse process would not violate the first law of thermodynamics—the thermal energy of the spoon would remain the same. Many processes that are consistent with the first law of thermodynamics have never been observed to occur spontaneously. However, there is more to modeling thermal events than making sure energy is conserved.

Energy spreads out Examine the melting ice pop and the cooling pizza in **Figure 20**. The first law of thermodynamics does not prohibit net thermal energy transfers from the cold ice pop to the air or from the air to the hot pizza. This does not occur, however, because of the second law of thermodynamics. When a hot object is placed in contact with cooler surroundings, the thermal energy in the hot object has the opportunity to disperse, or spread out more. Some of the thermal energy moves into the cold object, warming it and therefore cooling the originally hotter object. The **second law of thermodynamics** states that in an uncontrolled system whenever there is an opportunity for energy dispersal, the energy always spreads out toward more uniform energy distribution and to a more stable state.

Consider the cooling pizza. The particles in the pizza have a greater average kinetic energy than the particles in the air. Some of the pizza's original thermal energy disperses into the air. As a result, the pizza's temperature decreases and the air temperature increases a small amount. When the pizza and the air reach the same temperature, the average kinetic energy of the particles in the pizza and the air will be the same. That is, the energy spreads out among the particles. Similarly, if you leave the ice pop sitting on the counter, thermal energy from the air will be dispersed to the ice pop. The ice pop will heat up and melt, while the air will experience a small temperature decrease.



Get It?

Explain how the pizza in **Figure 20** represents an uncontrolled system and how the thermal energy became more uniformly distributed.

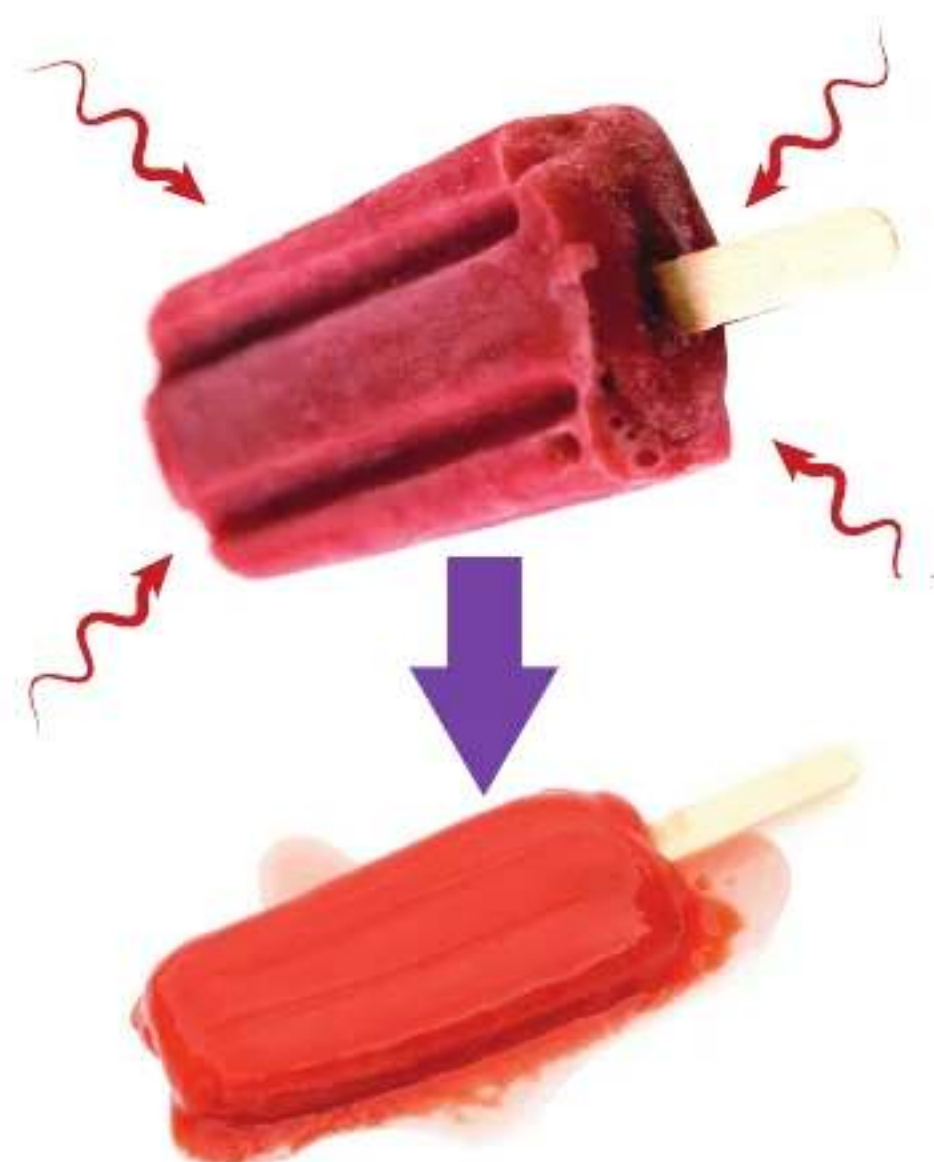


Figure 20 According to the second law of thermodynamics, thermal energy always spreads out. The red arrows represent thermal energy flow. The purple arrows represent time lapses. Thermal energy spontaneously flows from a warmer object to a colder object.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Using **Figure 20**, describe a system surrounding the pizza. Describe the system boundaries, inputs, and outputs in your defined system.

Entropy The measure of this dispersal of energy is known as **entropy** (S). A system in which the thermal energy is concentrated in one place is referred to as a system with low entropy. A system in which the thermal energy is spread throughout the system has high entropy.

Another way of stating the second law of thermodynamics is that natural processes go in a direction that maintains or increases the total entropy of the universe. That is, energy will naturally disperse unless some action is taken to localize it. Once a system is in a high-entropy state, it is highly unlikely that it will return to a lower entropy state on its own. Events that occur spontaneously, such as the melting ice pop or the cooling pizza, are events in which the entropy of the system increases. Processes that would decrease the entropy of a system do not tend to occur spontaneously but require work done by an external agent.



Get It?

State the second law of thermodynamics using the term *entropy*.

Entropy and heat engines How does entropy relate to heat engines? If heat engines completely converted thermal energy into mechanical energy with no waste heat, energy would still be conserved, and so the first law of thermodynamics would be obeyed. However, waste heat is always generated, dispersing thermal energy beyond the engine. In the nineteenth century, French engineer Sadi Carnot studied the ability of engines to convert thermal energy into mechanical energy. He developed a logical proof that even an ideal engine would generate some waste heat. Carnot's result was one of the first formal analyses leading to the development of the concept of entropy.

Changes in entropy Like energy, entropy is a property of a system. If thermal energy is added to a system, the entropy increases. If thermal energy is removed from a system, its entropy decreases. If a system does work on its surroundings without any transfer of thermal energy, the entropy does not change. For a reversible process, the change in entropy (ΔS) is expressed by the following equation. Entropy has units of J/K. The temperature, in kelvins, is constant.

Change in Entropy

For a reversible process, the change in entropy of a system is equal to the heat added to the system divided by the temperature of the system in kelvins.

$$\Delta S = \frac{Q}{T}$$

PHYSICS Challenge

Entropy has some interesting properties. Calculate the change in entropy for the following situations. Explain how and why these changes in entropy are different from each other. For these small temperature changes, you can use the original temperature to find the change in entropy.

1. Heating 1.0 kg of water from 273 K to 274 K.
2. Heating 1.0 kg of water from 353 K to 354 K.
3. Heating 1.0 kg of lead from 273 K to 274 K.
4. Completely melting 1.0 kg of ice at 273 K.

Entropy and the energy crisis The second law of thermodynamics and the increase in entropy are exemplified by the energy crisis. The energy crisis refers to the continued use of limited resources such as petroleum. When you use a resource, you do not use up the energy in the resource. For example, when you drive a car, such as those in **Figure 21**, the gas ignites and the chemical energy contained in the molecules of the gas is converted into the kinetic energy that runs the car and the thermal energy that heats the engine. The chemical energy of the unburned gas has dispersed into many more objects contained within a much larger volume. While it is mathematically possible for all of the dispersed energy to be brought back together in one object, the probability of this happening is very near zero. For this reason, entropy often is used as a measure of the unavailability of useful energy. The energy in the warmed air in a home is not as available to do mechanical work or to transfer thermal energy to other objects as the energy in the original gas molecules was. The lack of usable energy is actually a surplus of entropy.



Figure 21 Burning gasoline uses up natural resources and increases entropy but does not use up energy. The energy is no longer in a useful form.

Check Your Progress

29. **Heat Engine** Describe the energy transformations and transfers made by a heat engine, and explain why operating a heat engine causes an increase in entropy.
30. **Heat of Vaporization** Old heating systems sent water vapor into radiators in each room of a house. In the radiators, the water vapor condensed to water. Analyze this process and explain how it heated a room.
31. **Heat of Fusion** How much thermal energy is needed to change 50.0 g of ice at -20.0°C to water at 10.0°C ?
32. **Heat of Vaporization** How much energy is needed to heat 1.0 kg of mercury metal from 10.0°C to its boiling point (357°C) and vaporize it completely? For mercury, $C = 140 \text{ J/kg}\cdot^{\circ}\text{C}$ and $H_v = 3.06 \times 10^5 \text{ J/kg}$.
33. **Mechanical Energy and Thermal Energy** A man uses a 320-kg hammer moving at 5.0 m/s to smash a 3.0-kg block of lead against a 450-kg rock. When he measured the temperature of the lead block, he found that it had increased by 5.0°C . Explain how this happened.
34. **Mechanical Energy and Thermal Energy** James Joule carefully measured the difference in temperature of water at the top and the bottom of a waterfall. Why did he expect a difference?
35. **Mechanical Energy and Thermal Energy** For the waterfall in **Figure 22**, calculate the temperature difference between the water at the top and the bottom of the fall. Assume that the potential energy of the water is all converted to thermal energy.



Figure 22

36. **Entropy** Evaluate why heating a home with natural gas results in increased entropy.
37. **Critical Thinking** Many outdoor amusement parks and zoos have systems that spray a fine mist of water, which evaporates quickly. Explain why this process cools the surrounding air.

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Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

ENGINEERING & TECHNOLOGY

Under Pressure

Diesel engines, like gasoline engines, are internal combustion engines in which the fuel is burned inside the engine. They both convert the chemical energy in fuel into the mechanical energy that moves the vehicle. However, diesel engines are more powerful and efficient than gasoline engines because of the way the engine burns fuel.

Diesel Versus Gasoline Engines

In gasoline engines, air and fuel are injected into the engine's cylinders. The piston rises up the cylinder, compressing the air and fuel and making them explosive. When the piston reaches the top of the cylinder, the car's spark plugs provide the electricity needed to make the air and fuel explode. The explosion pushes the piston back down the cylinder. The piston is connected to the crankshaft, so when it travels downward, it pushes the crankshaft, which turns the wheels of the vehicle. As the crankshaft turns, it pushes the piston up the cylinder again, and the cycle repeats.

In a diesel engine, the process is slightly different. First, air is injected into the cylinder. The piston then compresses the air with much more force than it is compressed in a gasoline engine. The air produces so much heat as it is compressed that, when fuel is sprayed into the cylinder, there is no need for a spark plug to ignite it with electricity. Instead, the fuel instantly explodes, driving the piston back down the cylinder and turning the crankshaft.



Diesel engines differ from gasoline engines in several crucial ways that increase their efficiency and power.

Diesel Fuel Versus Gasoline

Both gasoline and diesel fuel are made from crude oil. However, the fuels are made from different hydrocarbons that are separated during the refining process. Gasoline is lighter and less oily than diesel fuel because it contains fewer carbon and hydrogen atoms, but it also has a lower energy density than diesel fuel. Diesel fuel's high energy density contributes to its greater power and efficiency.

Historically, one downside of diesel fuel is that it contributes to air pollution by releasing large amounts of particulate matter and nitrogen compounds into the atmosphere. However, improved refining techniques and new technologies are helping to lessen pollution caused by diesel fuel.



DEVELOP AND USE MODELS TO
ILLUSTRATE

Work with a partner to create a 2D or 3D model of the combustion cycle of a gasoline or diesel engine. Use sources for additional information. Display your model in your classroom.

MODULE 11

STUDY GUIDE

 **GO ONLINE** to study with your Science Notebook.

Lesson 1 TEMPERATURE, HEAT, AND THERMAL ENERGY

- Thermal energy is the sum of the kinetic and potential energies of an object's particles. An object's temperature is a measure of the average kinetic energy of its particles.
- When two objects are in thermal equilibrium, there is no net transfer of thermal energy between the objects, and the two objects are at the same temperature. A thermometer measures temperature by reaching thermal equilibrium with its surroundings. When an object's temperature is at absolute zero, the average kinetic energy of its particles is zero, and that object cannot transfer thermal energy.
- Heat is the transfer of thermal energy. Thermal energy is spontaneously transferred from a warm object to a cool object. Thermal energy is transferred by three processes: conduction, convection, and radiation.
- Substances heat differently, based on their specific heats. Specific heat (C) is the heat required to raise the temperature of 1 kg of a substance 1 K.

$$Q = mC \Delta T = mC(T_f - T_i)$$

- A calorimeter is a closed system used to measure changes in thermal energies. Specific heat is calculated by using measurements from a calorimeter.

- thermal conduction
- thermal equilibrium
- heat
- convection
- radiation
- specific heat

Lesson 2 CHANGES OF STATE AND THERMODYNAMICS

- Thermal energy transferred during a change of state does not change the temperature of a substance. The heat of fusion is the quantity of heat needed to change 1 kg of a substance from a solid state to a liquid state at its melting point.

$$Q = mH_f$$

- The heat of vaporization is the quantity of heat needed to change 1 kg of a substance from a liquid state to a gaseous state at its boiling point.

$$Q = mH_v$$

- The first law of thermodynamics states that the change in the thermal energy of an object is equal to the heat added to the object minus the work done by the object.

$$\Delta U = Q - W$$

- A heat engine converts thermal energy to mechanical energy. A heat pump and a refrigerator use mechanical energy to transfer thermal energy from a region of lower temperature to one of higher temperature.
- The second law of thermodynamics states that whenever there is an opportunity for energy dispersal, the energy always spreads out. Entropy (S) is a measure of the energy dispersal of a system. The second law of thermodynamics indicates that natural processes go in a direction that maintains or increases the total entropy of the universe. The change in entropy of an object is defined as the heat added to the object divided by the object's temperature.

- heat of fusion
- heat of vaporization
- first law of thermodynamics
- heat engine
- second law of thermodynamics
- entropy



THREE-DIMENSIONAL THINKING Module Wrap-Up

REVISIT THE PHENOMENON

Why do different parts of the pan appear to be different colors in this thermal image?



CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.



STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

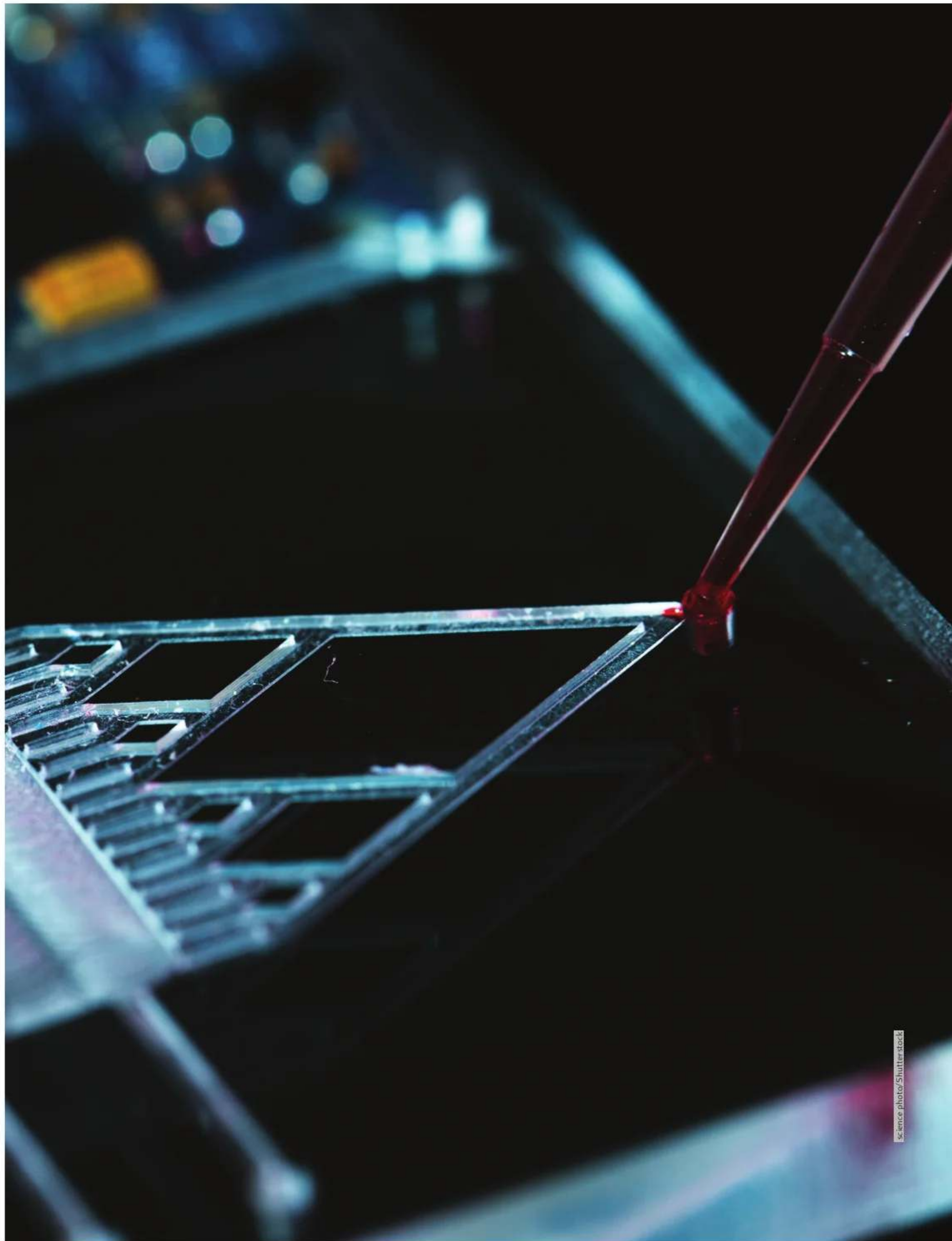
How do materials differ in how fast they heat?

The following materials are each placed in identical containers holding equal amounts of room-temperature methanol.

Material	Mass (g)	Initial Temperature (°C)
Aluminum	50	30
Aluminum	60	40
Glass	50	30
Silver	50	30
Zinc	50	30

CER Analyze and Interpret Data

- Claim** Rank the materials according to the amount of thermal energy they transfer to the methanol, from least to greatest. Specifically indicate any ties.
- Evidence and Reasoning** Explain how you decided your rankings.



MODULE 12

STATES OF MATTER

ENCOUNTER THE PHENOMENON

How will the fluid flow through this device?



 **GO ONLINE** to play a video about how air flows through a pipe.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about how the fluid will flow through the device. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

 **GO ONLINE** to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain:
Liquids and Gases



LESSON 4: Explore & Explain:
Solid Bodies



Additional Resources

LESSON 1

PROPERTIES OF FLUIDS

FOCUS QUESTION

What is a fluid and what are its properties?

Liquids and Gases

Water and air are common substances in the everyday lives of people. Both water and air flow, and unlike solids, neither one has a definite shape. Gases and liquids are two states of matter in which atoms and molecules have great freedom to move.

Fluids Like ice cubes, the chunks of ice in **Figure 1** have a certain mass and a certain shape that do not depend on the size or shape of the lake basin. When the ice melts, its mass remains the same, but its shape changes. The water flows to take the shape of the basin and forms a definite, flat, upper surface. As evaporation occurs, the liquid water changes into a gas in the form of water vapor. Like liquid water, water vapor flows and does not have any definite shape. Both liquids and gases are fluids. **Fluids** are materials that can flow and have no definite shape of their own.



Figure 1 The chunks of ice in this lake, which are solids, have definite shapes. However, the liquid water, a fluid, takes the shape of the lake basin.

Identify What fluid is filling the space above the water?

Glow Images/Superstock



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



Quick Investigation: Pressure

Analyze data to determine how pressure is affected by the force an object exerts over an area.



Review the News

Obtain information from a current news story about the properties or behavior of fluids. Evaluate your source and communicate your findings to your class.

Pressure

When considering fluids (as well as solids), it is often useful to think about pressure as well as force. You have probably heard people talk about water pressure and air pressure, and you might already have a sense that pressure and force are related. Pressure and force are not the same, however. **Pressure** is the perpendicular component of a force on a surface divided by the area of the surface. Since pressure is force exerted on a surface, anything that exerts pressure is capable of producing change and doing work. In **Figure 2**, both the astronaut and the legs of the landing module are exerting pressure on the Moon's surface.

Pressure

Pressure equals the perpendicular component of the force divided by the surface area to which it is applied.

$$P = \frac{F}{A}$$

Pressure is a scalar. In the SI system, the unit of pressure is the **pascal** (Pa), which is 1 N/m^2 . One pascal is a very small amount of pressure, about equal to the pressure that a flat dollar bill exerts on a tabletop. Thus the kilopascal (kPa), equal to 1000 Pa, is usually used. **Table 1** shows pressures in various locations.

Solids, liquids, and pressure Imagine you are standing on the surface of a frozen lake. The forces your feet exert on the ice are spread over the area of your shoes, resulting in pressure on the ice. Ice is a solid that is made up of vibrating water molecules, and the forces that hold the water molecules in place cause the ice to exert upward forces on your feet that equal your weight. If the ice melted, most of the bonds between the water molecules would be weakened.

Although the molecules would continue to vibrate and remain close to each other, they also would slide past one another, and you would break through the surface. The moving water molecules would continue to exert forces on your body.

Gases and pressure The pressure exerted by a gas can be understood by applying the kinetic-molecular theory of gases, which explains the properties of an ideal gas. In this model, particles are treated as taking up no space and having no intermolecular attractive forces. In spite of the fact that particles of a real gas take up space and exert attractive forces, an ideal gas is an accurate model of a real gas under most conditions.

NASA Johnson Space Center (NASA-JSC)



Figure 2 The landing module and the astronaut both exert pressure on the surface of the Moon.

Calculate If the lunar module weighed approximately 12,000 N and rested on four pads that were each 91 cm in diameter, what pressure did it exert on the Moon's surface? How could you estimate the pressure exerted by the astronaut?

Table 1 Some Typical Pressures

Location	Pressure (kPa)
The center of Earth	4×10^8
The deepest ocean trench	1.1×10^5
Standard atmosphere	1.01325×10^2
Blood pressure	1.6×10^1
Air pressure on top of Mt. Everest	3×10^1
The best vacuum	1×10^{-10}

CCC CROSSCUTTING CONCEPTS

Structure and Function The molecular structure of a material at a given temperature and pressure allows you to predict its physical state and some of its properties. Make a model to provide evidence that the properties of fluids are based on their molecular structures.

According to the kinetic-molecular theory, the particles in a gas are in random motion at high speeds and colliding elastically with each other. When a gas particle hits a container's surface, it rebounds, which changes its momentum. The impulses exerted by these collisions result in gas pressure on the surface.

Atmospheric pressure At sea level, gases of the atmosphere exert a force in all directions of approximately 10 N, about the weight of a 1-kg object, on each square centimeter of surface area. This atmospheric pressure on your body is so well balanced by your body's outward forces that you seldom notice it. You probably become aware of this pressure only when your ears pop as the result of pressure changes, as when you ride an elevator in a tall building or fly in an airplane. Atmospheric pressure is about 10 N per 1 cm² (10⁻⁴ m²), which is about 1.0×10⁵ N/m², or 100 kPa. Other planets in our solar system also have atmospheres that exert pressure. For example, the pressure at the surface of Venus is about 92 times that at Earth's surface.

EXAMPLE Problem 1

CALCULATING PRESSURE A child weighs 364 N and sits on a three-legged stool, which weighs 41 N. The bottoms of the stool's legs touch the ground over a total area of 19.3 cm².

- What is the average pressure that the child and the stool exert on the ground?
- How does the pressure change when the child leans over so that only two legs of the stool touch the floor?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the child and the stool, labeling the total force that they exert on the ground.
- List the variables, including the force that the child and the stool exert on the ground and the areas for parts **a** and **b**.

Known

$$F_{g \text{ child}} = 364 \text{ N}$$

$$F_{g \text{ stool}} = 41 \text{ N}$$

$$F_{g \text{ total}} = F_{g \text{ child}} + F_{g \text{ stool}}$$

$$= 364 \text{ N} + 41 \text{ N}$$

$$= 405 \text{ N}$$

$$A_a = 19.3 \text{ cm}^2$$

$$A_b = \frac{2}{3} \times 19.3 \text{ cm}^2$$

$$= 12.9 \text{ cm}^2$$

Unknown

$$P_a = ?$$

$$P_b = ?$$



2 SOLVE FOR THE UNKNOWN

Find each pressure.

$$P = \frac{F}{A}$$

$$\text{a. } P_a = \left(\frac{405 \text{ N}}{19.3 \text{ cm}^2} \right) \left(\frac{(100 \text{ cm})^2}{(1 \text{ m})^2} \right)$$

$$= 2.10 \times 10^2 \text{ kPa}$$

$$\text{b. } P_b = \left(\frac{405 \text{ N}}{12.9 \text{ cm}^2} \right) \left(\frac{(100 \text{ cm})^2}{(1 \text{ m})^2} \right)$$

$$= 3.14 \times 10^2 \text{ kPa}$$

$$\text{Substitute } F = F_{g \text{ total}} = 405 \text{ N}, A = A_a = 19.3 \text{ cm}^2.$$

$$\text{Substitute } F = F_{g \text{ total}} = 405 \text{ N}, A = A_b = 12.9 \text{ cm}^2.$$

3 EVALUATE THE ANSWER

- Are the units correct?** The units for pressure should be Pa, and 1 N/m² = 1 Pa.

PRACTICE Problems

ADDITIONAL PRACTICE

1. The atmospheric pressure at sea level is about 1.0×10^5 Pa. What is the force at sea level that air exerts on the top of a desk that is 152 cm long and 76 cm wide?
2. A car tire makes contact with the ground on a rectangular area of 12 cm by 18 cm. If the car's mass is 925 kg, what pressure does the car exert on the ground as it rests on all four tires?
3. A lead brick, $5.0 \text{ cm} \times 10.0 \text{ cm} \times 20.0 \text{ cm}$, rests on the ground on its smallest face. Lead has a density of 11.8 g/cm^3 . What pressure does the brick exert on the ground?
4. Suppose that during a storm, the atmospheric pressure suddenly drops by 15 percent outside. What net force would be exerted on a front door to a house that is 195 cm high and 91 cm wide? In what direction would this force be exerted?
5. **CHALLENGE** Large pieces of industrial equipment are placed on wide steel plates that spread the weight of the equipment over larger areas. If an engineer plans to install a 454-kg device on a floor that is rated to withstand additional pressure of 5.0×10^4 Pa, how large should the steel support plate be?

The Gas Laws

Think about a container of gas that is held at a constant temperature. If you reduced the volume, what would happen to the pressure of the gas? There would be more collisions between the particles and the container's walls, and so the pressure would increase. Similarly, if you increased the volume, there would be fewer collisions, decreasing the pressure. This inverse relationship was found by seventeenth-century chemist and physicist Robert Boyle. Because the product of inversely related variables is a constant, Boyle's law can be written $PV = \text{constant}$, or $P_1V_1 = P_2V_2$. The subscripts that you see in the gas laws help keep track of different variables, such as pressure and volume, as they change throughout a problem. The relationship between the pressure and the volume of a gas is critical to the scuba diver in **Figure 3**.

About 100 years after Boyle's work, Jacques Charles cooled a gas and found that the volume shrank by $\frac{1}{273}$ of its original volume for every degree cooled, which is a linear relationship. At the time, Charles could not cool gases to the extremely low temperatures achieved in modern laboratories. In order to see what lower limits might be possible, he extended, or extrapolated, the graph of his data to these temperatures. This extrapolation suggested that if the temperature were reduced to -273°C , a gas would have zero volume. The temperature at which a gas would have zero volume is now called absolute zero, which is represented by the zero of the Kelvin temperature scale.

These experiments indicated that under constant pressure, the volume of a sample of gas varies directly with its Kelvin temperature, a result that is now called Charles's law. Charles's law can be written $\frac{V}{T} = \text{constant}$, or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$.

Combined gas law Combining Boyle's law and Charles's law relates the pressure, temperature, and volume of a fixed amount of ideal gas, which leads to the **combined gas law**.

Combined Gas Law

For a fixed amount of an ideal gas, the pressure times the volume, divided by the Kelvin temperature equals a constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \text{constant}$$



Figure 3 The gas in the tank on a diver's back is at high pressure. The regulator in the diver's mouth reduces the pressure, making the pressure of the gas the diver breathes equal to that of the water pressure. Bubbles are emitted from the regulator as the diver exhales.

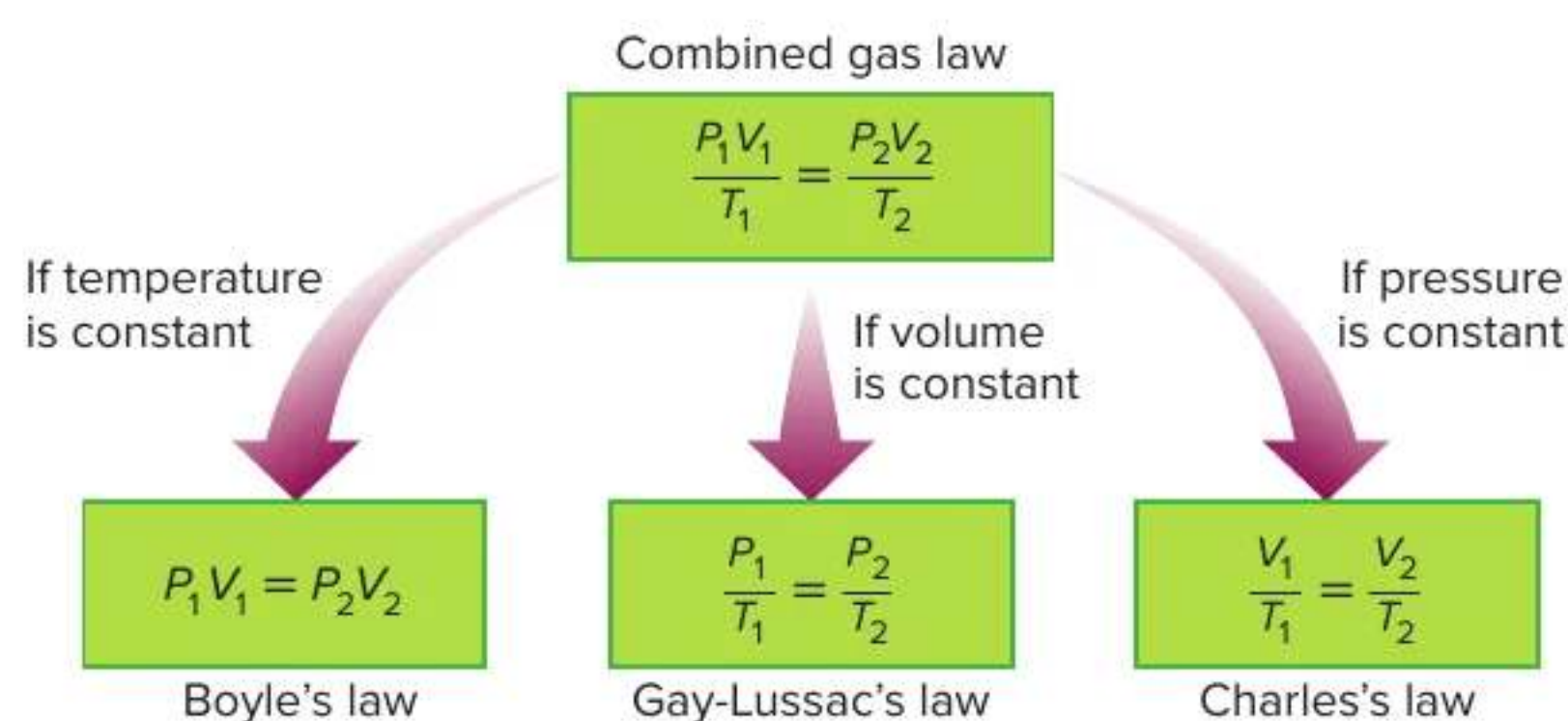


Figure 4 The combined gas law shows the relationship among pressure, temperature, and volume of a fixed amount of an ideal gas. Boyle's law and Charles's law can each be derived from the combined gas law under certain conditions.

Explain What happens if you hold volume constant?

As shown in **Figure 4**, the combined gas law reduces to Boyle's law under conditions of constant temperature and to Charles's law under conditions of constant pressure.

The ideal gas law What might the constant in the combined gas law depend on? Suppose the volume and temperature of an ideal gas are held constant while the number of particles (N) is increased. What happens to the pressure? The particles will have more collisions with the sides of the container, thus increasing the pressure. Removing particles decreases the number of collisions, and thus, decreases the pressure. Therefore, the constant in the combined gas law equation is proportional to N .

$$\frac{PV}{T} = \kappa N$$

The constant (κ) is called Boltzmann's constant, and its value is $1.38 \times 10^{-23} \text{ Pa} \cdot \text{m}^3/\text{K}$. In any practical application, the number of particles (N) is very large. Instead of using N , scientists often use a unit called a mole. One mole (abbreviated *mol*) is similar to one dozen, except that instead of representing 12 items, one mole represents 6.022×10^{23} particles. This number is called Avogadro's number, after Italian scientist Amedeo Avogadro.

Avogadro's number is numerically equal to the number of particles in a sample of matter whose mass equals the molar mass of the substance. You can use this relationship to convert between mass and n , the number of moles present. Using moles instead of the number of particles, however, changes Boltzmann's constant. This new constant is abbreviated R , and it has the value $8.31 \text{ Pa} \cdot \text{m}^3/(\text{mol} \cdot \text{K})$. Rearranging, you can write the ideal gas law in its most familiar form. The **ideal gas law** states that for an ideal gas, the pressure times the volume is equal to the number of moles multiplied by the constant R and the Kelvin temperature.

Ideal Gas Law

For an ideal gas, the pressure times the volume is equal to the number of moles times the constant R times the temperature.

$$PV = nRT$$

Note that with the given value of R , volume must be expressed in cubic meters, temperature in kelvins, and pressure in pascals. In practice, the ideal gas law predicts the behavior of gases remarkably well, except under conditions of high pressures or low temperatures.

EXAMPLE Problem 2

GAS LAWS A 20.0-L sample of argon gas at 273 K is at atmospheric pressure (101.3 kPa). The temperature is lowered to 120 K, and the pressure is increased to 145 kPa.

- What is the new volume of the argon sample?
- Find the number of moles of argon atoms in the argon sample.
- Find the mass of the argon sample. The molar mass (M) of argon is 39.9 g/mol.

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation. Indicate the conditions in the container of argon before and after the change in temperature and pressure.
- List the known and unknown variables.

Known

$$V_1 = 20.0 \text{ L}$$

$$P_1 = 101.3 \text{ kPa}$$

$$T_1 = 273 \text{ K}$$

$$P_2 = 145 \text{ kPa}$$

$$T_2 = 120 \text{ K}$$

$$R = 8.31 \text{ Pa}\cdot\text{m}^3/(\text{mol}\cdot\text{K})$$

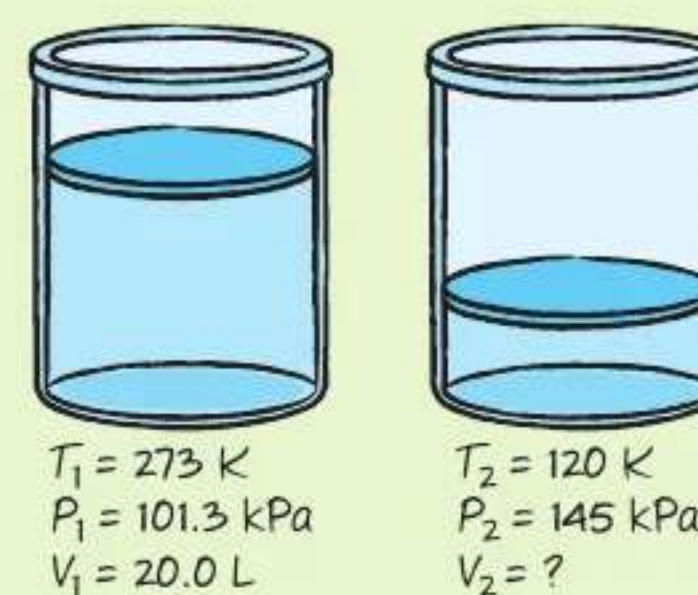
$$M_{\text{argon}} = 39.9 \text{ g/mol}$$

Unknown

$$V_2 = ?$$

$$\text{moles of argon} = ?$$

$$\text{mass of argon sample} = ?$$

**2 SOLVE FOR THE UNKNOWN**

- Use the combined gas law and solve for V_2 .

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$V_2 = \frac{P_1 V_1 T_2}{P_2 T_1}$$

$$= \frac{(101.3 \text{ kPa})(20.0 \text{ L})(120 \text{ K})}{(145 \text{ kPa})(273 \text{ K})}$$

$$= 6.1 \text{ L}$$

Substitute $P_1 = 101.3 \text{ kPa}$, $P_2 = 145 \text{ kPa}$, $V_1 = 20.0 \text{ L}$, $T_1 = 273 \text{ K}$, $T_2 = 120 \text{ K}$.

- Use the ideal gas law, and solve for n .

$$PV = nRT$$

$$n = \frac{PV}{RT}$$

$$= \frac{(101.3 \times 10^3 \text{ Pa})(0.0200 \text{ m}^3)}{(8.31 \text{ Pa}\cdot\text{m}^3/(\text{mol}\cdot\text{K}))(273 \text{ K})}$$

$$= 0.893 \text{ mol}$$

Substitute $P = 101.3 \times 10^3 \text{ Pa}$, $V = 0.0200 \text{ m}^3$, $R = 8.31 \text{ Pa}\cdot\text{m}^3/(\text{mol}\cdot\text{K})$, $T = 273 \text{ K}$.

- Use the molar mass to convert from moles of argon in the sample to mass of the sample.

$$m = Mn$$

$$m_{\text{argon sample}} = (39.9 \text{ g/mol})(0.893 \text{ mol})$$

$$= 35.6 \text{ g}$$

Substitute $M = 39.9 \text{ g/mol}$, $n = 0.893 \text{ mol}$.

3 EVALUATE THE ANSWER

- Are the units correct?** The volume (V_2) is in liters, and the mass of the sample is in grams.
- Is the magnitude realistic?** The change in volume is consistent with an increase in pressure and a decrease in temperature. The calculated mass of the argon sample is reasonable.

PRACTICE Problems

6. A tank of helium gas used to inflate toy balloons is at a pressure of 15.5×10^6 Pa and a temperature of 293 K. The tank's volume is 0.020 m^3 . How large a balloon would it fill at 1.00 atmosphere and 323 K?
7. What is the mass of the helium gas in the previous problem? The molar mass of helium gas is 4.00 g/mol.

**ADDITIONAL PRACTICE**

8. A tank containing 200.0 L of hydrogen gas at 0.0°C is kept at 156 kPa. The temperature is raised to 95°C , and the volume is decreased to 175 L. What is the new pressure of the gas?
9. **CHALLENGE** The average molar mass of the components of air (mainly diatomic nitrogen gas and diatomic oxygen gas) is about 29 g/mol. What is the volume of 1.0 kg of air at atmospheric pressure and 20.0°C ?

Thermal Expansion

As you applied the combined gas law, you discovered how gases expand as their temperatures increase. **Thermal expansion** is a property of all forms of matter that causes the matter to expand, becoming less dense, when heated. Thermal expansion has many useful applications, such as circulating air in a room.

Convection currents Figure 5 shows that when the air near the radiator of a room is heated, it becomes less dense and, therefore, rises. Gravity pulls the denser, colder air near the ceiling down. The cold air is subsequently warmed by the radiator, and air continues to circulate. This circulation of air within a room is called a convection current. Convection currents also occur in a pot of hot, but not boiling, water on a stove. When the pot is heated from the bottom, the colder and denser water sinks to the bottom where it is warmed and then pushed up by the continuous flow of cooler water from the top.

This thermal expansion occurs in most fluids. A good model for all liquids does not exist, but it is useful to think of a liquid as a finely ground solid. Groups of two, three, or more particles move together as if they were tiny pieces of a solid. When a liquid is heated, particle motion causes these groups to expand in the same way that particles in a solid are pushed apart. The spaces between groups increase. As a result, the whole liquid expands. With an equal change in temperature, liquids expand considerably more than solids but not as much as gases.

Why ice floats Because matter expands as it is heated, you might predict that ice would be more dense than water, and therefore, it should sink. However, when water is heated from 0°C to 4°C , instead of expanding, it contracts as the forces between particles increase and the ice crystals collapse. These forces between water molecules are strong, and the crystals that make up ice have a very open structure. Even when ice melts, tiny crystals remain. These remaining crystals are melting, and the volume of the water decreases until the temperature reaches 4°C . However, once the temperature of water moves above 4°C , its volume increases because of greater molecular motion. The practical result is that water is most dense at 4°C , and ice floats. This unique property of water is very important to our lives and environment. If ice sank, lakes would freeze from the bottom each winter and many would never melt completely in the summer.

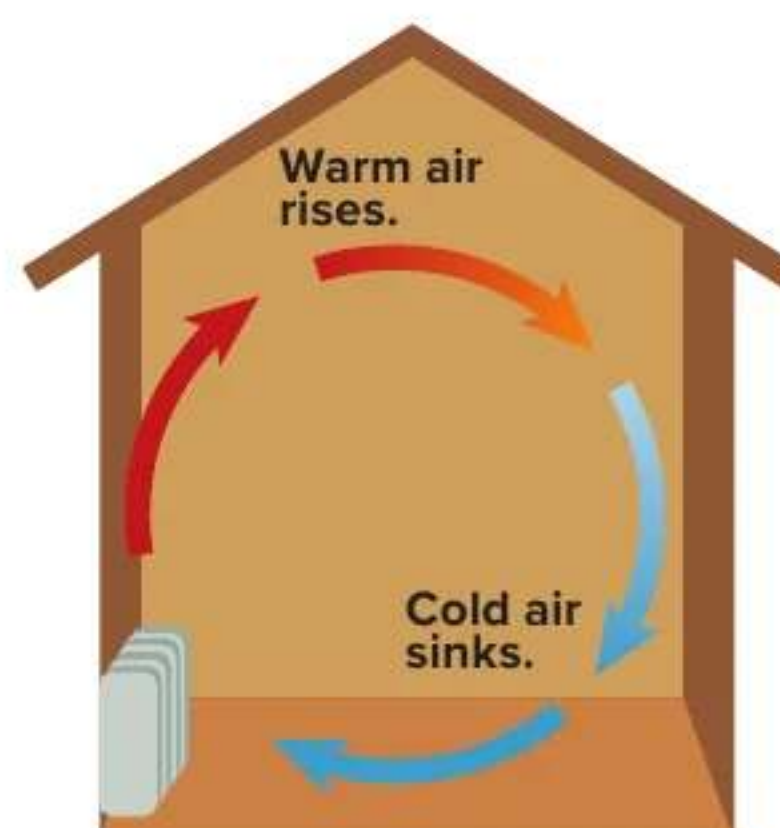


Figure 5 Convection currents occur as warmer, less dense air rises and cooler, denser air sinks.

Plasma

If you heat a solid, it melts to form a liquid. Further heating results in a gas. What happens if you increase the temperature still further? Collisions between the particles become violent enough to tear the electrons off the atoms, thereby producing positively charged ions. The gaslike state of negatively charged electrons and positively charged ions is called **plasma**. Plasma is considered to be another state of matter.

The plasma state may seem to be uncommon, but plasma is actually the most common state of matter in the universe. Stars consist mostly of plasma at extremely high temperatures. Much of the matter between stars and galaxies consists of energetic hydrogen that has no electrons. This hydrogen is in the plasma state. The primary difference between gas and plasma is that plasmas can conduct an electric current, whereas gases cannot. Lightning bolts are in the plasma state. Neon signs, such as the one shown in **Figure 6**, contain plasma. The fluorescent bulbs that probably light your school also contain plasma.



Figure 6 Plasma emits light as it conducts electricity. The color produced by glowing plasma depends on the gas inside the tube.

Check Your Progress

10. **Comparing States** Compare and contrast liquids, gases, and plasmas.
11. **Pressure and Force** Two boxes are each suspended by thin strings in midair. One is $20\text{ cm} \times 20\text{ cm} \times 20\text{ cm}$. The other is $20\text{ cm} \times 20\text{ cm} \times 40\text{ cm}$.
 - a. How does the pressure of the air on the outside of the two boxes compare?
 - b. How does the magnitude of the total force of the air on the two boxes compare?
12. **Meteorology** A weather balloon used by meteorologists is made of a flexible bag that allows the gas inside to freely expand. If a weather balloon containing 25.0 m^3 of helium gas is released from sea level, what is the volume of gas when the balloon reaches a height of 2100 m , where the pressure is $0.82 \times 10^5\text{ Pa}$? Assume the temperature is unchanged.
13. **Density and Temperature** Starting at 0°C , how will the density of water change as it is heated to 4°C ? To 8°C ?
14. **Gas Compression** In a certain internal-combustion engine, 0.0021 m^3 of air at atmospheric pressure and 303 K is rapidly compressed to a pressure of $20.1 \times 10^5\text{ Pa}$ and a volume of 0.0003 m^3 . What is the final temperature of the compressed gas?
15. **The Standard Molar Volume** What is the volume of 1.00 mol of a gas at atmospheric pressure and a temperature of 273 K ?
16. **The Air in a Refrigerator** How many moles of air are in a refrigerator with a volume of 0.635 m^3 at a temperature of 2.00°C ? If the average molar mass of air is 29 g/mol , what is the mass of the air in the refrigerator?
17. **Critical Thinking** Compared to the particles that make up carbon dioxide gas, the particles that make up helium gas are very small. What can you conclude about the number of particles in a 2.0-L sample of carbon dioxide gas compared to the number of particles in a 2.0-L sample of helium gas if both samples are at the same temperature and pressure?

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LESSON 2

FORCES WITHIN LIQUIDS

FOCUS QUESTION

How do the forces between particles in a fluid affect its behavior?

Cohesive Forces

Figure 7 shows a water strider walking across the surface of a pond. This lightweight insect can do this because of surface tension—the tendency of the surface of a liquid to contract to the smallest possible area. Surface tension results from the cohesive forces among the particles of a liquid. **Cohesive forces** are the forces of attraction that like particles exert on one another. Notice that beneath the liquid's surface in **Figure 7**, each particle of the liquid is attracted equally in all directions by neighboring particles. As a result, no net force acts on any of the particles beneath the surface. At the surface, however, the particles are attracted downward and to the sides but not upward. There is a net downward force, which acts on the top layers and causes the surface layer to be slightly compressed. The surface layer acts like a tightly stretched sheet that is strong enough to support the weight of very light objects, such as the water strider.

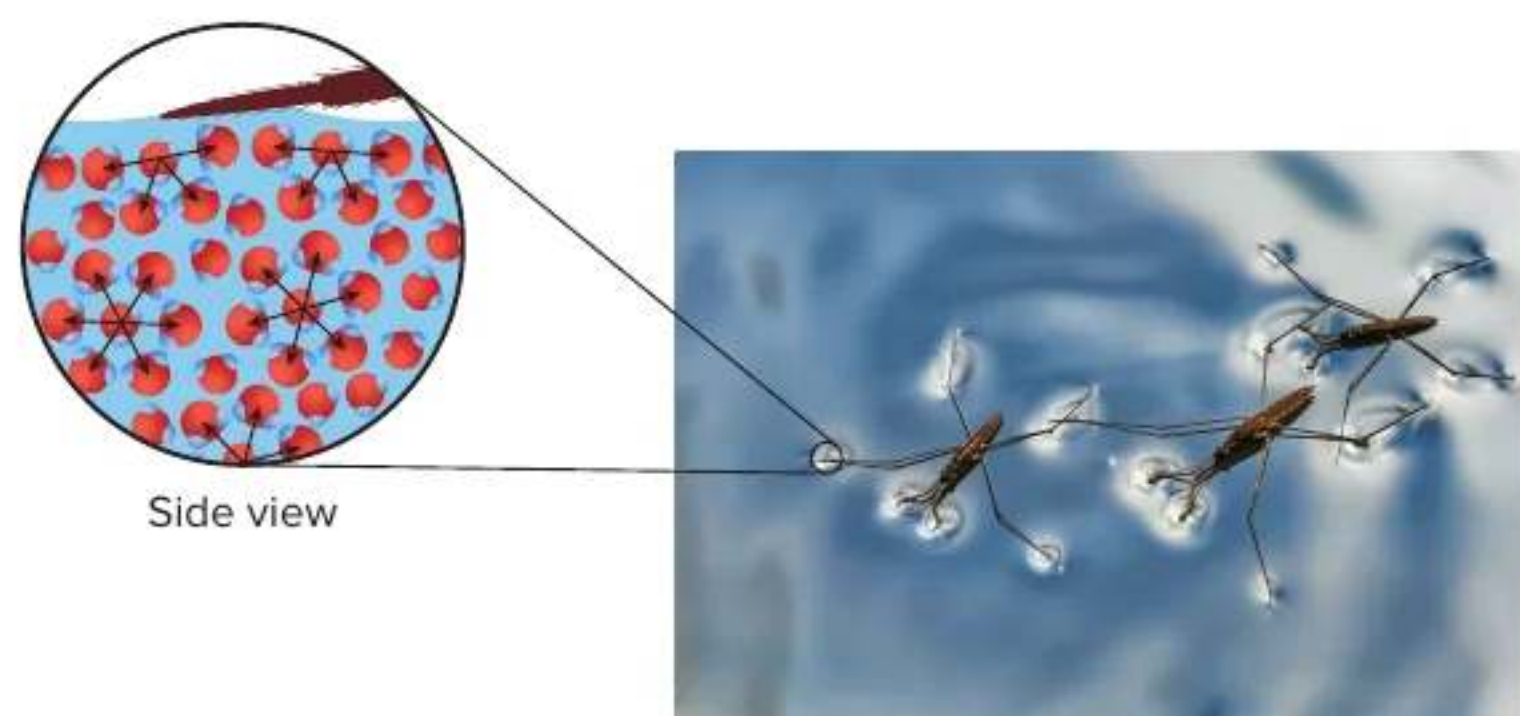


Figure 7 A water strider can walk on water because molecules at the surface experience a net downward force. Below the surface, each particle of liquid is equally attracted in all directions.

Jan Miko/Shutterstock.com



3D THINKING

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

INVESTIGATE

GO ONLINE to find these activities and more resources.



PhysicsLAB: Evaporative Cooling

Construct an explanation based on evidence to explain how cohesive forces affect the rate of evaporation of a liquid.

CCC Identify Crosscutting Concepts

Create a table of the crosscutting concepts and fill in examples you find as you read.

You might have seen beaded water droplets on a freshly washed and waxed car. Why do these spherical drops form? The force pulling the surface particles into a liquid causes the surface to become as small as possible, and the shape that has the least surface for a given volume is a sphere. The higher the surface tension of the liquid, the more resistant the liquid is to having its surface broken. For example, liquid mercury has much stronger cohesive forces than water does. Liquid mercury forms spherical drops, even when it is placed on a smooth surface. A drop of water flattens out on a smooth surface.

Viscosity In nonideal fluids, the cohesive forces and collisions between fluid molecules cause internal friction that slows the fluid flow and dissipates mechanical energy. The measure of this internal friction is called the viscosity of the liquid. Water is not very viscous, but motor oil is very viscous. As a result of its viscosity, motor oil flows slowly over the parts of an engine to coat the metal and reduce rubbing. Lava, molten rock that flows from a volcano or vent in Earth's surface, is one of the most viscous fluids. There are several types of lava, and the viscosity of each type varies with composition and temperature.

Adhesive Forces

Similar to cohesive forces, **adhesive forces** are attractive forces that act between particles of different substances. When a glass tube is placed in a beaker of water, the surface of the water climbs the outside of the tube, as shown in **Figure 8**. The adhesive forces between the particles that make up the glass and the water molecules are greater than the cohesive forces between the water molecules. In contrast, the cohesive forces between mercury atoms are greater than the adhesive forces between the mercury and the glass, so the liquid does not climb the tube. These forces also cause the mercury's surface to depress, as shown in **Figure 8**.



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Real-World Physics



PLANTS The combination of adhesive and cohesive forces acting on water molecules in plant tissues results in a certain amount of tension. As water molecules evaporate from cells in leaves, this tension pulls adjacent water molecules further into the leaves. Cohesive forces keep water from separating into individual droplets, and water is pulled the entire length of a plant, in some cases as much as 115 m.

Figure 8 Due to adhesive forces, water climbs the outside wall of the glass tube. In the mercury, however, the forces of attraction between mercury atoms are stronger than any adhesive forces between the mercury and the glass. Therefore, the mercury is depressed by the tube.

If a glass tube with a small inner diameter is placed in water, the water rises inside the tube. This happens because the adhesive forces between glass and water molecules are stronger than the cohesive forces between water molecules. The water continues to rise until the weight of the water that is lifted balances the total adhesive force between the glass and water molecules. If the radius of the tube increases, the volume and the weight of the water will increase proportionally faster than the surface area of the tube. Thus, water is lifted higher in a narrow tube than in a wider one. This phenomenon is called capillary action. It causes molten wax to rise in a candle's wick and water to move up through the soil and into the roots of plants.

**Get It?**

Compare How do the adhesive forces between water and glass compare with the cohesive forces between water molecules?

Evaporation and Condensation

Why does a puddle of water disappear on a hot, dry day? As you have previously read, the particles in a liquid are moving at random speeds. If a fast-moving particle can break through the surface layer, it will escape from the liquid. Because there is a net downward cohesive force at the surface, only the most energetic particles escape. This escape of particles is called evaporation.

Evaporative cooling Evaporation has a cooling effect. On a hot day, your body perspires, and the evaporation of your sweat cools you. In a puddle of water, evaporation causes the remaining liquid to cool. Each time a particle with higher-than-average kinetic energy escapes from the water, the average kinetic energy of the remaining particles decreases. As you learned earlier, a decrease in average kinetic energy is a decrease in temperature. Rubbing alcohol has a noticeable cooling effect when it evaporates from a person's skin. Alcohol molecules evaporate easily because they have weak cohesive forces. A liquid that evaporates quickly is called a volatile liquid.

Have you ever wondered why humid days feel warmer than dry days at the same temperature? On a humid day, the water vapor content of the air is high. Because there are already many water molecules in the air, the water molecules in perspiration are less likely to evaporate from the skin. Evaporation is the body's primary cooling mechanism, so the body is not able to cool itself as effectively on a humid day.

**Get It?**

Explain why evaporation has a cooling effect.

**CROSSCUTTING CONCEPTS**

Energy and Matter Prepare a skit that provides evidence to explain the changes between the liquid and gas states through a study of the flow of energy and matter.

Condensation Particles of liquid that have evaporated into the air can also return to the liquid phase if the kinetic energy or temperature decreases in a process called condensation.

What happens if you bring a cold glass into a hot, humid area? The outside of the glass soon becomes coated with condensed water. Water molecules moving randomly in the air surrounding the glass strike the cold surface, and if the water molecules lose enough energy, the cohesive forces between them become strong enough to prevent their escape.



Get It?

Define What happens during the process of condensation?

The air above any body of water, as shown in **Figure 9**, contains evaporated water vapor, which is water in the form of gas. If the temperature is reduced, the water vapor condenses around tiny dust particles in the air and produces droplets only 0.01 mm in diameter. A cloud of these droplets that forms at Earth's surface is called fog. Fog often forms when moist air is chilled by the cold ground. Fog also occurs briefly when a carbonated drink is opened. The sudden decrease in pressure causes the temperature of the gas in the container to drop, which condenses the water vapor dissolved in that gas.



Figure 9 Warm, moist, surface air rises. Clouds form when the air cools and the water vapor condenses.



Get It?

Identify What are two changes in conditions that can cause water vapor in the air to condense?



Check Your Progress

18. **Science Terms** The English language includes the term *adhesive tape* and the phrase *working as a cohesive group*. In these examples, are the words *adhesive* and *cohesive* being used in the same context as their meanings in physics? Explain your answer.
19. **Surface Tension** A paper clip, which has a density greater than that of water, can be made to stay on the surface of water. What procedures must you follow for this to happen? Explain.
20. **Floating** How can you tell that the paper clip in the previous problem was not floating?
21. **Adhesion and Cohesion** In terms of adhesion and cohesion, explain why alcohol clings to the surface of a glass rod but mercury does not.
22. **Evaporation and Cooling** In the past when a baby had a high fever, the doctor might have suggested gently sponging off the baby with a liquid that evaporates easily. Why would this help?
23. **Critical Thinking** On a hot, humid day, Beth took a glass of cold water outside. Soon, water coated the outside of the glass. Her friend, Sally, suggested that the water had leaked out through the glass. Design an experiment for Beth to show Sally where the water came from.

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LESSON 3

FLUIDS AT REST AND IN MOTION

FOCUS QUESTION

What principles describe the behavior and motion of fluids?

Fluids at Rest

If you have ever dived deep into a swimming pool or a lake, you likely felt pressure on your ears. Blaise Pascal, a French physician, found that the pressure at a point in a fluid depends on its depth in the fluid and is unrelated to the shape of the fluid's container. He also noted that any change in pressure applied at any point on a confined fluid is transferred undiminished throughout the fluid, a fact that is now known as **Pascal's principle**. One application of Pascal's principle is using fluids in machines to multiply forces. In the hydraulic system shown in **Figure 10**, a fluid is confined to two connecting chambers. Each chamber has a piston that is free to move, and the pistons have different surface areas. Recall that if a force (F_1) is exerted on the first piston with a surface area of A_1 , the pressure (P_1) exerted on the fluid is $P_1 = \frac{F_1}{A_1}$. The pressure exerted by the fluid on the second piston, with a surface area A_2 , is $P_2 = \frac{F_2}{A_2}$.

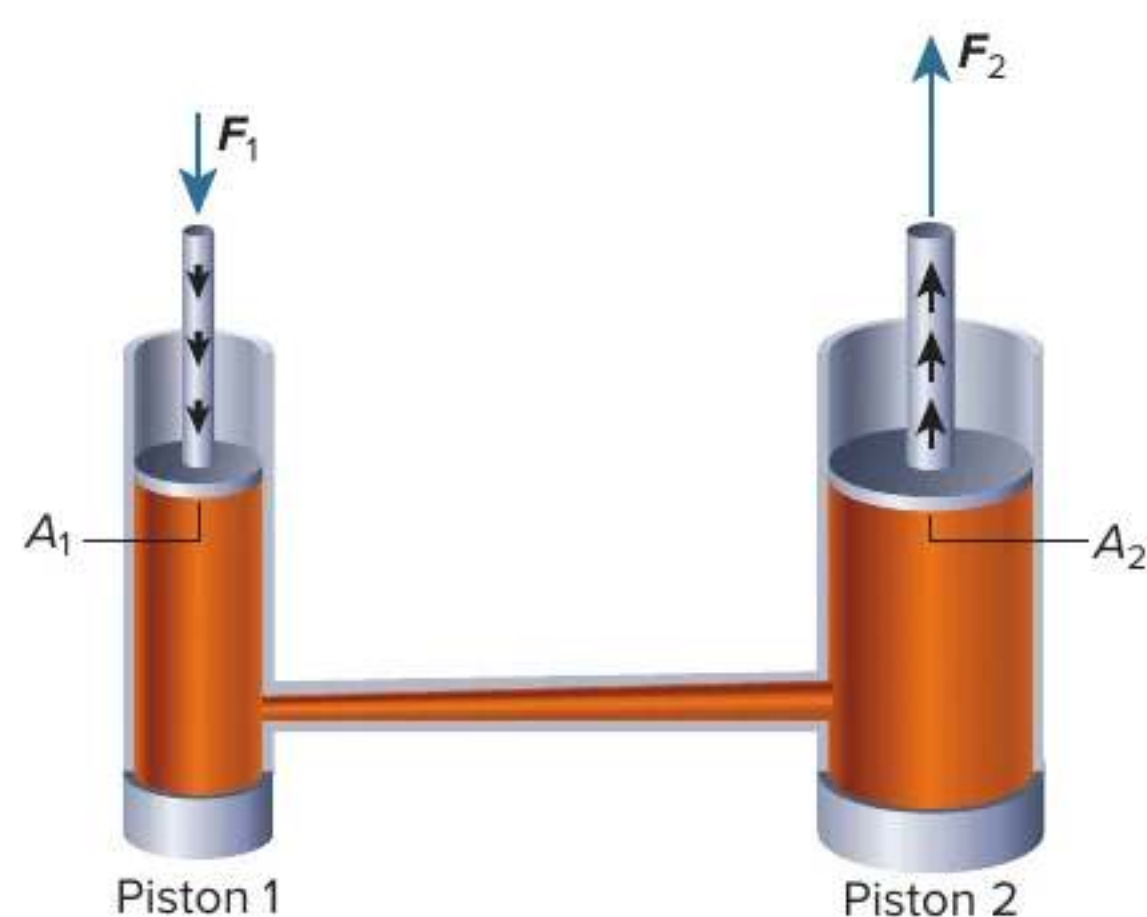


Figure 10 As F_1 exerts pressure on the smaller piston (piston 1), the pressure is transmitted throughout the fluid. As a result, a multiplied force (F_2) is exerted on the larger piston (piston 2).

Infer How would F_2 change if F_1 increased? Explain why.



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



PhysicsLAB: Under Pressure

Carry out an investigation to determine the effect of force in a fluid.



Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

PRACTICE Problems **ADDITIONAL PRACTICE**

- 24.** Dentists' chairs are examples of hydraulic-lift systems. If a chair weighs 1600 N and rests on a piston with a cross-sectional area of 1440 cm², what force must be applied to the smaller piston, with a cross-sectional area of 72 cm², to lift the chair?
- 25.** A mechanic exerts a force of 55 N on a 0.015 m² hydraulic piston to lift a small automobile. The piston the automobile sits on has an area of 2.4 m². What is the weight of the automobile?
- 26. CHALLENGE** By multiplying a force, a hydraulic system serves the same purpose as a lever or a seesaw. If a 400-N child standing on one piston is balanced by a 1100-N adult standing on another piston, what is the ratio of the areas of their pistons?

According to Pascal's principle, pressure is transmitted without change throughout a fluid, so pressure P_2 is equal in value to P_1 . You can determine the force exerted by the second piston by setting the pressures equal to each other and solving for F_2 .

Force Exerted by a Hydraulic Lift

The force exerted by the second piston is equal to the force exerted by the first piston multiplied by the ratio of the area of the second piston to the area of the first piston.

$$F_2 = \frac{F_1 A_2}{A_1}$$

Swimming Under Pressure

When you are swimming, you feel the pressure of the water increase as you dive deeper. This pressure is a result of gravity; it is related to the weight of the water above you. The deeper you go, the more water there is above you, and the greater the pressure. The pressure of the water is equal to the weight (F_g) of the column of water above you divided by the column's cross-sectional area (A). Even though gravity pulls only in the downward direction, the fluid transmits the pressure in all directions: up, down, and to the sides. As before, the pressure of the water is $P = \frac{F_g}{A}$.

The weight of the column of fluid is $F_g = mg$, and the mass is equal to the density (ρ) of the fluid times its volume, $m = \rho V$. You also know that the volume of the fluid is the area of the base of the column times its height, $V = Ah$. Therefore, $F_g = \rho Ahg$. Substituting ρAhg for F_g gives $P = \frac{F_g}{A} = \frac{\rho Ahg}{A}$. Divide A from the numerator and denominator to arrive at the simplest form of the equation for the pressure exerted by a column of fluid on a submerged body.

Pressure of Fluid on a Body

The pressure a column of fluid exerts on a body is equal to the density of the fluid times the height of the column times the free-fall acceleration.

$$P = \rho hg$$

The pressure of a fluid on a body depends on the density of the fluid, its depth, and g . As shown in **Figure 11**, submersibles have explored the deepest ocean trenches and encountered pressures in excess of 1000 times standard air pressure.

BIOLOGY Connection Scientists use submersibles to learn more about deep ocean ecosystems. In 1977 the first hydrothermal vents were discovered as the crewed submersible ALVIN cruised over the Pacific Ocean floor. Hydrothermal vents form when superheated water flows up from cracks in the seafloor.



Figure 11 Submersibles are built to withstand the crushing pressure exerted by the water column.

Because these vents are located thousands of meters below the ocean surface, the fluid pressure can be over one hundred times the standard atmospheric pressure. Despite the high pressure and the fact that sunlight does not reach them, hydrothermal vents thrive with life. Giant tube worms harbor bacteria in their tissues. The bacteria use hydrogen sulfide from the vent water to produce sugar, which provides the energy that supports the entire ecosystem. Other organisms that live at hydrothermal vents include fish, mussels, shrimp, clams, and octopuses. Submersibles have been used to explore hydrothermal vents in the Atlantic, Indian, and Arctic Oceans.

Buoyancy What produces the upward force that allows you to swim? The increase in pressure with increasing depth creates an upward force called the **buoyant force**. By comparing the buoyant force on an object with its weight, you can predict whether the object will sink or float.

Suppose that a box is immersed in water. It has a height of l , and its top and bottom each have a surface area of A . Its volume, then, is $V = lA$. Water pressure exerts forces on all sides, as shown in **Figure 12**. Will the box sink or float? As you know, the pressure on the box depends on its depth (h). To find out whether the box will float in water, you will need to analyze the forces acting on it, which are its weight and the forces on each side due to the pressure of the fluid. Compare these two equations:

$$F_{\text{top}} = P_{\text{top}} A = \rho h g A$$

$$F_{\text{bottom}} = P_{\text{bottom}} A = \rho(l + h)gA$$

On the four vertical sides, the forces are equal in all directions, so there is no net horizontal force. The upward force on the bottom is larger than the downward force on the top, so there is a net upward force. The buoyant force can now be determined.

$$\begin{aligned} F_{\text{buoyant}} &= F_{\text{bottom}} - F_{\text{top}} \\ &= \rho(l + h)gA - \rho h g A \\ &= \rho l g A = \rho V g \end{aligned}$$

These calculations show the net upward force to be proportional to the volume of the box. This volume equals the volume of the fluid displaced, or pushed out of the way, by the box. Therefore, the magnitude of the buoyant force ($\rho V g$) equals the weight of the fluid displaced by the object.

Buoyant Force

The buoyant force on an object is equal to the weight of the fluid displaced by the object, which is equal to the density of the fluid in which the object is immersed multiplied by the object's volume and the free-fall acceleration.

$$F_{\text{buoyant}} = \rho_{\text{fluid}} V g$$

Archimedes' principle The relationship between buoyant force and the weight of the fluid displaced by an object was discovered in the third century B.C. by Greek scientist and mathematician Archimedes. **Archimedes' principle** states that an object immersed in a fluid has an upward force on it that is equal to the weight of the fluid displaced by the object. The force does not depend on the weight of the object, only on the weight of the displaced fluid.

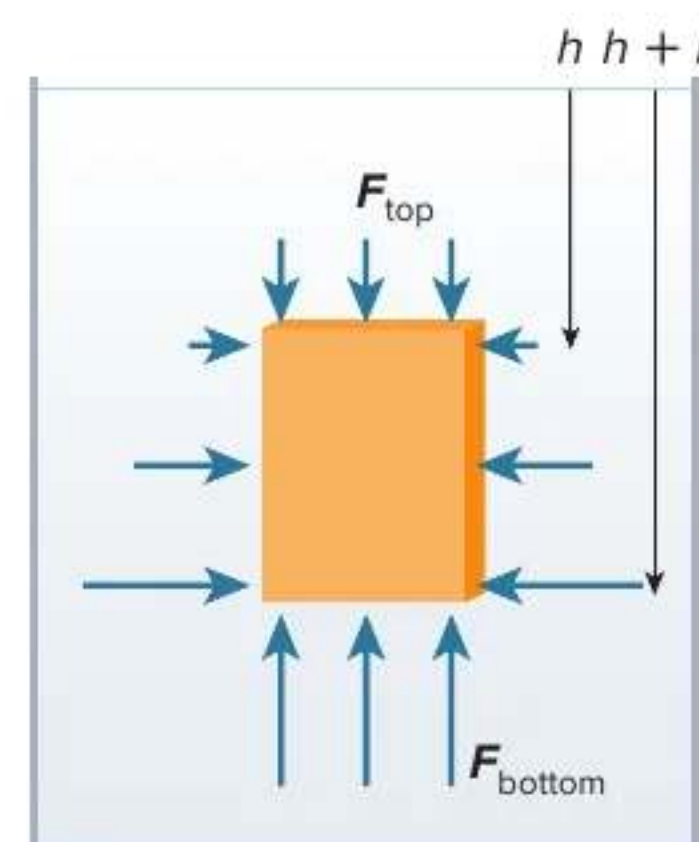


Figure 12 A fluid exerts a greater upward force on the bottom of an immersed object than the downward force on the top of the object. The net upward force is called the buoyant force.

Sink or Float? If you want to know whether an object sinks or floats, you have to take into account all of the forces acting on the object. The buoyant force pushes up, but the weight of the object pulls it down. The difference between the buoyant force and the object's weight determines whether an object sinks or floats.

Suppose you submerge three objects in a tank of water ($\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$). Each object has a volume of 400 cm^3 , or $4.00 \times 10^{-4} \text{ m}^3$. The first object is a steel block with a mass of 3.60 kg . The second is a can of soda with a mass of 0.40 kg . The third is an ice block with a mass of 0.36 kg . How will each item move when it is immersed in water and released? Since the three objects have the same volume, they displace the same volume of water, and the upward force on all three objects is the same, as shown in **Figure 13**. This buoyant force can be calculated as follows.

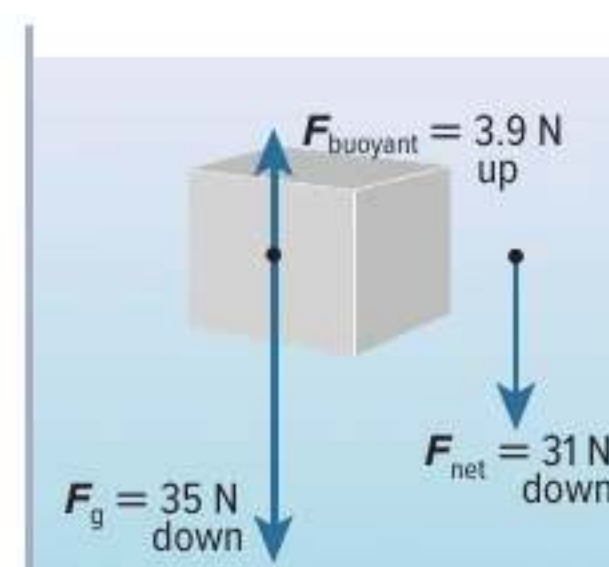
$$\begin{aligned} F_{\text{buoyant}} &= \rho_{\text{water}} V g \\ &= (1.00 \times 10^3 \text{ kg/m}^3)(4.00 \times 10^{-4} \text{ m}^3)(9.8 \text{ N/kg}) \\ &= 3.9 \text{ N} \end{aligned}$$

Figure 13 All of the forces on an object must be accounted for to determine whether an object will sink or float.

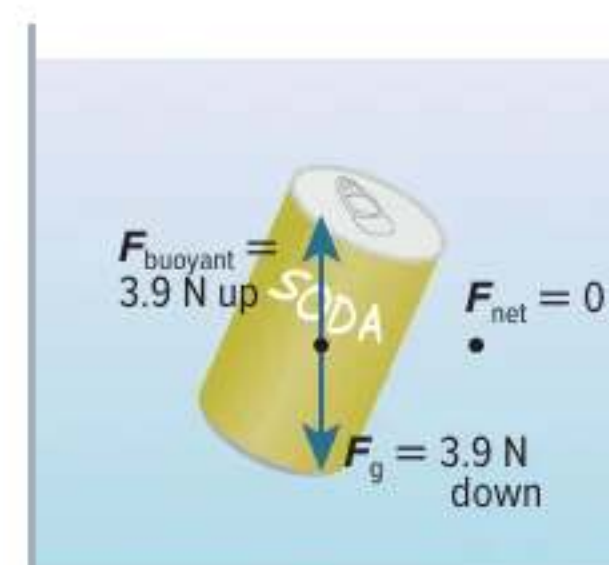
Describe the circumstances under which an object will float.

Buoyant Force

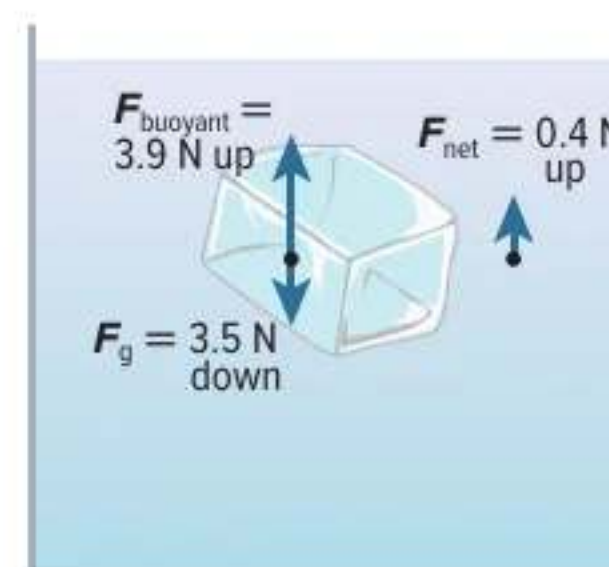
Sinking The weight of the block of steel is 35 N , much greater than the buoyant force. There is a net downward force, so the block will sink to the bottom of the tank. The net downward force is less than the object's weight. All objects in a liquid, even those that sink, experience a net force that is less than the net force when the object is in air.



Neutral The weight of the soda can is 3.9 N , the same as the weight of the water displaced. There is, therefore, no net force, and the can will remain wherever it is placed in the water. It is said to have neutral buoyancy.



Floating The weight of the ice cube is 3.5 N , less than the buoyant force, so there is a net upward force, and the ice cube will rise. An object will float if its density is less than the density of the fluid in which it is placed.



Note: Force vectors are not drawn to scale.

EXAMPLE Problem 3

ARCHIMEDES' PRINCIPLE A cubic decimeter ($1.00 \times 10^{-3} \text{ m}^3$) of a granite building block is submerged in water. The density of granite is $2.70 \times 10^3 \text{ kg/m}^3$.

- What is the magnitude of the buoyant force acting on the block?
- What is the net force on the block?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the cubic decimeter of granite immersed in water.
- Show the upward buoyant force and the downward force due to gravity acting on the granite.

Known

$$V = 1.00 \times 10^{-3} \text{ m}^3$$

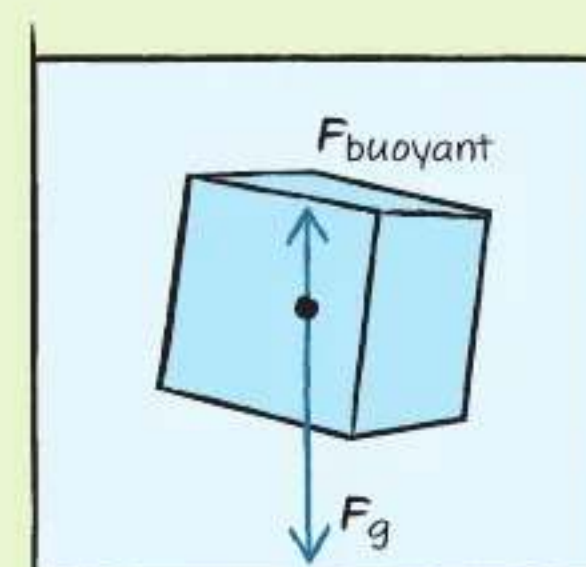
$$\rho_{\text{granite}} = 2.70 \times 10^3 \text{ kg/m}^3$$

$$\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$$

Unknown

$$F_{\text{buoyant}} = ?$$

$$F_{\text{net}} = ?$$

**2 SOLVE FOR THE UNKNOWN**

- a. Calculate the buoyant force on the granite block.

$$\begin{aligned} F_{\text{buoyant}} &= \rho_{\text{water}} Vg \\ &= (1.00 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^{-3} \text{ m}^3)(9.8 \text{ N/kg}) \\ &= 9.8 \text{ N} \end{aligned}$$

Substitute $\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$, $V = 1.00 \times 10^{-3} \text{ m}^3$, $g = 9.8 \text{ N/kg}$.

- b. Calculate the granite's weight, and then find its net force.

$$\begin{aligned} F_g &= mg = \rho_{\text{granite}} Vg \\ &= (2.70 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^{-3} \text{ m}^3)(9.8 \text{ N/kg}) \\ &= 26.5 \text{ N} \end{aligned}$$

Substitute $\rho_{\text{granite}} = 2.70 \times 10^3 \text{ kg/m}^3$, $V = 1.00 \times 10^{-3} \text{ m}^3$, $g = 9.8 \text{ N/kg}$.

$$\begin{aligned} F_{\text{net}} &= F_g - F_{\text{buoyant}} \\ &= 26.5 \text{ N} - 9.8 \text{ N} \\ &= 17 \text{ N} \end{aligned}$$

Substitute $F_g = 26.5 \text{ N}$, $F_{\text{buoyant}} = 9.8 \text{ N}$.

3 EVALUATE THE ANSWER

- **Are the units correct?** The forces and the weight are in newtons, as expected.
- **Is the magnitude realistic?** The buoyant force is about one-third the weight of the granite, a sensible answer because the density of water is about one-third that of granite.

PRACTICE Problems**ADDITIONAL PRACTICE**

- Common brick is about 1.8 times denser than water. What is the net force on a 0.20 m^3 block of bricks under water?
- A girl is floating in a freshwater lake with her head just above the water. If she weighs 610 N, what is the volume of the submerged part of her body?
- What is the tension in a wire supporting a 1250-N camera submerged in water? The volume of the camera is $16.5 \times 10^{-3} \text{ m}^3$.
- Plastic foam is about 0.10 times as dense as water. What weight of bricks could you stack on a $1.0\text{-m} \times 1.0\text{-m} \times 0.10\text{-m}$ slab of foam so that the slab of foam floats in water and is barely submerged, leaving the bricks dry?
- CHALLENGE** Canoes often have plastic foam blocks mounted under the seats for flotation in case the canoe fills with water. What is the approximate minimum volume of foam needed for flotation for a 480-N canoe?

Ships How can steel ships float? If you make a small boat out of folded aluminum foil, it should float easily. Add a cargo of pennies, and it will ride lower in the water. Crumple the foil into a tight ball, and it will sink. The boat floats when it is hollow and large enough so that its average density is less than water's density. As you add cargo, the density increases, and more of the boat is submerged. The crumpled boat has a density greater than the water's and sinks. Archimedes' principle also applies to submarines and fishes. Submarines can pump water into or out of chambers to change the submarine's average density, causing it to sink or rise. Fishes that have swim bladders move upward in the water by filling the bladder with gas to displace more water and increase the buoyant force. The fish moves downward by contracting the swim bladder.



Get It?

Explain how the submersible at the beginning of the module can rise, sink, or stay suspended in the water.



Figure 14 You can demonstrate Bernoulli's principle by narrowing the opening of the hose as water flows out. As the velocity of the water increases, the pressure it exerts decreases.

Bernoulli's Principle

Study the flow of water from the hose in **Figure 14**. In the top photo, the water flows from the hose unobstructed. In the bottom photo, the hose opening has been narrowed by a person's thumb. Notice that the streams of water look different. The velocity of the water stream in the bottom photo is greater compared to the velocity of the stream in the top photo. What you can't see is that pressure exerted by the water in the bottom photo decreased. The relationship between the velocity and pressure exerted by a moving fluid is named for Swiss scientist Daniel Bernoulli.

Bernoulli's principle states that as the velocity of a fluid increases, the pressure exerted by that fluid decreases. This principle is a statement of work and energy conservation as applied to fluids.

Another instance in which the velocity of water can change is in a stream. You might have seen the water in a stream speed up as it passed through narrowed sections of the stream bed. As the opening of the hose and the stream channel become wider or narrower, the velocity of the fluid changes to maintain the overall flow of water. The pressure of blood in our circulatory systems depends partly on Bernoulli's principle. Bernoulli's principle also helps explain how smoke is pulled up a fireplace chimney.

Consider a horizontal pipe completely filled with a smoothly flowing ideal fluid. If a certain mass of the fluid enters one end of the pipe, then an equal mass must come out the other end. What happens if the cross section becomes narrower, as shown in **Figure 15**? To keep the same mass of fluid moving through the narrow section in a fixed amount of time, the velocity of the fluid in the tube must increase. As the fluid's velocity increases, so does its kinetic energy. This means that net work has been done on the swifter fluid. This net work comes from the difference between the work that was done to move the mass of fluid into the pipe and the work that was done by the fluid pushing the same mass out of the pipe. The work is proportional to the force on the fluid, which, in turn, depends on the pressure. If the net work is positive, the pressure at the input end of the section, where the velocity is lower, must be larger than the pressure at the output end, where the velocity is higher.

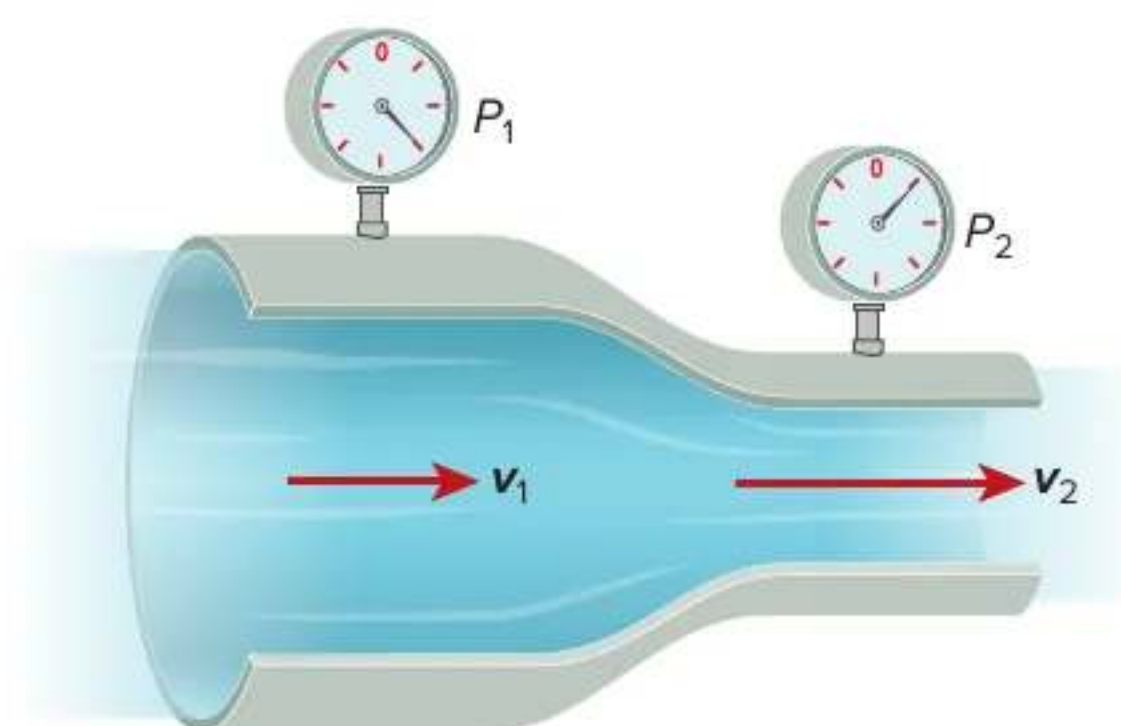


Figure 15 The fluid flowing through this pipe also demonstrates Bernoulli's principle. As the velocity of the fluid increases (v_2 is greater than v_1), the pressure it exerts decreases (P_2 is less than P_1).



Get It?

Describe the relationship between the velocity of a fluid and the pressure it exerts according to Bernoulli's principle.

Applications of Bernoulli's principle There are many common applications of Bernoulli's principle, such as paint sprayers and sprayers attached to garden hoses to apply fertilizers and pesticides to lawns and gardens. In a hose-end sprayer, a strawlike tube is sunk into the chemical solution in the sprayer. The sprayer is attached to a hose. A trigger on the sprayer allows water from the hose to flow at a high speed, producing an area of low pressure above the tube. The solution is then sucked up through the tube and into the stream of water.

A gasoline engine's carburetor, which is where air and gasoline are mixed, is another common application of Bernoulli's principle. Part of the carburetor is a tube with a constriction, as shown in the diagram in **Figure 16**. The pressure on the gasoline in the fuel supply is the same as the pressure in the thicker part of the tube. Air flowing through the narrow section of the tube, which is attached to the fuel supply, is at a lower pressure, so fuel is forced into the air flow. By regulating the flow of air in the tube, the amount of fuel mixed into the air can be varied. Carburetors are used in motorcycles, stock car race cars, and the motors of small gasoline-powered machines, such as lawn mowers.

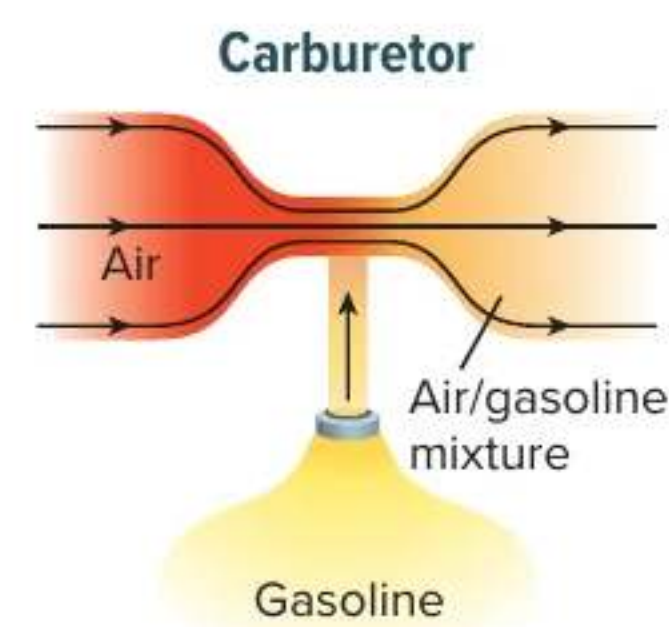


Figure 16 In a carburetor, low pressure in the narrow part of the tube draws fuel into the air flow.

SCIENCE USAGE v. COMMON USAGE

Streamlines

Science usage: lines representing the flow of fluids around objects

Simulations of the streamlines around the airplane indicated a design flaw.

Common usage: makes simpler or more efficient

The new computer system streamlines the registration process.

Streamlines Automobile and aircraft manufacturers spend a great deal of time and money testing new designs in wind tunnels to ensure the greatest efficiency of movement through air. The flow of fluids around objects is represented by **streamlines**, as shown in **Figure 17**. Objects require less energy to move through a smooth streamlined flow.

Streamlines can best be illustrated by a simple demonstration. Imagine carefully squeezing tiny drops of food coloring into a smoothly flowing fluid. If the colored lines that form stay thin and well defined, the flow is said to be streamlined. Notice that if the flow narrows, the streamlines move closer together. Closely spaced streamlines indicate greater velocity and, therefore, reduced pressure. If streamlines swirl and become diffused, the flow of the fluid is said to be turbulent. Bernoulli's principle does not apply to turbulent flow.



Figure 17 The streamlines show the air flowing above a car in a wind tunnel.

Check Your Progress

32. **Buoyancy** All soda cans contain the same volume of liquid, 354 mL, and displace the same volume of water. What might be a difference between a can that sinks and one that floats? *Hint: Place a full can of regular soda and a full can of diet soda in water.*
33. **Transmission of Pressure** A toy rocket launcher is designed so that a child stomps on a rubber cylinder, which increases the air pressure in a launching tube and pushes a foam rocket into the sky. If the child stomps with a force of 150 N on a $2.5 \times 10^{-3}\text{-m}^2$ area piston, what is the additional force transmitted to the $4.0 \times 10^{-4}\text{-m}^2$ launch tube?

34. **Floating in Air** A helium balloon rises because of the buoyant force of the air lifting it. The density of helium is 0.18 kg/m^3 , and the density of air is 1.3 kg/m^3 . How large a volume would a helium balloon need to lift the lead brick shown in **Figure 18**?

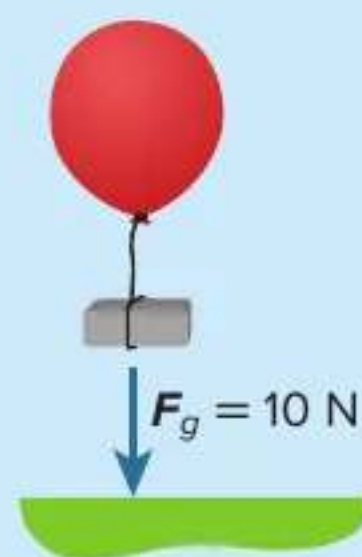


Figure 18

35. **Floating and Density** A fishing bobber made of cork floats with one-tenth of its volume below the water's surface. What is the density of cork?
36. **Pressure and Force** An automobile weighing $2.3 \times 10^4\text{ N}$ is lifted by a hydraulic cylinder with an area of 0.15 m^2 .
- What is the pressure in the hydraulic cylinder?
 - The pressure in the lifting cylinder is produced by pushing on a 0.0082-m^2 cylinder. What force must be exerted on this small cylinder to lift the automobile?
37. **Volume Displacement** Which displaces more water when placed in a pool?
- a 1.0-kg block of aluminum or a 1.0-kg block of lead
 - a 10-cm^3 block of aluminum or a 10-cm^3 block of lead
38. **Critical Thinking** A tornado passing over a house sometimes makes the house explode from the inside out. How might Bernoulli's principle explain this phenomenon? What could be done to reduce the danger of a door or window exploding outward?

LEARNSMART

Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

LESSON 4 SOLIDS

FOCUS QUESTION

How are solids different from fluids?

Solid Bodies

Solids are stiff, can be cut, and retain their shapes. Liquids flow and do not retain their shapes. When the temperature of a liquid is lowered, the average kinetic energy of the particles decreases. As the particles slow down, the cohesive forces have more effect, and for many solids, the particles become frozen into a fixed pattern called a **crystal lattice**, shown in **Figure 19**. Although the cohesive forces hold the particles in place, the particles in a crystalline solid do not stop moving completely. Rather, they vibrate around their fixed positions. In other materials, such as butter and glass, the particles do not form a fixed crystalline pattern. A substance that has no regular crystal structure but does have a definite volume and shape is called an **amorphous solid**.

Pressure and freezing As a liquid becomes a solid, its particles usually fit more closely together than in the liquid state, making solids more dense than liquids. Also, an increase in the pressure on the surface of the liquid increases its freezing point. However, water is an exception. It is most dense at 4°C. Because water expands as it freezes, an increase in pressure forces the molecules closer together and opposes the freezing. Therefore, higher pressure lowers the freezing point of water very slightly.

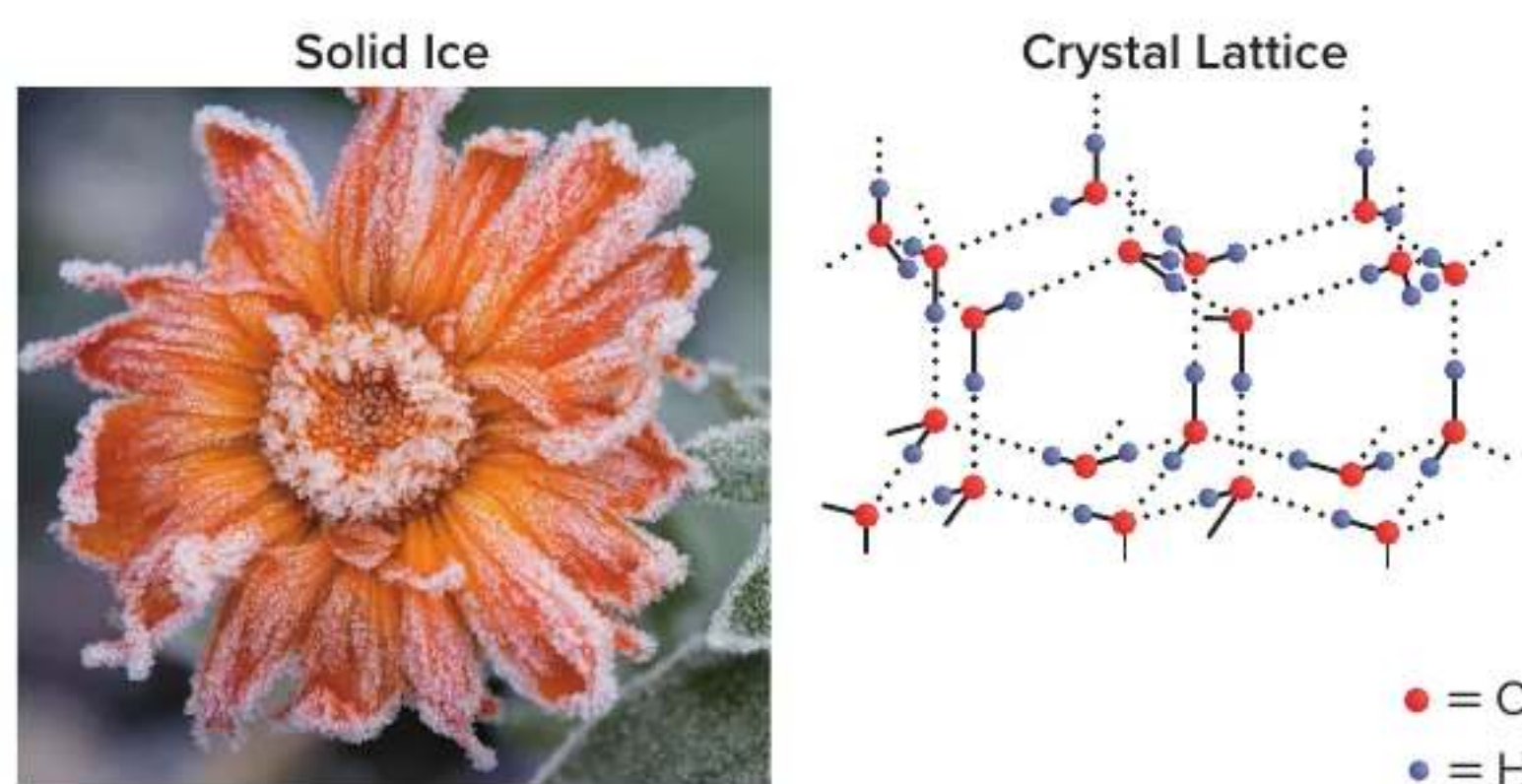


Figure 19 As the temperature of water is lowered and it changes from a liquid to a solid, the particles are frozen in a pattern called a crystal lattice.



Get It?

Compare For most substances, how do the densities of the solid and liquid compare?



3D THINKING

DCI Disciplinary Core Ideas

CCC Crosscutting Concepts

SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.



Quick Investigation: Jumpers

Carry out an investigation into the expansion **rates** for different **solids**.



Review the News

Obtain information from a current news story about the **properties or behavior of solids**.

Evaluate your source and **communicate** your findings to your class.

Elasticity of solids External forces applied to a solid object may twist, stretch, or bend it out of shape. The ability of a solid object to return to its original form when the external forces are removed is called the elasticity of the solid. If too much deformation occurs, the object will not return to its original shape because its elastic limit has been exceeded. Elasticity is a property of each substance and depends on the forces holding its particles together. Malleability and ductility are two properties that depend on the structure and elasticity of a substance. Because gold can be flattened and shaped into thin sheets, it is said to be malleable. Copper is a ductile metal because it can be pulled into thin strands of wire.



Get It?

Identify List one product or device that makes use of the property of malleability and one that makes use of the property of ductility.

Thermal Expansion of Solids

It is standard practice for engineers to design small gaps, called expansion joints, into concrete-and-steel highway bridges to allow for the expansion of parts in the heat of summer. Expansion joints are shown in **Figure 20**. Objects expand only a small amount when they are heated, but that small amount could be several centimeters in a 100-m-long bridge. If expansion gaps were not present, the bridge could buckle, or parts of it could break.

Some materials, such as the ovenproof glass used for laboratory experiments and cooking, are designed to have the least possible thermal expansion. Large telescope mirrors are made of a ceramic material that is designed to undergo essentially no thermal expansion.

To understand the expansion of heated solids, picture a solid as a collection of particles connected by springs that represent the attractive forces between the particles. When the particles get too close, the springs push them apart. When a solid is heated, the kinetic energy of the particles increases, and they vibrate rapidly and move farther apart, weakening the attractive forces between the particles. As a result, when the particles vibrate more violently with increased temperature, their average separation increases and the solid expands.

Tom Uhlman/Alamy Stock Photo



Figure 20 Expansion joints are included when bridges, roads, and train tracks are built.

Infer If there were no expansion joints in this road, what might happen during the summer?

STEM CAREER Connection

Industrial Designer

Have you bought a product because you liked its design? Did you ever have a product break and wonder what other material the manufacturer could have used? An industrial designer develops the products people use every day, such as toys, phones, and cars. They must consider both the aesthetics of the product and the durability of materials.

The change in length of a solid is proportional to the change in temperature, as shown in **Figure 21**. A solid will expand in length twice as much when its temperature is increased by 20°C than when it is increased by 10°C. The expansion also is proportional to its length. This means that a 2-m bar will expand twice as much as a 1-m bar with the same change in temperature. The length (L_2) of a solid at temperature T_2 can be found using the following relationship, where L_1 is the length at temperature T_1 and alpha (α) is the coefficient of linear expansion. The **coefficient of linear expansion** is equal to the change in length divided by the original length and the change in temperature.

$$L_2 = L_1 + \alpha L_1 (T_2 - T_1)$$

With some algebra, you can solve for α .

$$\begin{aligned} L_2 - L_1 &= \alpha L_1 (T_2 - T_1) \\ \Delta L &= \alpha L_1 \Delta T \end{aligned}$$

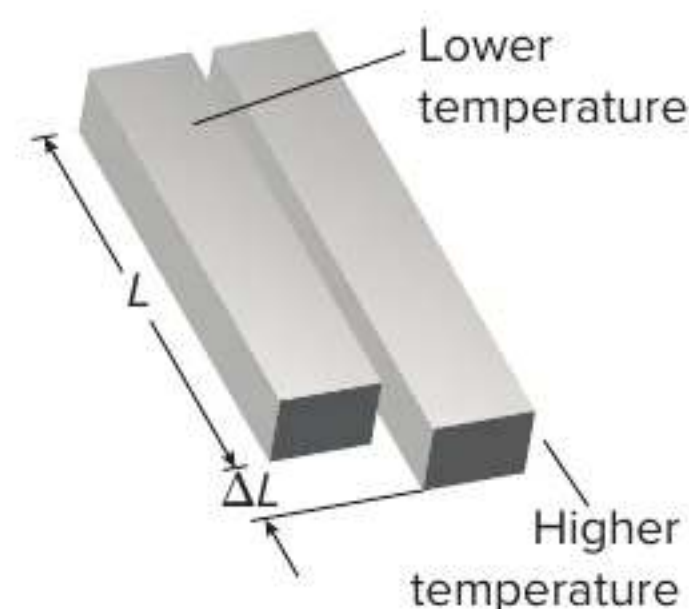


Figure 21 The change in length of a material is proportional to its original length and the change in temperature.

Coefficient of Linear Expansion

The coefficient of linear expansion is equal to the change in length divided by the product of the original length and the change in temperature.

$$\alpha = \frac{\Delta L}{L_1 \Delta T}$$

The unit for the coefficient of linear expansion is the reciprocal of degrees Celsius (which can be written as $1/^\circ\text{C}$ or $^\circ\text{C}^{-1}$). The **coefficient of volume expansion** is equal to the change in volume divided by the product of the original volume and the change in temperature. The coefficient of volume expansion (β) is generally about three times the coefficient of linear expansion.

Coefficient of Volume Expansion

The coefficient of volume expansion is equal to the change in volume divided by the product of the original volume and the change in temperature.

$$\beta = \frac{\Delta V}{V_1 \Delta T}$$

The unit for β is also $1/^\circ\text{C}$ ($^\circ\text{C}^{-1}$). The two coefficients of thermal expansion for a variety of materials are given in **Table 2**. (See reference table in Student Resources for additional materials.)

Table 2 Coefficients of Thermal Expansion at 20°C

Material	Coefficient of Linear Expansion, $\alpha(^\circ\text{C}^{-1})$	Coefficient of Volume Expansion, $\beta(^\circ\text{C}^{-1})$
Solids		
Aluminum	23×10^{-6}	69×10^{-6}
Glass (soft)	9×10^{-6}	27×10^{-6}
Glass (ovenproof)	3×10^{-6}	9×10^{-6}
Concrete	12×10^{-6}	36×10^{-6}
Copper	17×10^{-6}	51×10^{-6}
Liquids		
Methanol	Not Applicable	1200×10^{-6}
Gasoline	Not Applicable	950×10^{-6}
Water	Not Applicable	210×10^{-6}

EXAMPLE Problem 4

LINEAR EXPANSION A metal bar is 1.60 m long at room temperature (21°C). The bar is put into an oven and heated to a temperature of 84°C. It is then measured and found to be 1.7 mm longer. What is the coefficient of linear expansion of this material?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the bar, which is 1.7 mm longer at 84°C than at 21°C.
- Identify the initial length of the bar (L_1) and the change in length (ΔL).

Known

$$L_1 = 1.60 \text{ m}$$

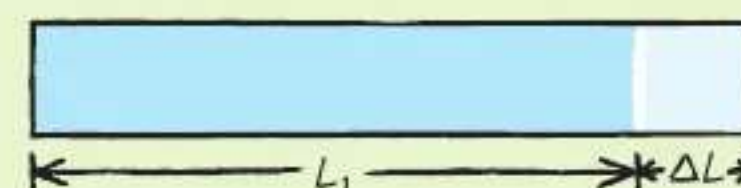
$$\Delta L = 1.7 \times 10^{-3} \text{ m}$$

$$T_1 = 21^\circ\text{C}$$

$$T_2 = 84^\circ\text{C}$$

Unknown

$$\alpha = ?$$

**2 SOLVE FOR THE UNKNOWN**

Calculate the coefficient of linear expansion using the relationship among known length, change in length, and change in temperature.

$$\alpha = \frac{\Delta L}{L_1 \Delta T}$$

$$= \frac{1.7 \times 10^{-3} \text{ m}}{(1.60 \text{ m})(84^\circ\text{C} - 21^\circ\text{C})}$$

$$= 1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$$

$$\text{Substitute } \Delta L = 1.7 \times 10^{-3} \text{ m, } L_1 = 1.60 \text{ m, } \Delta T = (T_2 - T_1) = 84^\circ\text{C} - 21^\circ\text{C}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** The units are correctly expressed in $^\circ\text{C}^{-1}$.
- **Is the magnitude realistic?** The magnitude of the coefficient is close to the accepted value for copper.

PRACTICE Problems**ADDITIONAL PRACTICE**

- 39.** A piece of aluminum house siding is 3.66 m long on a cold winter day of -28°C . How much longer is it on the hot summer day shown in **Figure 22**?

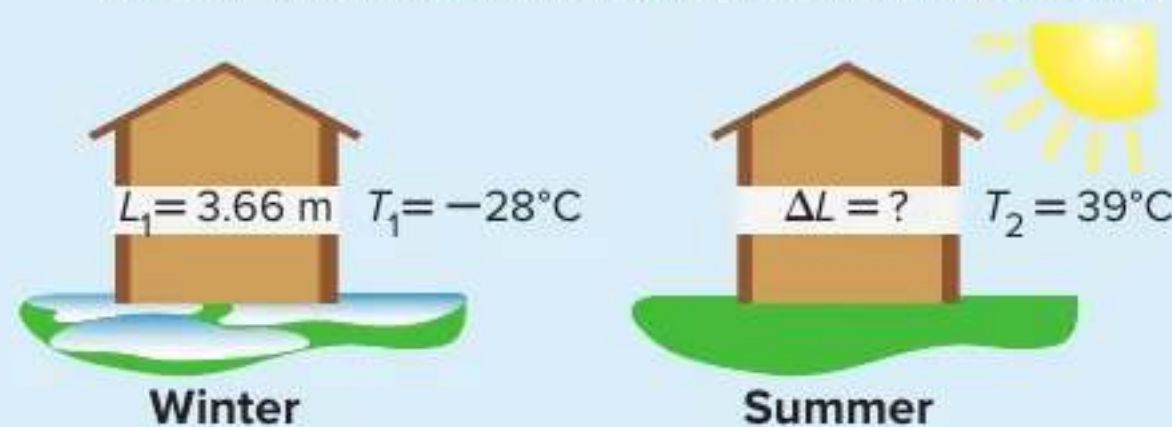
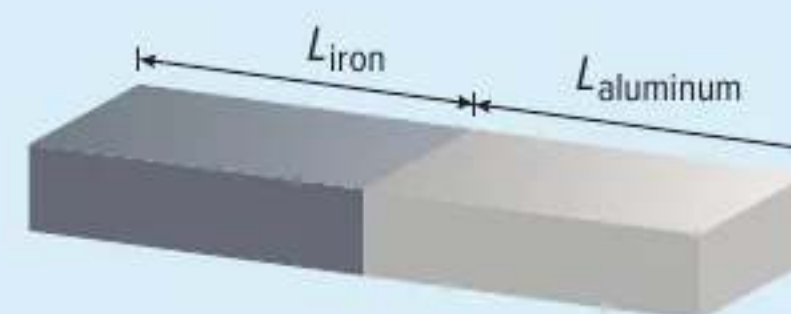


Figure 22

- 40.** A piece of steel is 11.5 cm long at 22°C . It is heated to 1221°C , close to its melting temperature. How long is it?
- 41.** A 400-mL glass beaker at room temperature is filled to the brim with cold water at 4.4°C . When the water warms up to 30.0°C , how much water will spill from the beaker?
- 42.** A tank truck takes on a load of 45,725 L of gasoline in Houston, where the temperature is 28.0°C . The truck delivers its load in Minneapolis, where the temperature is -12.0°C .
- How many liters of gasoline does the truck deliver?
 - What happened to the gasoline?
- 43.** A hole with a diameter of 0.85 cm is drilled into a steel plate. At 30.0°C , the hole exactly accommodates an aluminum rod of the same diameter. What is the spacing between the plate and the rod when they are cooled to 0.0°C ?
- 44. CHALLENGE** A steel ruler is marked in millimeters so that the ruler is absolutely correct at 30.0°C . By what percentage would the ruler be incorrect at -30.0°C ?

PHYSICS Challenge

You need to make a 1.00-m-long bar that expands with temperature in the same way as a 1.00-m-long bar of copper would. As shown in the figure at the right, your bar must be made from a bar of iron and a bar of aluminum attached end to end. How long should each of the bars be?



Applications of thermal expansion Engineers take the thermal expansion of materials into consideration as they design structures. You've already read about the expansion joints that are installed on concrete highways and bridges. The regular gaps between slabs of concrete in sidewalks also help keep sidewalks from buckling when the concrete expands during hot weather. Different materials expand at different rates, as indicated by the different coefficients of expansion given in **Table 2**. Engineers also consider different expansion rates when designing systems. Steel bars are often used to reinforce concrete. The steel and concrete must have the same expansion coefficient. Otherwise, the structure could crack on a hot day. Similarly, filling materials used to repair teeth must expand and contract at the same rate as tooth enamel. Different rates of expansion have useful applications. For example, engineers have taken advantage of these differences to construct a useful device called a bimetallic

strip, which is used in thermostats. A bimetallic strip consists of two strips of different metals welded or riveted together. Usually, one strip is brass, and the other is iron. When heated, brass expands more than iron does. Thus, when the bimetallic strip of brass and iron is heated, the brass part of the strip becomes longer than the iron part. As a result, the bimetallic strip bends with the brass on the outside of the curve. If the bimetallic strip is cooled, it bends in the opposite direction. The brass is then on the inside of the curve.

In a home thermostat, the bimetallic strip is installed so that it bends toward an electric contact as the room cools. When the room cools below the setting on the thermostat, the bimetallic strip bends enough to make electric contact with the switch, which turns on the heater. As the room warms, the bimetallic strip bends in the other direction. When the room's temperature reaches the setting on the thermostat, the electric circuit is broken, and the heater switches off.

 Check Your Progress

45. **Thermal Expansion** On a hot day, you are installing an aluminum screen door in a concrete doorframe. You want the door to fit well on a cold winter day. Should you make the door fit tightly in the frame or leave extra room?
46. **Types of Solids** What is the difference between the structure of candle wax and that of ice?
47. **Thermal Expansion** Can you heat a piece of copper enough to double its length?
48. **States of Matter** Does **Table 2** provide a way to distinguish between solids and liquids?

49. **Solids and Liquids** A solid can be defined as a material that can be bent and will resist bending. Explain how these properties relate to the binding of atoms in a solid but do not apply to a liquid.
50. **Critical Thinking** The iron ring in **Figure 23** was made by cutting a small piece from a solid ring. If the ring in the figure is heated, will the gap become wider or narrower? Explain your answer.



Figure 23

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SCIENTIFIC BREAKTHROUGHS

A Solid That Flows

Solids, liquids, and gases are the three most familiar and abundant states of matter on Earth. Plasma, a fourth state of matter that is essentially an ionized gas, is the state of matter most common throughout the universe. However, other states of matter exist. Recently, two different research groups succeeded in making an unusual state of matter that had previously been only hypothetical: a supersolid.

How to Make a Supersolid

You can lower the temperature of liquid water to make ice, but it is far more difficult to make a supersolid. A supersolid is a state of matter that has particles arranged in a regular, repeating pattern, like a solid, but are also frictionless and can flow. Scientists have been trying for decades to produce one. In 2004, a group of scientists thought they had produced one using helium-4, but later testing debunked the discovery.

A Case of Multiple Discovery

In 2017, two independent teams of scientists, one from MIT in Cambridge, Massachusetts, and one from a research institute in Zurich, Switzerland, published results in a scientific journal showing they had produced supersolids. Although they used different elements and different methods, the fact that they both achieved similar results provides strong evidence that they succeeded.

The MIT team used sodium, cooling it to nearly absolute zero. When sodium becomes this cold, it forms another state of matter called a Bose-Einstein condensate. A Bose-Einstein condensate is a



Sodium, shown here as a solid, was one of the metals used to make supersolids.

superfluid, which is a fluid with particles that have zero viscosity. The researchers then used an ultra-high-vacuum chamber and lasers to cause the sodium atoms to form a crystalline pattern.

The Swiss team also began with a Bose-Einstein condensate, but they used rubidium as a starting material instead of sodium. Their method for causing the particles of the superfluid to form a crystalline arrangement involved mirrors as well as lasers.

To provide evidence that the atoms had formed a supersolid, both teams used lasers to show that the material produced a diffraction pattern characteristic of a solid crystal.

No applications exist yet for the new state of matter, but the discovery may lead to insights that will improve existing superconductor technology. Further work by the research groups and others will aim to provide more evidence for and uncover further information about the properties of the supersolid state.



Two independent teams made supersolids nearly simultaneously. Research to find another example of multiple discovery. Share your findings by writing a blog post.

MODULE 12

STUDY GUIDE

 **GO ONLINE** to study with your Science Notebook.

Lesson 1 PROPERTIES OF FLUIDS

- Matter in the fluid state flows and has no definite shape of its own.
- The combined gas law represents the relationships among the pressure, volume, and temperature of gases.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

- The ideal gas law represents the relationships among pressure, volume, temperature, and the number of moles of a gas.
- Plasma is a gaslike state of negatively charged electrons and positively charged ions.

- fluids
- pressure
- pascal
- combined gas law
- ideal gas law
- thermal expansion
- plasma

Lesson 2 FORCES WITHIN LIQUIDS

- Surface tension is the tendency of the surface of a liquid to contract to the smallest possible area. Surface tension results from the attractive forces that like particles exert on one another.
- Capillary action occurs when a liquid rises in a thin tube because the adhesive forces between the tube and the liquid are stronger than the cohesive forces between the liquid's molecules.
- Clouds form when water vapor in the atmosphere cools and condenses, forming droplets around dust particles.

- cohesive forces
- adhesive forces

Lesson 3 FLUIDS AT REST AND IN MOTION

- Pascal's principle states that an applied pressure change is transmitted undiminished throughout a fluid.
- According to Archimedes' principle, the buoyant force equals the weight of the fluid displaced by an object.
- Bernoulli's principle states that the pressure exerted by a fluid decreases as its velocity increases.

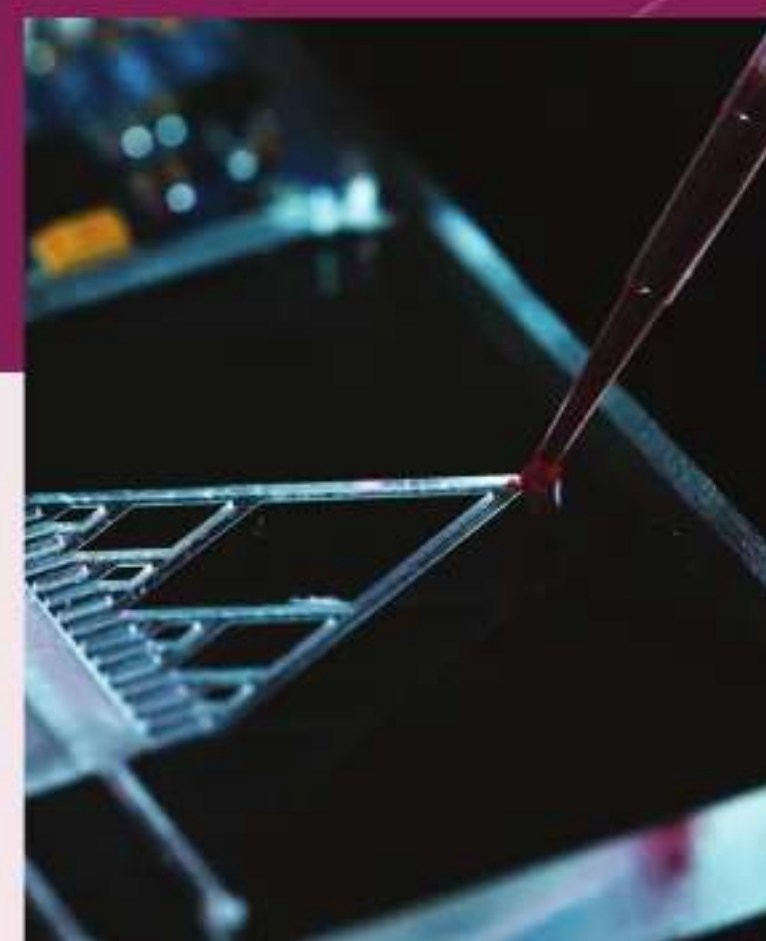
- Pascal's principle
- buoyant force
- Archimedes' principle
- Bernoulli's principle
- streamlines

Lesson 4 SOLIDS

- A crystalline solid has a regular pattern of particles, and an amorphous solid has an irregular pattern of particles. Malleability and ductility depend on structure type.
- As the temperature of a solid changes, the kinetic energy of its particles changes accordingly. As the vibrations of the particles change, a solid expands as temperature increases and contracts as temperature decreases.
- Expansion rates of different materials must be considered when designing structures.



THREE-DIMENSIONAL THINKING Module Wrap-Up



REVISIT THE PHENOMENON

How will the fluid flow through this device?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.



STEM UNIT PROJECT

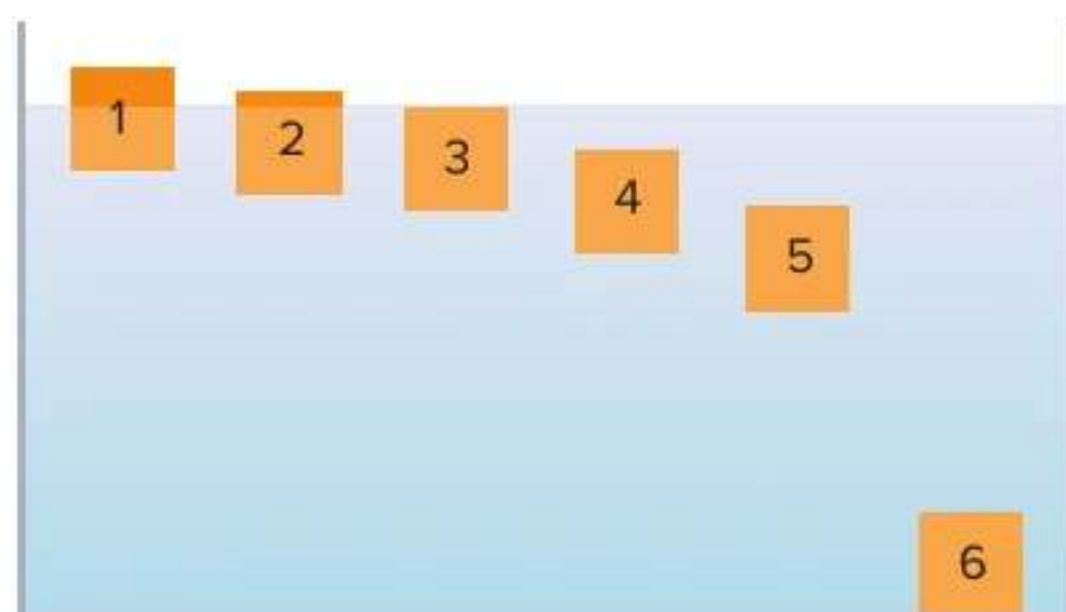
Now that you've completed the module, revisit your STEM unit project. You will apply your evidence from this module and complete your project.

GO FURTHER

SEP Data Analysis Lab

Sinking or floating?

Five objects with the following densities are placed in a tank of water. The diagram shows six possible positions of the objects.



Object	Density (g/cm ³)
A	0.85
B	0.95
C	1.05
D	1.15
E	1.25

CER Analyze and Interpret Data

- Claim** Select a position, from 1 to 6, for each of the five objects. Not all positions need to be selected.
- Evidence and Reasoning** Explain your selections.

Credits

1. Module 15 Fundamentals of Light: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 1
2. Module 16: Reflection and Refraction: *Chapter from Inspire Physics 9-12 Student Edition by McGraw-Hill, 2020* 26
3. Module 08 Rotational Motion: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 64
4. Module 09 Momentum and Its Conservation: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 91
5. Module 11 Thermal Energy: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 114
6. Module 12 States of Matter: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 140

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