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MODULE 1

A PHYSICS TOOLKIT

ENCOUNTER THE PHENOMENON

What tools and skills do physicists use?



GO ONLINE to play a video about the way Newton studied light.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about the tools and skills physicists use. Explain your reasoning.

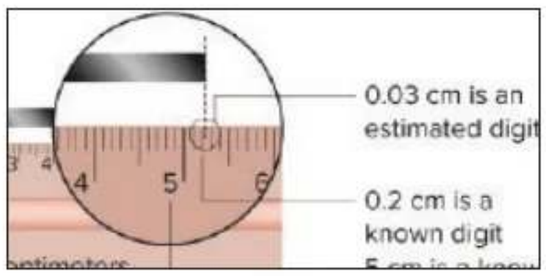
Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain: What is physics?



LESSON 2: Explore & Explain: Uncertainty in Data: Significant Figures

Smithsonian

Additional Resources

(t)Video Supplied by BBC Worldwide Learning, (b)Dusit/Shutterstock

LESSON 1

METHODS OF SCIENCE

FOCUS QUESTION

What do physicists do?

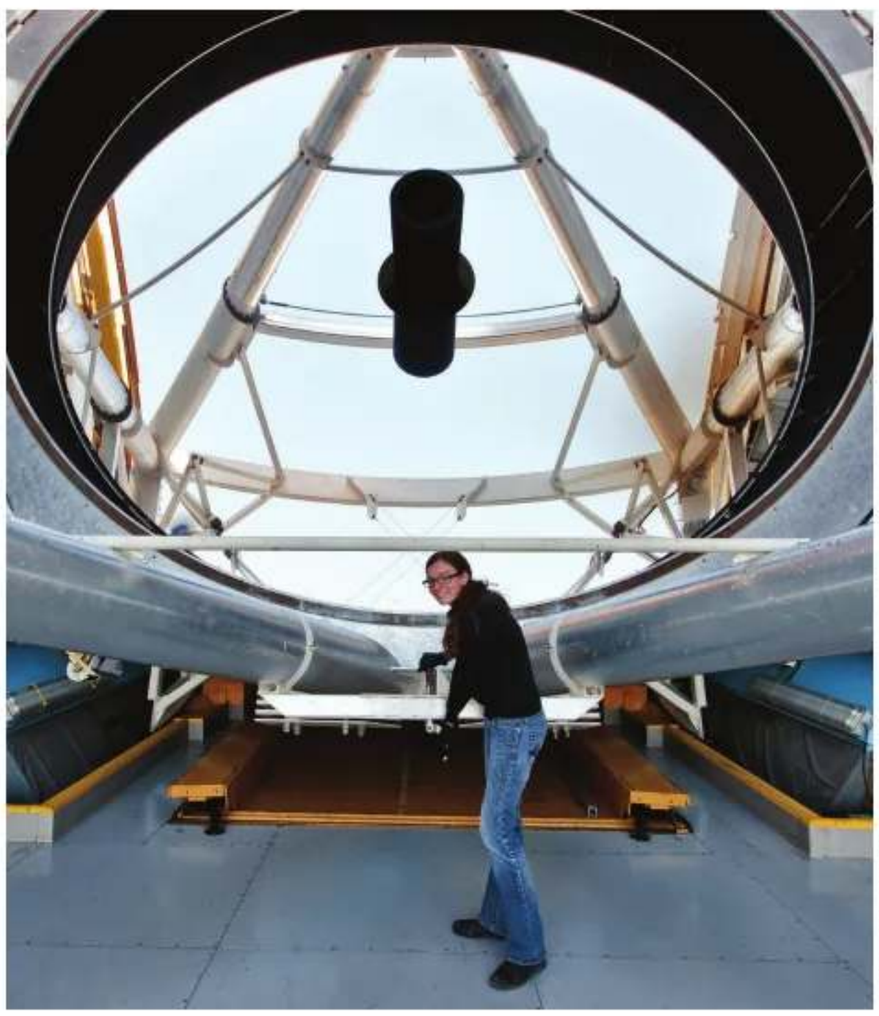
What is physics?

Science is not just a subject in school. It is a process based on inquiry that helps develop explanations about events in nature. **Physics** is a branch of science that involves the study of the physical world: energy, matter, and how they are related.

When you see the word *physics* you might picture a chalkboard full of formulas and mathematics: $E = mc^2$, $I = \frac{V}{R}$, $x = \left(\frac{1}{2}\right)at^2 + v_0t + x_0$. Maybe you picture scientists in white lab coats or well-known figures such as Marie Curie and Albert Einstein. Alternatively, you might think of the many modern technologies created with physics, such as weather satellites, laptop computers, and lasers.

Physicists investigate the motions of electrons and rockets, the energy in sound waves and electric circuits, and the structure of the proton and of the universe. The goal of this course is to help you better understand the physical world.

People who study physics go on to many different careers. Some become scientists at universities and colleges, at industries, or in research institutes. Others go into related fields, such as engineering, computer science, teaching, medicine, or astronomy, as shown in **Figure 1.** Still others use the problem-solving skills of physics to work in finance, construction, or other very different disciplines. In the last 50 years, research in the field of physics has led to many new technologies, including satellite-based communications and high-speed microscanners used to detect disease.



inga spence/Alamy Stock Photo

Figure 1 Physicists may choose from a variety of careers.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Quick Investigation: Measuring Change

Analyze data to find patterns that can be used for extrapolation.

Review the News

Obtain information from a current news story about current physics research. Evaluate your source and communicate your findings to your class.

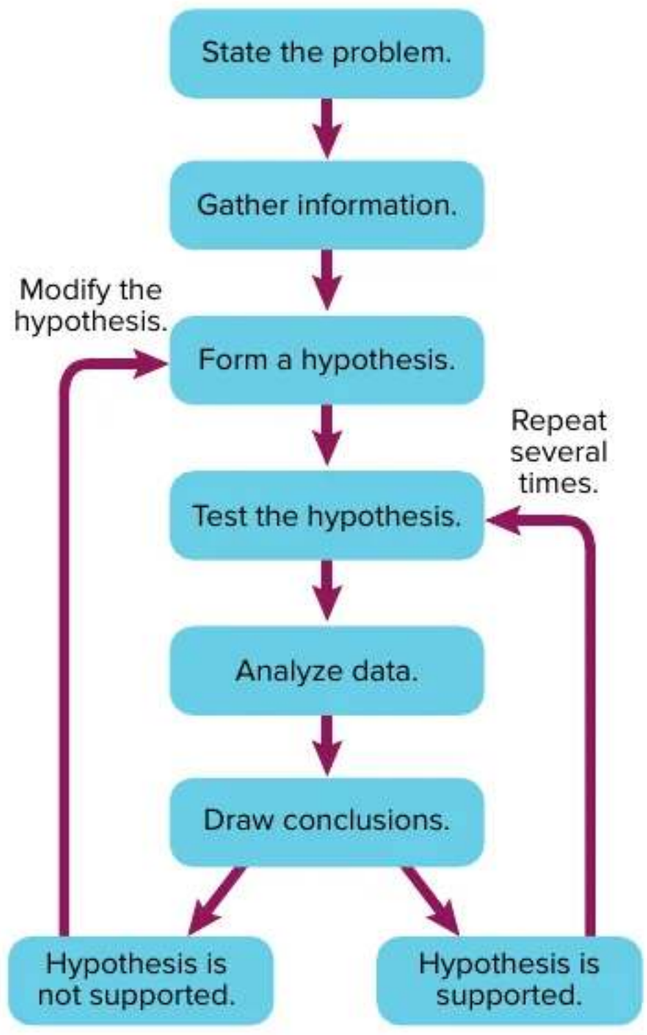


Figure 2 The series of procedures shown here is one way to use scientific methods to solve a problem.

Scientific Methods

Although physicists do not always follow a rigid set of steps, investigations often follow similar patterns called **scientific methods,** as shown in **Figure 2.** Depending on the particular investigation, a scientist might add new steps, repeat some steps, or skip steps altogether.

State the problem

Investigations can begin when one observes an event in nature and wonders why or how it occurs. The question of "why" or "how" is the problem to be stated.

Sometimes a new question is posed during an investigation, leading to a new statement of a problem. In the 1940s, researcher Percy Spencer was trying to answer the question of how to mass-produce the magnetron tubes used in radar systems. When he stood in front of an operating magnetron, which produces microwaves, a candy bar in his pocket melted. The new question of how the magnetron was cooking food was then asked.

Research and gather information

Before beginning an investigation, it is useful to research what is already known about the problem. Making and examining observations and interpretations from reliable sources helps fine-tune the question and form it into a hypothesis.

WORD ORIGINS

Science

comes from the Latin word *scientia*, which means *knowledge*

STEM CAREER Connection

Survey or Mapping Technician

Do you like being outside? Survey technicians spend most of their time outside in various weather conditions collecting geographic data such as elevation and contour. Mapping technicians use the data and sophisticated computer software to make maps of Earth's surface.

Form and test a hypothesis

A **hypothesis** is a possible explanation for a problem using what you know and have observed. A scientific hypothesis can be tested through experimentation and observation. Sometimes scientists must wait for new technologies before a hypothesis can be tested. For example, the first hypotheses about the existence of atoms were developed more than 2300 years ago, but the technologies to test these hypotheses were not available for many centuries.

Some hypotheses can be tested by making observations. Others can be tested by building a model and relating it to real-life situations. One common way to test a hypothesis is to perform an experiment. An experiment tests the effect of one thing on another, using a control. Sometimes it is not possible to perform experiments; in these cases, investigations become descriptive in nature. For example, physicists cannot conduct experiments in deep space. They can, however, collect and analyze valuable data to help us learn more about events occurring there.

Analyze the data

An important part of every investigation includes recording observations and organizing data into easy-to-read tables and graphs.

Later in this module, you will study ways to display data. When you are making and recording observations, you should include all your results, even unexpected ones. Many important discoveries have been made from unexpected results. Scientific inferences are based on scientific observations. All possible scientific explanations must be considered. If the data are not organized in a logical manner, incorrect conclusions can be drawn.

When a scientist communicates and shares data, other scientists will examine those data and how the data were analyzed, and compare the data to the work of others. Scientists, such as the planetary scientist in **Figure 3,** share their data and analyses through reports, journals, and conferences.

Draw conclusions

Based on the analysis of the data, the next step is to decide whether the hypothesis is supported. For the hypothesis to be considered valid and widely accepted, the results of the experiment must be the same every time it is repeated. If the experiment does not support the hypothesis, the hypothesis must be reconsidered. Perhaps the hypothesis needs to be revised, or maybe the experimenter's procedure needs to be refined.



FREDERIC J BROWN/AFP/Getty Images

Figure 3 An important part of scientific methods is to share data and results with other scientists. This scientist is sharing predicted planetary orbital data with colleagues.

Peer review Before it is made public, science-based information is reviewed by scientists' peers—scientists who are in the same field of study. Peer review is a process by which the procedures and results of an experiment are evaluated by peer scientists of those who conducted the research. Reviewing other scientists' work is a responsibility that many scientists have.

Being objective One also should be careful to reduce bias in scientific investigations. Bias can occur when the scientist's expectations affect how the results are analyzed or the conclusions are made. This might cause a scientist to select a result from one trial over those from other trials. Bias might also be found if the advantages of a product being tested are used in a promotion and the drawbacks are not presented. Scientists can lessen bias by running as many trials as possible and by keeping accurate notes of each observation made.

Models

Sometimes, scientists cannot see everything they are testing. They might be observing an object that is too large or too small, a process that takes too much time to see completely, or a material that is hazardous. In these cases, scientists use models. A **model** is a representation of an idea, event, structure, or object that helps people better understand it.

Models in history Models have been used throughout history. In the early 1900s, British physicist J.J. Thomson created a model of the atom that consisted of electrons embedded in a ball of positive charge. Several years later, physicist Ernest Rutherford created a model of the atom based on new research. Later in the twentieth century, scientists discovered that the nucleus is not a solid ball but is made of protons and neutrons. The present-day model of the atom is a nucleus made of protons and neutrons surrounded by an electron cloud. All three of these models are shown in **Figure 4.** Scientists use models of atoms to represent their current understanding because of the small size of an atom.

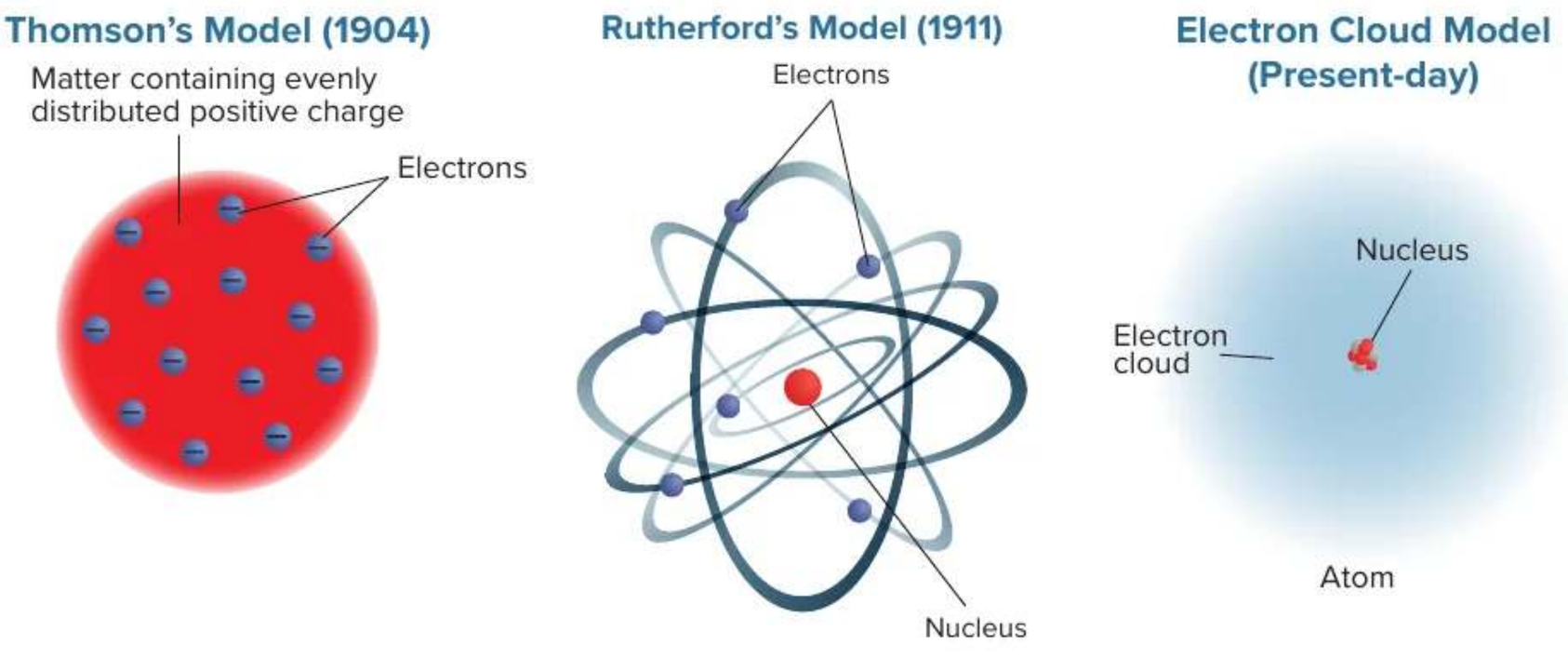


Figure 4 Throughout history, scientists have made models of the atom.

Infer *Why have models of the atom changed over the years?*

Get It?

Explain how the model shown at the beginning of this module is helpful.

High-tech models Both physical models and computers can be used in various ways to aid in the engineering design process. Computers are useful for a variety of purposes, such as running simulations to test different ways of solving a problem or to see which one is most efficient or economical; and in making a persuasive presentation to a client about how a given design will meet his or her needs. Computer software is designed to mimic the processes under study.

For instance, computer simulators, such as the one shown in **Figure 5,** help airplane pilots practice all aspects of flight without ever leaving the ground. In addition, computer simulations can simulate harsh weather conditions and other potentially dangerous in-flight challenges.



Figure 5 This is a flight simulator used to help train pilots. The image mimics what the pilot would see if flying a real plane.

Identify *other models around your classroom.*

Get It?

Discuss how computer simulations can help develop possible solutions to a problem.

Scientific Theories and Laws

A **scientific theory** is an explanation of things or events based on knowledge gained from many observations and investigations.

A theory is not a guess. If scientists repeat an investigation and the results always support the hypothesis, the hypothesis can be called a theory. Just because a scientific theory has data supporting it does not mean it will never change. As new information becomes available, theories can be refined or modified, as shown in **Figure 6** on the next page.

A **scientific law** is a statement about what happens in nature and seems to be true all the time. Laws tell you what will happen under certain conditions, but they don't explain why or how something happens. Gravity is an example of a scientific law. The law of gravity states that any one mass will attract another mass. To date, no experiments have been performed that disprove the law of gravity.

A theory can be used to explain a law, but theories do not become laws. For example, many theories have been proposed to explain how the law of gravity works. Even so, there are few accepted theories in science and even fewer laws.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Models can be used to simulate systems. Choose a model not mentioned in this lesson. Remember that a model isn't necessarily a physical model, something you can build and touch, although it could be. Prepare a poster that shows how your model can help test a process or a procedure. What type of model will you use? What evidence supports your explanation?

santofilme/iStock/Getty Images

Greek philosophers proposed that objects fall because they seek their natural places. The more massive the object, the faster it falls.

Revision

Galileo showed that the speed at which an object falls depends on the amount of time for which that object has fallen and not on the object's mass.

Revision

Newton provided an explanation for why objects fall. Newton proposed that objects fall because the object and Earth are attracted by a force. Newton also stated that there is a force of attraction between any two objects with mass.

Revision

Einstein suggested that the force of attraction between two objects is due to mass causing the space around it to curve.

Figure 6 If experiments provide new insight and evidence about a theory, such as the theory describing the behavior of falling objects, the theory is modified accordingly.

The Limitations of Science

Science can help you explain many things about the world, but science cannot explain or solve everything. Scientists make guesses, but the guesses must be tested and verified. Questions about opinions, values, and emotions are not scientific because they cannot be tested. For example, some people may find a particular piece of art beautiful while others do not. Some people might think that certain foods, such as pizza, taste delicious while others do not. You might take a survey to gather opinions about such questions, but that would not prove that the opinions are true for everyone.

Check Your Progress

1. **Summarize** the steps you might use to carry out an investigation using scientific methods.
2. **Define** the term *hypothesis*. Identify three ways to test a hypothesis.
3. **Describe** why it is important for scientists to avoid bias.
4. **Explain** why scientists use models. Give an example of a scientific model not mentioned in this lesson, and explain how it is useful.
5. **Analyze** Your friend finds that 90 percent of students surveyed in the cafeteria like pizza. She says this scientifically proves that everyone likes pizza. How would you respond?
6. **Critical Thinking** An accepted value for free-fall acceleration is 9.8 m/s^2. In an experiment with pendulums, you calculate a value to be 9.4 m/s^2. Should the accepted value be tossed out because of your finding? Explain.

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LESSON 2

MATHEMATICS AND PHYSICS

FOCUS QUESTION

How is math helpful to physicists?

Mathematics in Physics

Physicists often use the language of mathematics. In physics, equations are important tools for modeling observations and for making predictions. Equations are one way of representing relationships between measurements. Physicists rely on theories and experiments with numerical results to support their conclusions. For example, you can predict that if you drop a penny, it will fall, but can you predict how fast it will be going when it strikes the ground? Different models of falling objects give different answers to how the speed of the object changes as it falls or on what the speed depends. By measuring how an object falls, you can compare the experimental data with the results predicted by different models. This tests the models, allowing you to pick the best one or to develop a new model.

SI Units

To communicate results, it is helpful to use units that everyone understands. The worldwide scientific community uses an adaptation of the metric system for measurements. **Table 1** shows that the Système International d'Unités, or SI, uses seven base quantities. Other units, called derived units, are formed by combining the base units in various ways. Velocity is measured in meters per second (m/s). Often, derived units are given their own names. For example, electric charge is measured in ampere-seconds (A·s), which are also called coulombs (C).

SI is regulated by the International Bureau of Weights and Measures in Sèvres, France. In the past, this bureau and the National Institute of Science and Technology (NIST) in Gaithersburg, Maryland, kept physical samples on which the standards were based.

Table 1 SI Base Units

Base Quantity	Base Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Amount of a substance	mole	mol
Electric current	ampere	A
Luminous intensity	candela	cd

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

CCC **Identify Crosscutting Concepts**

Create a table of the crosscutting concepts and fill in examples you find as you read.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

However, the standards are being redefined in terms of phenomena. For example, the meter, as shown in **Figure 7,** is now defined as $\frac{1}{299,792,458}$, the distance that light travels in $\frac{1}{299,792,458}$ of a second.

The ease of switching between units is another convenient feature of SI. To convert between units, multiply or divide by the appropriate power of 10. Prefixes are used to change SI base units by powers of 10, as shown in **Table 2.** You often will encounter these prefixes in daily life, as in, for example, milligrams and centimeters.



Figure 7 You will use metersticks as your standard of measurement, but the official definition of a meter is $\frac{1}{299,792,458}$, the distance that light travels in $\frac{1}{299,792,458}$ of a second.

Describe *Why is it important to have standards for measurements?*

Dimensional Analysis

You often will need to manipulate a formula, or use a string of formulas, to solve a physics problem. One way to check whether you have set up a problem correctly is to write out the equation or set of equations you plan to use. Before doing calculations, check that the answer will be in the expected units. For example, if you are finding a car's speed and you see that your answer will have the units s/m or m/s^2, you have made an error in setting up the problem. This method of treating the units as algebraic quantities that can be canceled is called **dimensional analysis.** Knowing that your answer will be in the correct units is not a guarantee that your answer is right, but if you find that your answer has or will have the wrong units, you can be sure that you have made an error. Dimensional analysis also is used in choosing conversion factors. A conversion factor is a multiplier equal to 1.

Table 2 Prefixes Used with SI Units

Prefix	Symbol	Multiplier	Scientific Notation	Example
femto–	f	0.000000000000001	10^{-15}	femtosecond (fs)
pico–	p	0.000000000001	10^{-12}	picometer (pm)
nano–	n	0.000000001	10^{-9}	nanometer (nm)
micro–	μ	0.000001	10^{-6}	microgram (μg)
milli–	m	0.001	10^{-3}	milliamps (mA)
centi–	c	0.01	10^{-2}	centimeter (cm)
deci–	d	0.1	10^{-1}	deciliter (dL)
kilo–	k	1000	10^{3}	kilometer (km)
mega–	M	1,000,000	10^{6}	megagram (Mg)
giga–	G	1,000,000,000	10^{9}	gigameter (Gm)
tera–	T	1,000,000,000,000	10^{12}	terahertz (THz)

Get It?

Identify the prefix that would be used to express 2,000,000,000 bytes of computer memory.

Richard Hutchings/Digital Light Source/McGraw-Hill Education

For example, because 1 kg = 1000 g, you can construct the following conversion factors:

$$1 = \frac{1\,\text{kg}}{1000\,\text{g}} \quad \text{and} \quad 1 = \frac{1000\,\text{g}}{1\,\text{kg}}$$

Choose a conversion factor that will make the initial units cancel, leaving the answer in the desired units. For example, to convert a mass of 1.34 kg to grams, set up the conversion as shown below.

$$1.34\ \cancel{\text{kg}}\left(\frac{1000\,\text{g}}{1\ \cancel{\text{kg}}}\right) = 1340\ \text{g}$$

You also might need to do a series of conversions. To convert 43 km/h to m/s, do the following:

$$\left(\frac{43\ \cancel{\text{km}}}{1\ \cancel{\text{h}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right) = 12\ \text{m/s}$$

Significant Figures

Suppose you measure a pen and find that the end of the pen is just past 138 mm, as shown in **Figure 8.** You estimate that the pen is one-tenth of a millimeter past the last tick mark on the ruler and record the pen as being 138.1 mm long. This measurement has four valid digits: the first three digits are certain, and the last one is uncertain. The valid digits in a measurement are called **significant figures.** The last digit given for any measurement is the uncertain digit. All nonzero digits in a measurement are significant.

Are all zeros significant? No. For example, in the measurement 0.0860 m, the first two zeros serve only to locate the decimal point and are not significant. The last zero, however, is the estimated digit and is significant. The measurement 172,000 m could have 3, 4, 5, or 6 significant figures. This ambiguity is one reason to use scientific notation. It is clear that the measurement 1.7200×10^5 m has five significant figures.

Arithmetic with significant figures When you do any arithmetic operation, remember that the result never can be more precise than the least-precise measurement.

To add or subtract measurements, first do the operation, then round off the result to correspond to the least-precise value involved. For example, 3.86 m + 2.4 m = 6.3 m, not 6.26 m, because the least-precise measure is to one-tenth of a meter.

To multiply or divide measurements, first do the calculation, and then round to the same number of significant figures as the least-precise measurement. For example, $\frac{409.2\ \text{km}}{11.4\ \text{L}} = \frac{35.9\ \text{km}}{\text{L}}$, because the least-precise measurement has three significant figures. Some calculators display several additional digits, while others round at different points. Be sure to record your answers with the correct number of digits.

Solving Problems

Most practice problems in this course will be complex and require a strategy to solve. This textbook includes many example problems, each of which is solved using a three-step process. Example Problem 1 on the next page follows the steps to calculate a car's average speed using distance and time.

Figure 8 You recorded the length of this pen as 138.1 mm.

Infer *Why is the last digit uncertain?*



Richard Hutchings/Digital Light Source

EXAMPLE Problem 1

USING DISTANCE AND TIME TO FIND SPEED When a car travels 434 km in 4.5 h, what is the car's average speed?

1 ANALYZE AND SKETCH THE PROBLEM

The car's speed is unknown. The known values include the distance the car traveled and the time. Use the relationship among speed, distance, and time to solve for the car's speed.

Known
distance = 434 km
time = 4.5 h

Unknown
speed = ?

2 SOLVE FOR THE UNKNOWN

distance = **speed** × time — State the relationship as an equation.

$\textbf{speed} = \frac{\text{distance}}{\text{time}}$ — Solve the equation for speed.

$\textbf{speed} = \frac{434\ \text{km}}{4.5\ \text{h}}$ — Substitute distance = 434 km and time = 4.5 h.

speed = 96.4 km/h — Calculate, and specify the units.

3 EVALUATE THE ANSWER

Check your answer by using it to calculate the distance the car traveled.

distance = speed × time = 96.4 km/~~h~~ × 4.5 ~~h~~ = 434 km

The calculated distance matches the distance stated in the problem. This means that the calculated average speed is correct.

THE PROBLEM

1. Read the problem carefully.
2. Be sure you understand what is being asked.

ANALYZE AND SKETCH THE PROBLEM

1. Read the problem again.
2. Identify what you are given, and list the known data. If needed, gather information from graphs, tables, or figures.
3. Identify and list the unknowns.
4. Determine whether you need a sketch to help solve the problem.
5. Plan the steps you will follow to find the answer.

SOLVE FOR THE UNKOWN

1. If the solution is mathematical, write the equation and isolate the unknown factor.
2. Substitute the known quantities into the equation.
3. Solve the equation.
4. Continue the solution process until you solve the problem.

EVALUATE THE ANSWER

1. Reread the problem. Is the answer reasonable?
2. Check your math. Are the units and significant figures correct?

Check Your Progress

7. **Modeling** Why are concepts in physics described with formulas?
8. **Significant Figures** Solve the following problems, using the correct number of significant figures each time.
   a. 10.8 g – 8.264 g
   b. 4.75 m – 0.4168 m
   c. 139 cm × 2.3 cm
   d. 13.78 g / 11.3 mL
   e. 1.6 km + 1.62 m + 1200 cm
9. **Dimensional Analysis** How many seconds are in a leap year?
10. **Solving Problems** Rewrite $F = Bqv$ to find v in terms of F, q, and B.
11. **Critical Thinking** Using values given in a problem and the equation for distance, distance = speed × time, you calculate a car's speed to be 290 km/h. Is this answer reasonable? Explain. Under what circumstances might this be a reasonable answer?

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LESSON 3

MEASUREMENT

FOCUS QUESTION

Why is it important to make careful measurments?

What is measurement?

A **measurement** is a comparison between an unknown quantity and a standard. For example, if you measure the mass of a rolling cart used in an experiment, the unknown quantity is the mass of the cart and the standard is the gram, as defined by the balance or spring scale you use. If you do an experiment with a spring for which the length is unknown, the centimeter is the standard you might use for length. Measurements quantify our observations. For example, a person's blood pressure isn't just "pretty good"; it's $\frac{110}{60}$, the low end of the good range.

Comparing Results

As you learned in Lesson 1, scientists share their results. Before new data are fully accepted, other scientists examine the experiment, look for possible sources of error, and try to reproduce the results. Results often are reported with an uncertainty. A new measurement that is within the margin of uncertainty is in agreement with the old measurement.

For example, archaeologists use radiocarbon dating to determine the age of cave paintings, such as those from the Niaux cave in France, in **Figure 9,** and the Chauvet cave, also in France. Each radiocarbon date is reported with an uncertainty. Three radiocarbon ages from a panel in the Chauvet cave are 30,940 ± 610 years, 30,790 ± 600 years, and 30,230 ± 530 years. While none of the measurements matches, the uncertainties in all three overlap, and the measurements agree with each other.



CAROLUS/Pixtal/age fotostock

Figure 9 These drawings are from the Niaux cave in France. Scientists estimate that the drawings were made about 17,000 years ago.

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COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Mass and Volume

Carry out an investigation to determine the relationship between mass, volume, and density.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Suppose three students performed an experiment several times starting with springs of the same length. With two washers on the spring, student 1 made repeated measurements, which ranged from 14.4 cm to 14.8 cm. The average of student 1's measurements was 14.6 cm, as shown in **Figure 10.** This result was reported as (14.6 ± 0.2) cm. Student 2 reported finding the spring's length to be (14.8 ± 0.3) cm. Student 3 reported a length of (14.0 ± 0.1) cm.

Could you conclude that the three measurements are in agreement? Is student 1's result reproducible? The ranges of the results of students 1 and 2 overlap between 14.5 cm and 14.8 cm. However, there is no overlap and, therefore, no agreement, between their results and the result of student 3.

Get It?

Explain Is student 3's result reproducible? Why or why not?

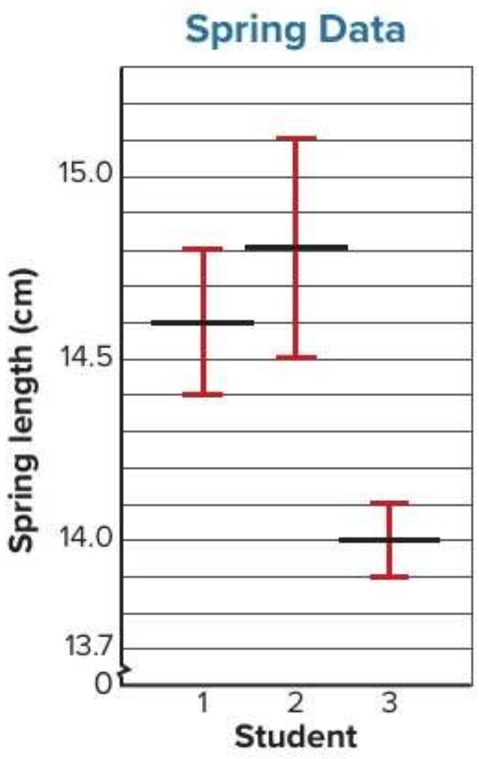


Figure 10 Three students took multiple measurements. The red bars show the uncertainty of each student's measurement.

Precision Versus Accuracy

Both precision and accuracy are characteristics of measured values, as shown in **Figure 11.** How precise and accurate are the measurements of the three students above? The degree of exactness of a measurement is called its **precision.** In the example above, student 3's measurements are the most precise, within ± 0.1 cm. Both the measurements of student 1 and student 2 are less precise because they have a larger uncertainty (student 1 = ± 0.2 cm, student 2 = ± 0.3 cm).

Precision depends on the instrument and technique used to make the measurement. Generally, the device that has the finest division on its scale produces the most precise measurement. The precision of a measurement is one-half the smallest division of the instrument.

For example, suppose a graduated cylinder has divisions of 1 mL. You could measure an object to within 0.5 mL with this device. However, if the smallest division on a beaker is 50 mL, how precise would your measurements be compared to those taken with the graduated cylinder?

The significant figures in an answer show its precision. A measure of 67.100 g is precise to the nearest thousandth of a gram. Say you add 1.2 mL of acid to a beaker containing 2.4×10^2 mL of water—you cannot say you now have 2.412×10^2 mL of fluid because the volume of water was not measured to the nearest tenth of a milliliter, but to the nearest 10 mL.

Accuracy describes how well the results of a measurement agree with the "real" value; that is, the accepted value as measured by competent experimenters. **Figure 11** illustrates accuracy and precision.

If the length of the spring that the three students above measured had been 14.8 cm, then student 2 would have been most accurate and student 3 least accurate. What might have led someone to make inaccurate measurements? How could you check the accuracy of measurements?

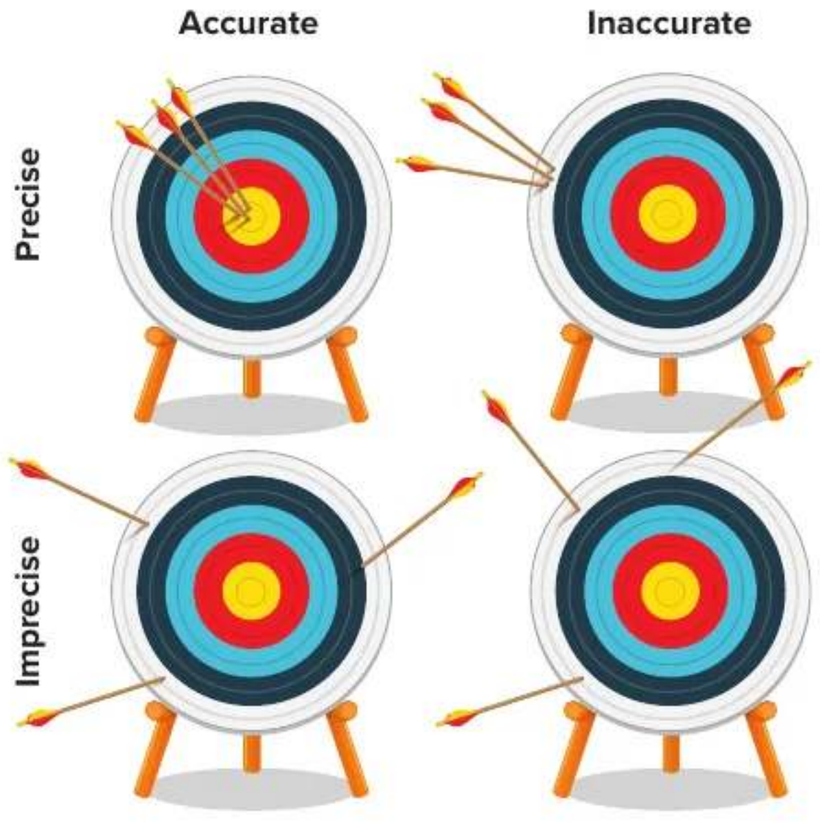


Figure 11 The yellow area in the center of each target represents an accepted value for a particular measurement. The arrows represent measurements taken by a scientist during an experiment.

A common method for checking the accuracy of an instrument is called the two-point calibration. First, does the instrument read zero when it should, as shown in **Figure 12?** Second, does it give the correct reading when it is measuring an accepted standard? Regular checks for accuracy are performed on critical measuring instruments, such as the radiation output of the machines used to treat cancer.

Get It?

Compare and contrast precision and accuracy.

Techniques of Good Measurement

To assure accuracy and precision, instruments also have to be used correctly. Measurements have to be made carefully if they are to be as precise as the instrument allows. One common source of error comes from the angle at which an instrument is read. Scales should be read with one's eye directly in front of the measure, as shown on the left of **Figure 13.** If the scale is read from an angle, as shown on the right of **Figure 13,** a different, less accurate, value will be obtained. The difference in the readings is caused by parallax, which is the apparent shift in the position of an object when it is viewed from different angles. To experiment with parallax, place your pen on a ruler and read the scale with your eye directly over the tip, then read the scale with your head shifted far to one side.



Figure 12 Accuracy is checked by zeroing an instrument before measuring.

Infer *Is this instrument accurate? Why or why not?*



Figure 13 By positioning the scale head-on (left), your results will be more accurate than if you read your measurements at an angle (right).

Identify *How far did parallax shift the measurement on the right?*

(t)McGraw-Hill Education; (b)Hutchings Photography/Digital Light Source

GPS

The Global Positioning System, or GPS, offers an illustration of accuracy and precision in measurement. The GPS consists of 24 satellites with transmitters in orbit and several receivers on Earth. The satellites send signals with the time, measured by highly accurate atomic clocks.

The receiver uses the information from at least four satellites to determine latitude, longitude, and elevation. (The clocks in the receivers are not as accurate as those on the satellites.)

Receivers have different levels of precision. A device in an automobile might give your position to within a few meters. Devices used by geophysicists and geologists, as in **Figure 14,** can measure movements of millimeters in Earth's crust.

The GPS was developed by the United States Department of Defense. It uses atomic clocks, which were developed to test Einstein's theories of relativity and gravity. The GPS eventually was made available for civilian use.

GPS signals now are provided worldwide free of charge and are used in navigation on land, at sea, and in the air, for mapping and surveying, by telecommunications and satellite networks, and for scientific research into earthquakes and plate tectonics.



Figure 14 This geologist in the Mali Desert is using a highly accurate GPS receiver to record and analyze the movements of continental plates. His findings will help in the search for oil deposits.

Greenshoots Communications/Alamy Stock Photo

Check Your Progress

12. **Precision and Accuracy** You find a micrometer (a tool used to measure objects to the nearest 0.001 mm) that has been bent. How does it compare to a new, high-quality meter-stick in its precision and accuracy?
13. **Accuracy** Some wooden rulers do not start with 0 at the edge, but have it set in a few millimeters. How could this improve the accuracy of the ruler?
14. **Parallax** Does parallax affect the precision of a measurement that you make? Explain.
15. **Uncertainty** Your friend tells you that his height is 182 cm. In your own words, explain the range of heights implied by this statement.
16. **Precision** A box has a length of 18.1 cm, a width of 19.2 cm, and is 20.3 cm tall.
    a. What is its volume?
    b. How precise is the measurement of length? Of volume?
    c. How tall is a stack of 12 of these boxes?
    d. How precise is the measurement of the height of one box? Of 12 boxes?
17. **Critical Thinking** Your friend states in a report that the average time required for a car to circle a 2.4-km track was 65.414 s. This was measured by timing 7 laps using a clock with a precision of 0.1 s. How much confidence do you have in the results of the report? Explain.

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LESSON 4

GRAPHING DATA

FOCUS QUESTION

How do graphs help scientists analyze data?

Identifying Variables

When you perform an experiment, it is important to change only one factor at a time. For example, **Table 3** gives the length of a spring with different masses attached. Only the mass varies; if different masses were hung from different types of springs, you wouldn't know how much of the difference between two data pairs was due to the different masses and how much was due to the different springs.

Independent and dependent variables A variable is any factor that might affect the behavior of an experimental setup. The factor that is manipulated during an investigation is the **independent variable.** In the experiment that gave the data in **Table 3,** the mass was the independent variable. The factor that depends on the independent variable is the **dependent variable.** In this investigation, the amount the spring stretched depended on the mass, so the amount of stretch was the dependent variable.

Line of best fit A line graph shows how the dependent variable changes with the independent variable. The data from **Table 3** are graphed in **Figure 15** on the next page. The line in blue, drawn as close to all the data points as possible, is called a **line of best fit.** The line of best fit is a better model for predictions than any one point along the line. **Figure 15** gives detailed instructions on how to construct a graph, plot data, and sketch a line of best fit.

A well-designed graph allows patterns that are not immediately evident in a list of numbers to be seen quickly and simply. The graph in **Figure 15** shows that the length of the spring increases as the mass suspended from the spring increases.

Table 3 Length of a Spring for Different Masses

Mass Attached to Spring (g)	Length of Spring (cm)
0	13.7
5	14.1
10	14.5
15	14.9
20	15.3
25	15.7
30	16.0
35	16.4

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Forensics Lab: It's in the Blood

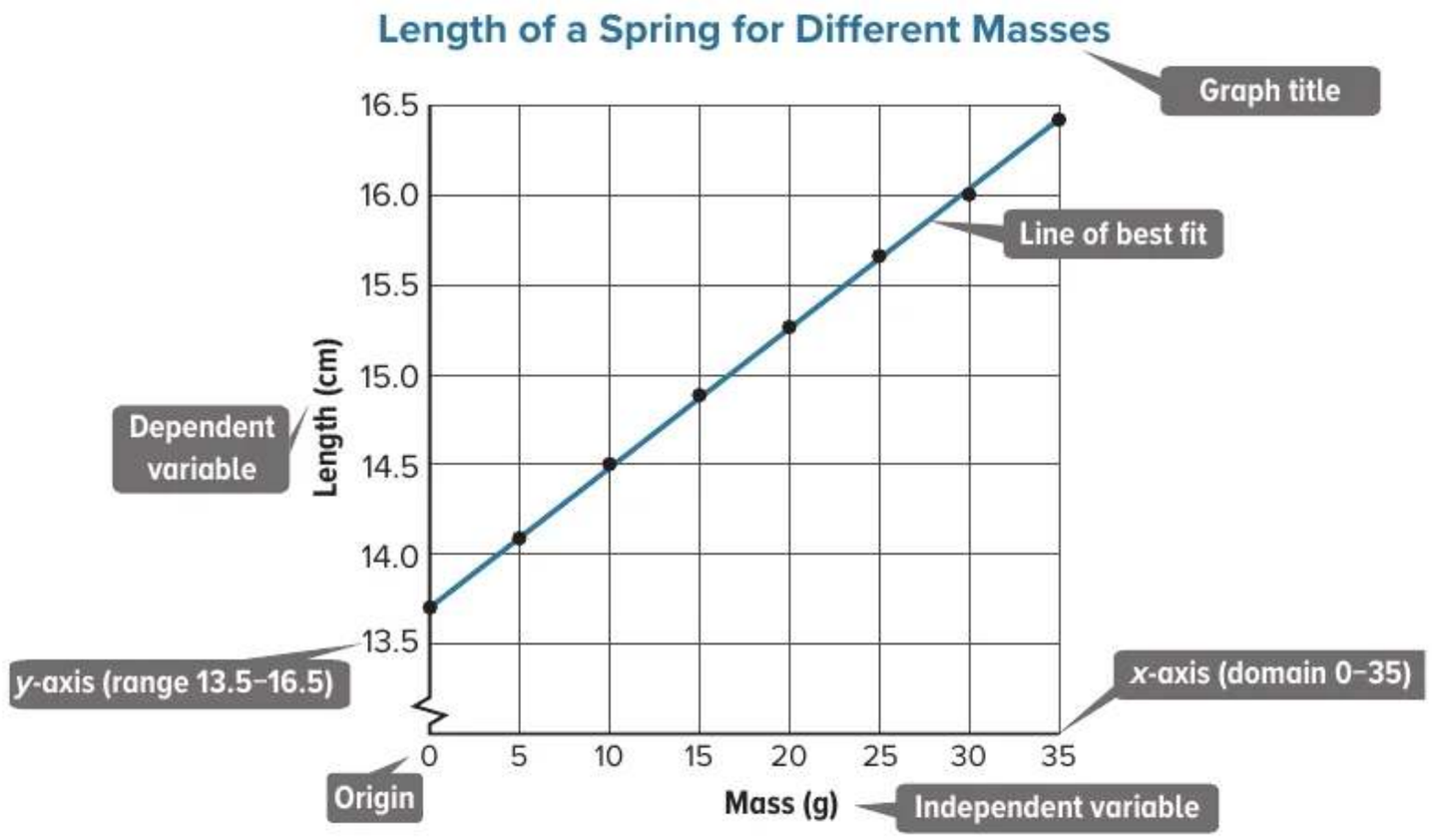
Analyze and interpret data to determine cause and effect at a crime scene.

Review the News

Obtain information from a current news story where graphs are used to present data.

Evaluate your source and communicate your findings to your class.

Figure 15 Use the steps outlined here to plot line graphs from data tables.



1. Identify the independent variable and the dependent variable in your data. In this example, the independent variable is mass (g) and the dependent variable is length (cm). The independent variable is plotted on the horizontal axis, the *x*-axis. The dependent variable is plotted on the vertical axis, the *y*-axis.
2. Determine the range of the independent variable to be plotted. In this case the range is 0–35.
3. Decide whether the origin (0, 0) is a valid data point.
4. Spread the data out as much as possible. Let each division on the graph paper stand for a convenient unit. This usually means units that are multiples of 2, 5, or 10.
5. Number and label the horizontal axis. The label should include the name of the variable and its units, for example, Mass (g).
6. Repeat steps 2–5 for the dependent variable.
7. Plot the data points on the graph.
8. Draw the best-fit straight line or smooth curve that passes through as many data points as possible. This is sometimes called *eyeballing*. Do not use a series of straight-line segments that connect the dots. The line that looks like the best fit to you may not be exactly the same as someone else's. There is a formal procedure, which many graphing calculators use, called the least-squares technique, that produces a unique best-fit line, but that is beyond the scope of this textbook.
9. Give the graph a title that clearly tells what the graph represents.

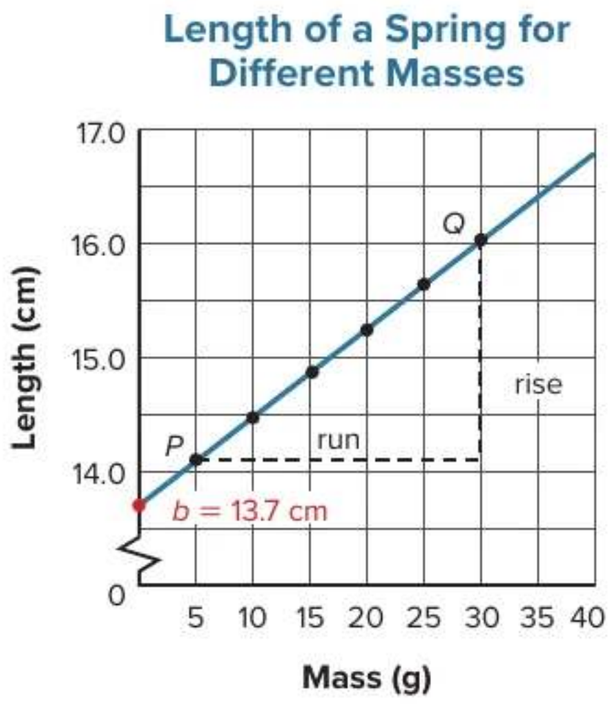


Figure 16 In a linear relationship, the dependent variable—in this case, length—varies linearly with the independent variable. The independent variable in this experiment is mass.

Describe *What happens to the length of the spring as mass decreases?*

Linear Relationships

Scatter plots of data take many different shapes, suggesting different relationships. Three of the most common relationships are linear relationships, quadratic relationships, and inverse relationships. You probably are familiar with them from math class.

When the line of best fit is a straight line, as in **Figure 15,** there is a linear relationship between the variables. In a **linear relationship,** the dependent variable varies linearly with the independent variable. The relationship can be written as the following equation.

Linear Relationship Between Two Variables

$$y = mx + b$$

Here, m is the slope of the line, or the ratio of the vertical change to the horizontal change, and b is the y-intercept, the point at which the line crosses the vertical axis. To find the slope, select two points, P and Q, far apart on the line—they may or may not be data points. The vertical change, or rise (Δy), is the difference between the vertical values of P and Q, as shown in **Figure 16.** The horizontal change, or run (Δx), is the difference between the horizontal values of P and Q.

Slope

The slope of a line is equal to the rise divided by the run, which also can be expressed as the vertical change divided by the horizontal change.

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x}$$

In **Figure 16:** $m = \frac{(16.0\text{ cm} - 14.1\text{ cm})}{(30\text{ g} - 5\text{ g})} = 0.08\text{ cm/g}$

If y gets smaller as x gets larger, then $\frac{\Delta y}{\Delta x}$ is negative, and the line slopes downward from left to right. The y-intercept, or the y-value when the value of x is zero, in this example is $b = 13.7$ cm (**Figure 16**). So, when no mass is suspended by the spring, it has a length of 13.7 cm. When $b = 0$, or $y = mx$, y is said to vary directly with x. In physics, the slope of the line and the y-intercept always contain information about the physical system that is described by the graph.

Nonlinear Relationships

Figure 17 graphs the distance a brass ball falls versus time. Note that the graph is not a straight line, meaning the relationship is not linear. There are many types of nonlinear relationships in science. Two of the most common are quadratic and inverse relationships.

Quadratic relationships

The graph in **Figure 17** is a quadratic relationship, represented by the equation below. A **quadratic relationship** exists when one variable depends on the square of another.

Quadratic Relationship Between Two Variables

$$y = ax^2 + bx + c$$

A computer program or graphing calculator can easily find the values of the constants a, b, and c in the above equation. In **Figure 17,** the equation is $d = 5t^2$. See the Math Skill Handbook in the back of this book or online for more on making and using line graphs.

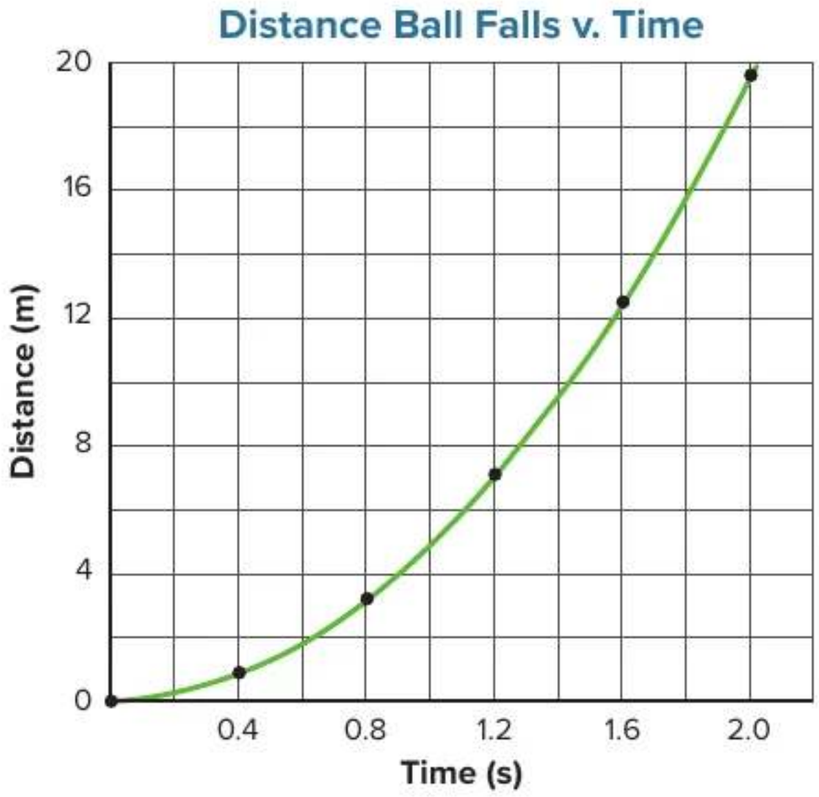


Figure 17 The quadratic, or parabolic, relationship shown here is an example of a nonlinear relationship.

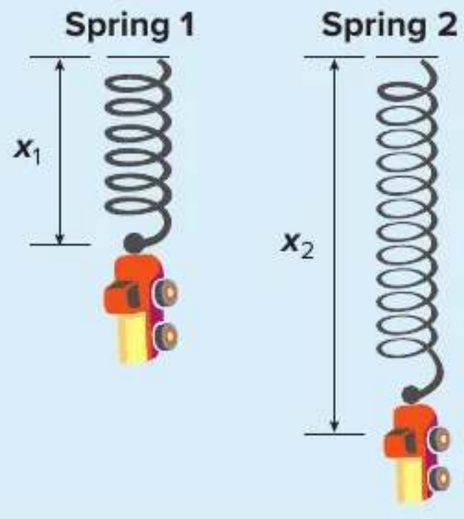
Get It?

Explain how two variables are related to each other in a quadratic relationship.

PHYSICS Challenge

An object is suspended from spring 1, and the spring's elongation (the distance it stretches) is x_1. Then the same object is removed from the first spring and suspended from a second spring. The elongation of spring 2 is x_2. x_2 is greater than x_1.

1. On the same axes, sketch the graphs of the mass versus elongation for both springs.
2. Should the origin be included in the graph? Why or why not?
3. Which slope is steeper?
4. At a given mass, $x_2 = 1.6\ x_1$. If $x_2 = 5.3$ cm, what is x_1?



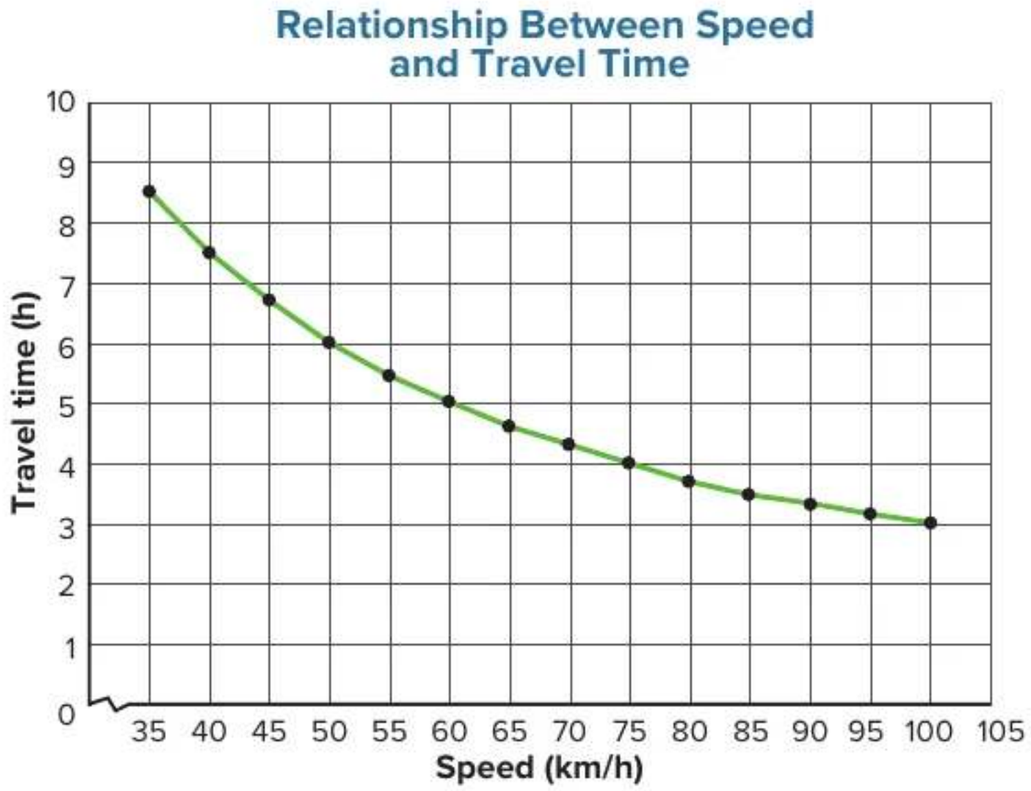


Figure 18 This graph shows the inverse relationship between speed and travel time.

Describe *How does travel time change as speed increases?*

Inverse relationships

The graph in **Figure 18** shows how the time it takes to travel 300 km varies as a car's speed increases. This is an example of an inverse relationship, represented by the equation below. An **inverse relationship** is a hyperbolic relationship in which one variable depends on the inverse of the other variable.

Inverse Relationship Between Two Variables

$$y = \frac{a}{x}$$

The three relationships you have learned about are a sample of the relations you will most likely investigate in this course. Many other mathematical models are used. Important examples include sinusoids, used to model cyclical phenomena, and exponential growth and decay, used to study radioactivity. Combinations of different mathematical models represent even more complex phenomena.

Get It?

Explain how two variables are related to each other in an inverse relationship.

PRACTICE Problems

ADDITIONAL PRACTICE

18. Refer to the data listed in **Table 4.**

a. Plot mass versus volume, and draw the curve that best fits all points. Describe the curve.

b. What type of relationship exists between the mass of the gold nuggets and their volume?

c. What is the value of the slope of this graph? Include the proper units.

d. Write the equation showing mass as a function of volume for gold.

e. Write a word interpretation for the slope of the line.

Table 4 Mass of Pure Gold Nuggets

Volume (cm^3)	Mass (g)
1.0	19.4
2.0	38.6
3.0	58.1
4.0	77.4
5.0	96.5

Predicting Values

When scientists discover relationships like the ones shown in the graphs in this lesson, they use them to make predictions. For example, the equation for the linear graph in **Figure 16** is as follows:

$$y = (0.08 \text{ cm/g})x + 13.7 \text{ cm}$$

Relationships, either learned as formulas or developed from graphs, can be used to predict values you haven't measured directly. How far would the spring in **Table 3** stretch with 49 g of mass?

$$y = (0.08 \text{ cm/g})(49 \text{ g}) + 13.7 \text{ cm}$$
$$= 18 \text{ cm}$$

It is important to decide how far you can extrapolate from the data you have. For example, 90 g is a value far outside the ones measured and displayed in **Table 3,** and the spring might break rather than stretch that far.

Physicists use models to accurately predict how systems will behave: what circumstances might lead to a solar flare (an immense outburst of material from the Sun's surface into space) or how changes to a grandfather clock's pendulum will change its ability to keep accurate time. People in all walks of life use models in many ways. One example is shown in **Figure 19.** With the tools you have learned in this module, you can answer questions and produce models for the physics questions you will encounter in the rest of this textbook.

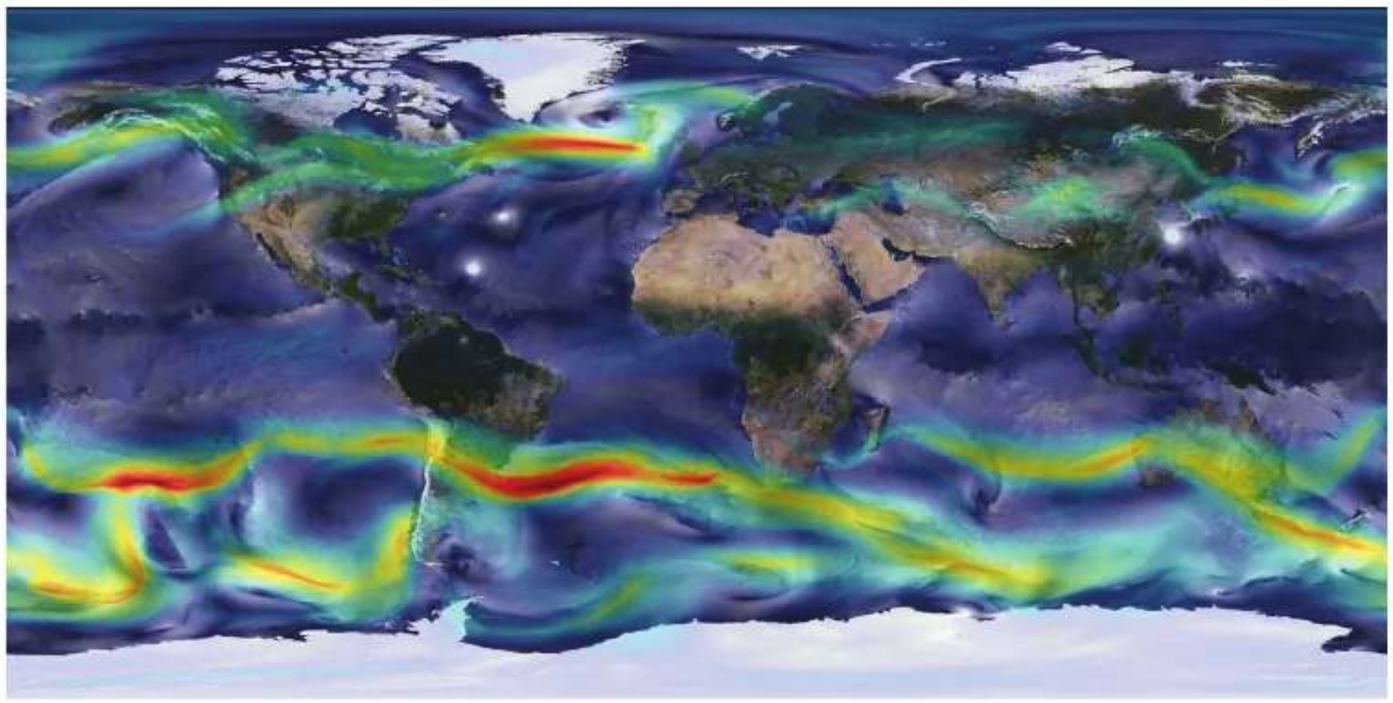


Figure 19 In order to create a realistic model, computer animators use mathematical models of the real world to help visualize global atmospheric conditions.

William Putman/NASA Goddard Space Flight Center

Check Your Progress

19. **Make a Graph** Graph the following data. Time is the independent variable.

Time (s)	0	5	10	15	20	25	30	35
Speed (m/s)	12	10	8	6	4	2	2	2

20. **Interpret a Graph** What would be the meaning of a nonzero *y*-intercept in a graph of total mass versus volume?

21. **Predict** Use the relationship illustrated in **Figure 16** to determine the mass required to stretch the spring 15 cm.

22. **Predict** Use the relationship shown in **Figure 18** to predict the travel time when speed is 110 km/h.

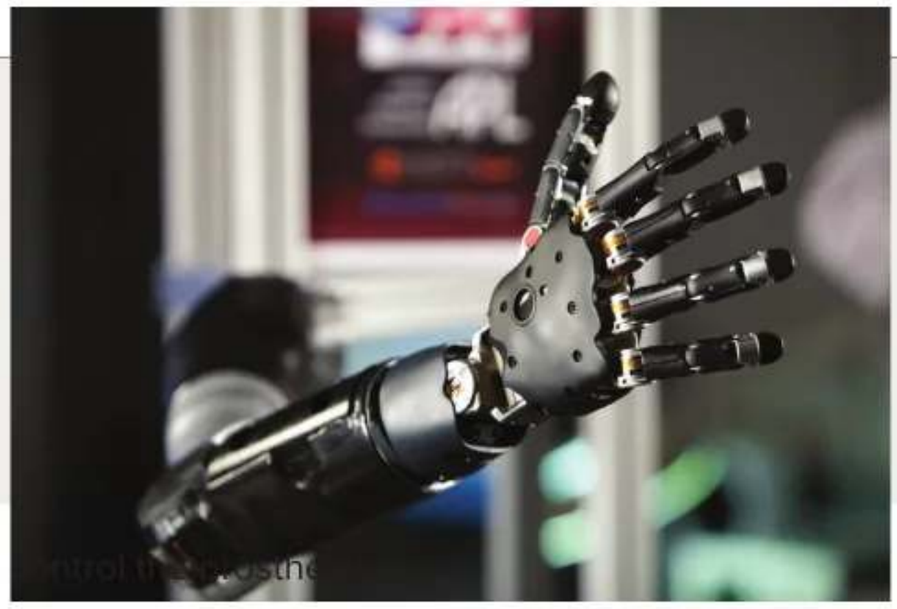
23. **Critical Thinking** Look again at the graph in **Figure 16.** In your own words, explain how the spring would be different if the line in the graph were shallower or had a smaller slope.

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ENGINEERING & TECHNOLOGY

A Step in the Right Direction

Prosthetics, or artificial body parts, have been in use since ancient Greek times. Most prostheses of the past, however, were more decorative than functional. They were made mainly of metal and wood, and were attached to the body with straps or harnesses made of leather. Prosthetics have come a long way over the last few decades. Today's prosthetic limbs can transform people's lives.



Some prosthetics can be attached directly to the patient's body, rather than being attached with slings or harnesses.

New materials, new look

The prostheses of today look and function more like biological limbs than ever before. The inner structure of a prosthesis, called a pylon, is now commonly made of carbon-fiber composites and new types of plastics. These materials make the prosthesis stronger, but also lighter, than metal or wood prostheses.

Electronic components give users better control over a prosthesis. For example, myoelectric prostheses use the electric signals generated by muscles to control a prosthesis. A person with a myoelectric prosthetic hand can move muscles in his or her arm to signal the hand to move or grip in different ways, with varying amounts of force.

Targeted muscle reinnervation (TMR) is another new method of giving people more control over cutting-edge prostheses, specifically arms. The nerves in a person's residual limb, which once controlled the amputated limb, are surgically "reassigned" to

Another game-changing advance in prosthetic technology is a change in the socket, the part of a prosthesis that connects to a person's residual limb. When prostheses are attached with slings or harnesses, the socket is sometimes uncomfortable. It can cause sores, blisters, and pain, and it can damage the tissues under the skin. To combat this problem, some new prostheses are mounted directly into the bone marrow of a person's residual limb. This method of attachment increases the person's comfort, as well as his or her control of the prosthesis.

Some prosthetic advances are still making the transition from prototypes to patient care. But scientists are determined to speedily move these new technologies into the everyday lives of patients.



COMMUNICATE TECHNICAL INFORMATION

Use print or online sources to research an advance in prosthetics that was not discussed in this feature. Share the results of your research with your class.

Food Collection/Alamy Stock Photo

STEM AT WORK

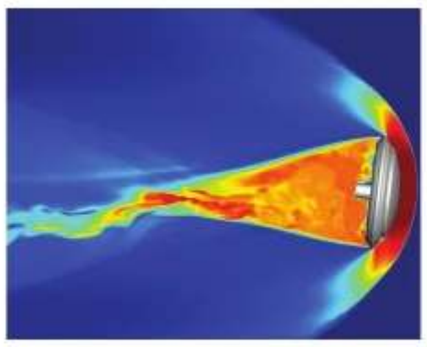
What do physicists study?

Physicist study a wide range of topics from the tiniest particles to stars, galaxies, and the universe itself.



Mechanics is the branch of physics that studies motion and forces.

Physicists often work with other branches of science. For example, geophysics is the study of Earth's physical properties and processes. Another area of collaboration is biomechanics, which is the study of the motion of living things. The researcher in the photo at the right is collecting data about the motion of an athlete.

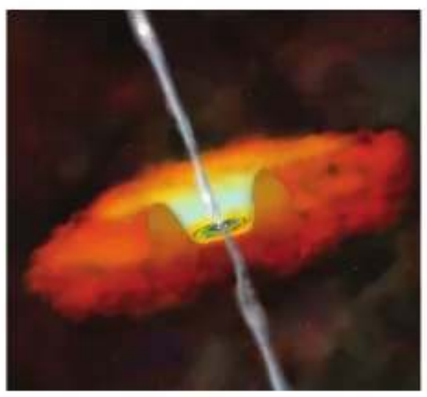


Thermodynamics is the study of the energy of a system's particles. It includes the study of temperature, heat, and the relationship between the characteristics of a system's particles and the macroscopic characteristics of the system. Because systems contain very large numbers of particles, physics often use computer models to study thermodynamics. The NASA model on the left allows scientists to predict how heat shields will perform in the atmosphere.

Electromagnetism is the study of electricity, magnetism, and the relationship between them. By understanding how currents produce magnetic fields, physicists and engineers can design and build electromagnets, like the one on the right.



Nuclear and Particle Physics Nuclear physicists study the atomic nucleus. Particle physicists will use this superconducting electromagnet in a particle accelerator to study even smaller structures—the elementary particles that make up universe.



Optics is the study of the generation, transmission, and detection of visible, infrared, ultraviolet, and microwave radiation.

Astrophysics and Cosmology are, respectively, the study of the physical properties of celestial bodies such as the active galactic nucleus on the left and the development and overall structure of the universe.

(t to b, l to r)DC Studio/Shutterstock; Joe Brock, NASA Ames Research Center; Cultura Creative(RF)/Alamy Stock Photo; NASA/CXC/M.Weiss



OBTAIN AND COMMUNICATE INFORMATION

Choose one of the fields of physics that interests you. Research your chosen field, and identify a related career, what a person does in that career, and the types of education and training required. Develop a presentation in which you are recruiting candidates for a job opening in that career.

MODULE 1
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 METHODS OF SCIENCE

- Scientific methods include making observations and asking questions about the natural world.
- Scientists use models to represent things that may be too small or too large, processes that take too much time to see completely, or a material that is hazardous.
- A scientific theory is an explanation of things or events based on knowledge gained from observations and investigations. A scientific law is a statement about what happens in nature, which seems to be true all the time.
- Science can't explain or solve everything. Questions about opinions and values cannot be tested.

- physics
- scientific methods
- hypothesis
- model
- scientific theory
- scientific law

Lesson 2 MATHEMATICS AND PHYSICS

- Using SI, an adaptation of the metric system, helps scientists around the world communicate more easily.
- Dimensional analysis is used to check that an answer is in the correct units.
- Significant figures are the valid digits in a measurement.

- dimensional analysis
- significant figures

Lesson 3 MEASUREMENT

- Measurements are reported with uncertainty because a new measurement that is within the margin of uncertainty confirms the old measurement.
- Precision is the degree of exactness with which a quantity is measured. Accuracy is the extent to which a measurement matches the true value.
- A common source of error that occurs when making a measurement is the angle at which an instrument is read. If the scale of an instrument is read at an angle, as opposed to at eye level, the measurement will be less accurate.

- measurement
- precision
- accuracy

Lesson 4 GRAPHING DATA

- Graphs contain information about the relationships between variables. Patterns that are not immediately evident in a list of numbers are seen more easily when the data are graphed.
- Common relationships shown in graphs are linear relationships, quadratic relationships, and inverse relationships. In a linear relationship, the dependent variable varies linearly with the independent variable. A quadratic relationship occurs when one variable depends on the square of another. In an inverse relationship, one variable depends on the inverse of the other variable.
- Scientists use models and relationships between variables to make predictions.

- independent variable
- dependent variable
- line of best fit
- linear relationship
- quadratic relationship
- inverse relationship

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

What tools and skills do physicists use?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

GO FURTHER

SEP Data Analysis Lab

How is string length and square of the period of a pendulum related?

A group of students planned and carried out an investigation into the relationship between string length L and the time it took for one complete swing of a pendulum (its period) T.

Data and Observations Their measurements are given in the table.

String Length L (m)	Period T (s)
0.07	0.53
0.10	0.63
0.20	0.90
0.40	1.27
0.55	1.49
0.70	1.68
0.90	1.90

CER Analyze and Interpret Data

One of the students used the data to investigate the relationship between the string length L and the square of the period T^2.

1. **Calculate and record** the values of T^2.
2. **Construct a Model** Plot the values of L and T^2 and draw the best-fit curve. Write an equation that describes the curve.
3. **Claim** What is the relationship between the square of the period of a pendulum and the length of the string?
4. **Evidence and Reasoning** Use your graph as evidence and explain your reasoning to justify your claim.

FatCamera/E+/Getty Images


Burnie
Penguin
Ulverstone
Devonport
Launceston
Deloraine
Tarraleah
GPS
Aurora Photos/Alamy Stock Photo

MODULE 2

REPRESENTING MOTION

ENCOUNTER THE PHENOMENON

How does a GPS unit know where you are?



GO ONLINE to play a video about how GPS works.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about how a GPS unit knows where you are. Explain your reasoning.

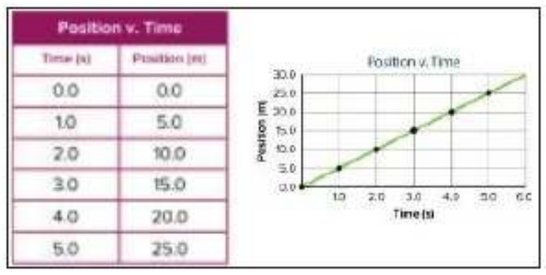
Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 2: Explore & Explain: Coordinates and Vectors



LESSON 3: Explore & Explain: Velocity and Speed

Smithsonian

Additional Resources

(t)Video Supplied by BBC Worldwide Learning, (b)vectorfusionart/Shutterstock

LESSON 1

PICTURING MOTION

FOCUS QUESTION

How do you know that something is moving?

All Kinds of Motion

You have learned about scientific processes that will be useful in your study of physics. You will now begin to use these tools to analyze motion. In subsequent modules, you will apply these processes to many kinds of motion. You will use words, sketches, diagrams, graphs, and equations.

Changes in position What comes to your mind when you hear the word motion? A spinning ride at an amusement park? A baseball soaring over a fence for a home run? Motion is all around you—from fast trains and speedy skiers to slow breezes and lazy clouds. Objects move in many different ways, such as the straight-line path of a bowling ball in a bowling lane's gutter, the curved path of a car rounding a turn, the spiral of a falling kite, and swirls of water circling a drain. When an object is in motion, such as the train in **Figure 1,** its position changes.

Some types of motion are more complicated than others. When beginning a new area of study, it is generally a good idea to begin with the least complicated situation, learn as much as possible about it, and then gradually add more complexity to that simple model. In the case of motion, you will begin your study with movement along a straight line.



Figure 1 The train appears blurry in the photograph because its position changed during the time the camera shutter was open.

Ingram Publishing

Get It?

Describe how the picure in **Figure 1** would be different if the train were sitting still.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Motion Diagrams

Use a model to identify patterns in motion.

Review the News

Obtain information from a current news story about motion capture technology. Evaluate your source and communicate your findings to your class.

Movement along a straight line

In general, an object can move along many different kinds of paths, but straight-line motion follows a path directly between two points without turning left or right. For example, you might describe an object's motion as forward and backward, up and down, or north and south. In each of these cases, the object moves along a straight line.

Suppose you are reading this textbook at home. As you start to read, you glance over at your pet hamster and see that it is sitting in a corner of the cage. Sometime later you look over again, and you see that it now is sitting next to the food dish in the opposite corner of the cage. You can infer that your hamster has moved from one place to another in the time between your observations. What factors helped you make this inference about the hamster's movement?

The description of motion is a description of place and time. You must answer the questions of where an object is located and when it is at that position in order to describe its motion.

Get It?

Identify two factors you must know to describe the motion of an object along a straight line.

Motion Diagrams

Consider the following example of straight-line motion: a runner jogs along a straight path. One way of representing the runner's motion is to create a series of images showing the runner's position at equal time intervals. You can do this by photographing the runner in motion to obtain a sequence of pictures. Each photograph will show the runner at a point that is farther along the straight path.

Consecutive images

Suppose you point a camera in a direction and a runner crosses the camera's field of view. Then you take a series of photographs of the runner at equal time intervals, without moving the camera. **Figure 2** shows what a series of consecutive images for a runner might look like. Notice that the runner is in a different position in each image, but everything in the background remains in the same position. This indicates that, relative to the camera and the ground, only the runner is in motion.



Richard Hutchings/Digital Light Source

Figure 2 You can tell that the runner is in motion because her position changes relative to the tree and the ground.

Combining images Suppose that you layered the four images of the runner from **Figure 2** one on top of the other, as shown in **Figure 3**. A series of images showing the positions of a moving object at equal time intervals is called a **motion diagram.**

Particle Models

Keeping track of the runner's motion is easier if you disregard the movement of her arms and her legs and instead concentrate on a single point at the center of her body. In a **particle model,** you replace the object or objects with single points.

To use the particle model, the object's size must be much less than the distance it moves. In the photographic motion diagram, you could identify a central point at her waistline, and draw a dot to represent the position at different times. The bottom of **Figure 3** shows the particle model for the runner's motion.

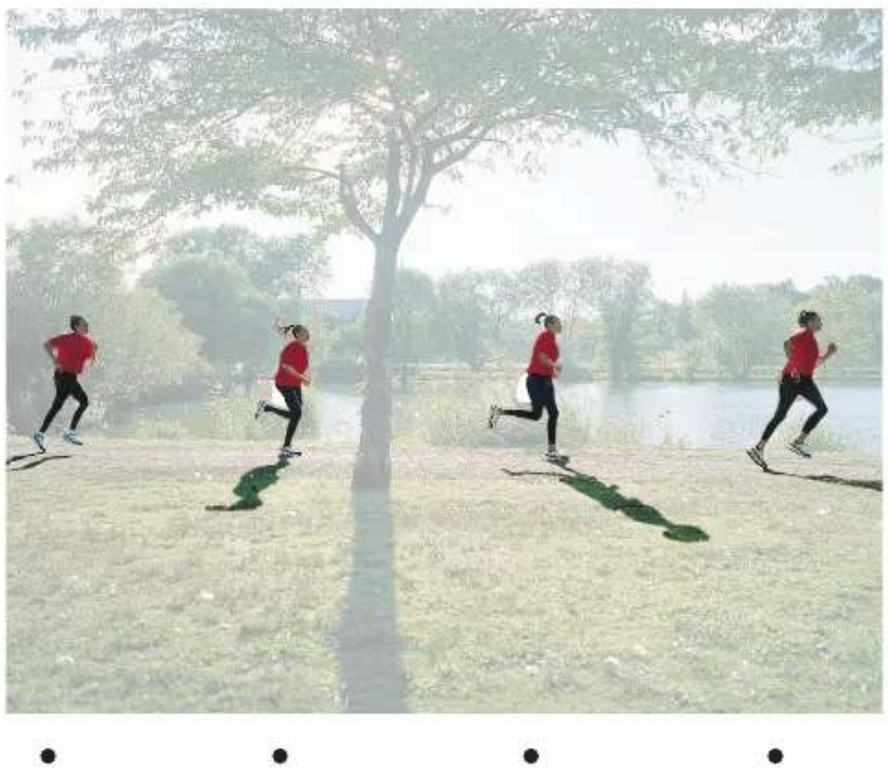


Figure 3 Combining the images from **Figure 2** produces this motion diagram of the runner's movement. The series of dots at the bottom of the figure is a particle model that corresponds to the motion diagram.

Explain *how the particle model shows that the runner's speed is not changing.*

Get It?

Describe how you would model the motion of the hiker at the beginning of this module.

Check Your Progress

1. **Representing Motion** How does a motion diagram represent an object's motion?
2. **Bike Motion Diagram** Draw a particle model motion diagram for a bike rider moving at a constant pace along a straight path.
3. **Car Motion Diagram** Draw a particle motion diagram corresponding to the motion in **Figure 4** for a car coming to a stop at a stop sign. What point on the car did you use to represent the car?

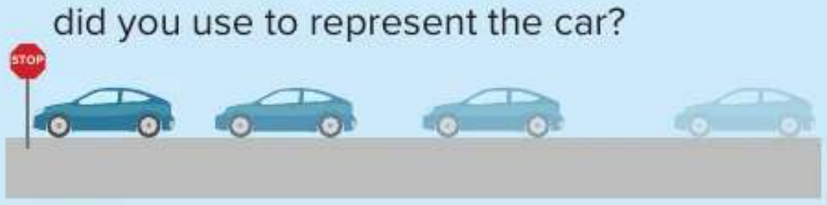


Figure 4

4. **Bird Motion Diagram** Draw a particle model motion diagram corresponding to the motion diagram in **Figure 5.** What point on the bird did you choose to represent the bird?



Figure 5

5. **Critical Thinking** Draw particle model motion diagrams for two runners during a race in which the first runner crosses the finish line as the other runner is three-fourths of the way to the finish line.

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Richard Hutchings/Digital Light Source

LESSON 2

WHERE AND WHEN?

FOCUS QUESTION

What are some different ways of describing and representing motion?

Coordinate Systems

Is it possible to measure distance and time on a motion diagram? Before photographing a runner, you could place a long measuring tape on the ground to show where the runner is in each image. A stopwatch within the camera's view could show the time. But where should you place the end of the measuring tape? When should you start the stopwatch?

Position and distance It is useful to identify a system in which you have chosen where to place the zero point of the measuring tape and when to start the stopwatch. A **coordinate system** gives the location of the zero point of the variable you are studying and the direction in which the values of the variable increase, as shown in the diagram in **Figure 6.**

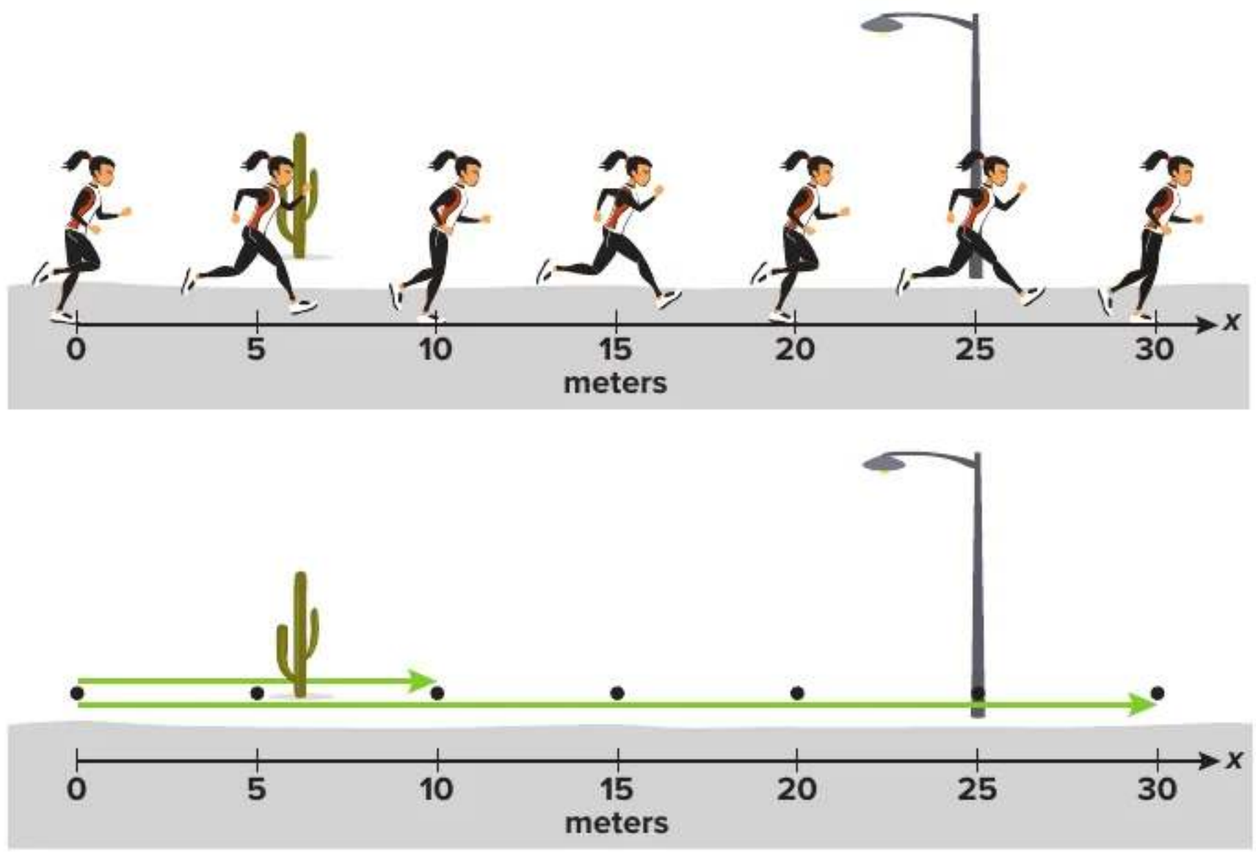


Figure 6 A simplified motion diagram uses dots to represent a moving object and arrows to indicate positions.

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COLLECT EVIDENCE

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INVESTIGATE

GO ONLINE to find these activities and more resources.

Quick Investigation: Vector Models

Develop and use a model to visualize the result of adding vector quantities in one dimension.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

The **origin** is the point at which all variables in a coordinate system have the value zero. In the example of the runner shown in **Figure 6,** the origin, which is the zero point of the measuring tape, could be 6 m to the left of the cactus. Because the motion is in a straight line, your measuring tape should lie along this line. The straight line is an axis of the coordinate system.

You can indicate how far the runner in **Figure 6** is from the origin at a certain time on the motion diagram by drawing an arrow from the origin to the point that represents the runner, shown at the bottom of **Figure 6**. This arrow represents the runner's **position,** the distance and direction from the origin to the object. In general, **distance** is the entire length of an object's path, even if the object moves in many directions. Because the motion in **Figure 6** is in one direction, the arrow lengths represent distance.

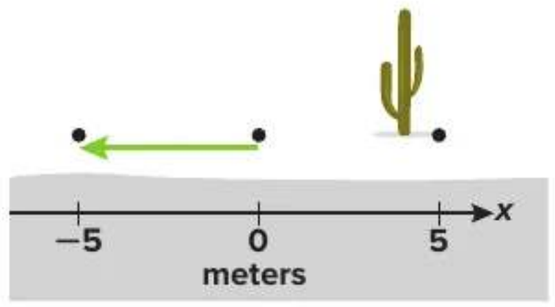


Figure 7 The green arrow indicates a negative position of −5 m, if the direction right of the origin is chosen as positive.

Infer *What position would the arrow indicate if you chose the direction left of the origin as positive?*

Negative position

Is there such a thing as a negative position? Suppose you chose the coordinate system just described but this time placed the origin 4 m left of the cactus with the *x*-axis extending in a positive direction to the right. A position 9 m left of the cactus, or 5 m left of the origin, would be a negative position, as shown in **Figure 7.**

Get It?
Explain how positive and negative positions are determined.

Vectors and Scalars

As you might imagine, there are many kinds of measurements and numbers used to represent or describe motion. If you needed to describe how far you ran, you might say that you ran 1.6 km. If you needed to run to a specific location, you might say that you need to run 1.6 km north. Many quantities in physics have both size, also called **magnitude,** and direction. A quantity that has both magnitude and direction is called a **vector.** You can represent a vector with an arrow. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector. A quantity that is just a number without any direction, such as distance, time, or temperature, is called a **scalar.** In this textbook, we will use boldface letters to represent vector quantities and regular letters to represent scalars.

Get It?
Describe the difference between a vector and a scalar.

SCIENCE USAGE V. COMMON USAGE

Magnitude
Science usage: a measure of size
When drawing vectors, the magnitude of a vector is proportional to that vector's length.
Common usage: great size or extent
The magnitude of the Grand Canyon is difficult to capture in photographs.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Vectors represent a form of modeling. With a partner, create a vector drawing representing motion from one location to another. Add explanations to your drawing that explain how your motion was modeled.

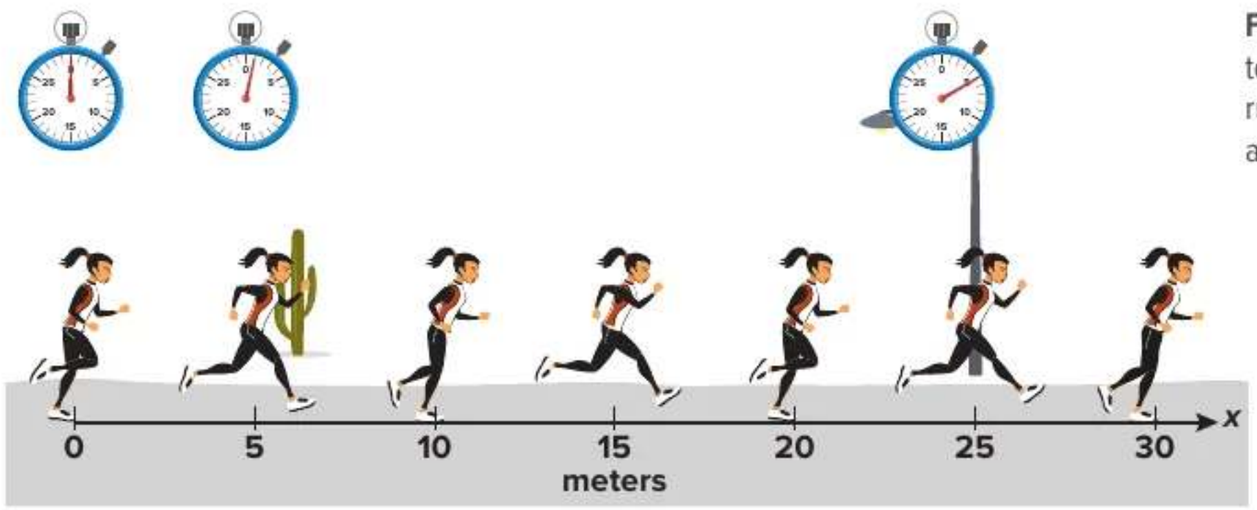


Figure 8 You can use the clocks in the figure to calculate the time interval (Δt) for the runner's movement from one position to another.

Time intervals are scalars. When analyzing the runner's motion in **Figure 8,** you might want to know how long it took her to travel from the cactus to the lamppost. You can obtain this value by finding the difference between the stopwatch reading at the cactus and the stopwatch reading at the lamppost. **Figure 8** shows these stopwatch readings. The difference between two times is called a **time interval.**

A common symbol for a time interval is Δt, where the Greek letter delta (Δ) is used to represent a change in a quantity. Let t_i represent the initial (starting) time, when the runner was at the cactus. Let t_f represent the final (ending) time of the interval, when the runner was at the lamppost. We define a time interval mathematically as follows.

Time Interval

The time interval is equal to the change in time from the initial time to the final time.

$$\Delta t = t_f - t_i$$

The subscripts i and f represent the initial and final times, but they can be the initial and final times of any time interval you choose. In the example of the runner, the time it takes for her to go from the cactus to the lamppost is $t_f - t_i = 5.0\ \text{s} - 1.0\ \text{s} = 4.0\ \text{s}$. You could instead describe the time interval for the runner to go from the origin to the lamppost. In this case, the time interval would be $t_f - t_i = 5.0\ \text{s} - 0.0\ \text{s} = 5.0\ \text{s}$. The time interval is a scalar because it has no direction. Is the runner's position a scalar?

Positions and displacements are vectors. You have already seen how a position can be described as negative or positive in order to indicate whether that position is to the left or the right of a coordinate system's origin. This suggests that position is a vector because position has direction—either right or left in this case.

STEM CAREER Connection

Geographic Information Systems (GIS) Manager

Would you like to work with computers, maps, and logistics? Then, a GIS manager might be a career for you. GIS managers work with teams that produce geographical information systems that are used for road traffic management, health care delivery, defense planning, market research, and community services such as garbage collection and utilities delivery.

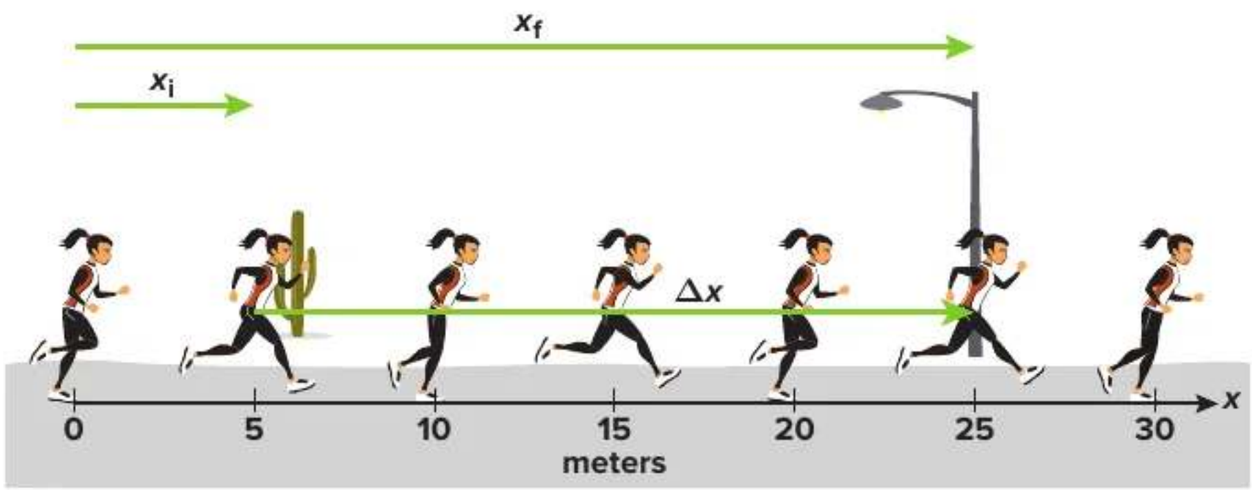


Figure 9 The vectors x_i and x_f represent positions. The vector Δx represents displacement from x_i to x_f.

Describe *the displacement from the lamppost to the cactus.*

Figure 9 shows the position of the runner at both the cactus and the lamppost. Notice that you can draw an arrow from the origin to the location of the runner in each case. These arrows have magnitude and direction. In common speech, a position refers to a certain place, but in physics, the definition of a position is more precise. A position is a vector with the arrow's tail at the origin of a coordinate system and the arrow's tip at the place.

You can use the symbol x to represent position vectors mathematically. In **Figure 9**, the symbol x_i represents the position at the cactus, and the symbol x_f represents the position at the lamppost. The symbol Δx represents the change in position from the cactus to the lamppost. Because a change in position is described and analyzed so often in physics, it has a special name. In physics, a change in position is called a **displacement.** Because displacement has both magnitude and direction, it is a vector.

What was the runner's displacement when she ran from the cactus to the lamppost? By looking at **Figure 9**, you can see that this displacement is 20 m to the right. Notice also, that the displacement from the cactus to the lamppost (Δx) equals the position at the lamppost (x_f) minus the position at the cactus (x_i). This is true in general; displacement equals final position minus initial position.

Displacement

Displacement is the change in position from initial position to final position.

$$\Delta x = x_f - x_i$$

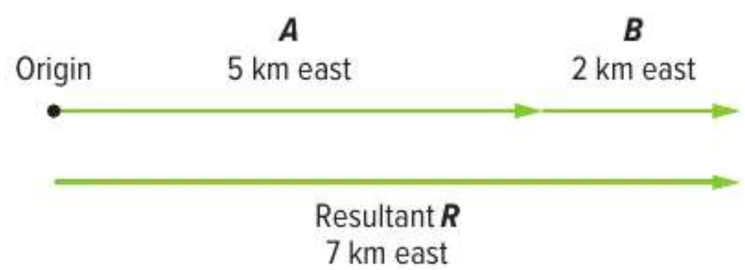
Remember that the initial and final positions are the start and the end of any interval you choose. Although position is a vector, sometimes the magnitude of a position is described without the boldface. In this case, a plus or minus sign might be used to indicate direction.

Vector addition and subtraction You will learn about many different types of vectors in physics, including velocity, acceleration, and momentum. Often, you will need to find the sum of two vectors or the difference between two vectors. A vector that represents the sum of two other vectors is called a **resultant. Figure 10** on the next page shows how to add and subtract vectors in one dimension. In a later module, you will learn how to add and subtract vectors in two dimensions.

Figure 10 You can use a diagram or an equation to combine vectors.
Analyze *What is the sum of a vector 12 m north and a vector 8 m north?*

COLOR CONVENTION
Displacement (x) ⟷ green

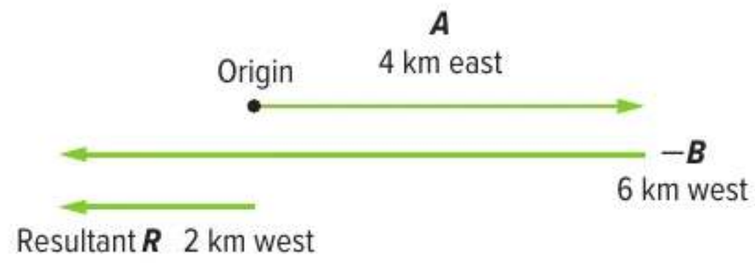
Example of Vector Addition



$R = A + B$
$= 5\text{ km} + 2\text{ km}$
$= 7\text{ km}$

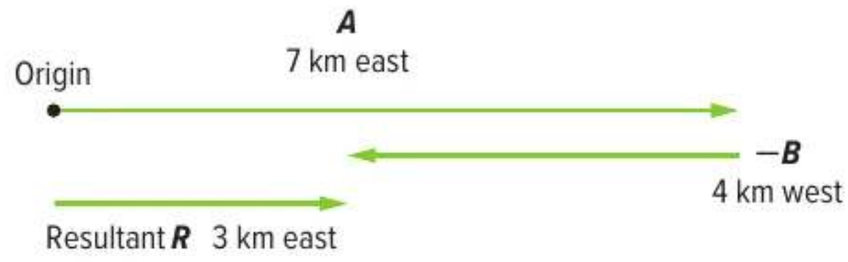
$\boldsymbol{R = A + B}$
$= 7\text{ km east}$

Examples of Vector Subtraction



$R = A - B$
$= 4\text{ km} - 6\text{ km}$
$= -2\text{ km}$

$\boldsymbol{R = A - B}$
$= \boldsymbol{A} + (-\boldsymbol{B})$
$= 2\text{ km west}$



$R = A - B$
$= 7\text{ km} - 4\text{ km}$
$= 3\text{ km}$

$\boldsymbol{R = A - B}$
$= \boldsymbol{A} + (-\boldsymbol{B})$
$= 3\text{ km east}$

Check Your Progress

6. **Coordinate System** Identify a coordinate system you could use to describe the motion of a girl swimming across a rectangular pool.
7. **Displacement** The motion diagram for a car traveling on an interstate highway is shown below. The starting and ending points are indicated.

   Start • • • • • • End

   Make a copy of the diagram. Draw a vector to represent the car's displacement from the starting time to the end of the third time interval.
8. **Position** Two students added a vector for a moving object's position at $t = 2$ s to a motion diagram. When they compared their diagrams, they found that their vectors did not point in the same direction. Explain.
9. **Displacement** The motion diagram for a boy walking to school is shown below.

   Home • • • • • • • • • • School

   Make a copy of this motion diagram, and draw vectors to represent the displacement between each pair of dots.

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LESSON 3

POSITION-TIME GRAPHS

FOCUS QUESTION

What can you learn from position-time graphs?

Finding Positions

When analyzing complex motion, it often is useful to represent the motion in a variety of ways. A motion diagram contains information about an object's position at various times. Tables and graphs can also show this same information. Review the motion diagrams in **Figure 8** and **Figure 9**. You can use these diagrams to organize the times and corresponding positions of the runner, as in **Table 1.**

Plotting data The data listed in **Table 1** can be presented on a **position-time graph,** in which the time data is plotted on a horizontal axis and the position data is plotted on a vertical axis. The graph of the runner's motion is shown in **Figure 11**. To draw this graph, first plot the runner's positions. Then, draw a line that best fits the points.

Table 1 Position v. Time

Time (s)	Position (m)
0.0	0.0
1.0	5.0
2.0	10.0
3.0	15.0
4.0	20.0
5.0	25.0

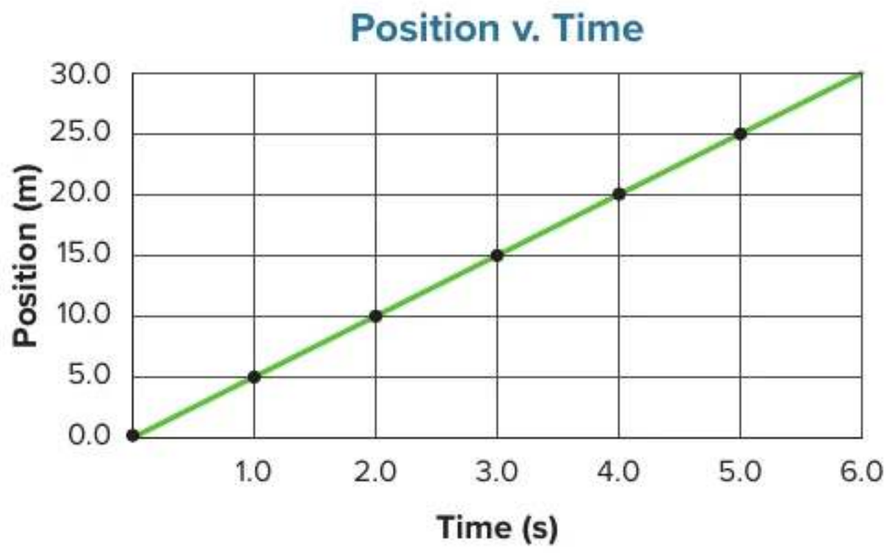


Figure 11 You can create a position-time graph by plotting the positions and times from the table. By drawing a best-fit line, you can estimate other times and positions.

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COLLECT EVIDENCE

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INVESTIGATE

GO ONLINE to find these activities and more resources.

CCC **Identify Crosscutting Concepts**

Create a table of the crosscutting concepts and fill in examples you find as you read.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Estimating time and position

Notice that the graph is not a picture of the runner's path—the graphed line is sloped, but the runner's path was horizontal. Instead, the line represents the most likely positions of the runner at the times between the recorded data points. Even though there is no data point exactly when the runner was 12.0 m beyond her starting point or where she was at $t = 4.5$ s, you can use the graph to estimate the time or her position. The example problem on the next page shows how.

Table 1 Position v. Time

Time (s)	Position (m)
0.0	0.0
1.0	5.0
2.0	10.0
3.0	15.0
4.0	20.0
5.0	25.0

Instantaneous position

How long did the runner spend at any location? Each position has been linked to a time, but how long did that time last? You could say "an instant," but how long is that? If an instant lasts for any finite amount of time, then the runner would have stayed at the same position during that time, and she would not have been moving. An instant is not a finite period of time, however. It lasts zero seconds. The symbol x represents the runner's **instantaneous position**—the position at a particular instant. Instantaneous position is usually simply called position.

Equivalent representations

As shown in **Figure 12**, you now have several different ways to describe motion. You might describe motion using words, pictures (or pictorial representations), motion diagrams, data tables, or position-time graphs. All of these representations contain the same information about the runner's motion. However, depending on what you want to learn about an object's motion, some types of representations will be more useful than others.

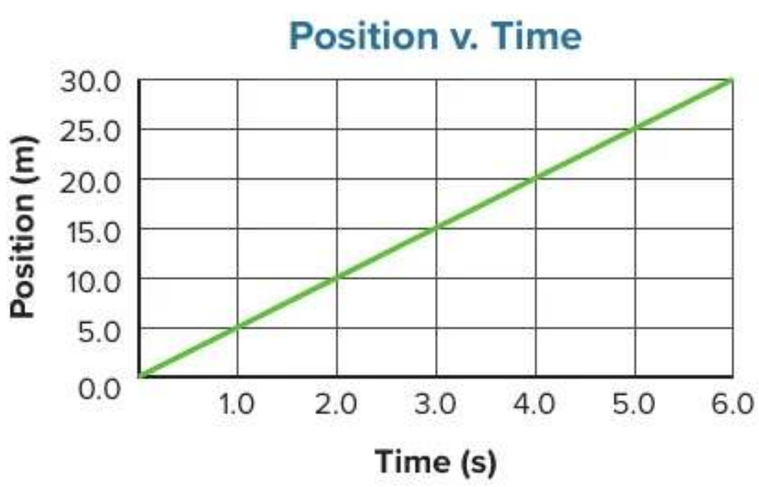




Figure 12 You can describe the runner's motion using the data table, the graph, and the motion diagram.

Identify *one benefit the table has over the graph.*

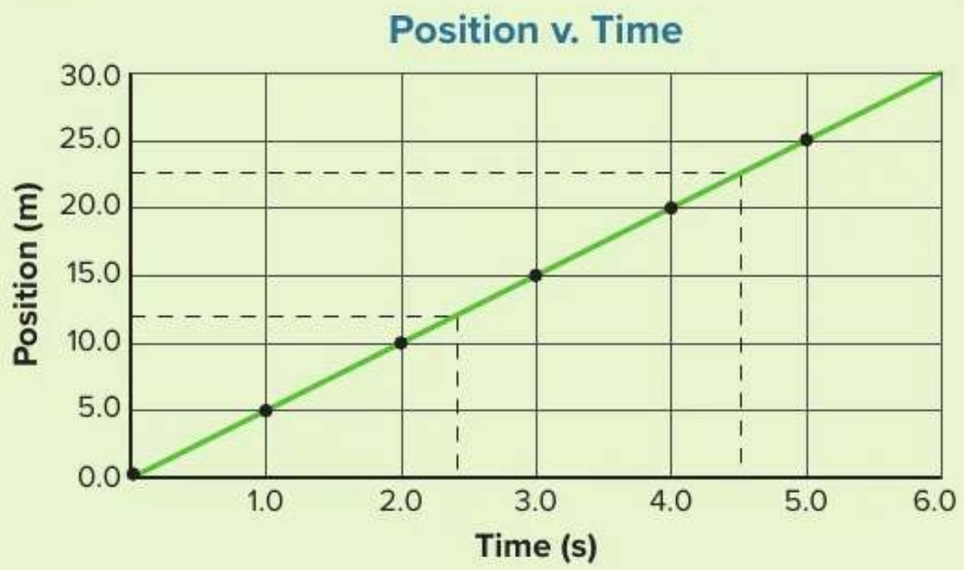
PHYSICS Challenge

POSITION-TIME GRAPHS Natana, Olivia, and Phil all enjoy exercising and often go to a path along the river for this purpose. Natana bicycles at a very consistent 40.25 km/h, Olivia runs south at a constant speed of 16.0 km/h, and Phil walks south at a brisk 6.5 km/h. Natana starts biking north at noon from the waterfalls. Olivia and Phil both start at 11:30 A.M. at the canoe dock, 20.0 km north of the falls.

1. Draw position-time graphs for each person.
2. At what time will the three exercise enthusiasts be located within the smallest distance interval from each other?
3. What is the length of that distance interval?

EXAMPLE Problem 1

ANALYZE A POSITION-TIME GRAPH When did the runner whose motion is described in **Figure 11** reach 12.0 m beyond the starting point? Where was she after 4.5 s?



1 ANALYZE THE PROBLEM

Restate the questions.

Question 1: At what time was the magnitude of the runner's position (x) equal to 12.0 m?

Question 2: What was the runner's position at time $t = 4.5$ s?

2 SOLVE FOR THE UNKNOWN

Question 1

Examine the graph to find the intersection of the best-fit line with a horizontal line at the 12.0 m mark. Next, find where a vertical line from that point crosses the time axis. The value of t there is 2.4 s.

Question 2

Find the intersection of the graph with a vertical line at 4.5 s (halfway between 4.0 s and 5.0 s on this graph). Next, find where a horizontal line from that point crosses the position axis. The value of x is approximately 22.5 m.

PRACTICE Problems

ADDITIONAL PRACTICE

*For problems 10–12, refer to **Figure 13.***

10. The graph in **Figure 13** represents the motion of a car moving along a straight highway. Describe in words the car's motion.

11. Draw a particle model motion diagram that corresponds to the graph.

12. Answer the following questions about the car's motion. Assume that the positive x-direction is east of the origin and the negative x-direction is west of the origin.

a. At what time was the car's position 25.0 m east of the origin?

b. Where was the car at time $t = 1.0$ s?

c. What was the displacement of the car between times $t = 1.0$ s and $t = 3.0$ s?

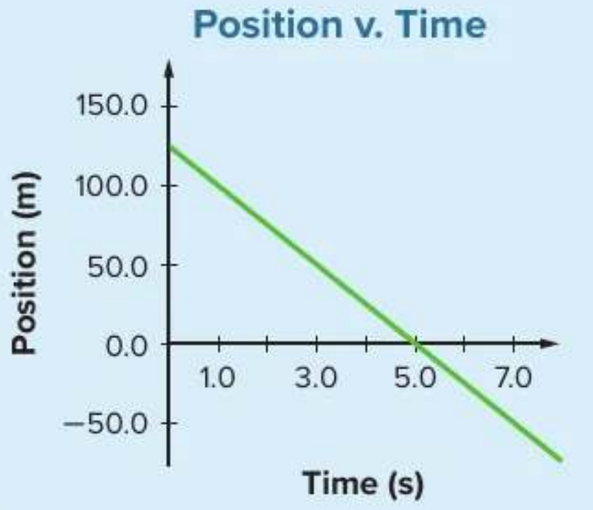


Figure 13

13. The graph in **Figure 14** represents the motion of two pedestrians who are walking along a straight sidewalk in a city. Describe in words the motion of the pedestrians. Assume that the positive direction is east of the origin.

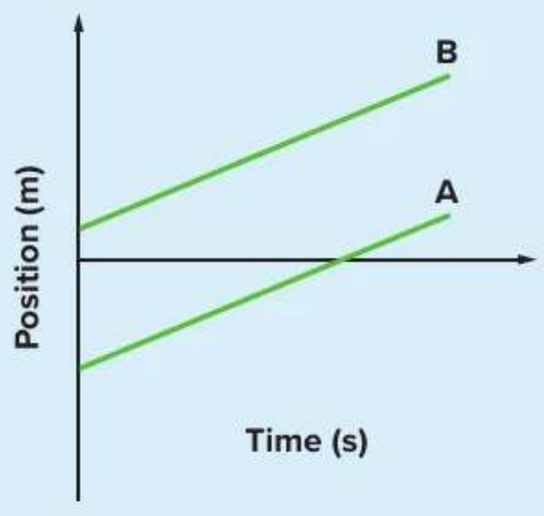


Figure 14

14. CHALLENGE Ari walked down the hall at school from the cafeteria to the band room, a distance of 100.0 m. A class of physics students recorded and graphed his position every 2.0 s, noting that he moved 2.6 m every 2.0 s. When was Ari at the following positions?

a. 25.0 m from the cafeteria

b. 25.0 m from the band room

c. Create a graph showing Ari's motion.

Multiple Objects on a Position-Time Graph

A position-time graph for two different runners is shown in Example Problem 2 below. Notice that runner A is ahead of runner B at time $t = 0$, but the motion of each runner is different. When and where does one runner pass the other? First, you should restate this question in physics terms: At what time are the two runners at the same position? What is their position at this time? You can evaluate these questions by identifying the point on the position-time graph at which the lines representing the two runners' motions intersect.

The intersection of two lines on a position-time graph tells you when objects have the same position, but does this mean that they will collide? Not necessarily. For example, if the two objects are runners and if they are in different lanes, they will not collide, even though they might be the same distance from the starting point.

Get It?

Explain what the intersection of two lines on a position-time graph means.

What else can you learn from a position-time graph? Notice in Example Problem 2 that the lines on the graph have different slopes. What does the slope of the line on a position-time graph tell you? In the next lesson, you will use the slope of a line on a position-time graph to determine the velocity of an object. When you study accelerated motion, you will draw other motion graphs and learn to interpret the areas under the plotted lines. In later studies, you will continue to refine your skills with creating and interpreting different types of motion graphs.

EXAMPLE Problem 2

INTERPRETING A GRAPH The graph to the right describes the motion of two runners moving along a straight path. The lines representing their motion are labeled A and B. When and where does runner B pass runner A?

1 ANALYZE THE PROBLEM

Restate the questions.

Question 1: At what time are runner A and runner B at the same position?

Question 2: What is the position of runner A and runner B at this time?

2 SOLVE FOR THE UNKNOWN

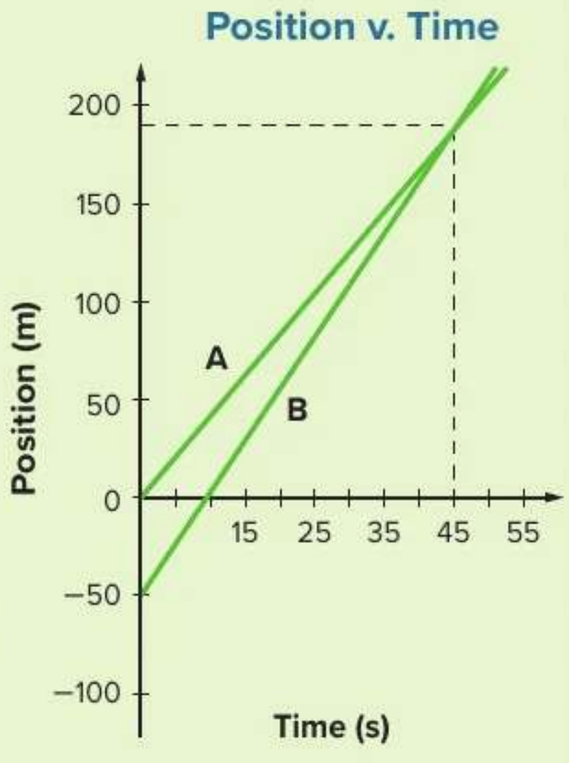
Question 1

Examine the graph to find the intersection of the line representing the motion of runner A with the line representing the motion of runner B. These lines intersect at time 45 s.

Question 2

Examine the graph to determine the position when the lines representing the motion of the runners intersect. The position of both runners is about 190 m from the origin.

Runner B passes runner A about 190 m beyond the origin, 45 s after A has passed the origin.



PRACTICE Problems

ADDITIONAL PRACTICE

For problems 15–18, refer to the figure in Example Problem 2 on the previous page.

15. Where was runner A located at $t = 0$ s?

16. Which runner was ahead at $t = 48.0$ s?

17. When runner A was at 0.0 m, where was runner B?

18. How far apart were runners A and B at $t = 20.0$ s?

19. CHALLENGE Juanita goes for a walk to the north. Later her friend Heather starts to walk after her. Their motions are represented by the position-time graph in **Figure 15.**

a. How long had Juanita been walking when Heather started her walk?

b. Will Heather catch up to Juanita? How can you tell?

c. What was Juanita's position at $t = 0.2$ h?

d. At what time was Heather 5.0 km from the start?

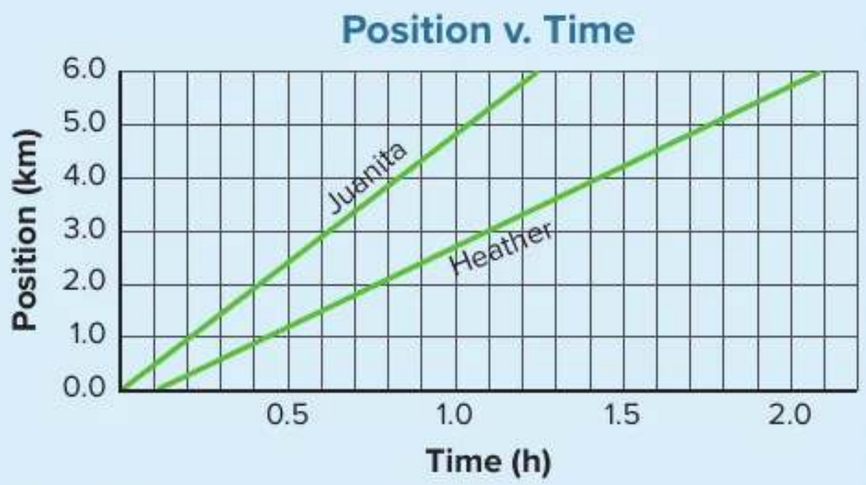


Figure 15

Check Your Progress

20. **Particle Diagram** Using the particle model motion diagram in **Figure 16** of a baby crawling across a kitchen floor, plot a position-time graph to represent the motion. The time interval between dots on the diagram is 1 s.

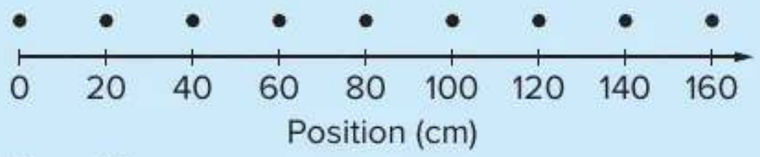


Figure 16

*For problems 21–24, refer to **Figure 17.***

21. **Particle Model** Create a particle model motion diagram from the position-time graph of a hockey puck gliding across the ice.

22. **Time** Use the hockey puck's position-time graph to determine the time when the puck was 10.0 m beyond the origin.

23. **Distance** Use the position-time graph to determine how far the hockey puck moved between 0.0 s and 5.0 s.

24. **Time Interval** Use the position-time graph for the hockey puck to determine the time it took for the puck to go from 40.0 m beyond the origin to 80.0 m beyond the origin.

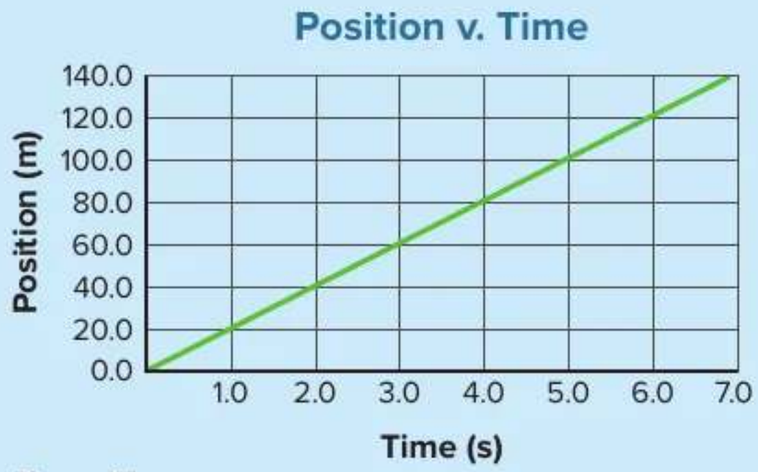


Figure 17

25. **Critical Thinking** Look at the diagram and graph shown in **Figure 18**. Do they describe the same motion? Explain. The time intervals in the particle model diagram are 2 s.

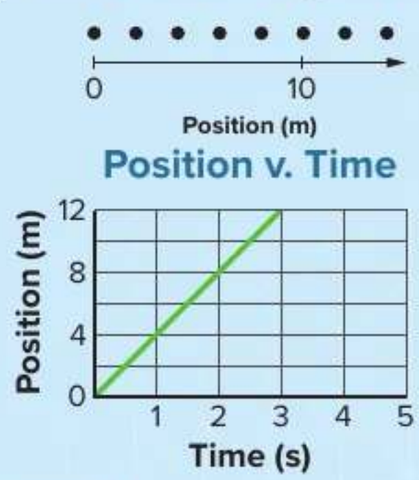


Figure 18

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LESSON 4

HOW FAST?

FOCUS QUESITON

How do you describe how fast something is moving?

Velocity and Speed

Suppose you recorded the motion of two joggers on one diagram, as shown by the graph in **Figure 19.** The position of the jogger wearing red changes more than that of the jogger wearing blue. For a fixed time interval, the magnitude of the displacement (Δx) is greater for the jogger in red because she is moving faster. Now, suppose that each jogger travels 100 m. The time interval (Δt) for the 100 m would be smaller for the jogger in red than for the one in blue.

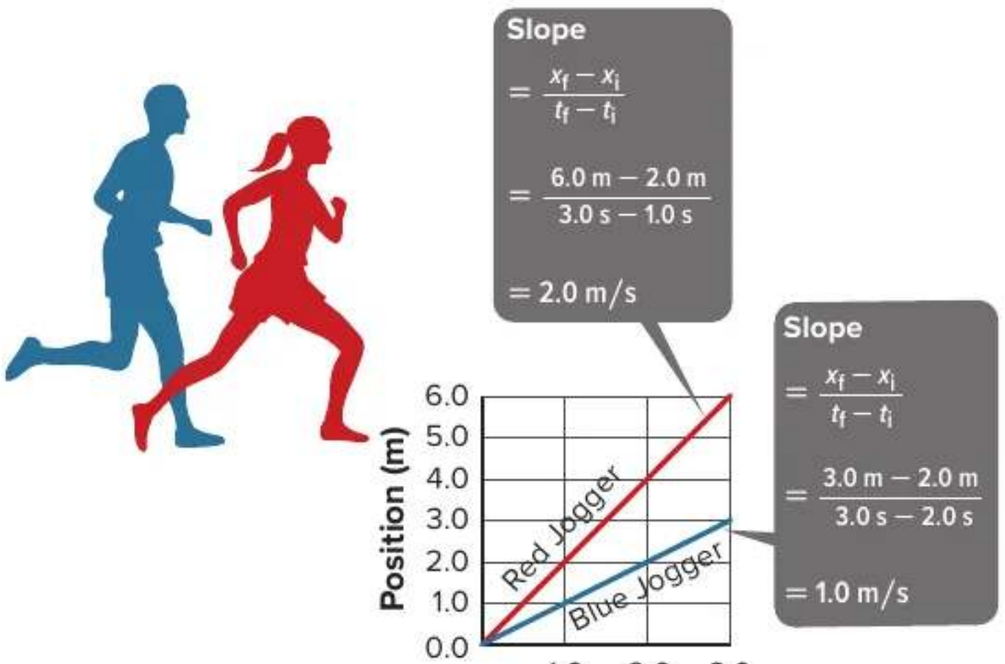


Figure 19 A greater slope shows that the red jogger traveled faster.

Analyze *How much farther did the red jogger travel than the blue jogger in the 3 s interval described by the graph?*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Constant Speed

Plan and carry out an investigation to relate distance, speed, and time interval.

Review the News

Obtain information from a current news story about speed or velocity. Evaluate your source and communicate your findings to your class.

Slope on a position-time graph Compare the lines representing the joggers in the graph in **Figure 19.** The slope of the red jogger's line is steeper, indicating a greater change in position during each time interval. Recall that you find the slope of a line by first choosing two points on the line. Next, you subtract the vertical coordinate (x in this case) of the first point from the vertical coordinate of the second point to obtain the rise of the line. After that, you subtract the horizontal coordinate (t in this case) of the first point from the horizontal coordinate of the second point to obtain the run. The rise divided by the run is the slope.

Get It?

Explain In the graph in **Figure 19**, the graph for each jogger starts at the coordinates (0, 0). Explain what this means in terms of position.

Average velocity Notice that the slope of the faster runner's line in **Figure 19** is a greater number. A greater slope indicates a faster speed. Also notice that the slope's units are meters per second. Looking at how the slope is calculated, you can see that slope is the change in the magnitude of the position divided by the time interval during which that change took place: $\frac{x_f - x_i}{t_f - t_i}$ or $\frac{\Delta x}{\Delta t}$. When Δx gets larger, the slope gets larger; when Δt gets larger, the slope gets smaller. This agrees with the interpretation given on the previous page of the speeds of the red and blue joggers. **Average velocity** is the ratio of an object's change in position to the time interval during which the change occurred. If the object is in uniform motion, so that its speed does not change, then its average velocity is the slope of its position-time graph.

Average velocity

Average velocity is defined as the change in position divided by the time during which the change occurred.

$$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The symbol ≡ means that the left-hand side of the equation is defined by the right-hand side.

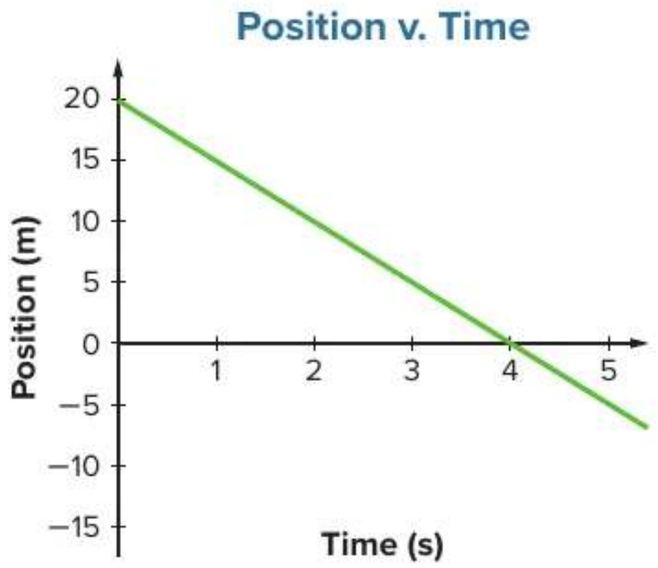
Interpreting slope The position-time graph's slope in **Figure 20** is −5.0 m/s. Notice that the slope of the graph indicates both magnitude and direction. By calculating the slope from the rise divided by the run between two points, you find that the object whose motion is represented by the graph has an average velocity of −5.0 m/s. The object started out at a positive position and moves toward the origin. After 4 s, it passes the origin and continues moving in the negative direction at a rate of 5.0 m/s.

Get It?

Explain the meaning of an upward and a downward slope of a position-time graph both above and below the x-axis.

Figure 20 The downward slope of this position-time graph shows that the motion is in the negative direction.

Analyze *What would the graph look like if the motion were at the same speed, but in the positive direction?*



Average speed The slope's absolute value is the object's **average speed,** 5.0 m/s, which is the distance traveled divided by the time taken to travel that distance. For uniform motion, average speed is the absolute value of the slope of the object's position-time graph. The combination of an object's average speed ($\overline{v}$) and the direction in which it is moving is the average velocity ($\overline{v}$). Remember that if an object moves in the negative direction, its change in position is negative. This means that an object's displacement and velocity are both always in the same direction.

Instantaneous velocity

Why do we call the quantity $\frac{\Delta x}{\Delta t}$ average velocity? Why don't we just call it velocity? A motion diagram shows the position of a moving object at the beginning and end of a time interval. It does not, however, indicate what happened within that time interval. During the time interval, the object's speed could have remained the same, increased, or decreased. The object may have stopped or even changed direction. You can find the average velocity for each time interval in the motion diagram, but you cannot find the speed and the direction of the object at any specific instant. The speed and the direction of an object at a particular instant is called the **instantaneous velocity.** In this textbook, the term velocity will refer to instantaneous velocity, represented by the symbol v.

Get It?
Explain how average velocity is different from velocity.

Average velocity on motion diagrams

When an object moves between two points, its average velocity is in the same direction as its displacement. The two quantities are also proportional—when displacement is greater during a given time interval, so is average velocity. A motion diagram indicates the average velocity's direction and magnitude.

Imagine two cars driving down the road at different speeds. A video camera records the motion of the cars at the rate of one frame every second. Imagine that each car has a paintbrush attached to it that automatically descends and paints a red line on the ground for half a second every second. The faster car would paint a longer line on the ground. The vectors you draw on a motion diagram to represent the velocity are like the lines that the paintbrushes make on the ground below the cars. In this book, we use red to indicate velocity vectors on motion diagrams. **Figure 21** shows motion diagrams with velocity vectors for two cars. One is moving to the right, and the other is moving to the left.

Get It?
Identify what the lengths of velocity vectors mean.

Real-World Physics



The fastest bird is the peregrine falcon. It can dive at a speed of 240 mph when hunting. The fastest land animal is the cheetah. It can run between 60–75 mph in short bursts.

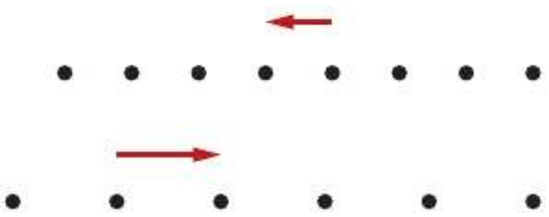
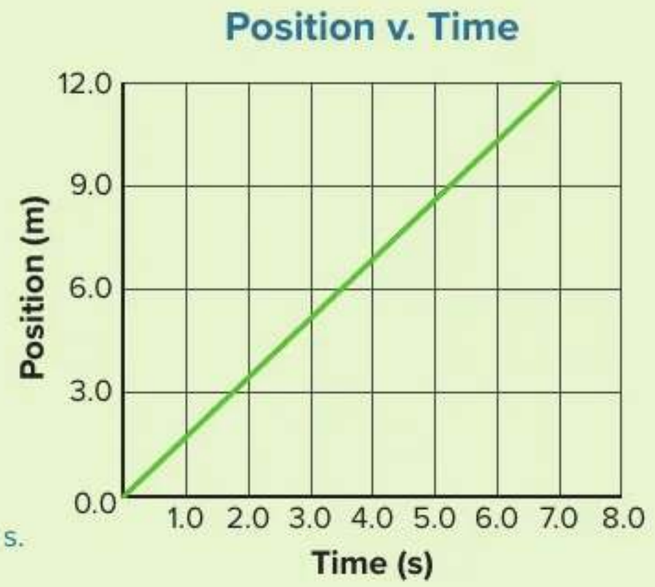


Figure 21 The length of each velocity vector is proportional to the magnitude of the velocity that it represents.

LebronCaptures/Shutterstock

EXAMPLE Problem 3

AVERAGE VELOCITY The graph at the right describes the straight-line motion of a student riding her skateboard along a smooth, pedestrian-free sidewalk. What is her average velocity? What is her average speed?



1 ANALYZE AND SKETCH THE PROBLEM

Identify the graph's coordinate systems.

UNKNOWN

$\bar{v} = ?$ $\bar{v} = ?$

2 SOLVE FOR THE UNKNOWN

Find the average velocity using two points on the line.

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

$$= \frac{x_f - x_i}{t_f - t_i}$$

$$= \frac{12.0\text{ m} - 0.0\text{ m}}{7.0\text{ s} - 0.0\text{ s}}$$ Substitute $x_f = 12.0$ m, $x_i = 0.0$ m, $t_f = 7.0$ s, $t_i = 0.0$ s.

$\bar{v} = 1.7$ m/s in the positive direction

The average speed ($\bar{v}$) is the absolute value of the average velocity, or 1.7 m/s.

3 EVALUATE THE ANSWER

- **Are the units correct?** The units for both velocity and speed are meters per second.
- **Do the signs make sense?** The positive sign for the velocity agrees with the coordinate system. No direction is associated with speed.

PRACTICE Problems

26. The graph in **Figure 22** describes the motion of a cruise ship drifting slowly through calm waters. The positive *x*-direction (along the vertical axis) is defined to be south.
   a. What is the ship's average speed?
   b. What is its average velocity?

27. Describe, in words, the cruise ship's motion in the previous problem.

28. What is the average velocity of an object that moves from 6.5 cm to 3.7 cm relative to the origin in 2.3 s?

29. The graph in **Figure 23** represents the motion of a bicycle.
   a. What is the bicycle's average speed?
   b. What is its average velocity?

30. Describe, in words, the bicycle's motion in the previous problem.

31. **CHALLENGE** When Marshall takes his pet dog for a walk, the dog walks at a very consistent pace of 0.55 m/s. Draw a motion diagram and a position-time graph to represent Marshall's dog walking the 19.8-m distance from in front of his house to the nearest stop sign.

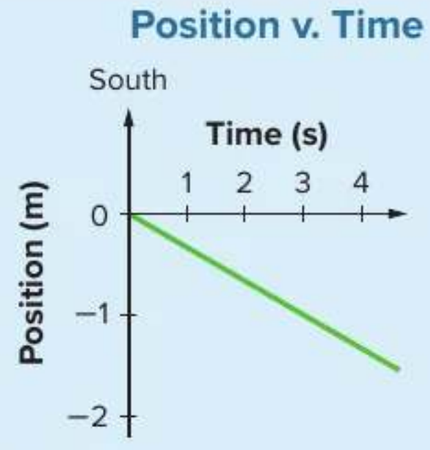


Figure 22

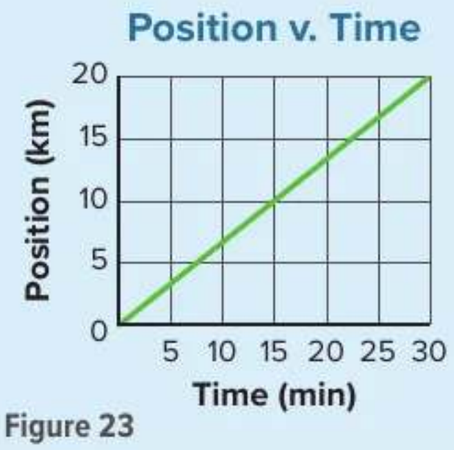


Figure 23

Equation of Motion

Often it is more efficient to use an equation, rather than a graph, to solve problems. Any time you graph a straight line, you can find an equation to describe it. Take another look at the graph in **Figure 20** for the object moving with a constant velocity of −5.0 m/s. Recall that you can represent any straight line with the equation $y = mx + b$, where y is the quantity plotted on the vertical axis, m is the line's slope, x is the quantity plotted on the horizontal axis, and b is the line's y-intercept.

For the graph in **Figure 20**, the quantity plotted on the vertical axis is position, represented by the variable $\boldsymbol{x}$. The line's slope is −5.0 m/s, which is the object's average velocity ($\vec{v}$). The quantity plotted on the horizontal axis is time (t). The y-intercept is 20.0 m. What does this 20.0 m represent? This shows that the object was at a position of 20.0 m when $t = 0.0$ s. This is called the initial position of the object and it is designated $\boldsymbol{x_i}$.

A summary is given below of how the general variables in the straight-line formula are changed to the specific variables you have been using to describe motion. The table also shows the numerical values for the average velocity and initial position. Consider the graph shown in **Figure 20.** The mathematical equation for the line graphed is as follows:

$$y = (-5.0 \text{ m/s})x + 20.0 \text{ m}$$

You can rewrite this equation, using $\boldsymbol{x}$ for position and t for time.

$$x = (-5.0 \text{ m/s})t + 20.0 \text{ m}$$

It might be confusing to use y and x in math but use $\boldsymbol{x}$ and t in physics. You do this because there are many types of graphs in physics, including position v. time graphs, velocity v. time graphs, and force v. position graphs. For a position v. time graph, the math equation $y = mx + b$ can be rewritten as follows:

Position

An object's position is equal to the average velocity multiplied by time plus the initial position.

$$\boldsymbol{x = \bar{v}t + x_i}$$

This equation gives you another way to represent motion. Note that a graph of x v. t would be a straight line.

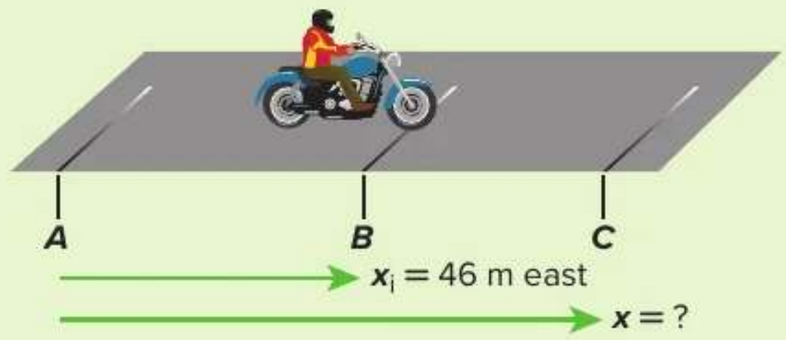
CONNECTING MATH to Physics

Lines and Graphs Symbols used in the point-slope equation of a line relate to symbols used for motion variables on a position-time graph.

General Variable	Specific Motion Variable	Value in Figure 20
y	$\boldsymbol{x}$	
m	$\boldsymbol{\bar{v}}$	−5.0 m/s
x	t	
b	$\boldsymbol{x_i}$	20.0 m

EXAMPLE Problem 4

POSITION The figure shows a motorcyclist traveling east along a straight road. After passing point ***B,*** the cyclist continues to travel at an average velocity of 12 m/s east and arrives at point ***C*** 3.0 s later. What is the position of point ***C***?



1 ANALYZE THE PROBLEM

Choose a coordinate system with the origin at ***A.***

KNOWN	UNKNOWN
$\bar{v} = 12$ m/s east	$x = ?$
$x_i = 46$ m east	
$t = 3.0$ s	

2 SOLVE FOR THE UNKNOWN

$x = \bar{v}t + x_i$ Use magnitudes for the calculations.

$= (12 \text{ m/s})(3.0 \text{ s}) + 46 \text{ m}$ Substitute $\bar{v} = 12$ m/s, $t = 3.0$ s, and $x_i = 46$ m.

$= 82$ m

$x = 82$ m east

3 EVALUATE THE ANSWER

- **Are the units correct?** Position is measured in meters.
- **Does the direction make sense?** The motorcyclist is traveling east the entire time.

PRACTICE Problems

For problems 32–35, refer to ***Figure 24.***

32. The diagram at the right shows the path of a ship that sails at a constant velocity of 42 km/h east. What is the ship's position when it reaches point ***C***, relative to the starting point, ***A***, if it sails from point ***B*** to point ***C*** in exactly 1.5 h?

33. Another ship starts at the same time from point ***B***, but its average velocity is 58 km/h east. What is its position, relative to ***A***, after 1.5 h?

34. What would a ship's position be if that ship started at point ***B*** and traveled at an average velocity of 35 km/h west to point ***D*** in a time period of 1.2 h?

35. CHALLENGE Suppose two ships start from point ***B*** and travel west. One ship travels at an average velocity of 35 km/h for 2.2 h. Another ship travels at an average velocity of 26 km/h for 2.5 h. What is the final position of each ship?

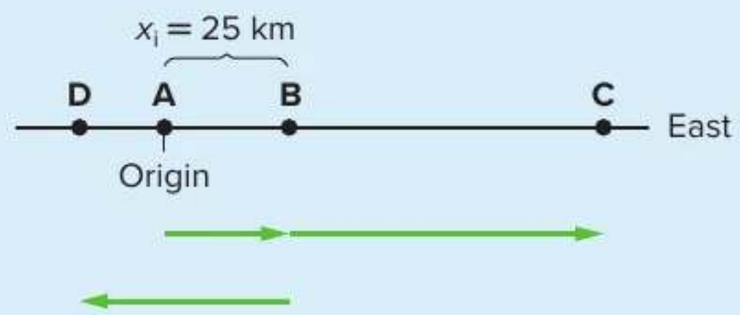


Figure 24

After your study of this lesson, you probably realize how closely linked physics is to mathematics. Algebraic thinking is used to examine scientific data and predict the effect of a change in one variable on another. For example, you can predict the change in average velocity of an object by examining the change in position over time. Using mathematics to model motion is a useful tool for physicists and physics students.

Check Your Progress

36. **Velocity and Position** How is an object's velocity related to its position?

For problems 37–39, refer to ***Figure 25.***

37. **Ranking Task** Rank the position-time graphs according to the average speed, from greatest average speed to least average speed. Specifically indicate any ties.
38. **Contrast Average Velocities** Describe differences in the average velocities shown on the graph for objects A and B. Describe differences in the average velocities shown on the graph for objects C and D.
39. **Ranking Task** Rank the graphs in **Figure 25** according to each object's initial position, from most positive position to most negative position. Specifically indicate any ties. Would your ranking be different if you ranked according to initial distance from the origin?

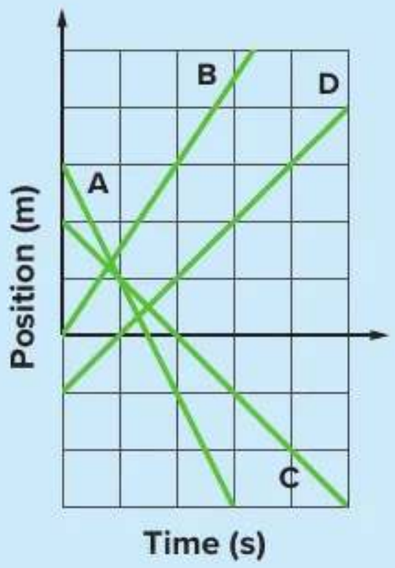


Figure 25

40. **Average Speed and Average Velocity** Explain how average speed and average velocity are related to each other for an object in uniform motion.
41. **Position** Two cars are traveling along a straight road, as shown in **Figure 26.** They pass each other at point B and then continue in opposite directions. The orange car travels for 0.25 h from point B to point C at a constant velocity of 32 km/h east. The blue car travels for 0.25 h from point B to point D at a constant velocity of 48 km/h west. How far has each car traveled from point B? What is the position of each car relative to the origin, point A?

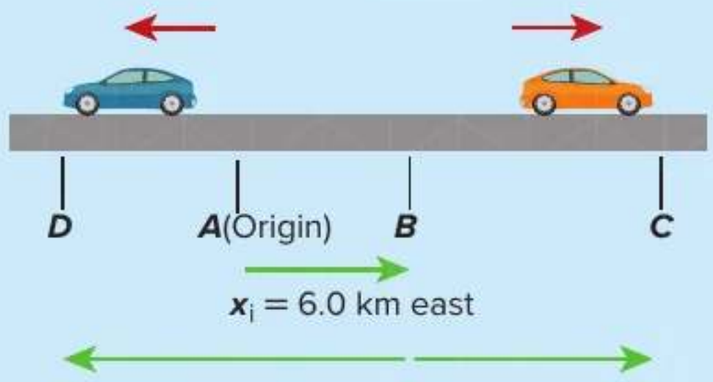


Figure 26

42. **Position** A car travels north along a straight highway at an average speed of 85 km/h. After driving 2.0 km, the car passes a gas station and continues along the highway. What is the car's position relative to the start of its trip 0.25 h after it passes the gas station?
43. **Critical Thinking** In solving a physics problem, why is it important to create pictorial and physical models before trying to solve an equation?

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SCIENTIFIC BREAKTHROUGHS

In the Nick of Time

Global Positioning System (GPS) and other navigation systems rely on satellites with extremely accurate atomic clocks to keep track of time. However, some navigation systems must operate in environments—such as inside buildings, in the depths of canyons, or deep under water—where radio signals from satellites cannot reach them. Navigation systems that rely on satellites are also susceptible to intentional interference due to hacking or jamming. To address this problem, scientists have developed tiny atomic clocks for navigation systems small enough to be carried by a person.



Chip-scale atomic clocks (CSACs) allow for astoundingly accurate and precise time-keeping in a very small package.

Chip-scale atomic clocks

Old-fashioned clocks count the seconds by the swings of a pendulum. Mechanical clocks—and even digital clocks, which use the electrical oscillations in power lines or the oscillations of quartz crystals to keep time—are not completely accurate and precise. So, in 1949, scientists at the National Institute of Standards and Technology (NIST) took a step toward solving this problem by developing the first atomic clock.

Atomic clocks count the seconds by oscillations between atomic energy states of various elements, including cesium, rubidium, and strontium. Over the years, advances in technology have led to the creation of atomic clocks that are more and more accurate and precise. The most current models can keep perfect time for the next 15 billion years.

In the early 2000s, NIST began working with the U.S. Department of Defense's Defense Advanced Research Projects Agency (DARPA) to create atomic clocks small enough to work in portable navigation systems. In 2004, they introduced the first chip-scale atomic clock (CSAC) which, as the name implies, is the size of a computer chip. Today, the CSAC is used in portable navigation systems, telecommunications systems, and other applications, both civilian and military.

Even CSACs have vulnerabilities, such as temperature fluctuations that cause variations in atomic frequencies. In 2016, DARPA kicked off the Atomic Clocks with Enhanced Stability (ACES) project with the goal of creating an atomic clock that performs 1,000 times better than the original CSAC.



USE A MODEL TO ILLUSTRATE

Use print or online sources to find a diagram of an atomic clock or a CSAC. Study the diagram and then use it to write a short text explaining how the device works.

NIST

MODULE 2
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 PICTURING MOTION

- A motion diagram shows the position of an object at successive equal time intervals.
- In a particle model motion diagram, an object's position at successive times is represented by a series of dots. The spacing between dots indicates whether the object is moving faster or slower.

- motion diagram
- particle model

Lesson 2 WHERE AND WHEN?

- A coordinate system gives the location of the zero point of the variable you are studying and the direction in which the values of the variable increase.
- A vector drawn from the origin of a coordinate system to an object indicates the object's position in that coordinate system. The directions chosen as positive and negative on the coordinate system determine whether the objects' positions are positive or negative in the coordinate system.
- A time interval is the difference between two times.

$$\Delta t = t_f - t_i$$

- Change in position is displacement, which has both magnitude and direction.

$$\Delta x = x_f - x_i$$

- On a motion diagram, the displacement vector's length represents how far the object was displaced. The vector points in the direction of the displacement, from x_i to x_f

- coordinate system
- origin
- position
- distance
- magnitude
- vector
- scalar
- time interval
- displacement
- resultant

Lesson 3 POSITION-TIME GRAPHS

- Position-time graphs provide information about the motion of objects. They also might indicate where and when two objects meet.
- The line on a position-time graph describes an object's position at each time.
- Motion can be described using words, motion diagrams, data tables, or graphs.

- position-time graph
- instantaneous position

Lesson 4 HOW FAST?

- An object's velocity tells how fast it is moving and in what direction it is moving.
- Speed is the magnitude of velocity.
- Slope on a position-time graph describes the average velocity of the object.

$$\boldsymbol{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

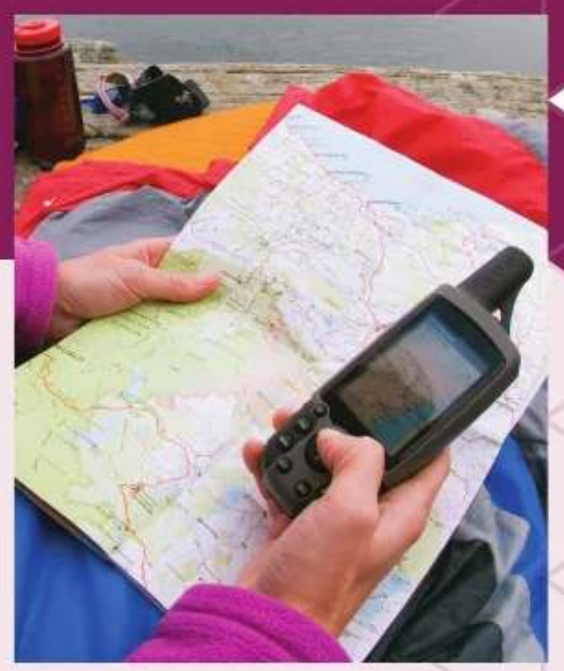
- A simple equation relates an object's initial position (x_i), its constant average velocity ($\boldsymbol{v}$), its position (x), and the time (t) since the object was at its initial position.

$$x = \bar{v}t + x_i$$

- average velocity
- average speed
- instantaneous velocity

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

How does a GPS unit know where you are?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

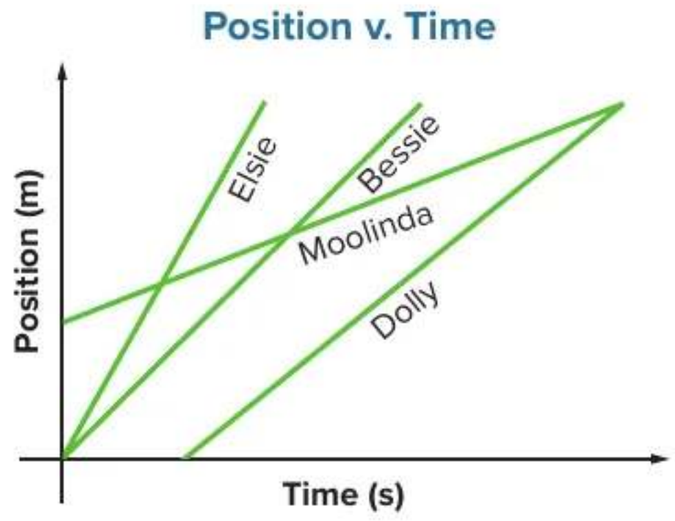
Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

How can you rank velocity from a graph?

Four cows were walking back to the barn at the end of the day. The position-time graph shows the motion of the four cows.



CER Analyze and Interpret Data

1. **Compare and contrast the average velocities of the cows.**
2. **Claim** Rank the cows according to their average velocities.
3. **Evidence and Reasoning** Explain the evidence you obtained from the position-time graph and your reasoning to justify your ranking.

Aurora Photos/Alamy Stock Photo



Steve Whittington/Moment/Getty Images

MODULE 3

ACCELERATED MOTION

MODULE 3

ACCELERATED MOTION

ENCOUNTER THE PHENOMENON

Why do sudden changes in the direction or speed of jet planes affect pilots?



GO ONLINE to play a video about the effects of acceleration on pilots.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about why sudden changes in the direction or speed of jet planes affect pilots. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain: Calculating Acceleration



LESSON 3: Explore & Explain: Galileo's Discovery

Smithsonian

Additional Resources

(t)Video Supplied by BBC Worldwide Learning, (b)ImagineGolf/iStock/Getty Images

LESSON 1

ACCELERATION

FOCUS QUESTION

What are two ways velocity can change?

Nonuniform Motion Diagrams

An object in uniform motion moves along a straight line with an unchanging velocity, but few objects move this way all the time. More common is nonuniform motion, in which velocity is changing. In this module, you will study nonuniform motion along a straight line. Examples include balls rolling down hills, cars braking to a stop, and falling objects. In later modules you will analyze nonuniform motion that is not confined to a straight line, such as motion along a circular path and the motion of thrown objects, such as baseballs.

Describing nonuniform motion Uniform motion feels smooth. If you close your eyes, it feels as if you are not moving at all. In contrast, when you move around a curve or up and down a roller coaster hill, you feel pushed or pulled.

In the first diagram in **Figure 1**, the single image indicates that the person is motionless, like a runner waiting for the signal at the start of a race. In the other diagrams, the distances between successive positions change in different ways. In the second diagram, the distances are the same, which indicates that the jogger is in uniform motion at a constant velocity. In the third diagram, the distance increases because the jogger speeds up. In the fourth diagram, the distance decreases because the jogger slows down.

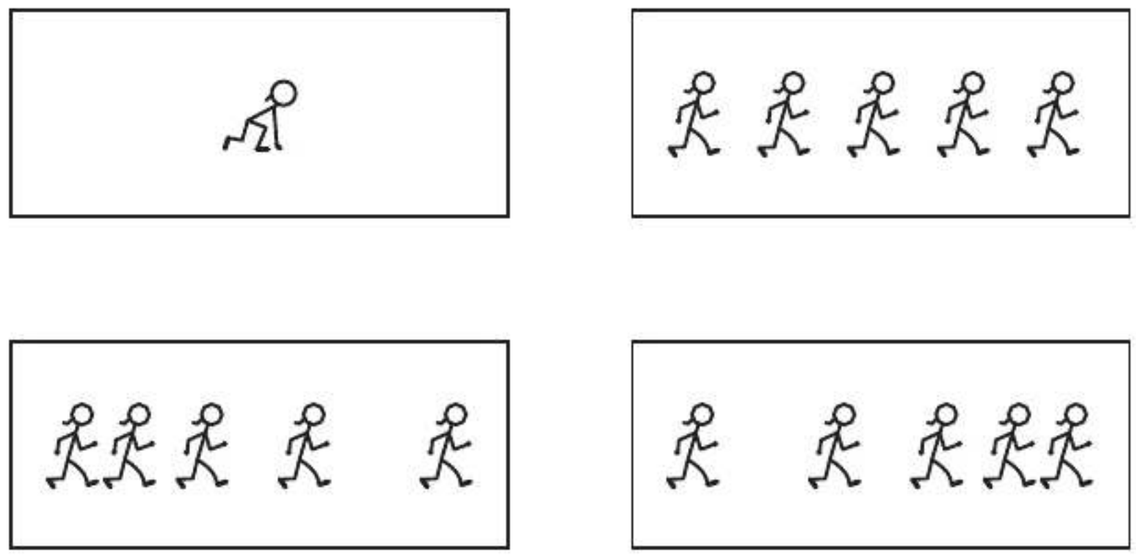


Figure 1 The distance the jogger moves in each time interval indicates the type of motion.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Virtual Investigation: Accelerated Motion

Use a computer model to relate acceleration, change in velocity, and time.

Probeware Lab: Tossed Ball Motion

Carry out an investigation to determine how the position, velocity, and acceleration of a tossed ball change over time.

Particle model diagram

What does a particle model motion diagram look like for an object with changing velocity? **Figure 2** shows particle model motion diagrams below the motion diagrams of the jogger when she is speeding up and slowing down. There are two major indicators of the change in velocity in this form of the motion diagram. The change in the spacing of the dots and the differences in the lengths of the velocity vectors indicate the changes in velocity. If an object speeds up, each subsequent velocity vector is longer, and the spacing between dots increases. If the object slows down, each vector is shorter than the previous one, and the spacing between dots decreases. Both types of motion diagrams indicate how an object's velocity is changing.

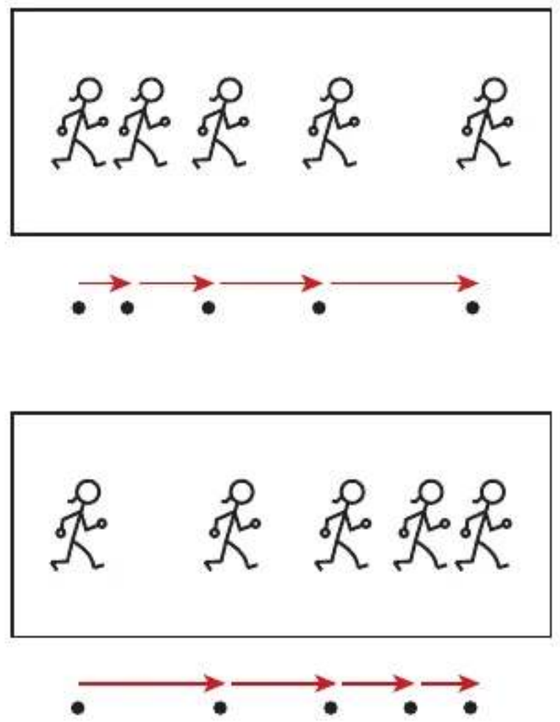


Figure 2 The change in length of the velocity vectors on these motion diagrams indicates whether the jogger is speeding up or slowing down.

Get It?

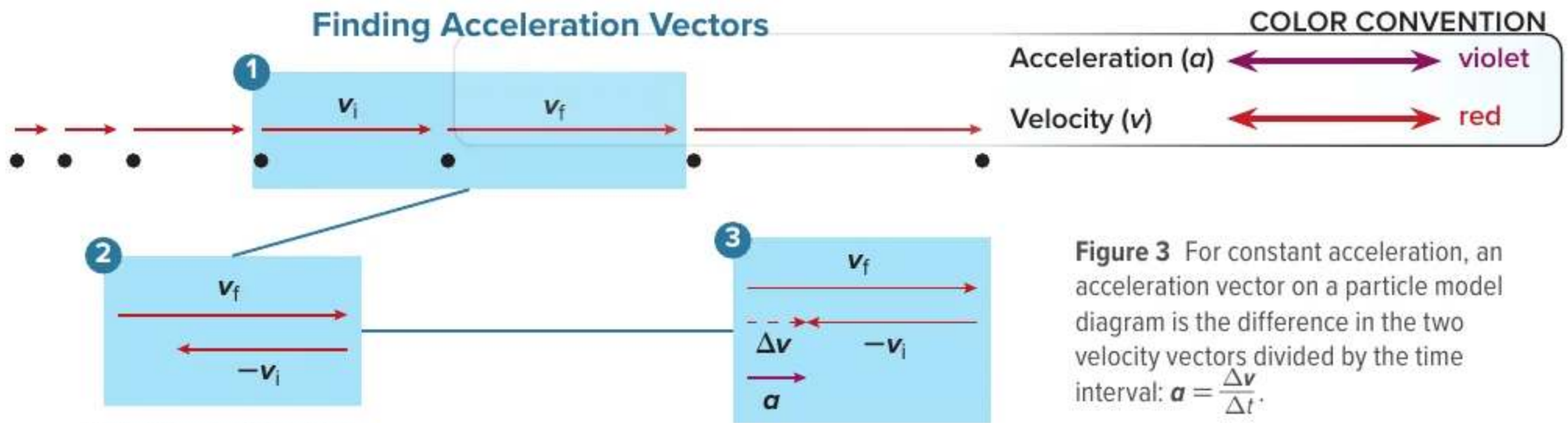
Analyze What do increasing and decreasing lengths of velocity vectors indicate on a motion diagram?

Displaying acceleration on a motion diagram

For a motion diagram to give a full picture of an object's movement, it should contain information about the rate at which the object's velocity is changing. The rate at which an object's velocity changes is called the **acceleration** of the object. By including acceleration vectors on a motion diagram, you can indicate the rate of change for the velocity.

Figure 3 shows a particle motion diagram for an object with increasing velocity. Notice that the lengths of the red velocity vectors get longer from left to right along the diagram. The figure also describes how to use the diagram to draw an acceleration vector for the motion. The acceleration vector that describes the increasing velocity is shown in violet on the diagram.

Notice in the figure that if the object's acceleration is constant, you can determine the length and direction of an acceleration vector by subtracting two consecutive velocity vectors and dividing by the time interval. That is, first find the change in velocity, $\Delta v = v_f - v_i = v_f + (-v_i)$, where v_i and v_f refer to the velocities at the beginning and the end of the chosen time interval. Then divide by the time interval (Δt). The time interval between each dot in **Figure 3** is 1 s. You can draw the acceleration vector from the tail of the final velocity vector to the tip of the initial velocity vector.



First, draw v_f. Below that, draw v_i with its tail aligned with the tip of v_f.

Next, draw the vector Δv from the tail of v_f to the tip of v_i. The acceleration vector a is the same as Δv divided by the time interval.

Figure 3 For constant acceleration, an acceleration vector on a particle model diagram is the difference in the two velocity vectors divided by the time interval: $a = \frac{\Delta v}{\Delta t}$.

Analyze *Can you draw an acceleration vector for two successive velocity vectors that are the same length and direction? Explain.*

Direction of Acceleration

Consider the four situations shown in **Figure 4** in which an object can accelerate by changing speed. The first motion diagram shows the car moving in the positive direction and speeding up. The second motion diagram shows the car moving in the positive direction and slowing down. The third shows the car speeding up in the negative direction, and the fourth shows the car slowing down as it moves in the negative direction. The figure also shows the velocity vectors for the second time interval of each diagram, along with the corresponding acceleration vectors. Note that Δt is equal to 1 s.

In the first and third situations, when the car is speeding up, the velocity and acceleration vectors point in the same direction. In the other two situations, in which the acceleration vector is in the opposite direction from the velocity vectors, the car is slowing down. In other words, when the car's acceleration is in the same direction as its velocity, the car's speed increases. When they are in opposite directions, the speed of the car decreases.

Both the direction of an object's velocity and its direction of acceleration are needed to determine whether it is speeding up or slowing down. An object has a positive acceleration when the acceleration vector points in the positive direction and a negative acceleration when the acceleration vector points in the negative direction. It is important to notice that the sign of acceleration alone does not indicate whether the object is speeding up or slowing down.

Get It?

Describe the motion of an object if its velocity and acceleration vectors have opposite signs.

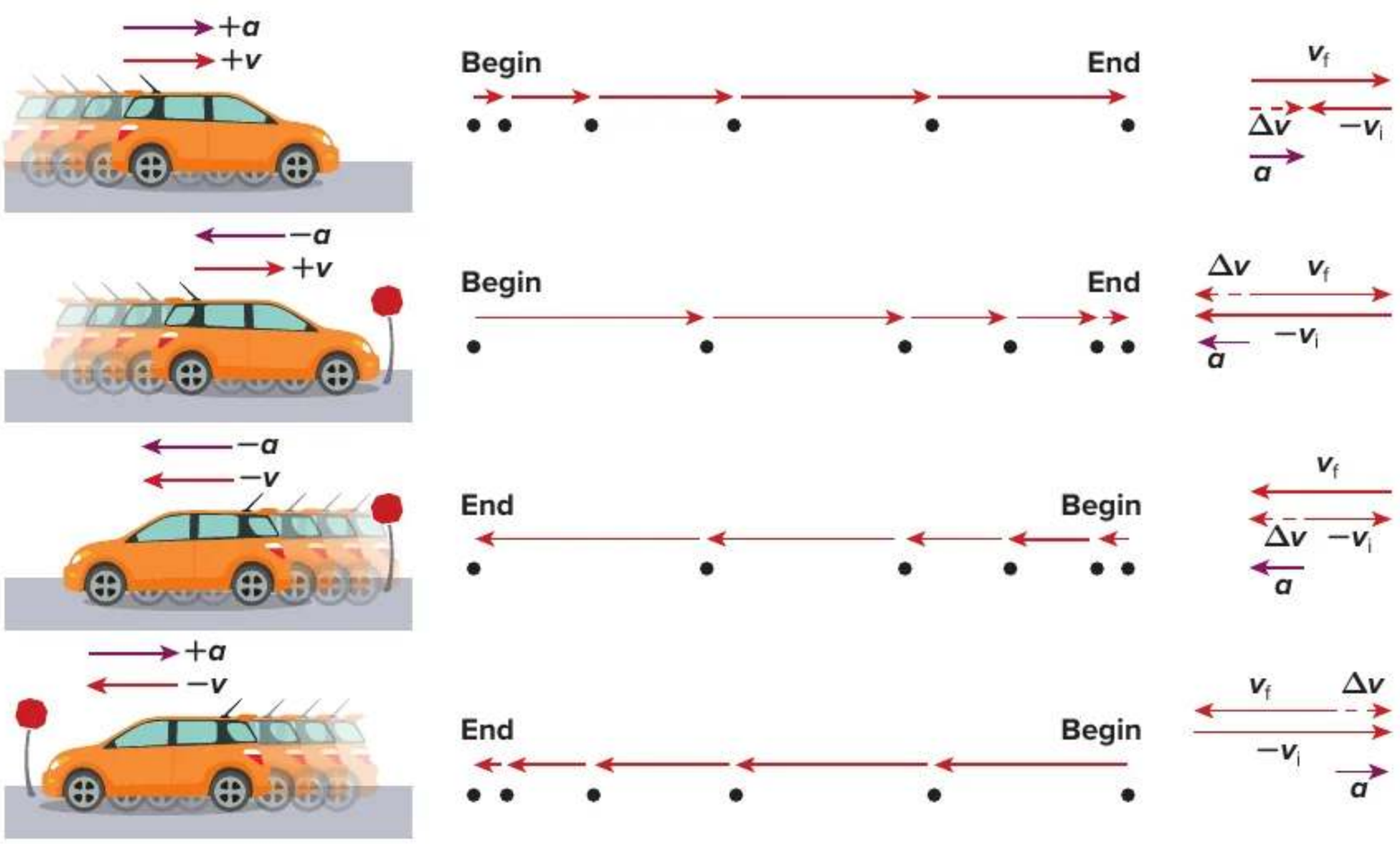


Figure 4 You need to know the direction of both the velocity and acceleration vectors in order to determine whether an object is speeding up or slowing down.

Velocity-Time Graphs

Just as it was useful to plot position versus time, it also is useful to plot velocity versus time to analyze an object's motion. On a **velocity-time graph,** or *v*-*t* graph, velocity is plotted on the vertical axis and time is plotted on the horizontal axis.

Slope The velocity-time graph for a car that started at rest and sped up along a straight stretch of road is shown in **Figure 5.** The positive direction has been chosen to be the same as that of the car's motion. Notice that the graph is a straight line. This means the car sped up at a constant rate. The rate at which the car's velocity changed can be found by calculating the slope of the velocity-time graph

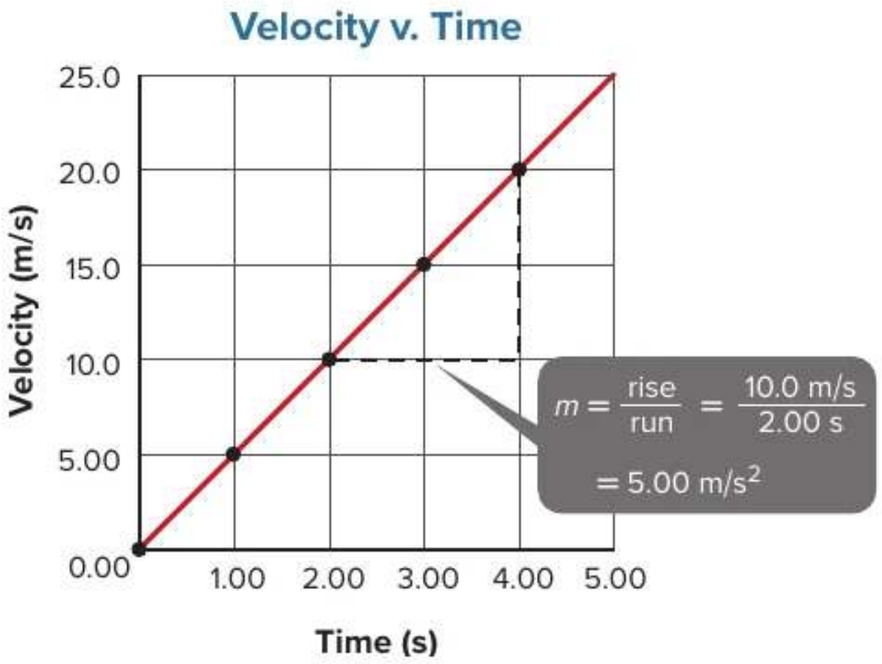


Figure 5 You can determine acceleration from a velocity-time graph by calculating the slope of the data. The slope is the rise divided by the run using any two points on the line.

Get It?

Identify What can you conclude about the acceleration of an object if the graph of its motion is a straight line on a velocity-time graph?

The graph shows that the slope is 5.00 (m/s)/s, which is often written as 5.00 m/s^2. Consider the time interval between 4.00 s and 5.00 s. At 4.00 s, the car's velocity was 20.0 m/s in the positive direction. At 5.00 s, the car was traveling at 25.0 m/s in the same direction. Thus, in 1.00 s, the car's velocity increased by 5.0 m/s in the positive direction. When the velocity of an object changes at a constant rate, it has a constant acceleration.

Reading velocity-time graphs The motions of five runners are shown in **Figure 6.** Assume that the positive direction is east. The slopes of Graphs A and E are zero. Thus, the accelerations are zero. Both graphs show motion at a constant velocity—Graph A to the east and Graph E to the west. Graph B shows motion with a positive velocity eastward. Its slope indicates a constant, positive acceleration. You can infer that the speed increases because velocity and acceleration are positive. Graph C has a negative slope. It shows motion that begins with a positive velocity, slows down, and then stops. This means the acceleration and the velocity are in opposite directions. The point at which Graphs C and B cross shows that the runners' velocities are equal at that time. It does not, however, identify their positions.

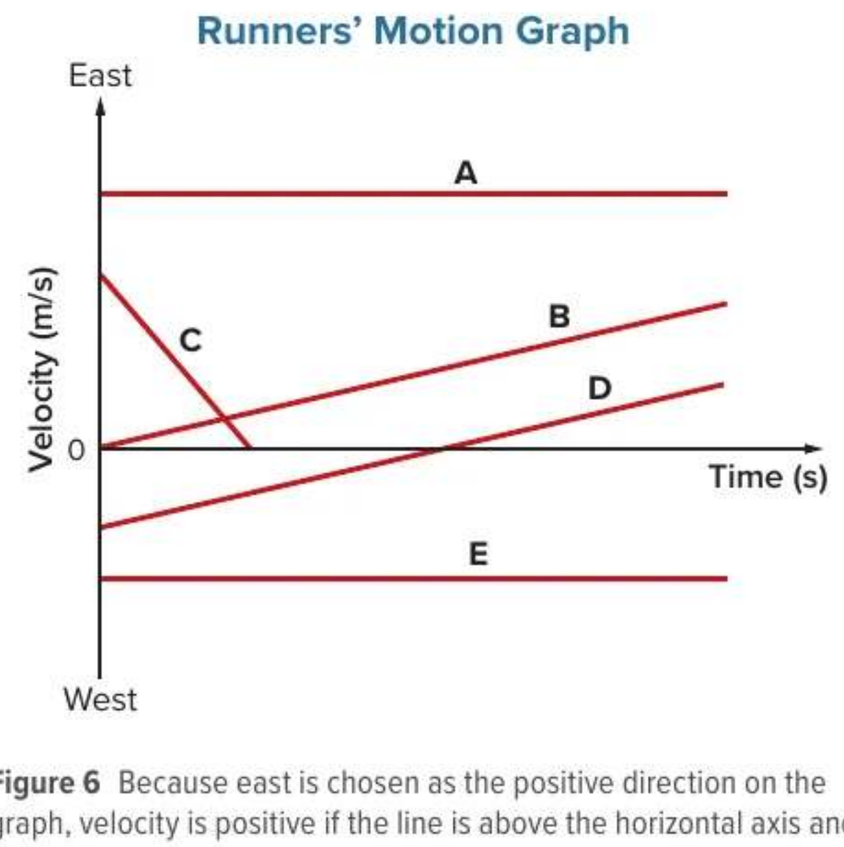


Figure 6 Because east is chosen as the positive direction on the graph, velocity is positive if the line is above the horizontal axis and negative if the line is below it. Acceleration is positive if the line is slanted upward on the graph. Acceleration is negative if the line is slanted downward on the graph. A horizontal line indicates constant velocity and zero acceleration.

Graph D indicates motion that starts out toward the west, slows down, for an instant has zero velocity, and then moves east with increasing speed. The slope of Graph D is positive. Because velocity and acceleration are initially in opposite directions, the speed decreases to zero at the time the graph crosses the *x*-axis. After that time, velocity and acceleration are in the same direction, and the speed increases.

Get It?
Describe the meaning of a line crossing the *x*-axis in a velocity-time graph.

Average and Instantaneous Acceleration

How does it feel differently if the car you ride in accelerates a little or a lot? As with velocity, the acceleration of most moving objects continually changes. If you want to describe an object's acceleration, it is often more convenient to describe the overall change in velocity during a certain time interval rather than describing the continual change.

The **average acceleration** of an object is its change in velocity during some measurable time interval divided by that time interval. Average acceleration is measured in meters per second per second (m/s/s), or simply meters per second squared (m/s^2). A car might accelerate quickly at times and more slowly at times. Just as average velocity depends only on the starting and ending displacement, average acceleration depends only on the starting and ending velocity during a time interval.

Figure 7 shows a graph of motion in which the acceleration is changing. The average acceleration during a certain time interval is determined just as it is in **Figure 5** for constant acceleration. Notice, however, that because the line is curved, the average acceleration in this graph varies depending on the time interval that you choose.

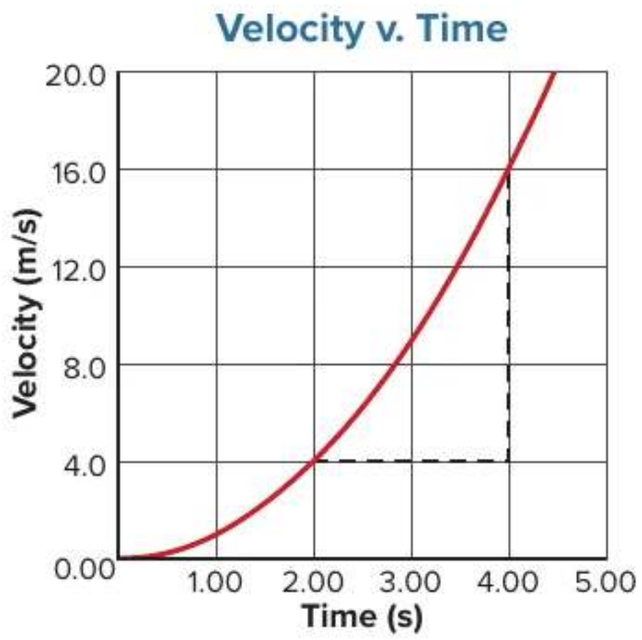


Figure 7 A curved line on a velocity-time graph shows that the acceleration is changing. The slope indicates the average acceleration during a time interval that you choose.

Calculate *How large is the average acceleration between 0.00 s and 2.00 s?*

The change in an object's velocity at an instant of time is called **instantaneous acceleration.** You can determine the instantaneous acceleration of an object by drawing a tangent line on the velocity-time graph at the point of time in which you are interested. A tangent line is a line that intersects the graph at one and only one point. The slope of this line is equal to the instantaneous acceleration. Most of the situations considered in this textbook assume an ideal case of constant acceleration. When the acceleration is the same at all points during a time interval, the average acceleration and the instantaneous accelerations are equal.

Get It?

Contrast How is instantaneous acceleration different from average acceleration?

Calculating Acceleration

How can you describe the acceleration of an object mathematically? Recall that the acceleration of an object is the slope of that object's velocity v. time graph. On a velocity v. time graph, slope equals $\frac{\Delta v}{\Delta t}$.

Average Acceleration

Average acceleration is defined as the change in velocity divided by the time it takes to make that change.

$$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

Suppose you run wind sprints back and forth across the gym. You run toward one wall, turn, then run toward the opposite wall. You first run at a speed of 4.0 m/s toward the wall. Then, 10.0 s later, your speed is 4.0 m/s as you run away from the wall. What is your average acceleration if the positive direction is toward the wall?

$$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$= \frac{-4.0\ \text{m/s} - 4.0\ \text{m/s}}{10.0\ \text{s}} = -0.80\ \text{m/s}^2$$

Get It?

Explain why the final velocity used in the equation is −4.0 m/s.

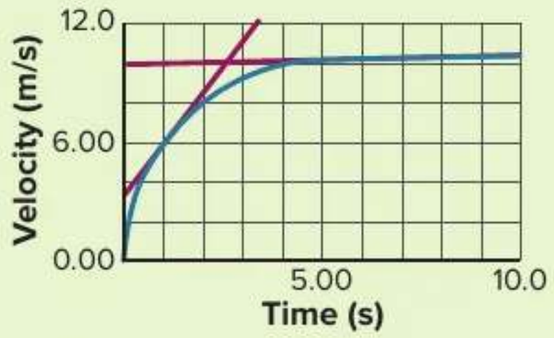
STEM CAREER Connection

Delivery Truck Driver

With more and more online sales, companies must have a way to get the products to your door. Delivery truck drivers pick up products at a distribution center and drive them to businesses and homes for delivery. Acceleration, velocity, and time are important for delivery truck drivers as they try to make their deliveries on a tight schedule.

EXAMPLE Problem 1

VELOCITY AND ACCELERATION How would you describe the sprinter's velocity and acceleration as shown on the graph?



1 ANALYZE AND SKETCH THE PROBLEM

From the graph, note that the magnitude of the sprinter's velocity starts at zero, increases rapidly for the first few seconds, and then, after reaching about 10.0 m/s, remains almost constant.

KNOWN	UNKNOWN
v = varies	a = ?

2 SOLVE FOR THE UNKNOWN

Draw tangents to the curve at two points. Choose $t = 1.00$ s and $t = 5.00$ s. Solve for the magnitude of the instantaneous acceleration at 1.00 s:

$$a = \frac{\text{rise}}{\text{run}}$$ The slope of the line at 1.00 s is equal to the acceleration at that time.

$$= \frac{10.0 \text{ m/s} - 6.00 \text{ m/s}}{2.4 \text{ s} - 1.00 \text{ s}}$$

$$= 2.9 \text{ m/s/s} = 2.9 \text{ m/s}^2$$

Solve for the magnitude of the instantaneous acceleration at 5.00 s:

$$a = \frac{\text{rise}}{\text{run}}$$ The slope of the line at 5.00 s is equal to the acceleration at that time.

$$= \frac{10.3 \text{ m/s} - 10.0 \text{ m/s}}{10.0 \text{ s} - 0.00 \text{ s}}$$

$$= 0.030 \text{ m/s/s} = 0.030 \text{ m/s}^2$$

The acceleration is not constant because its magnitude changes from 2.9 m/s^2 at 1.00 s to 0.030 m/s^2 at 5.00 s.

The acceleration is in the direction chosen to be positive because both values are positive.

3 EVALUATE THE ANSWER

- **Are the units correct?** Acceleration is measured in m/s^2.

PRACTICE Problems

ADDITIONAL PRACTICE

1. The velocity-time graph in **Figure 8** describes Steven's motion as he walks along the midway at the state fair. Sketch the corresponding motion diagram. Include velocity vectors in your diagram.
2. Use the *v-t* graph of the toy train in **Figure 9** to answer these questions.
   - **a.** When is the train's speed constant?
   - **b.** During which time interval is the train's acceleration positive?
   - **c.** When is the train's acceleration most negative?
3. Refer to **Figure 9** to find the average acceleration of the train during the following time intervals.
   - **a.** 0.0 s to 5.0 s **b.** 15.0 s to 20.0 s **c.** 0.0 s to 40.0 s
4. **CHALLENGE** Plot a *v-t* graph representing the following motion: An elevator starts at rest from the ground floor of a three-story shopping mall. It accelerates upward for 2.0 s at a rate of 0.5 m/s^2, continues up at a constant velocity of 1.0 m/s for 12.0 s, and then slows down with a constant downward acceleration of 0.25 m/s^2 for 4.0 s as it reaches the third floor.

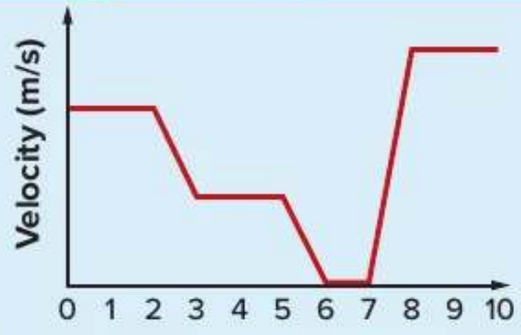


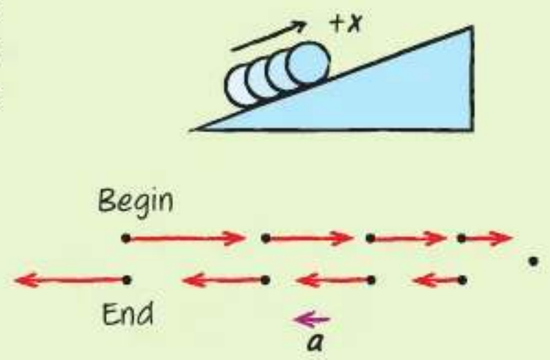
Figure 8



Figure 9

EXAMPLE Problem 2

ACCELERATION Describe a ball's motion as it rolls up a slanted driveway. It starts at 2.50 m/s, slows down for 5.00 s, stops for an instant, and then rolls back down. The positive direction is chosen to be up the driveway. The origin is where the motion begins. What are the sign and the magnitude of the ball's acceleration as it rolls up the driveway?



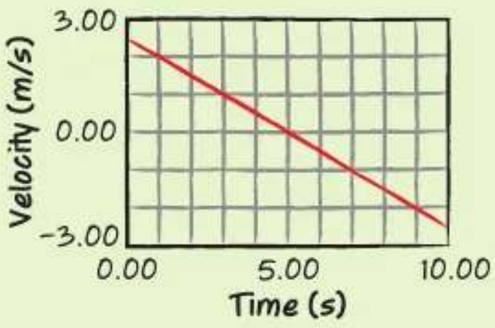
1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Draw the coordinate system based on the motion diagram.

KNOWN **UNKNOWN**

$v_i = +2.50$ m/s $a = ?$

$v_f = 0.00$ m/s at $t = 5.00$ s



2 SOLVE FOR THE UNKNOWN

Find the acceleration from the slope of the graph.

Solve for the change in velocity and the time taken to make that change.

$\Delta v = v_f - v_i$

$= 0.00 \text{ m/s} - 2.50 \text{ m/s} = -2.50 \text{ m/s}$ Substitute $v_f = 0.00$ m/s at $t_f = 5.00$ s, $v_i = 2.50$ m/s at $t_i = 0.00$ s.

$\Delta t = t_f - t_i$

$= 5.00 \text{ s} - 0.00 \text{ s} = 5.00 \text{ s}$ Substitute $t_f = 5.00$ s, $t_i = 0.00$ s.

Solve for the acceleration.

$\bar{a} \equiv \frac{\Delta v}{\Delta t} = (-2.50 \text{ m/s}) / 5.00 \text{ s}$ Substitute $\Delta v = -2.50$ m/s, $\Delta t = 5.00$ s.

$= -0.500 \text{ m/s}^2$ or 0.500 m/s^2 down the driveway

3 EVALUATE THE ANSWER

- **Are the units correct?** Acceleration is measured in m/s^2.
- **Do the directions make sense?** As the ball slows down, the direction of acceleration is opposite that of velocity.

PRACTICE Problems

ADDITIONAL PRACTICE

5. A race car's forward velocity increases from 4.0 m/s to 36 m/s over a 4.0-s time interval. What is its average acceleration?
6. The race car in the previous problem slows from 36 m/s to 15 m/s over 3.0 s. What is its average acceleration?
7. A bus is moving west at 25 m/s when the driver steps on the brakes and brings the bus to a stop in 3.0 s.
   a. What is the average acceleration of the bus while braking?
   b. If the bus took twice as long to stop, how would the acceleration compare with what you found in part **a?**
8. A car is coasting backward downhill at a speed of 3.0 m/s when the driver gets the engine started. After 2.5 s, the car is moving uphill at 4.5 m/s. If uphill is chosen as the positive direction, what is the car's average acceleration?
9. Rohith has been jogging east toward the bus stop at 3.5 m/s when he looks at his watch and sees that he has plenty of time before the bus arrives. Over the next 10.0 s, he slows his pace to a leisurely 0.75 m/s. What was his average acceleration during this 10.0 s?
10. **CHALLENGE** If the rate of continental drift were to abruptly slow from 1.0 cm/y to 0.5 cm/y over the time interval of a year, what would be the average acceleration?

Acceleration with Constant Speed

Think again about running wind sprints across the gym. You first run at a speed of 4.0 m/s toward the wall. Then, 10.0 s later, you are running away from the wall and your speed is 4.0 m/s. Notice that your speed is the same as you move toward the wall of the gym and as you move away from it. In both cases, you are running at a speed of 4.0 m/s. How is it possible for you to be accelerating?

Acceleration can occur even when speed is constant. The average acceleration for the entire trip you make toward the wall of the gym and back again is $-0.80\ m/s^2$. The negative sign indicates that the direction of your acceleration is away from the wall because the positive direction was chosen as toward the wall. The velocity changes from positive to negative when the direction of motion changes. A change in velocity results in acceleration. Thus, acceleration can also be associated with a change in the direction of motion.

Get It?

Describe the evidence that the planes at the beginning of this module are accelerating even if they are traveling at a constant speed.

Check Your Progress

11. **Describing Motion** What are three ways an object can accelerate?
12. **Position-Time and Velocity-Time Graphs** Two joggers run at a constant velocity of 7.5 m/s east. **Figure 10** shows the positions of both joggers at time $t = 0$.
    a. What would be the difference(s) in the position-time graphs of their motion?
    b. What would be the difference(s) in their velocity-time graphs?



Figure 10

13. **Velocity-Time Graph** Sketch a velocity-time graph for a car that goes east at 25 m/s for 100 s, then west at 25 m/s for another 100 s.
14. **Average Velocity and Average Acceleration** A canoeist paddles upstream at a velocity of 2.0 m/s for 4.0 s and then floats downstream at 4.0 m/s for 4.0 s.
    a. What is the average velocity of the canoe during the 8.0-s time interval?
    b. What is the average acceleration of the canoe during the 8.0-s time interval?
15. **Critical Thinking** A police officer clocked a driver going 32 km/h over the speed limit just as the driver passed a slower car. When the officer stopped the car, the driver argued that the other driver should get a ticket as well. The driver said that the cars must have been going the same speed because they were observed next to each other. Is the driver correct? Explain with a sketch and a motion diagram.

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LESSON 2

MOTION WITH CONSTANT ACCELERATION

FOCUS QUESTION

How are position, velocity, acceleration, and time related?

Position with Constant Acceleration

If an object experiences constant acceleration, its velocity changes at a constant rate. How does its position change? The positions at different times of a car with constant acceleration are graphed in **Figure 11.** The graph shows that the car's motion is not uniform. The displacements for equal time intervals on the graph get larger and larger. As a result, the slope of the line in **Figure 11** gets steeper as time goes on. For an object with constant acceleration, the position-time graph is a parabola.

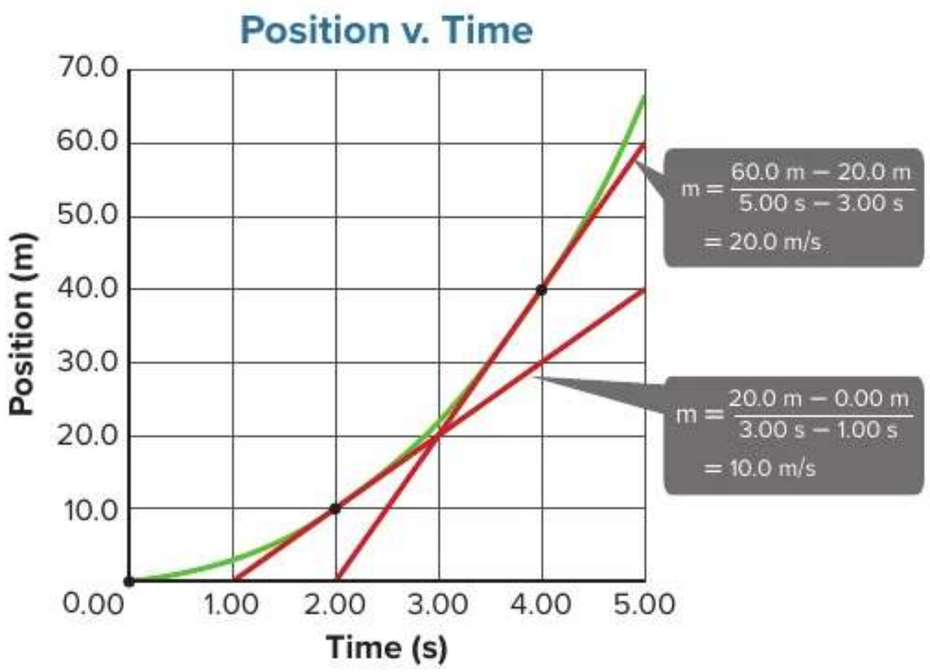


Figure 11 The slope of a position-time graph changes with time for an object with constant acceleration.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Probeware Lab: Measuring Acceleration

Analyze data to determine how the displacement, velocity, and acceleration of an object change over time.

CCC Identify Crosscutting Concepts

Create a table of the crosscutting concepts and fill in examples you find as you read.

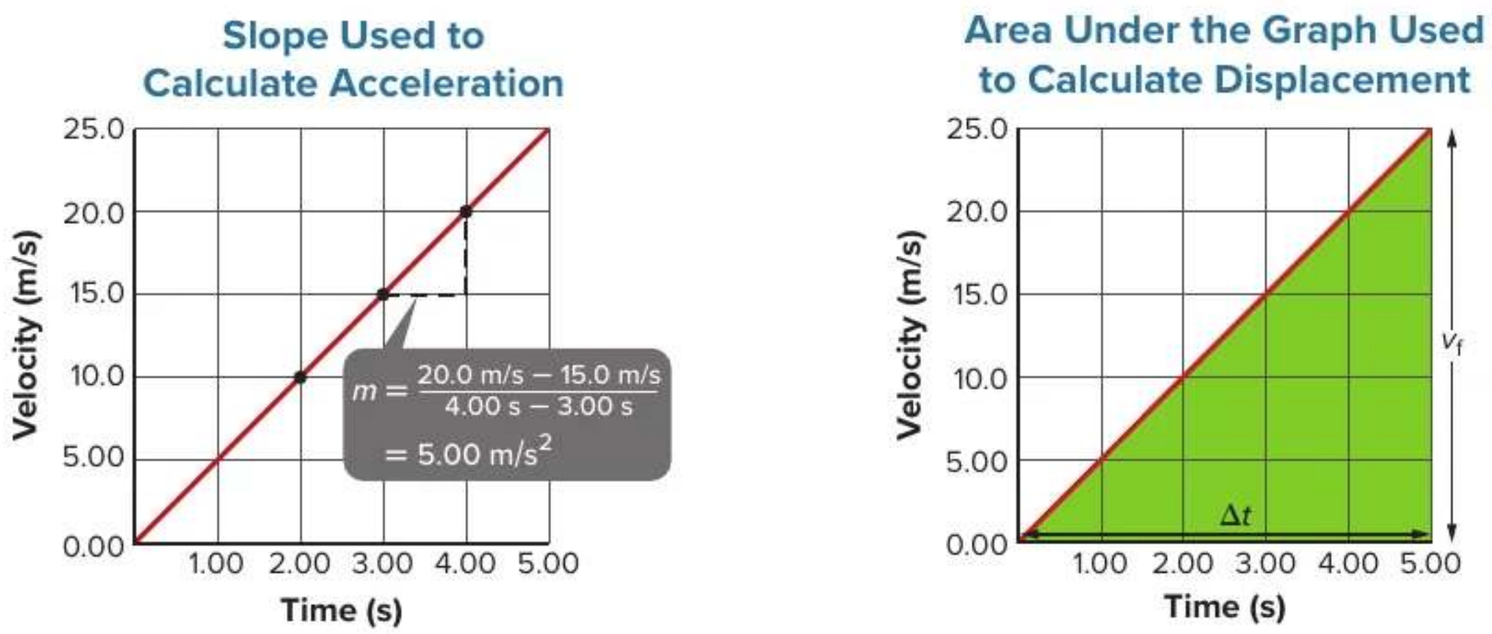


Figure 12 The slopes of the position-time graph in **Figure 11** are shown in these velocity-time graphs. The rise divided by the run gives the acceleration on the left. The area under the curve gives the displacement on the right.

Calculate *What is the slope of the velocity-time graph on the left between $t = 2.00$ s and $t = 5.00$ s?*

The slopes from the position-time graph in **Figure 11** have been used to create the velocity-time graph on the left in **Figure 12.** For an object with constant acceleration, the velocity-time graph is a straight line.

Get It?

Identify What is the shape of a position-time graph of an object traveling with constant acceleration?

A unique position-time graph cannot be created using a velocity-time graph because it does not contain information about position. It does, however, contain information about displacement. Recall that for an object moving at a constant velocity, the velocity is the displacement divided by the time interval. The displacement is then the product of the velocity and the time interval. On the right graph in **Figure 12**, v is the height of the plotted line above the horizontal axis, and Δt is the width of the shaded triangle. The area is $\left(\frac{1}{2}\right)v\Delta t$, or Δx. Thus, the area under the v-t graph equals the displacement.

Get It?

Identify What is the shape of a velocity-time graph of an object traveling with constant acceleration?

Velocity with Average Acceleration

You have read that the equation for average velocity can be algebraically rearranged to show the new position after a period of time, given the initial position and the average velocity. The definition of average acceleration can be manipulated similarly to show the new velocity after a period of time, given the initial velocity and the average acceleration.

If you know an object's average acceleration during a time interval, you can use it to determine how much the velocity changed during that time.

You can rewrite the definition of average acceleration $\left(\bar{a} \equiv \frac{\Delta v}{\Delta t}\right)$ as follows:

$$\Delta v = \bar{a}\Delta t$$

$$v_f - v_i = \bar{a}\Delta t$$

The equation for final velocity with average acceleration can be written:

Final Velocity with Average Acceleration

The final velocity is equal to the initial velocity plus the product of the average acceleration and the time interval.

$$v_f = v_i + \bar{a}\Delta t$$

In cases when the acceleration is constant, the average acceleration ($\bar{a}$) is the same as the instantaneous acceleration (a) at any point within the time interval. This equation can be rearranged to find the time at which an object with constant acceleration has a given velocity. You can also use it to calculate the initial velocity of an object when both a velocity and the time at which it occurred are given.

Real-World Physics



DRAG RACING A dragster driver tries to obtain maximum acceleration over a course. The fastest U.S. National Hot Rod Association time on record for the 402-m course is 3.771 s. The highest final speed on record is 145.3 m/s (324.98 mph).

PRACTICE Problems

ADDITIONAL PRACTICE

16. A golf ball rolls up a hill toward a miniature-golf hole. Assume the direction toward the hole is positive.

- **a.** If the golf ball starts with a speed of 2.0 m/s and slows at a constant rate of 0.50 m/s², what is its velocity after 2.0 s?
- **b.** What is the golf ball's velocity if the constant acceleration continues for 6.0 s?
- **c.** Describe the motion of the golf ball in words and with a motion diagram.

17. A bus traveling 30.0 km/h east has a constant increase in speed of 1.5 m/s². What is its velocity 6.8 s later?

18. If a car accelerates from rest at a constant rate of 5.5 m/s² north, how long will it take for the car to reach a velocity of 28 m/s north?

19. CHALLENGE A car slows from 22 m/s to 3.0 m/s at a constant rate of 2.1 m/s². How many seconds are required before the car is traveling at a forward velocity of 3.0 m/s?

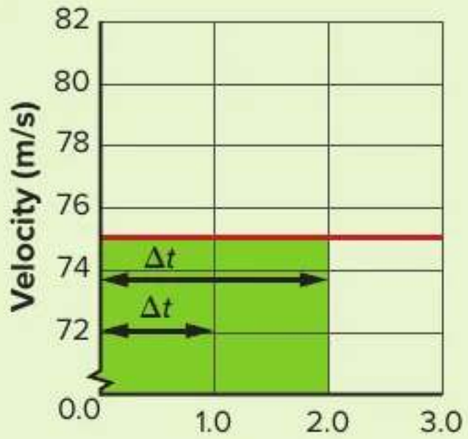
CCC CROSSCUTTING CONCEPTS

Scale, Proportion, and Quantity Using algebraic thinking, draw a conclusion about how changes in the initial velocity of an object affects the final position of the object. Make a table that provides evidence to support your conclusions.

Lucie Lang/Alamy Stock Photo

EXAMPLE Problem 3

FINDING DISPLACEMENT FROM A VELOCITY-TIME GRAPH The velocity-time graph at the right shows the motion of an airplane. Find the displacement of the airplane for $\Delta t = 1.0$ s and for $\Delta t = 2.0$ s. Let the positive direction be forward.



1 ANALYZE AND SKETCH THE PROBLEM

- The displacement is the area under the *v*-*t* graph.
- The time intervals begin at $t = 0.0$ s.

KNOWN	UNKNOWN
$\boldsymbol{v} = +75$ m/s	$\Delta \boldsymbol{x} = ?$
$\Delta t = 1.0$ s	
$\Delta t = 2.0$ s	

2 SOLVE FOR THE UNKNOWN

Use the relationship among displacement, velocity, and time interval to find $\Delta \boldsymbol{x}$ during $\Delta t = 1.0$ s.

$\Delta x = v\Delta t$

$= (+75 \text{ m/s})(1.0 \text{ s})$ Substitute $\boldsymbol{v} = +75$ m/s, $\Delta t = 1.0$ s.

$= +75$ m

Use the same relationship to find $\Delta \boldsymbol{x}$ during $\Delta t = 2.0$ s.

$\Delta x = v\Delta t$

$= (+75 \text{ m/s})(2.0 \text{ s})$ Substitute $\boldsymbol{v} = +75$ m/s, $\Delta t = 2.0$ s.

$= +150$ m

3 EVALUATE THE ANSWER

- **Are the units correct?** Displacement is measured in meters.
- **Do the signs make sense?** The positive sign agrees with the graph.
- **Is the magnitude realistic?** Moving a distance of about one football field per second is reasonable for an airplane.

PRACTICE Problems

ADDITIONAL PRACTICE

20. The graph in **Figure 13** describes the motion of two bicyclists, Akiko and Brian, who start from rest and travel north, increasing their speed with a constant acceleration. What was the total displacement of each bicyclist during the time shown for each?

Hint: Use the area of a triangle: area $= \left(\frac{1}{2}\right)$*(base)(height).*

21. The motion of two people, Carlos and Diana, moving south along a straight path is described by the graph in **Figure 14.** What is the total displacement of each person during the first 4.0-s interval shown on the graph?

22. CHALLENGE A car, just pulling onto a straight stretch of highway, has a constant acceleration from 0 m/s to 25 m/s west in 12 s.

a. Draw a *v*-*t* graph of the car's motion.

b. Use the graph to determine the car's displacement during the 12.0-s time interval.

c. Another car is traveling along the same stretch of highway. It travels the same distance in the same time as the first car, but its velocity is constant. Draw a *v*-*t* graph for this car's motion.

d. Explain how you knew this car's velocity.

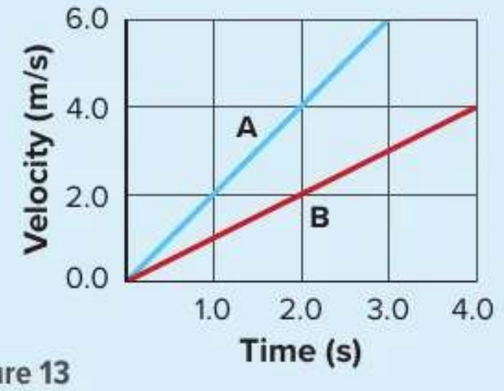


Figure 13

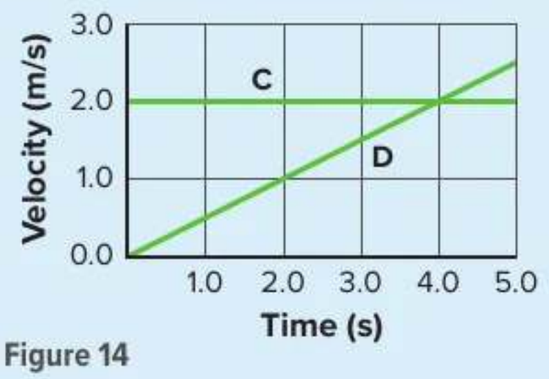


Figure 14

Motion with an initial nonzero velocity The graph in **Figure 15** describes constant acceleration that started with an initial velocity of v_i. To determine the displacement, you can divide the area under the graph into a rectangle and a triangle. The total area is then:

$$\Delta x = \Delta x_{\text{rectangle}} + \Delta x_{\text{triangle}} = v_i(\Delta t) + \left(\frac{1}{2}\right)\Delta v \Delta t$$

Substituting $a\Delta t$ for the change in velocity in the equation yields:

$$\Delta x = \Delta x_{\text{rectangle}} + \Delta x_{\text{triangle}} = v_i(\Delta t) + \left(\frac{1}{2}\right)a(\Delta t)^2$$

When the initial or final position of the object is known, the equation can be written as follows:

$$x_f - x_i = v_i(\Delta t) + \left(\frac{1}{2}\right)a(\Delta t)^2 \text{ or } x_f = x_i + v_i(\Delta t) + \left(\frac{1}{2}\right)a(\Delta t)^2$$

If the initial time is $t_i = 0$, the equation then becomes the following.

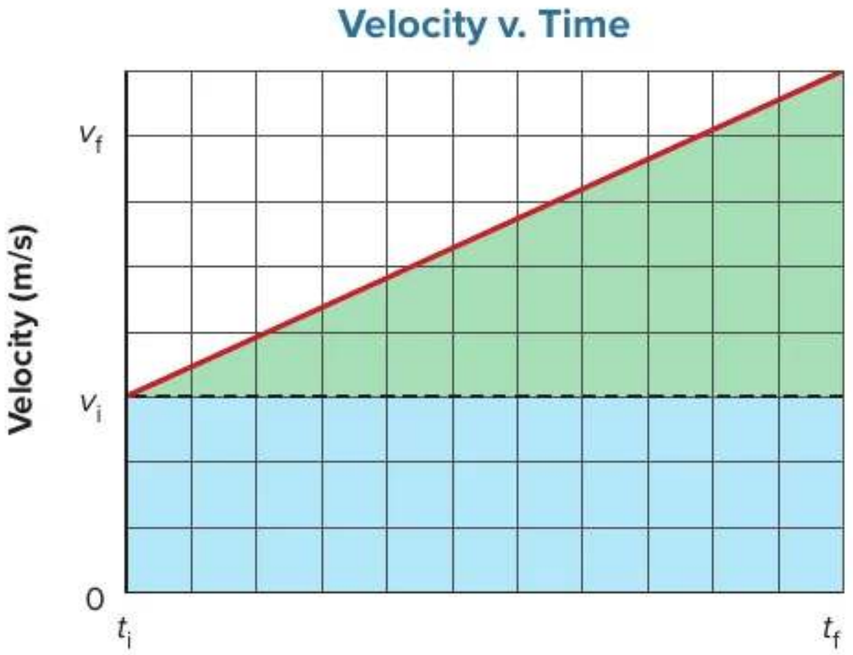


Figure 15 For motion with constant acceleration, if the initial velocity on a velocity-time graph is not zero, the area under the graph is the sum of a rectangular area and a triangular area.

Position with Average Acceleration

An object's final position is equal to the sum of its initial position, the product of the initial velocity and the final time, and half the product of the acceleration and the square of the final time.

$$x_f = x_i + v_i t_f + \left(\frac{1}{2}\right)a t_f^2$$

An Alternative Equation

Often, it is useful to relate position, velocity, and constant acceleration without including time.

Rearrange the equation $v_f = v_i + at_f$ to solve for time: $t_f = \frac{v_f - v_i}{a}$.

You can then rewrite the position with average acceleration equation by substituting t_f to obtain the following:

$$x_f = x_i + v_i\left(\frac{v_f - v_i}{a}\right) + \left(\frac{1}{2}\right)a\left(\frac{v_f - v_i}{a}\right)^2$$

This equation can be solved for the velocity (v_f) at any position (x_f).

Velocity with Constant Acceleration

The square of the final velocity equals the sum of the square of the initial velocity and twice the product of the acceleration and the displacement since the initial time.

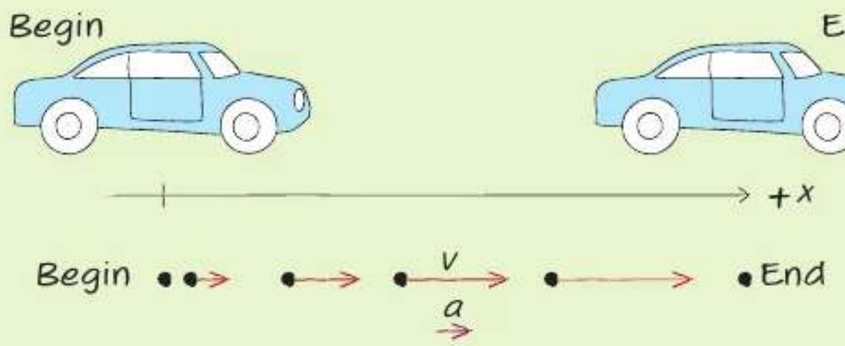
$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

EXAMPLE Problem 4

DISPLACEMENT An automobile starts at rest and accelerates at 3.5 m/s² after a traffic light turns green. How far will it have gone when it is traveling at 25 m/s?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Establish coordinate axes. Let the positive direction be to the right.
- Draw a motion diagram.



KNOWN

$x_i = 0.00$ m

$v_i = 0.00$ m/s

$v_f = +25$ m/s

$\overline{a} = a = +3.5$ m/s²

UNKNOWN

$x_f = ?$

2 SOLVE FOR THE UNKNOWN

Use the relationship among velocity, acceleration, and displacement to find x_f.

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$x_f = x_i + \frac{v_f^2 - v_i^2}{2a}$$

$$= 0.00 \text{ m} + \frac{(+25 \text{ m/s})^2 - (0.00 \text{ m/s})^2}{2(+3.5 \text{ m/s}^2)}$$

Substitute $x_i = 0.00$ m, $v_f = +25$ m/s, $v_i = 0.00$ m/s, $a = +3.5$ m/s².

$$= +89 \text{ m}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Position is measured in meters.
- **Do the signs make sense?** The positive sign agrees with both the pictorial and physical models.
- **Is the magnitude realistic?** The displacement is almost the length of a football field. The result is reasonable because 25 m/s (about 55 mph) is fast.

PRACTICE Problems

ADDITIONAL PRACTICE

23. A skateboarder is moving at a constant speed of 1.75 m/s when she starts up an incline that causes her to slow down with a constant acceleration of –0.20 m/s². How much time passes from when she begins to slow down until she begins to move back down the incline?

24. A race car travels on a straight racetrack with a forward velocity of 44 m/s and slows at a constant rate to a velocity of 22 m/s over 11 s. How far does it move during this time?

25. A car accelerates at a constant rate from 15 m/s to 25 m/s while it travels a distance of 125 m. How long does it take to achieve the final speed?

26. A bike rider pedals with constant acceleration to reach a velocity of 7.5 m/s north over a time of 4.5 s. During the period of acceleration, the bike's displacement is 19 m north. What was the initial velocity of the bike?

27. CHALLENGE The car in **Figure 16** travels west with a forward acceleration of 0.22 m/s². What was the car's velocity ($\boldsymbol{v}_i$) at point $\boldsymbol{x}_i$ if it travels a distance of 350 m in 18.4 s?

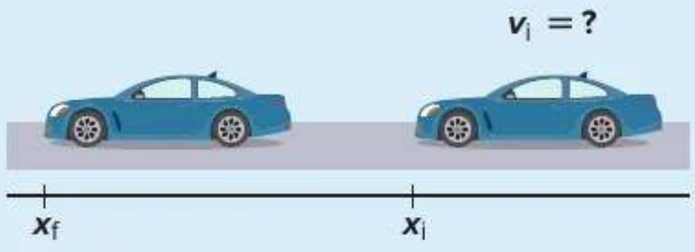


Figure 16

EXAMPLE Problem 5

TWO-PART MOTION You are driving a car, traveling at a constant velocity of 25 m/s along a straight road, when you see a child suddenly run onto the road. It takes 0.45 s for you to react and apply the brakes. As a result, the car slows with a steady acceleration of 8.5 m/s² in the direction opposite your motion and comes to a stop. What is the total displacement of the car before it stops?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Choose a coordinate system with the motion of the car in the positive direction.
- Draw the motion diagram, and label ***v*** and ***a***.

KNOWN

$\boldsymbol{v}_{\text{reacting}} = +25 \text{ m/s}$

$t_{\text{reacting}} = 0.45 \text{ s}$

$\boldsymbol{a}_{\text{braking}} = -8.5 \text{ m/s}^2$

$\boldsymbol{v}_{\text{i, braking}} = +25 \text{ m/s}$

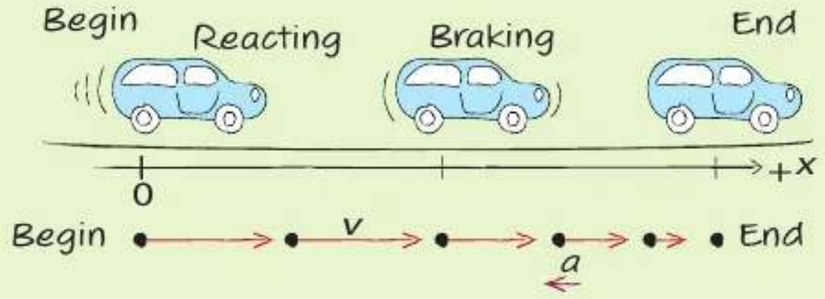
$\boldsymbol{v}_{\text{f, braking}} = 0.00 \text{ m/s}$

UNKNOWN

$\boldsymbol{x}_{\text{reacting}} = ?$

$\boldsymbol{x}_{\text{braking}} = ?$

$\boldsymbol{x}_{\text{total}} = ?$



2 SOLVE FOR THE UNKNOWN

Reacting:

Use the relationship among displacement, velocity, and time interval to find the displacement of the car as it travels at a constant speed.

$\boldsymbol{x}_{\text{reacting}} = \boldsymbol{v}_{\text{reacting}} t_{\text{reacting}}$

$\boldsymbol{x}_{\text{reacting}} = (+25 \text{ m/s})(0.45 \text{ s})$ Substitute $\boldsymbol{v}_{\text{reacting}} = +25 \text{ m/s}$, $t_{\text{reacting}} = 0.45 \text{ s}$.

$= +11 \text{ m}$

Braking:

Use the relationship among velocity, acceleration, and displacement to find the displacement of the car while it is braking.

$v_{\text{f, braking}}^2 = v_{\text{reacting}}^2 + 2a_{\text{braking}}(x_{\text{braking}})$

Solve for x_{braking}.

$$x_{\text{braking}} = \frac{v_{\text{f, braking}}^2 - v_{\text{reacting}}^2}{2a_{\text{braking}}}$$

$$= \frac{(0.00 \text{ m/s})^2 - (+25 \text{ m/s})^2}{2(-8.5 \text{ m/s}^2)}$$ Substitute $v_{\text{f, braking}} = 0.00 \text{ m/s}$, $v_{\text{reacting}} = +25 \text{ m/s}$, $a_{\text{braking}} = -8.5 \text{ m/s}^2$.

$= +37 \text{ m}$

The total displacement is the sum of the reacting displacement and the braking displacement.

Solve for $\boldsymbol{x}_{\text{total}}$.

$\boldsymbol{x}_{\text{total}} = \boldsymbol{x}_{\text{reacting}} + \boldsymbol{x}_{\text{braking}}$

$= +11 \text{ m} + 37 \text{ m}$ Substitute $\boldsymbol{x}_{\text{reacting}} = +11 \text{ m}$, $\boldsymbol{x}_{\text{braking}} = +37 \text{ m}$.

$= +48 \text{ m}$

3 EVALUATE THE ANSWER

- **Are the units correct?** Displacement is measured in meters.
- **Do the signs make sense?** Both xreacting and xbraking are positive, as they should be.
- **Is the magnitude realistic?** The braking displacement is small because the magnitude of the acceleration is large.

PRACTICE Problems

ADDITIONAL PRACTICE

28. A car with an initial velocity of 24.5 m/s east has an acceleration of 4.2 m/s² west. What is its displacement at the moment that its velocity is 18.3 m/s east?

29. A man runs along the path shown in **Figure 17.** From point A to point B, he runs at a forward velocity of 4.5 m/s for 15.0 min. From point B to point C, he runs up a hill. He slows down at a constant rate of 0.050 m/s² for 90.0 s and comes to a stop at point C. What was the total distance the man ran?

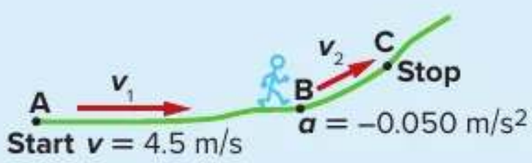


Figure 17

30. You start your bicycle ride at the top of a hill. You coast down the hill at a constant acceleration of 2.00 m/s². When you get to the bottom of the hill, you are moving at 18.0 m/s, and you pedal to maintain that speed. If you continue at this speed for 1.00 min, how far will you have gone from the time you left the hilltop?

31. Sunee is training for a 5.0-km race. She starts out her training run by moving at a constant pace of 4.3 m/s for 19 min. Then she accelerates at a constant rate until she crosses the finish line 19.4 s later. What is her acceleration during the last portion of the training run?

32. CHALLENGE Sekazi is learning to ride a bike without training wheels. His father pushes him with a constant acceleration of 0.50 m/s² east for 6.0 s. Sekazi then travels at 3.0 m/s east for another 6.0 s before falling. What is Sekazi's displacement? Solve this problem by constructing a velocity-time graph for Sekazi's motion and computing the area underneath the graphed line.

Check Your Progress

33. **Displacement** Given initial and final velocities and the constant acceleration of an object, what mathematical relationship would you use to find the displacement?

34. **Acceleration** A woman driving west along a straight road at a speed of 23 m/s sees a deer on the road ahead. She applies the brakes when she is 210 m from the deer. If the deer does not move and the car stops right before it hits the deer, what is the acceleration provided by the car's brakes?

35. **Distance** The airplane in **Figure 18** starts from rest and accelerates east at a constant 3.00 m/s² for 30.0 s before leaving the ground.
    a. What was the plane's displacement (Δx)?
    b. How fast was the airplane going when it took off?

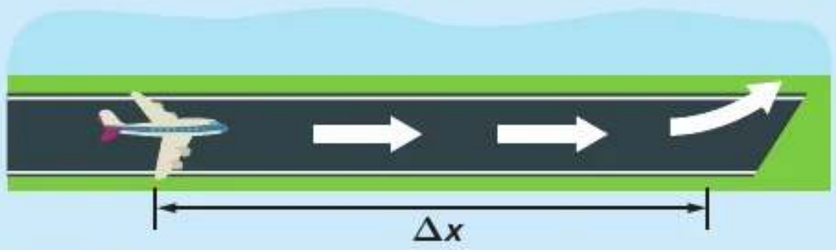


Figure 18

36. **Distance** An in-line skater accelerates from 0.0 m/s to 5.0 m/s in 4.5 s, then continues at this constant speed for another 4.5 s. What is the total distance traveled by the in-line skater?

37. **Final Velocity** A plane travels 5.0×10^2 m north while accelerating uniformly from rest at 5.0 m/s². What final velocity does it attain?

38. **Final Velocity** An airplane accelerated uniformly from rest at the rate of 5.0 m/s² south for 14 s. What final velocity did it attain?

39. **Graphs** A sprinter walks to the starting blocks at a constant speed, then waits. When the starting pistol sounds, she accelerates rapidly until she attains a constant velocity. She maintains this velocity until she crosses the finish line, and then she slows to a walk, taking more time to slow down than she did to speed up at the beginning of the race. Sketch a velocity-time and a position-time graph to represent her motion. Draw them one above the other using the same time scale. Indicate on your position-time graph where the starting blocks and finish line are.

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LESSON 3

FREE FALL

FOCUS QUESTION

How does an object's speed change as it falls?

Galileo's Discovery

Do heavier objects accelerate more as they fall? If you release a paper and a book, the book hits the ground first. But if you put the paper flat on the book, they fall together. Collisions with particles of air have a greater effect on the paper. To understand falling objects, first consider the case in which air does not have an appreciable effect on motion. Recall that gravity is an attraction between objects. **Free fall** is the motion of an object when gravity is the only significant force acting on it.

About 400 years ago, Galileo Galilei discovered that, neglecting the effect of the air, all objects in free fall have the same acceleration. It doesn't matter what they are made of or how much they weigh. The acceleration of an object due only to the effect of gravity is known as **free-fall acceleration.** **Figure 19** depicts the results of a 1971 free-fall experiment on the Moon in which astronauts verified Galileo's results.

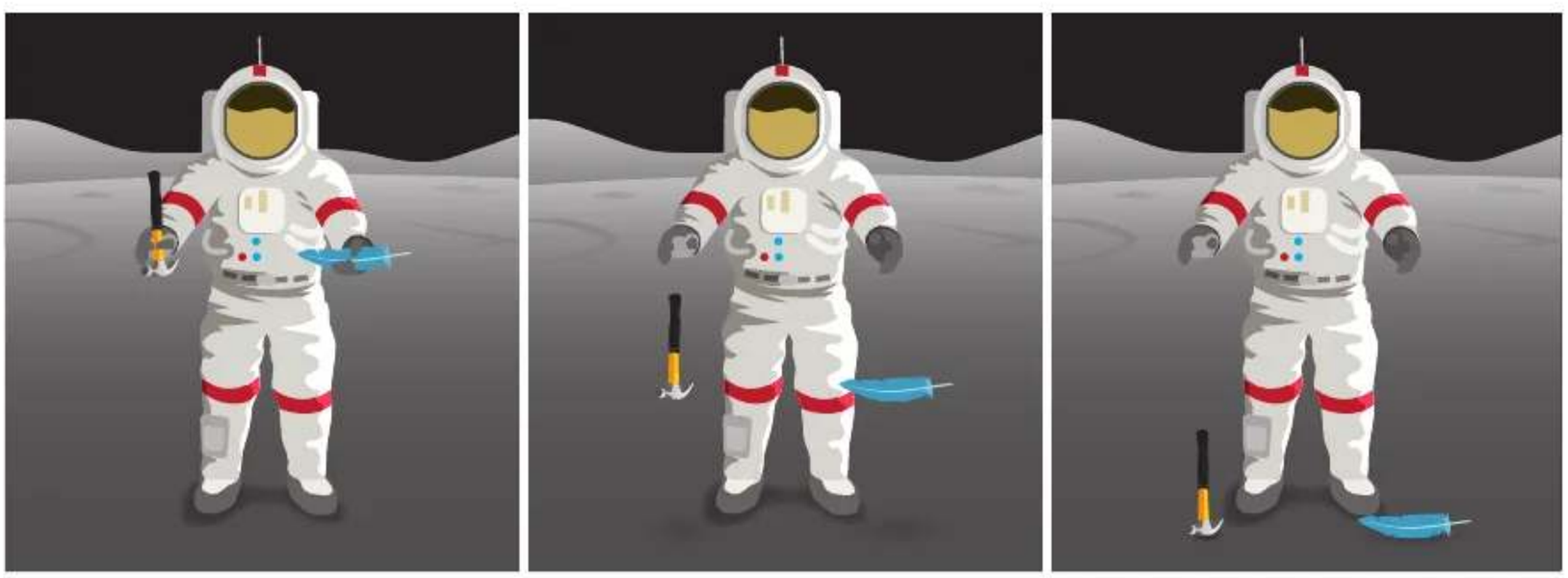


Figure 19 In 1971, astronaut David Scott dropped a hammer and a feather at the same time from the same height above the Moon's surface. The hammer's mass was greater, but both objects hit the ground at the same time because the Moon has gravity but no air.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

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INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Free-Fall Acceleration

Calculate the acceleration due to gravity for a system in free-fall.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Near Earth's surface, free-fall acceleration is about 9.8 m/s^2 downward (which is equal to about 22 mph/s downward). Think about skydivers. Each second skydivers fall, their downward velocity increases by 9.8 m/s. When analyzing free fall, whether you treat the acceleration as positive or negative depends on the coordinate system you use. If you define upward as the positive direction, then the free-fall acceleration is negative. If you decide that downward is the positive direction, then free-fall acceleration is positive.

Get It?

Identify Why is it important to clearly define the coordinate system you want to use when analyzing objects in free fall?

Free-Fall Acceleration

Galileo's discovery explains why parachutists can form a ring in midair. Regardless of their masses, they fall with the same acceleration. To understand the acceleration that occurs during free fall, look at the multiflash photo of a dropped ball in **Figure 20.**

When the photograph was taken, the time interval between flashes was set to 0.06 s. So, the same amount of time passes between one image of the ball and the next. Look at the distance that the ball travels during each time interval. The distance between each pair of images increases as the ball falls. So, the speed of the ball is increasing. If the upward direction is positive, then the velocity is becoming more and more negative.

Ball thrown upward Instead of a dropped ball, could this photo also illustrate a ball thrown upward? Suppose you throw a ball upward with a speed of 20.0 m/s. If you choose upward to be positive, then the ball starts at the bottom of the photo with a positive velocity. The acceleration is $a = -9.8$ m/s^2. Because velocity and acceleration are in opposite directions, the speed of the ball decreases. If you think of the bottom of the photo as the start, this agrees with the multiflash photo.

Rising and falling motion After 1 s, the ball's velocity is reduced by 9.8 m/s, so it now is traveling at +10.2 m/s. After 2 s, the velocity is +0.4 m/s, and the ball still is moving upward. What happens during the next second? The ball's velocity is reduced by another 9.8 m/s and equals −9.4 m/s. The ball now is moving downward. After 4 s, the velocity is −19.2 m/s, meaning the ball is falling even faster.



Figure 20 Because of free-fall acceleration, the speed of this falling ball increases 9.8 m/s each second.

Get It?

Analyze During which second does the rising ball stop and reverse direction? How can you tell?

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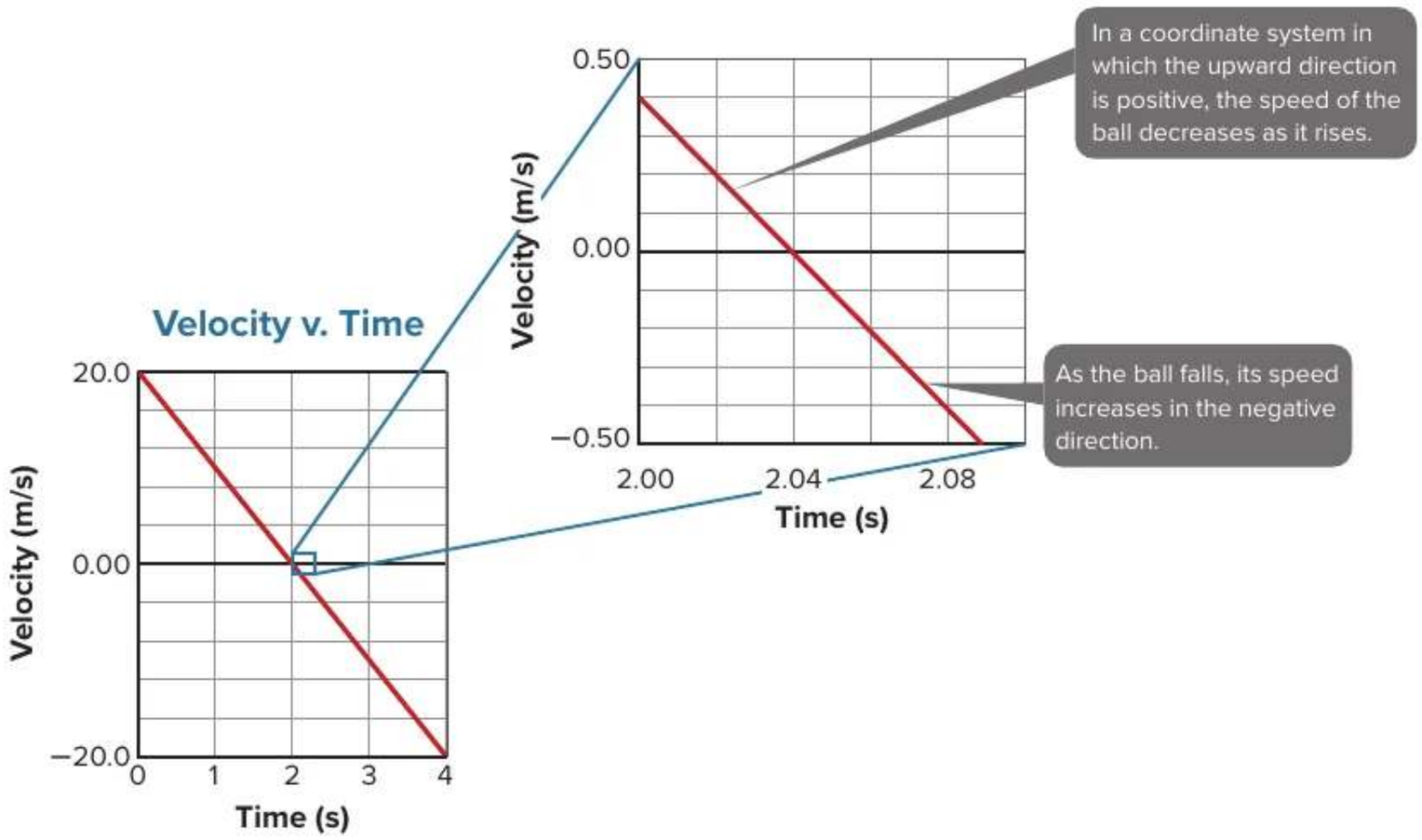


Figure 21 The velocity-time graph describes the change in the ball's speed as it rises and falls. The graph on the right gives a close-up view of the change in velocity at the top of the ball's trajectory.

Analyze *What would the graph look like if downward were chosen as the positive direction?*

Velocity-time graph The v-t graph for the ball as it goes up and down is shown in **Figure 21.** The straight line sloping downward does not mean that the speed is always decreasing. The speed decreases as the ball rises and increases as it falls. At around 2 s, the velocity changes smoothly from positive to negative. As the ball falls, its speed increases in the negative direction. The figure also shows a closer view of the v-t graph. At an instant of time, near 2.04 s, the velocity is zero.

Position-time graph Look at the position-time graphs in **Figure 22** on the next page. These graphs show how the ball's height changes as it rises and falls. If an object is moving with constant acceleration, its position-time graph forms a parabola. Because the ball is rising and falling, its graph is an inverted parabola. The shape of the graph shows the progression of time. It does not mean that the ball's path was in the shape of a parabola. The close-up graph on the right shows that at about 2.04 s, the ball reaches its maximum height.

Get It?

Describe If you throw a ball straight up, what would the shape of its position-time graph look like?

Maximum height Compare the close-up graphs in **Figure 21** and **Figure 22.** Just before the ball reaches its maximum height, its velocity is decreasing in the negative direction. At the instant of time when its height is maximum, its velocity is zero. Just after it reaches its maximum height, the ball's velocity is increasing in the negative direction.

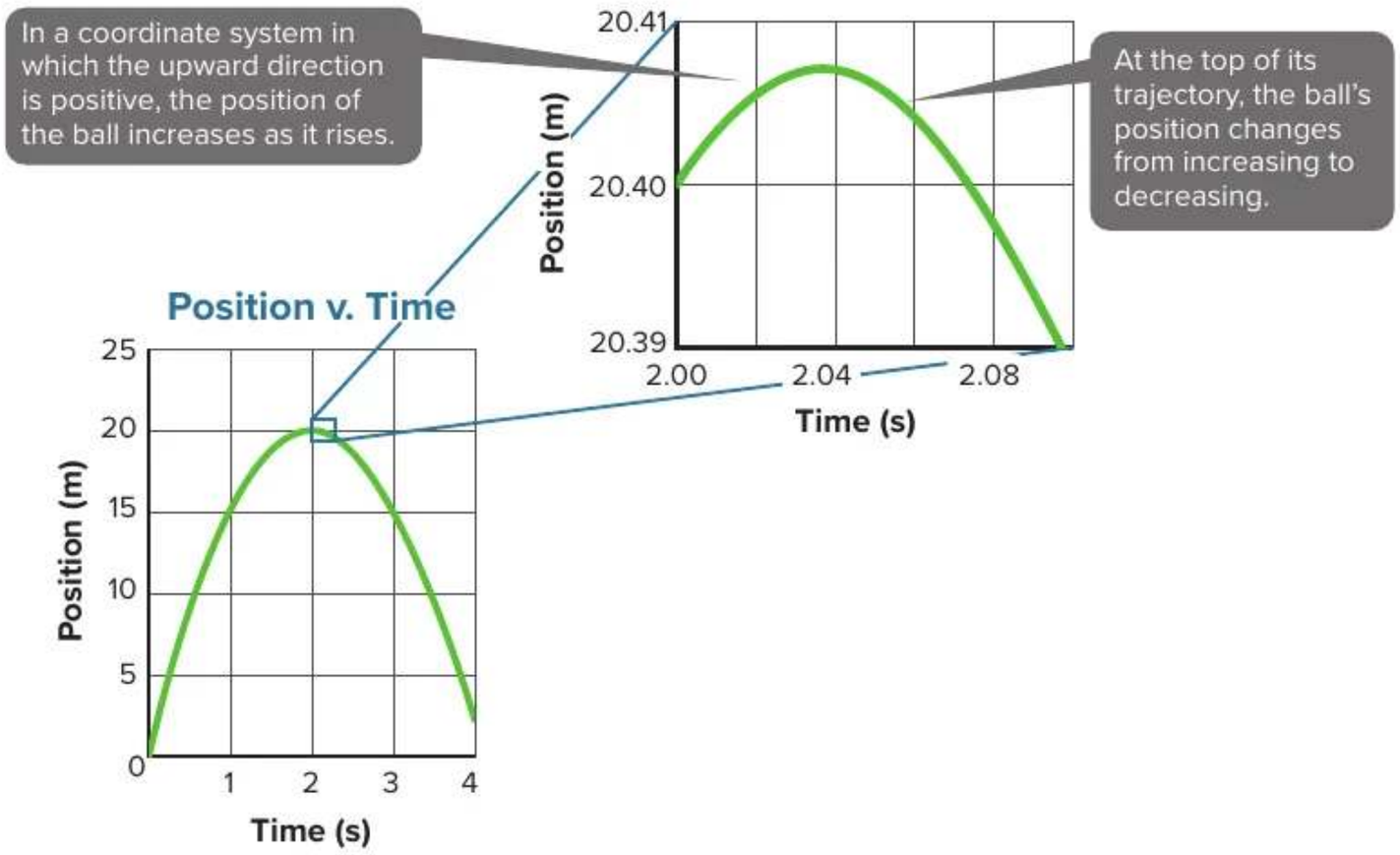


Figure 22 A position-time graph shows how the ball's position changes as it rises and falls. The graph at the right shows a close-up view of how the position changes at the top of the ball's trajectory.

Acceleration The slope of the line on the velocity-time graph in **Figure 21** is constant at $-9.8\ m/s^2$. This shows that the ball's free-fall acceleration is $9.8\ m/s^2$ in the downward direction the entire time the ball is rising and falling.

It may seem that the acceleration should be zero at the top of the trajectory, but this is not the case. At the top of the flight, the ball's velocity is 0 m/s. If its acceleration were also zero, the ball's velocity would not change and would remain at 0 m/s. The ball would not gain any downward velocity and would simply hover in the air. Have you ever seen that happen? Objects tossed in the air on Earth always fall, so you know the acceleration of an object at the top of its flight must not be zero. Further, because the object falls down, you know the acceleration must be downward.

Get It?

Analyze If you throw a ball straight up, what are its velocity and acceleration at the uppermost point of its path?

SCIENCE USAGE v. COMMON USAGE

Free fall

Science usage: motion of a body when air resistance is negligible and the acceleration can be considered due to gravity alone.
Acceleration during free fall is $9.8\ m/s^2$ downward.

Common usage: a rapid and continuing drop or decline
The stock market's free fall in 1929 marked the beginning of the Great Depression.



Figure 23 The people on this amusement-park ride experience free-fall acceleration.

Free-fall rides Amusement parks use the concept of acceleration to design rides that give the riders the sensation of free fall. These types of rides usually consist of three parts: the ride to the top, momentary suspension, and the fall downward. Motors provide the force needed to move the cars to the top of the ride. When the cars are in free fall, the most massive rider and the least massive rider will have the same acceleration.

Suppose the free-fall ride shown in **Figure 23** starts from the top at rest and is in free fall for 1.5 s. What would be its velocity at the end of 1.5 s? Choose a coordinate system with a positive axis upward and the origin at the initial position of the car. Because the car starts at rest, v_i would be equal to 0.0 m/s. To calculate the final velocity, use the equation for velocity with constant acceleration.

$$\begin{aligned} v_f &= v_i + \bar{a}t_f \\ &= 0.0\ \text{m/s} + (-9.8\ \text{m/s}^2)(1.5\ \text{s}) \\ &= -15\ \text{m/s} \end{aligned}$$

How far do people on the ride fall during this time? Use the equation for displacement when time and constant acceleration are known.

$$\begin{aligned} x_f &= x_i + v_i t_f + \left(\tfrac{1}{2}\right)\bar{a}t_f^2 \\ &= 0.0\ \text{m} + (0.0\ \text{m/s})(1.5\text{s}) + \left(\tfrac{1}{2}\right)(-9.8\,\text{m/s}^2)(1.5\ \text{s})^2 \\ &= -11\,\text{m} \end{aligned}$$

PRACTICE Problems

ADDITIONAL PRACTICE

40. A construction worker accidentally drops a brick from a high scaffold.

a. What is the velocity of the brick after 4.0 s?

b. How far does the brick fall during this time?

41. Suppose for the previous problem you choose your coordinate system so that the opposite direction is positive.

a. What is the brick's velocity after 4.0 s?

b. How far does the brick fall during this time?

42. A student drops a ball from a window 3.5 m above the sidewalk. How fast is it moving when it hits the sidewalk?

43. A tennis ball is thrown straight up with an initial speed of 22.5 m/s. It is caught at the same distance above the ground.

a. How high does the ball rise?

b. How long does the ball remain in the air?

Hint: The time it takes the ball to rise equals the time it takes to fall.

44. You decide to flip a coin to determine whether to do your physics or English homework first. The coin is flipped straight up.

a. What are the velocity and acceleration of the coin at the top of its trajectory?

b. If the coin reaches a high point of 0.25 m above where you released it, what was its initial speed?

c. If you catch it at the same height as you released it, how much time was it in the air?

45. CHALLENGE A basketball player is holding a ball in her hands at a height of 1.5 m above the ground. She drops the ball, and it bounces several times. After the first bounce, the ball only returns to a height of 0.75 m. After the second bounce, the ball only returns to a height of 0.25 m.

a. Suppose downward is the positive direction. What would the shape of a velocity-time graph look like for the first two bounces?

b. What would be the shape of a position-time graph for the first two bounces?

Oleksiy Maksymenko Photography/Alamy Stock Photo

Variations in Free Fall

When astronaut David Scott performed his free-fall experiment on the Moon, the hammer and the feather did not fall with an acceleration of magnitude 9.8 m/s^2. The value 9.8 m/s^2 is free-fall acceleration only near Earth's surface. The magnitude of free-fall acceleration on the Moon is approximately 1.6 m/s^2, which is about one-sixth its value on Earth.

When you study force and motion, you will learn about factors that affect the value of free-fall acceleration. One factor is the mass of the object, such as Earth or the Moon, that is responsible for the acceleration. Free-fall acceleration is not as great near the Moon as near Earth because the Moon has much less mass.

Free-fall acceleration also depends on the distance from the object responsible for it. The rings drawn around Earth in **Figure 24** show how free-fall acceleration decreases with distance from Earth. It is important to understand, however, that variations in free-fall acceleration at different locations on Earth's surface are very small, even with great variations in elevation. In New York City, for example, the magnitude of free-fall acceleration is about 9.81 m/s^2. In Denver, Colorado, it is about 9.79 m/s^2, despite a change in elevation of almost 1600 m greater. For calculations in this book, a value of 9.8 m/s^2 will be used for free-fall acceleration.

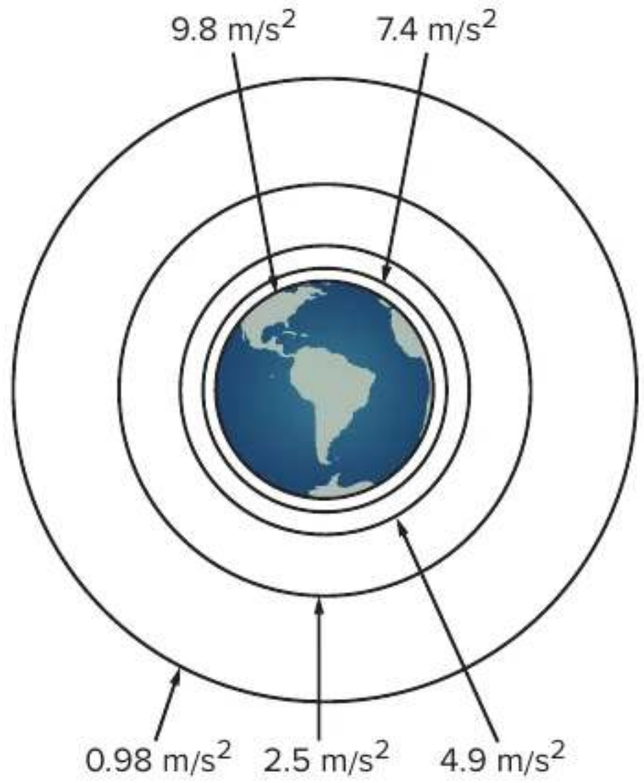


Figure 24 As the distance from Earth increases, the effect of free-fall acceleration decreases.

Analyze *According to the diagram, what is the magnitude of free-fall acceleration a distance above Earth's surface equal to Earth's radius?*

Check Your Progress

46. **Free Fall** Suppose you hold a book in one hand and a flat sheet of paper in your other hand. You drop them both, and they fall to the ground. Explain why the falling book is a good example of free fall, but the paper is not.
47. **Final Velocity** Your sister drops your house keys down to you from the second-floor window, as shown in **Figure 25.** What is the velocity of the keys when you catch them?
48. **Free-Fall Ride** Suppose a free-fall ride at an amusement park starts at rest and is in free fall. What is the velocity of the ride after 2.3 s? How far do people on the ride fall during the 2.3-s time period?

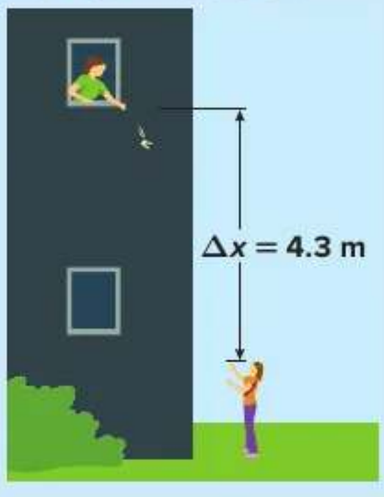


Figure 25

49. **Maximum Height and Flight Time** The free-fall acceleration on Mars is about one-third that on Earth. Suppose you throw a ball upward with the same velocity on Mars as on Earth.
    a. How would the ball's maximum height compare to that on Earth?
    b. How would its flight time compare?
50. **Velocity and Acceleration** Suppose you throw a ball straight up into the air. Describe the changes in the velocity of the ball. Describe the changes in the acceleration of the ball.
51. **Critical Thinking** A ball thrown vertically upward continues upward until it reaches a certain position, and then falls downward. The ball's velocity is instantaneously zero at that highest point. Is the ball accelerating at that point? Devise an experiment to prove or disprove your answer.

LEARNSMART® Go online to follow your personalized learning path to review, practice, and reinforce your understanding.

STEM AT WORK

Designing Fun

Think about the last amusement park attraction that made your pulse quicken. Perhaps it was a heart-pounding roller coaster or a chilling haunted house. Did you ever wonder how theme park rides are created? The people who design the attractions at theme parks have many different titles—theme park designer, creative design engineer, industrial designer, building information modeling manager, and many others. No matter their title, they all concentrate on creating exciting attractions that draw visitors to the parks.



Roller coasters are just one of many types of attractions created by theme park designers.

Tools of the Trade

Most theme park designers have a background in physics, engineering, and mathematics. Some are also graphic designers or writers. They use a variety of tools to create the attractions that delight park visitors.

CAD (computer-aided design) drafting software, for example, allows designers to create 2D and 3D blueprints for attractions. Increasingly, designers are also employing BIM (building information modeling) systems, used to create 3D models that can be viewed at different levels of complexity. There are other modeling software programs that allow designers to work in 3D as well. Theme park designers also use computer graphics software to create graphics and animations, and they use computer programs to alter images.

Forces and Safety

Theme park designers work with acceleration and gravitational forces to produce thrills for riders.

Roller coasters, for example, offer both changes in speed (such as slowly grinding up a hill and then barreling down it) and in direction (such as suddenly banking to the left or right). Attractions that drop riders from great heights and then slow their descent before they reach the ground make use of free-fall acceleration.

Designers must consider the attractions' effects on the human inner ear, heart, and other body systems. They want riders to be exhilarated, but they must also ensure that riders are safe. They must be mindful, for instance, that gravitational forces do not exceed the amount an average person can tolerate without suffering ill effects. Designers create elaborate systems of seatbelts and other restraints, headrests, and padding to protect riders from injury. They design improved braking systems to maximize safety.



DEVELOP AND USE MODELS TO ILLUSTRATE

With a partner, brainstorm, design, and build a model of a theme park ride that uses acceleration and gravitational forces. Use modeling software, if available.

David Kleyn/Alamy Stock Photo

MODULE 3
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 ACCELERATION

- Acceleration is the rate at which an object's velocity changes.
- Velocity and acceleration are not the same thing. An object moving with constant velocity has zero acceleration. When the velocity and the acceleration of an object are in the same direction, the object speeds up; when they are in opposite directions, the object slows down.
- You can use a velocity-time graph to find the velocity and the acceleration of an object. The average acceleration of an object is the slope of its velocity-time graph.

$$\bar{a} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- acceleration
- velocity-time graph
- average acceleration
- instantaneous acceleration

Lesson 2 MOTION WITH CONSTANT ACCELERATION

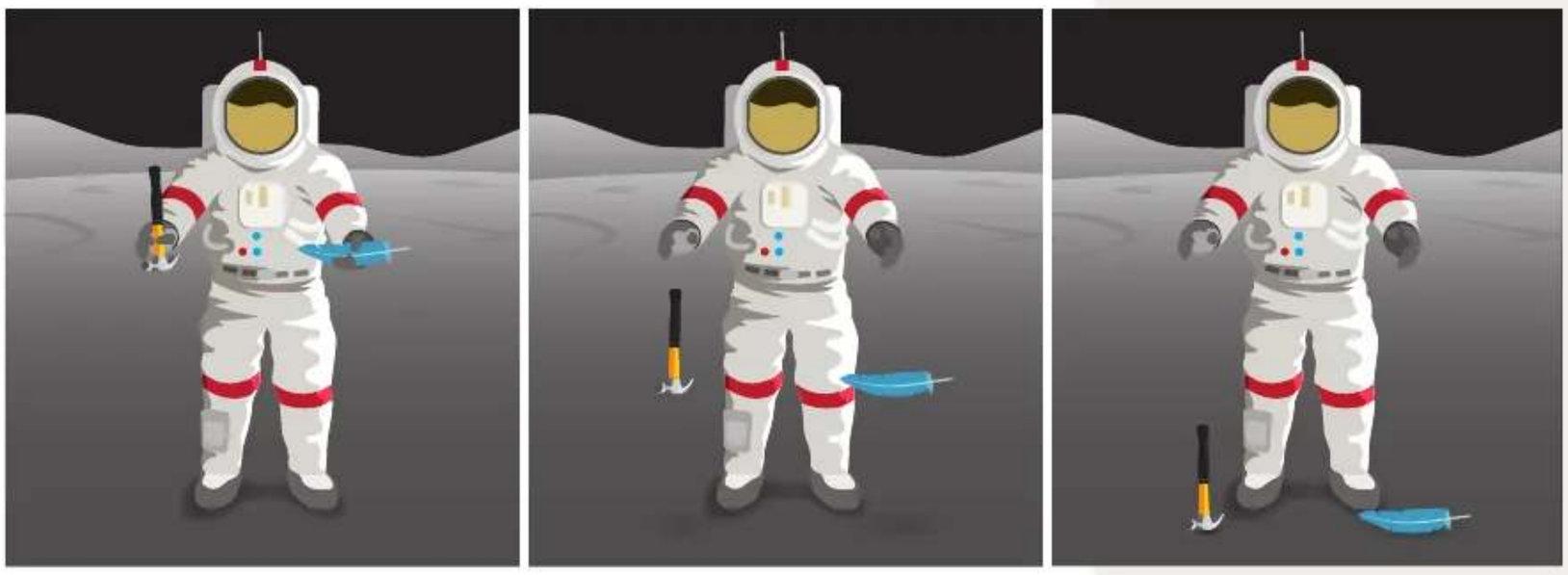
- If an object is moving with constant acceleration, its position-time graph is a parabola, and its velocity-time graph is a straight line.
- The area under an object's velocity-time graph is its displacement.
- In motion with constant acceleration, position, velocity, acceleration, and time are related:

$$v_f = v_i + \bar{a}\Delta t$$
$$x_f = x_i + v_i t_f + \frac{1}{2}\bar{a}t_f^2$$
$$v_f^2 = v_i^2 + 2\bar{a}(x_f - x_i)$$

Lesson 3 FREE FALL

- Free-fall acceleration on Earth is about 9.8 m/s² downward. The sign associated with free-fall acceleration in equations depends on the choice of the coordinate system.
- When an object is in free fall, gravity is the only force acting on it. Equations for motion with constant acceleration can be used to solve problems involving objects in free fall.

- free fall
- free-fall acceleration



THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

Why do sudden changes in the direction or speed of jet planes affect pilots?

CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

How do free fall motion on Earth and Jupiter compare?

Suppose a ball could be thrown vertically upward with the same initial velocity on Earth and on Jupiter.

Data and Observations An object on the planet Jupiter has about three times the free-fall acceleration as on Earth. Neglect the effects of atmospheric resistance and assume gravity is the only force on the ball.

CER Analyze and Interpret Data

1. **Claim** How would the maximum height reached by the ball on Jupiter compare to the maximum height reached on Earth?
2. **Evidence and Reasoning** Use sketches and mathematics to explain your approach to this problem and your reasoning to justify your claim.
3. How would your claim change if the initial velocity of the ball on Jupiter were three times greater? Explain your reasoning.

Steve Whittington/Moment/Getty Images



MODULE 4

FORCES IN ONE DIMENSION

Oliver Furrer/Cultura/Getty Images

MODULE 4

FORCES IN ONE DIMENSION

ENCOUNTER THE PHENOMENON

How do wing suits help BASE jumpers control their velocity?



GO ONLINE to play a video about da Vinci's parachute.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

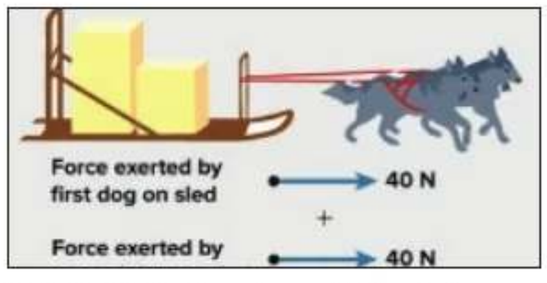
CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about how BASE jumpers control their velocity with wing suits. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain: Newton's Second Law



LESSON 2: Explore & Explain: Drag Force

Smithsonian

Additional Resources

(t)Video Supplied by BBC Worldwide Learning, (b)U.S. Army Photo by Sgt. Aaron Ricca, 115th Mobile Public Affairs Detachment

LESSON 1

FORCE AND MOTION

FOCUS QUESTION

What causes a change in motion?

Force

Consider a textbook resting on a table. To cause it to move, you could either push or pull on it. In physics, a push or a pull is called a **force.** If you push or pull harder on an object, you exert a greater force on the object. In other words, you increase the magnitude of the applied force. The direction in which the force is exerted also matters—if you push the resting book to the right, the book will start moving to the right. If you push the book to the left, it will start moving to the left. Because forces have both magnitude and direction, forces are vectors. The symbol $\boldsymbol{F}$ is vector notation that represents the size and direction of a force, while F represents only the magnitude. The magnitude of a force is measured in units called newtons (N).

Unbalanced forces change motion Recall that motion diagrams describe the positions of an object at equal time intervals. For example, the motion diagram for the book in **Figure 1** shows the distance between the dots increasing. This means the speed of the book is increasing. At $t = 0$, it is at rest, but after 2 seconds it is moving at 1.5 m/s. This change in speed means it is accelerating. What is the cause of this acceleration? The book was at rest until you pushed it, so the cause of the acceleration is the force exerted by your hand. In fact, all accelerations are the result of an unbalanced force acting on an object.

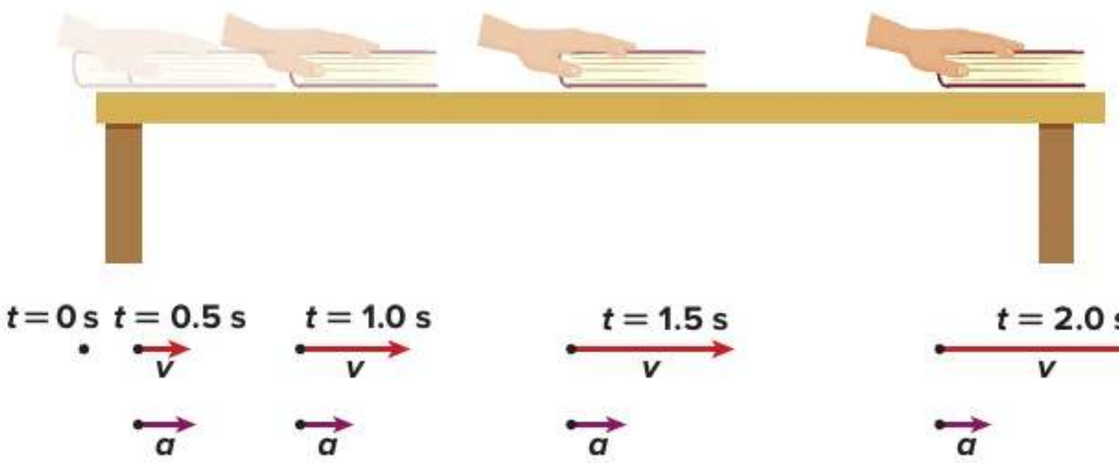


Figure 1 The hand pushing on the book exerts a force that causes the book to accelerate in the direction of the unbalanced force.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Applying Practices: Newton's Second Law

HS-PS2-1. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

Systems and external world When considering how a force affects motion, it is important to identify the object or objects of interest, called the **system.** Everything around the system with which the system can interact is called the external world. In **Figure 2**, the book is the system. Your hand, Earth, string and the table are parts of the external world that interact with the book by pushing or pulling on it.

Contact forces Again, think about the different ways in which you could move a textbook. You could push or pull it by directly touching it, or you could tie a string around it and pull on the string. These are examples of contact forces. A contact force exists when an object from the external world touches a system, exerting a force on it. If you are holding this physics textbook right now, your hands are exerting a contact force on it. If you place the book on a table, you are no longer exerting a contact force on the book. The table, however, is exerting a contact force because the table and the book are in contact.

Field forces There are other ways in which the motion of the textbook can change. You could drop it, and as you learned in a previous chapter, it would accelerate as it falls to the ground. The gravitational force of Earth acting on the book causes this acceleration. This force affects the book whether or not Earth is actually touching it. Gravitational force is an example of a field force. Field forces are exerted without contact. Can you think of other kinds of field forces? If you have ever investigated magnets, you know that they exert forces without touching. You will investigate magnetism and other field forces in future chapters. For now, the only field force you need to consider is the gravitational force.

Agents Forces result from interactions; every contact and field force has a specific and identifiable cause, called the agent. You should be able to name the agent exerting each force as well as the system upon which the force is exerted. For example, when you push your textbook, your hand (the agent) exerts a force on the textbook (the system). If there are not both an agent and a system, a force does not exist. What about the gravitational force? The agent is the mass of Earth exerting a field force on the book. The labels on the forces in **Figure 2** are good examples of how to identify a force's agent and the system upon which the force acts.

Get It?

Explain how contact forces are different from field forces.

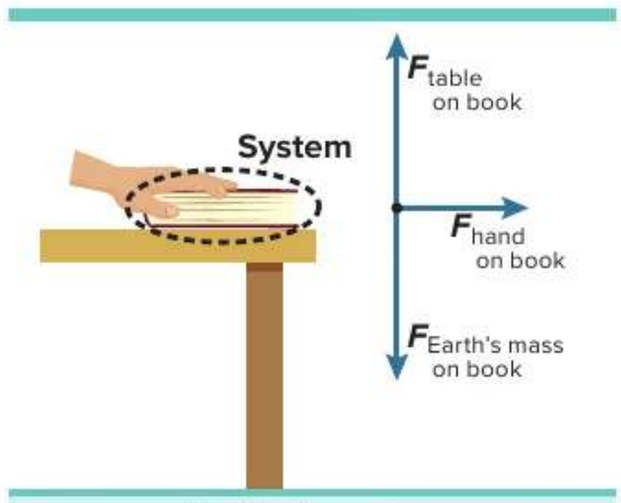


Figure 2 The book is the system in each of these situations.

Classify *each force in the first panel as either a contact force or a field force.*

SCIENCE USAGE V. COMMON USAGE

Force

Science usage: a push or pull exerted on an object
The force of gravity exerted by the Sun on Earth pulls Earth into orbit around the Sun.

Common usage: to compel by physical, moral, or intellectual means
Emi forced her younger brother to wash the dishes.

STEM CAREER Connection

Astronaut

On Earth, we are used to accounting for the force exerted by Earth's gravitational field. To prepare for missions in space, astronauts must become familiar with the effects of forces on objects in a zero-gravity environment.

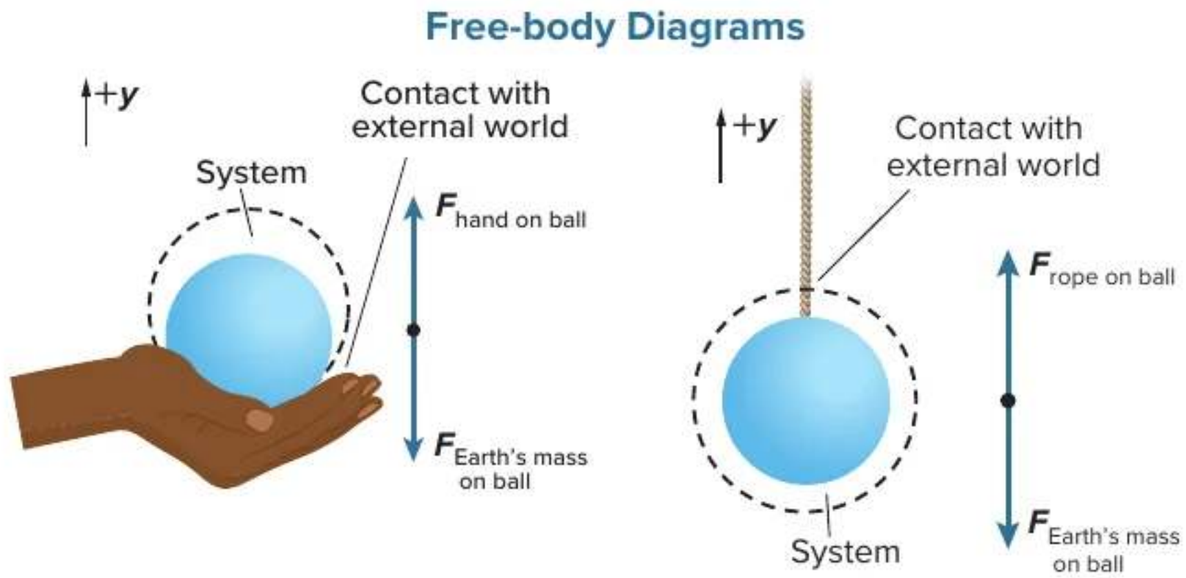


Figure 3 The drawings of the ball in the hand and the ball hanging from the string are both pictorial models. The free-body diagram for each situation is shown next to each pictorial model.

COLOR CONVENTION

Force (*F*) ⟷ blue

Free-body diagrams Just as pictorial representations and motion diagrams are useful in solving problems about motion, similar representations will help you analyze how forces affect motion. The first step is to sketch the situation, as shown in **Figure 3**. Circle the system, and identify every place where the system touches the external world. It is at these places that an agent exerts a contact force. Then identify any field forces on the system. This gives you the pictorial representation.

A **free-body diagram** is a physical representation that shows the forces acting on a system. Follow these guidelines when drawing a free-body diagram:

- The free-body diagram is drawn separately from the sketch of the problem situation.
- Apply the particle model, and represent the object with a dot.
- Represent each force with an arrow that points in the direction the force is applied. Always draw the force vectors pointing away from the particle, even when the force is a push.
- Make the length of each arrow proportional to the size of the force. Often you will draw these diagrams before you know the magnitudes of all the forces. In such cases, make your best estimate.
- Label each force. Use the symbol F with a subscript label to identify both the agent and the object on which the force is exerted.
- Choose a direction to be positive, and indicate this on the diagram.

Using free-body diagrams and motion diagrams Recall that all accelerations are the result of unbalanced forces. If a motion diagram shows that an object is accelerating, a free-body diagram of that object should have an unbalanced force in the same direction as the acceleration. Notice that the falling ball in **Figure 3** shows an unbalanced force downward, in the direction of the acceleration of the ball.

Get It?

Compare the direction of an object's acceleration with the direction of the unbalanced force exerted on the object.

CCC CROSSCUTTING CONCEPTS

Cause and Effect Study **Figure 3**. With a partner, plan a demonstration to show the specific cause and effect relationship between the motion of an object and the magnitudes of two forces acting on it in opposite directions. Vary the forces to show the different effects when the forces are unbalanced. Gather empirical evidence to show that the acceleration (or lack of acceleration) is caused directly by the imbalance (or balance) of the forces and not simply correlation.

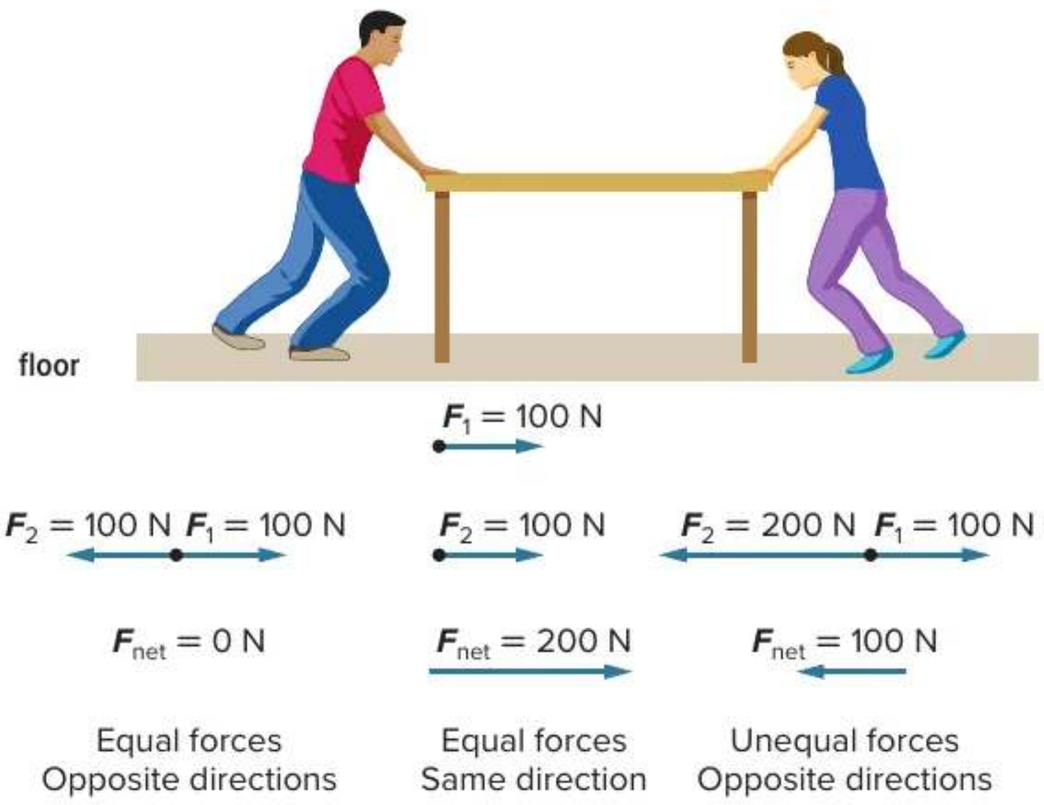


Figure 4 The net force acting on the table is the vector sum of all the forces acting on the table. This case only considers the horizontal forces acting on the table.

Combining Forces

What happens if you and a friend each push a table and exert 100 N of force on it? When you push together in the same direction, you give the table twice the acceleration that it would have if just one of you applied 100 N of force. When you push on the table in opposite directions with the same amount of force, as in **Figure 4,** there is no unbalanced force, so the table does not accelerate but remains at rest.

Net force The bottom portion of **Figure 4** shows free-body diagrams for these two situations. The third diagram in **Figure 4** shows the free-body diagram for a third situation in which your friend pushes on the table twice as hard as you in the opposite direction. Below each free-body diagram is a vector representing the resultant of the two forces. When the force vectors are in the same direction, they can be replaced by one vector with a length equal to their combined length. When the forces are in opposite directions, the resultant is the length of the difference between the two vectors. Another term for the vector sum of all the forces on an object is the **net force.**

You also can analyze the situation mathematically. Call the positive direction the direction in which you are pushing the table with a 100 N force. In the first case, your friend is pushing with a negative force of 100 N. Adding them together gives a total force of 0 N, which means there is no acceleration. In the second case, your friend's force is 100 N, so the total force is 200 N in the positive direction and the table accelerates in the positive direction. In the third case, your friend's force is −200 N, so the total force is −100 N and the table accelerates in the negative direction.

PRACTICE Problems

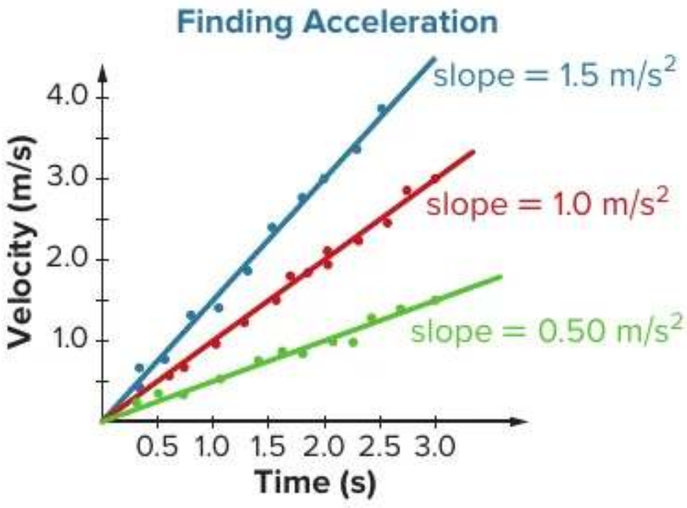
ADDITIONAL PRACTICE

For each of the following situations, specify the system and draw a motion diagram and a free-body diagram. Label all forces with their agents, and indicate the direction of the acceleration and of the net force. Draw vectors of appropriate lengths. Ignore air resistance unless otherwise indicated.

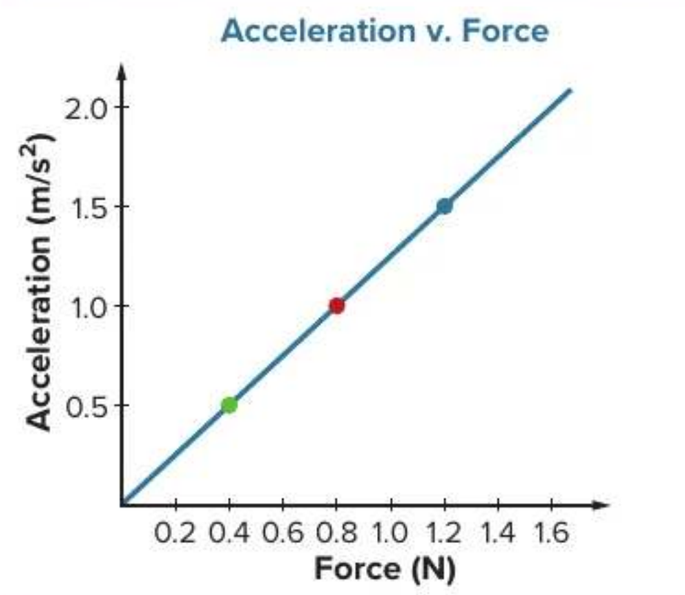
1. A skydiver falls downward through the air at constant velocity. (The air exerts an upward force on the person.)
2. You hold a softball in the palm of your hand and toss it up. Draw the diagrams while the ball is still touching your hand.
3. After the softball leaves your hand, it rises, slowing down.
4. After the softball reaches its maximum height, it falls down, speeding up.
5. **CHALLENGE** You catch the ball in your hand and bring it to rest.



Applying a Constant Force



Velocity-Time Graphs for Constant Forces



Various Forces on the Same Mass

Figure 5 A spring scale exerts a constant unbalanced force on the cart. Repeating the investigation with different forces produces velocity-time graphs with different slopes.

Richard Hutchings/Digital Light Source

Acceleration and Force

To explore how forces affect an object's motion, think about doing a series of investigations. Consider the simple situation shown in the top photo of **Figure 5** in which we exert one force horizontally on an object. Starting with the horizontal direction is helpful because gravity does not act horizontally. To reduce complications resulting from the object rubbing against the surface, the investigations should be done on a smooth surface, such as a well-polished table. We'll also use a cart with wheels that spin easily.

Apply constant force How can you exert a constant unbalanced force? One way is to use a device called a spring scale. Inside the scale is a spring that stretches proportionally to the magnitude of the applied force. The front of the scale is calibrated to read the force in newtons. If you pull on the scale so that the reading on the front stays constant, the applied force is constant. The top photo in **Figure 5** shows a spring scale pulling a low-resistance cart with a constant unbalanced force.

If you perform this investigation and measure the cart's velocity for a period of time, you could construct a graph like the green line shown in the velocity-time graphs for constant forces in the middle panel of **Figure 5**. The constant slope of the line in the velocity-time graph indicates the cart's velocity increases at a constant rate. The constant rate of change of velocity means the acceleration is constant. This constant acceleration is a result of the constant unbalanced force applied by the spring scale to the cart.

How does acceleration depend on force? Repeat the investigation with a larger constant force. Repeat it again with an even greater force. For each force, plot a velocity-time graph like the red and blue lines in the middle panel of **Figure 5.** Recall that the line's slope is the cart's acceleration. Calculate the slope of each line and plot the results for each force to make an acceleration-force graph, as shown in the bottom panel of **Figure 5**.

The graph indicates the relationship between force and acceleration is linear. Because the relationship is linear, you can apply the equation for a straight line:

$$y = kx + b$$

The y-intercept is 0, so the linear equation simplifies to $y = kx$. The y-variable is acceleration. The x-variable is force. Therefore, acceleration is equal to the slope of the line multiplied by the applied net force.

Get It?

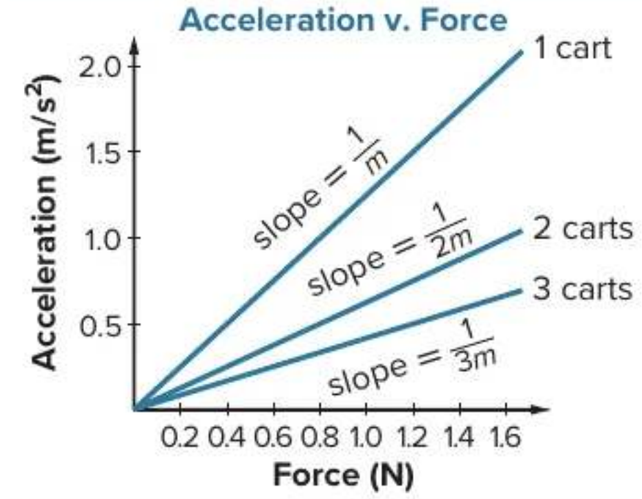
Describe the relationship between applied net force and acceleration.

Interpreting slope What is the physical meaning of the slope of the acceleration-force graph? Does it describe something about the object that is accelerating? To see, change the object. Suppose that a second, identical cart is placed on top of the first, and then a third cart is added as in **Figure 6.** The spring scale would be pulling two carts and then three. A plot of the force versus acceleration for one, two, and three carts is shown in the graph in **Figure 6.**

The graph shows that if the same force is applied in each case, the acceleration of two carts is $\frac{1}{2}$ the acceleration of one cart, and the acceleration of three carts is $\frac{1}{3}$ the acceleration of one cart. This means that as the number of carts increases, the acceleration decreases. In other words, a greater force is needed to produce the same acceleration. The slopes of the lines in **Figure 6** depend upon the number of carts; that is, the slope depends on the total mass of the carts. In fact, the slope is the reciprocal of the mass (slope $= \frac{1}{\text{mass}}$). Using this value for slope, the mathematical equation $y = kx$ becomes the physics equation: $a = \frac{F_{net}}{m}$. What information is contained in the equation $a = \frac{F_{net}}{m}$? It tells you that a net force applied to an object causes that object to experience a change in motion—the force causes the object to accelerate. It also tells you that for the same object, if you double the force, you will double the object's acceleration. Lastly, if you apply the same force to objects with different masses, the one with the most mass will have the smallest acceleration and the one with the least mass will have the greatest acceleration.



Changing the Mass of the System



Same Force on Different Masses

Figure 6 Changing an object's mass affects that object's acceleration.

Compare *the acceleration of one cart to the acceleration of two carts for an applied force of 1 N.*

Get It?

Determine how the force exerted on an object must be changed to reduce the object's acceleration by half.

Recall that forces are measured in units called newtons. Because $F_{net} = ma$, a newton has the units of mass times the units of acceleration. So one newton is equal to one kg·m/s². To get an approximate idea of the size of 1 N, think about the downward force you feel when you hold an apple in your hand. The force exerted by the apple on your hand is approximately one newton. **Table 1** shows the magnitudes of some other common forces.

Table 1 Common Forces

Description	*F*(N)
Force of gravity on a coin (nickel)	0.05
Force of gravity on a 0.45-kg bag of sugar	4.5
Force of gravity on a 70-kg person	686
Force exerted by road on an accelerating car	3000
Force of a rocket engine	5,000,000

Richard Hutchings/Digital Light Source



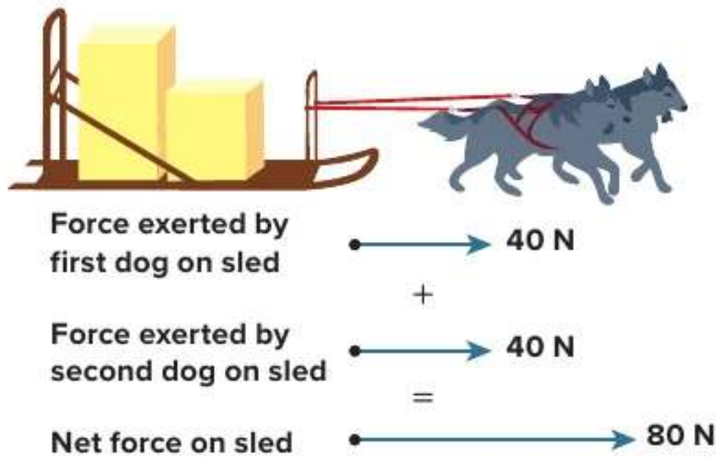


Figure 7 The net force acting on an object is the vector sum of all the forces acting on that object.

Newton's Second Law

Figure 7 shows two dogs pulling a sled. Each dog pulls with a force of 40 N. From the cart and spring-scale investigations, you know that the sled accelerates as a result of the unbalanced force acting it. Would the acceleration change if instead of two dogs each exerting a 40-N force, there was one bigger, stronger dog exerting a single 80-N force on the sled? When considering forces and acceleration, it is important to find the sum of all forces, called the net force, acting on a system.

Newton's second law states that the acceleration of an object is proportional to the net force and inversely proportional to the mass of the object being accelerated. This law is based on observations of how forces affect masses and is represented by the following equation. Newton's second law accurately predicts changes in the motion of macroscopic objects.

Newton's Second Law
The acceleration of an object is equal to the sum of the forces acting on the object divided by the mass of the object.

$$a = \frac{F_{net}}{m}$$

Solving problems using Newton's second law One of the most important steps in correctly applying Newton's second law is determining the net force acting on the object. Often, more than one force acts on an object, so you must add the force vectors to determine the net force. Draw a free-body diagram showing the direction and relative strength of each force acting on the system. Then, add the force vectors to find the net force. Next, use Newton's second law to calculate the acceleration. Finally, if necessary, you can use what you know about accelerated motion to find the velocity or position of the object.

PRACTICE Problems — ADDITIONAL PRACTICE

6. Two horizontal forces, 225 N and 165 N, are exerted on a canoe. If these forces are applied in the same direction, find the net horizontal force on the dance.
7. If the same two forces as in the previous problem are exerted on the canoe in opposite directions, what is the net horizontal force on the canoe? Be sure to indicate the direction of the net force.
8. **CHALLENGE** Three confused sled dogs are trying to pull a sled across the Alaskan snow. Alutia pulls east with a force of 35 N, Seward also pulls east but with a force of 42 N, and big Kodiak pulls west with a force of 53 N. What is the net force on the sled? Explain how Newton's second law accurately predicts the sled's change in motion.

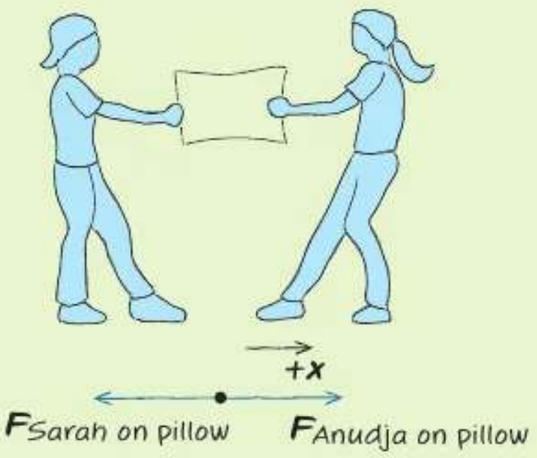
Zero Creatives/Cultura/Getty Images

EXAMPLE Problem 1

FIGHTING OVER A PILLOW Anudja is holding a pillow with a mass of 0.30 kg when Sarah decides that she wants it and tries to pull it away from Anudja. If Sarah pulls horizontally on the pillow with a force of 10.0 N and Anudja pulls with a horizontal force of 11.0 N, what is the horizontal acceleration of the pillow?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Identify the pillow as the system, and the direction in which Anudja pulls as positive.
- Draw the free-body diagram. Label the forces.



KNOWN **UNKNOWN**

$m = 0.30\ \text{kg}$ $a = ?$

$F_{\text{Anudja on pillow}} = 11.0\ \text{N}$

$F_{\text{Sarah on pillow}} = 10.0\ \text{N}$

2 SOLVE FOR THE UNKNOWN

$$F_{\text{net}} = F_{\text{Anudja on pillow}} + (-F_{\text{Sarah on pillow}})$$

Use Newton's second law.

$$a = \frac{F_{\text{net}}}{m}$$

$$= \frac{F_{\text{Anudja on pillow}} + (-F_{\text{Sarah on pillow}})}{m}$$

$$= \frac{11.0\ \text{N} - 10.0\ \text{N}}{0.30\ \text{kg}}$$ Substitute $F_{\text{Anudja on pillow}} = 11.0\ \text{N}$, $F_{\text{Sarah on pillow}} = 10.0\ \text{N}$, $m = 0.30\ \text{kg}$.

$$= 3.3\ \text{m/s}^2$$

$\boldsymbol{a} = 3.3\ \text{m/s}^2$ toward Anudja

3 EVALUATE THE ANSWER

- **Are the units correct?** m/s^2 is the correct unit for acceleration.
- **Does the sign make sense?** The acceleration is toward Anudja because Anudja is pulling toward herself with a greater force than Sarah is pulling in the opposite direction.
- **Is the magnitude realistic?** The net force is 1 N and the mass is 0.3 kg, so the acceleration is realistic.

PRACTICE Problems

ADDITIONAL PRACTICE

9. A spring scale is used to exert a net force of 2.7 N on a cart. If the cart's mass is 0.64 kg, what is the cart's acceleration?

10. Kamaria is learning how to ice skate. She wants her mother to pull her along so that she has an acceleration of 0.80 m/s^2. If Kamaria's mass is 27.2 kg, with what force does her mother need to pull her? (Neglect any resistance between the ice and Kamaria's skates.)

11. CHALLENGE Two horizontal forces are exerted on a large crate. The first force is 317 N to the right. The second force is 173 N to the left.

a. Draw a force diagram for the horizontal forces acting on the crate.

b. What is the net force acting on the crate?

c. The box is initially at rest. Five seconds later, its velocity is 6.5 m/s to the right. What is the crate's mass?

Newton's First Law

What is the motion of an object when the net force acting on it is zero? Newton's second law says that if $F_{net} = 0$, then acceleration equals zero. Recall that if acceleration equals zero, then velocity does not change. Thus a stationary object with no net force acting on it will remain at rest. What about a moving object, such as a ball rolling on a surface? How long will the ball continue to roll? The answer depends on the surface on which the ball is rolling. If the ball is rolled on a thick carpet that exerts a force on the ball, it will come to rest quickly. If it is rolled on a hard, smooth surface that exerts very little force, such as a bowling alley, the ball will roll for a long time with little change in velocity.

Galileo did many experiments and he concluded that if he could remove all forces opposing motion, horizontal motion would never stop. Galileo was the first to recognize that the general principles of motion could be found by extrapolating experimental results to an ideal case.

In the absence of a net force, the velocity of the moving ball and the lack of motion of the stationary object do not change. Newton recognized this and generalized Galileo's results into a single statement. Newton's statement, "an object that is at rest will remain at rest, and an object that is moving will continue to move in a straight line with constant speed, if and only if the net force acting on that object is zero," is called **Newton's first law.**

Inertia

Newton's first law is sometimes called the law of inertia because **inertia** is the tendency of an object to resist changes in velocity. The car and the red block in **Figure 8** demonstrate the law of inertia. In the left panel, both objects are moving to the right. In the right panel, the wooden box applies a force to the car, causing it to stop. The red block does not experience the force applied by the wooden box. It continues to move to the right with the same velocity as in the left panel.

Is inertia a force? No. Forces are results of interactions between two objects; they are not properties of single objects, so inertia cannot be a force. Remember that because velocity includes both the speed and direction of motion, a net force is required to change either the speed or the direction of motion. If the net force is zero, Newton's first law means the object will continue with the same speed and direction.



Figure 8 The car and the block approach the wooden box at the same speed. After the collision, the block continues on with the same horizontal speed.

Identify *the forces that will eventually cause the block to stop moving.*

Matt Meadows





Figure 9 An object is in equilibrium if its velocity isn't changing. In both cases pictured here, velocity isn't changing, so the net force must be zero.

Equilibrium According to Newton's first law, a net force causes the velocity of an object to change. If the net force on an object is zero, then the object is in **equilibrium.** An object is in equilibrium if it is moving at a constant velocity. Note that being at rest is simply of the state of constant velocity, $v = 0$. Newton's first law identifies a net force as something that disturbs a state of equilibrium. Thus, if there is no net force acting on the object, then the object does not experience a change in speed or direction and is in equilibrium. **Figure 9** indicates, at least in terms of net forces, there is no difference between lying on a sofa and falling at a constant velocity while skydiving—velocity isn't changing, so the net force is zero.

Keep in mind that the real world is full of forces that resist motion. Newton's ideal, friction-free world is not easy to obtain. If you analyze a situation and find that the result is different from your own experience, ask yourself if this s is because of the presence of frictional forces.

Check Your Progress

12. **Forces** Identify each of the following as either a, b, or c: mass, inertia, the push of a hand, friction, air resistance, spring force, gravity, and acceleration.
    a. contact force
    b. a field force
    c. not a force
13. **Free-Body Diagram** Draw a free-body diagram of a bag of sugar being lifted by your hand at an increasing speed. Specifically identify the system. Use subscripts to label all forces with their agents. Remember to make the arrows the correct lengths.
14. **Free-Body Diagram** Draw a free-body diagram of a water bucket being lifted by a rope at a decreasing speed. Specifically identify the system. Label all forces with their agents and make the arrows the correct lengths.
15. **Critical Thinking** A force of 1 N is the only horizontal force exerted on a block, and the horizontal acceleration of the block is measured. When the same horizontal force is the only force exerted on a second block, the horizontal acceleration is three times as large. What can you conclude about the masses of the two blocks?

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LESSON 2

WEIGHT AND DRAG FORCE

FOCUS QUESTION

How does the drag force change after a skydiver deploys their parachute?

Weight

From Newton's second law, the fact that the ball in **Figure 10** is accelerating means there must be unbalanced forces acting on the ball. The only force acting on the ball is the gravitational force due to Earth's mass. An object's **weight** is the gravitational force experienced by that object. This gravitational force is a field force whose magnitude is directly proportional to the mass of the object experiencing the force. In equation form, the gravitational force, which equals weight, can be written $F_g = mg$. The mass of the object is m, and g, called the **gravitational field,** is a vector quantity that relates an object's mass to the gravitational force it experiences at a given location.

Near Earth's surface, g is 9.8 N/kg toward Earth's center. Objects near Earth's surface experience 9.8 N of force for every kilogram of mass.

Scales When you stand on a scale as shown in the right panel of **Figure 10**, the scale exerts an upward force on you. Because you are not accelerating, the net force acting on you must be zero. Therefore the magnitude of the force exerted by the scale ($F_{\text{scale on you}}$) pushing up must equal the magnitude of F_g pulling down on you. Inside the scale, springs provide the upward force necessary to make the net force equal zero. The scale is calibrated to convert the stretch of the springs to a weight. If you were on a different planet with a different g, the scale would exert a different force to keep you in equilibrium, and consequently, the scale's reading would be different. Because weight is a force, the proper unit used to measure weight is the newton.

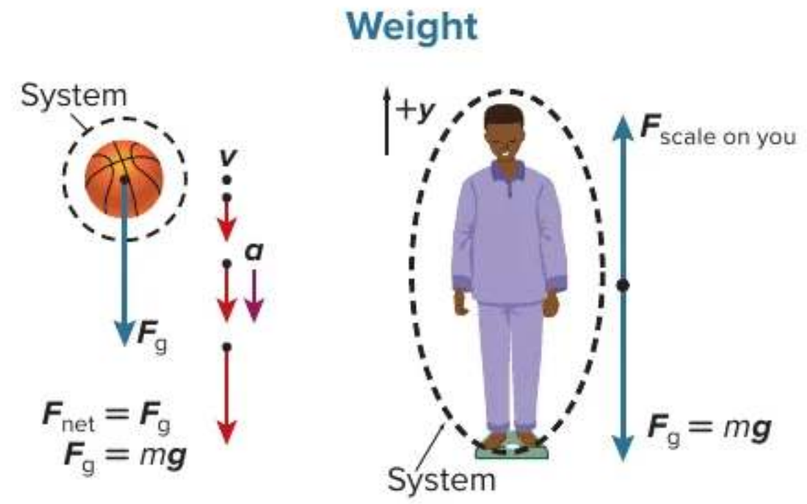


Figure 10 The gravitational force exerted by Earth's mass on an object equals the object's mass times the gravitational field, $F_g = mg$.

Identify *the forces acting on you when you are in equilibrium while standing on a scale.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Probeware Lab: Terminal Velocity

Analyze and interpret data to determine which factors affect the size of the drag force on a falling object.

Revisit the Encounter the Phenomenon Question

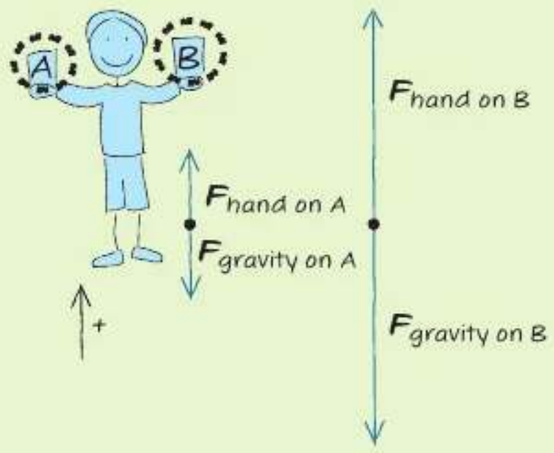
What information from this lesson can help you answer the Unit and Module questions?

EXAMPLE Problem 2

COMPARING WEIGHTS Kiran holds a brass cylinder in each hand. Cylinder A has a mass of 100.0 g and cylinder B has a mass of 300.0 g. What upward forces do his two hands exert to keep the cylinders at rest? If he then drops the two, with what acceleration do they fall? (Ignore air resistance.)

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Identify the two cylinders as the systems, and choose the upward direction as positive.
- Draw the free-body diagrams. Label the forces.



KNOWN

$m_A = 0.1000\ kg$

$m_B = 0.3000\ kg$

$g = -9.8\ N/kg$

UNKNOWNS

$F_{Hand\ on\ A} = ?$

$F_{Hand\ on\ B} = ?$

$a_A = ?$ $a_B = ?$

2 SOLVE FOR THE UNKNOWNS

For cylinder A:

$$F_{Net\ on\ A} = F_{Hand\ on\ A} + F_{Gravity\ on\ A}$$

$$0 = F_{Hand\ on\ A} + F_{Gravity\ on\ A}$$

$$F_{Hand\ on\ A} = -F_{Gravity\ on\ A}$$

$$F_{Hand\ on\ A} = -m_A g$$

$$= -(0.1000\ kg)(-9.8\ N/kg)$$

$$= 0.98\ N\ up$$

For cylinder B:

$$F_{Net\ on\ B} = F_{Hand\ on\ B} + F_{Gravity\ on\ B}$$

$$= F_{Hand\ on\ B} + F_{Gravity\ on\ B}$$

$$F_{Hand\ on\ B} = -F_{Gravity\ on\ B}$$

$$F_{Hand\ on\ B} = m_B g$$

$$= -(0.3000\ kg)(-9.8\ N/Kg)$$

$$= 2.9\ N\ up$$

After the cylinders are dropped, the only force on each is the force of gravity. Use Newton's second law.

$$a_A = \frac{F_{Net\ on\ A}}{m_A} \qquad a_B = \frac{F_{Net\ on\ B}}{m_B}$$

$$a_A = \frac{m_A g}{m_A} = g \qquad a_B = \frac{m_B g}{m_B} = g$$

Substitute $F_{Net\ on\ A} = m_A g$ and $F_{Net\ on\ B} = m_B g$

$$= -9.8\ m/s^2 \qquad = -9.8\ m/s^2$$

Substitute $g = -9.8\ N/kg = -9.8\ m/s^2$.

3 EVALUATE THE ANSWER

- **Are the units correct?** N is the correct unit for force; m/s^2 is the correct unit for acceleration.
- **Does the sign make sense?** The direction of the fall is downward, the negative direction, and the object is speeding up, so the acceleration should be negative.
- **Is the magnitude realistic?** Forces are 1–5 N, typical of that exerted by objects that have a mass of one kg or less. The accelerations are both equal to free fall acceleration.

PRACTICE Problems

ADDITIONAL PRACTICE

16. You place a 4.0-kg watermelon on a spring scale that measures in newtons. What is the scale's reading?

17. You place a 22.50-kg television on a spring scale. If the scale reads 235.2 N, what is the gravitational field?

18. A 0.50-kg guinea pig is lifted up from the ground. What is the smallest force needed to lift it? Describe the particular motion resulting from this minimum force.

19. CHALLENGE A grocery sack can withstand a maximum of 230 N before it rips. Will a bag holding 15 kg of groceries that is lifted from the checkout counter at an acceleration of 7.0 m/s^2 hold?

Apparent weight

What is weight? Because the weight force is defined as $F_g = mg$, F_g changes when g varies. On or near the surface of Earth, g is approximately constant, so an object's weight does not change appreciably as it moves around near Earth's surface. If a bathroom scale provides the only upward force on you, then it reads your weight. What would it read if you stood with one foot on the scale and one foot on the floor? What if a friend pushed down on your shoulders or lifted up on your elbows? Then there would be other contact forces on you, and the scale would not read your weight.

What happens if you stand on a scale in an elevator? As long as you are not accelerating, the scale will read your weight. What would the scale read if the elevator accelerated upward? **Figure 11** shows the pictorial and physical representations for this situation. You are the system, and upward is the positive direction. Because the acceleration of the system is upward, the net force must be upward. The upward force of the scale must be greater than the downward force of your weight. Therefore, the scale reading is greater than your weight.

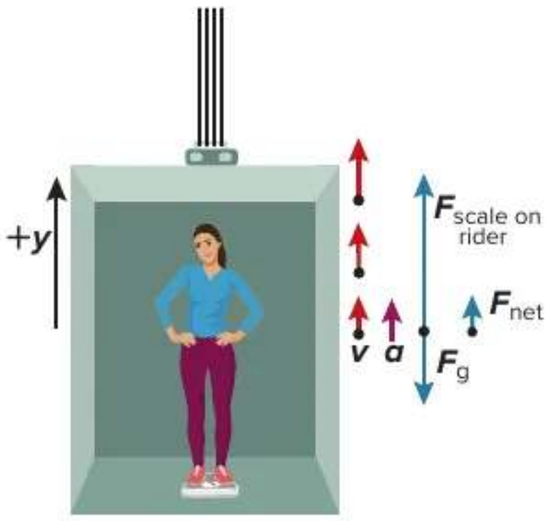


Figure 11 If you are accelerating upward, the net force acting on you must be upward. The scale must exert an upward force greater than the downward force of your weight.

Get It?

Describe the reading on the scale as the elevator accelerates upward from rest, reaches a constant speed, and then comes to a stop.

If you ride in an elevator accelerating upward, you feel as if you are heavier because the floor presses harder on your feet. On the other hand, if the acceleration is downward, then you feel lighter, and the scale reads less than your weight. The force exerted by the scale is an example of **apparent weight,** which is the support force exerted on an object.

Imagine that the cable holding the elevator breaks. What would the scale read then? The scale and you would both accelerate at $a = g$. According to this formula, the scale would read zero and your apparent weight would be zero. That is, you would be weightless. However, **weightlessness** does not mean that an object's weight is actually zero; rather, it means that there are no contact forces acting to support the object, and the object's *apparent weight* is zero. Similar to the falling elevator, astronauts experience weightlessness in orbit because they and their spacecraft are in free fall.

PROBLEM-SOLVING STRATEGY

Force and Motion

When solving force and motion problems, use the following strategies.

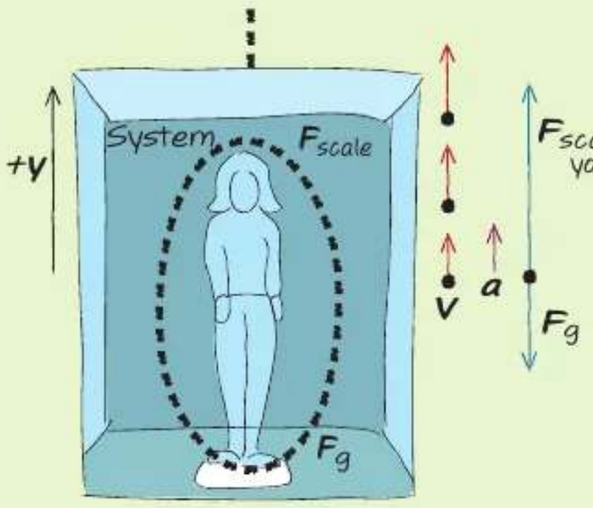
1. Read the problem carefully, and sketch a pictorial model.
2. Circle the system and choose a coordinate system.
3. Determine which quantities are known and which are unknown.
4. Create a physical model by drawing a motion diagram showing the direction of the acceleration.
5. Create a free-body diagram showing all the forces acting on the object.
6. Use Newton's laws to link acceleration and net force.
7. Rearrange the equation to solve for the unknown quantity.
8. Substitute known quantities with their units into the equation and solve.
9. Check your results to see whether they are reasonable.

EXAMPLE Problem 3

REAL AND APPARENT WEIGHT Your mass is 75.0 kg, and you are standing on a bathroom scale in an elevator. Starting from rest, the elevator accelerates upward at 2.00 m/s² for 2.00 s and then continues at a constant speed. Is the scale reading during acceleration greater than, equal to, or less than the scale reading when the elevator is at rest?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Choose a coordinate system with the positive direction as upward.
- Draw the motion diagram. Label *v* and *a*.
- Draw the free-body diagram. The net force is in the same direction as the acceleration, so the upward force is greater than the downward force.



KNOWN	UNKNOWN
$m = 75.0$ kg	$F_{scale} = ?$
$a = 2.00$ m/s²	
$t = 2.00$ s	
$g = 9.8$ N/kg	

2 SOLVE FOR THE UNKNOWN

$F_{net} = ma$

$F_{net} = F_{scale} + (-F_g)$ F_g is negative because it is in the negative direction defined by the coordinate system.

Solve for F_{scale}.

$F_{scale} = F_{net} + F_g$

Elevator at rest:

$F_{scale} = F_{net} + F_g$ The elevator is not accelerating. Thus, $F_{net} = 0.00$ N.

$= F_g$ Substitute $F_{net} = 0.00$ N.

$= mg$ Substitute $F_g = mg$.

$= (75.0\text{ kg})(9.8\text{ N/kg})$ Substitute $m = 75.0$ kg, $g = 9.8$ N/kg.

$= 735$ N

Elevator accelerating upward:

$F_{scale} = F_{net} + F_g$

$= ma + mg$ Substitute $F_{net} = ma$, $F_g = mg$

$= (75.0\text{ kg})(2.00\text{ m/s}^2) + (75.0\text{ kg})(9.8\text{ N/kg})$ Substitute $m = 75.0$ kg, $a = 2.00$ m/s², $g = 9.8$ N/kg

$= 885$ N

The scale reading when the elevator is accelerating (885 N) is larger than when it is at rest (735 N).

3 EVALUATE THE ANSWER

- **Are the units correct?** kg•m/s² is the force unit, N.
- **Does the sign make sense?** The positive sign agrees with the coordinate system.
- **Is the magnitude realistic?** $F_{Scale} = 885$ N is larger than it would be at rest when F_{scale} would be 735 N. The increase is 150 N, which is about 20 percent of the rest weight. The upward acceleration is about 20 percent of that due to gravity, so the magnitude is reasonable.

PRACTICE Problems — ADDITIONAL PRACTICE

20. On Earth, a scale shows that you weigh 585 N.

a. What is your mass?

b. What would the scale read on the Moon ($g = 1.60$ N/kg)?

21. CHALLENGE Use the results from Example Problem 3 to answer questions about a scale in an elevator on Earth. What force would be exerted by the scale on a person in the following situations?

a. The elevator moves upward at constant speed.

b. It slows at 2.0 m/s^2 while moving downward.

c. It speeds up at 2.0 m/s^2 while moving downward.

d. It moves downward at constant speed.

e. In what direction is the net force as the elevator slows to a stop as it is moving down?

Drag Force

The particles in the air around an object exert forces on that object. In fact, air exerts huge forces, but in most cases, it exerts balanced forces on all sides, and therefore has no net effect. So far, you have neglected the force of air on an object moving through the air. In actuality, when an object moves through any fluid, such as air or water, the fluid exerts a force on the moving object in the direction opposite the object's motion. A **drag force** is the force exerted by a fluid on an object opposing motion through the fluid. This force is dependent on the motion of the object, the properties of the object, and the properties of the fluid that the object is moving through. For example, as the speed of the object increases, so does the magnitude of the drag force. The size and shape of the object also affect the drag force. The fluid's properties, such as its density and viscosity, also affect the drag force.

Get It?

Describe how the wingsuits shown in the photo at the beginning of the module affect the drag force experienced by the skydivers.

PHYSICS Challenge

A 415-kg container of food and water is dropped from an airplane at an altitude of 300 m. First, consider the situation ignoring air resistance. Then calculate the more realistic situation involving a drag force provided by a parachute.

1. If you ignore air resistance, how long will it take the container to fall 300 m to the ground?

2. Again, ignoring air resistance, what is the speed of the container just before it hits the ground?

3. The container is attached to a parachute designed to produce a drag force that allows the container to reach a constant downward velocity of 6 m/s. What is the magnitude of the drag force when the container is falling at a constant 6 m/s down?



U.S. Army Photo by Sgt. Aaron Ricca, 115th Mobile Public Affairs Detachment

Terminal velocity If you drop a tennis ball, as in **Figure 12,** it has very little velocity at the start and thus only a small drag force. The downward force of gravity is much stronger than the upward drag force, so there is a downward acceleration. As the ball's velocity increases, so does the drag force. Soon the drag force equals the force of gravity. When this happens, there is no net force, and so there is no acceleration. The constant velocity that is reached when the drag force equals the force of gravity is called the **terminal velocity.**

When light objects with large surface areas fall, the drag force has a substantial effect on their motion, and they quickly reach terminal velocity. Heavier, more compact objects are not affected as much by the drag force. For example, the terminal velocity of a table-tennis ball in air is 9 m/s, and that of a baseball is 42 m/s. Skydivers can increase or decrease their terminal velocity by changing their body orientation and shape. A horizontal, spreadeagle shape produces the slowest terminal velocity, about 60 m/s. After the parachute opens, the skydiver becomes part of a large object with a correspondingly large drag force and a terminal velocity of about 5 m/s.

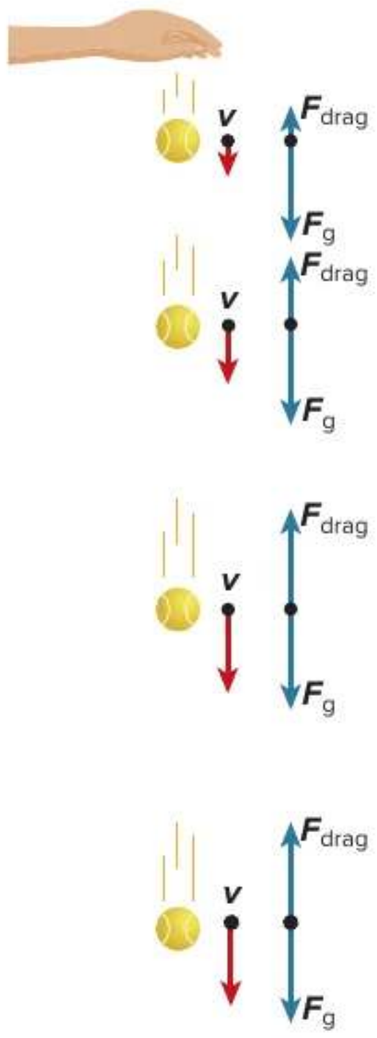


Figure 12 The drag force on an object increases as its velocity increases. When the drag force equals the gravitational force, the object is in equilibrium.

Check Your Progress

22. **Terminal Velocity** The skydiver in **Figure 13** falls at a constant speed in the spread-eagle position. Immediately after opening the parachute, is the skydiver accelerating? If so, in which direction? Explain your answer.

Figure 13



23. **Lunar Gravity** Compare the force holding a 10.0-kg rock on Earth and on the Moon. The gravitational field on the Moon is 1.6 N/kg.
24. **Motion of an Elevator** You are riding in an elevator holding a spring scale with a 1-kg mass suspended from it. You look at the scale and see that it reads 9.3 N. What does this tell you about the elevator's motion?
25. **Apparent Weight** You take a ride in a fast elevator to the top of a tall building and ride back down. Compare your apparent and real weights at each part of the journey. Sketch free-body diagrams to support your answers.
26. **Acceleration** Tecle, with a mass of 65.0 kg, is standing on an ice-skating rink. His friend applies a force of 9.0 N to him. What is Tecle's resulting acceleration?
27. **Critical Thinking** You have a job loading inventory onto trucks at a meat warehouse. Each truck has a weight limit of 10,000 N of cargo. You push each crate of meat along a low-resistance roller belt to a scale and weigh it before moving it onto the truck. One night, right after you weigh a 1000-N crate, the scale breaks. Describe a way in which you could apply Newton's laws to approximate the masses of the remaining crates.

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LESSON 3

NEWTON'S THIRD LAW

FOCUS QUESTION

If you push on a wall, what force does the wall exert on you?

Interaction Pairs

Figure 14 illustrates the idea of forces as interaction pairs. There is a force from the woman on the dog's toy, and there is a force from the dog's toy on the woman. Forces always come in pairs similar to this example. Consider the woman (A) as one system and the toy (B) as another. What forces act on each of the two systems? Looking at the force diagrams in **Figure 14**, you can see that each system exerts a force on the other. The two forces, $F_{A \text{ on } B}$ and $F_{B \text{ on } A}$, are an example of an **interaction pair,** which is a set of two forces that are in opposite directions, have equal magnitudes, and act on different objects. Sometimes, an interaction pair is called an action-reaction pair. This might suggest that one causes the other; however, this is not true. For example, the force of the woman pulling on the toy doesn't cause the toy to pull on the woman. The two forces either exist together or not at all.

Definition of Newton's third law

In **Figure 14**, the force exerted by the woman on the toy is equal in magnitude and opposite in direction to the force exerted by the toy on the woman. Such an interaction pair is an example of **Newton's third law,** which states that all forces come in pairs. The two forces in a pair act on different objects and are equal in strength and opposite in direction.



FatCamera/E+/Getty Images

Figure 14 The force that the toy exerts on the woman and the force that the woman exerts on the toy are an interaction pair.

3D THINKING

DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Newton's Third Law

Plan and carry out a investigation that applies Newton's laws of motion to different systems.

Review the News

Obtain information from a current news story about forces and motion. Evaluate your source and communicate your findings to your class.

Newton's Third Law

The force of A on B is equal in magnitude and opposite in direction of the force of B on A.

$$F_{\text{A on B}} = -F_{\text{B on A}}$$

Using Newton's third law Consider the situation of holding a book in your hand. You can draw one free-body diagram for you and one for the book. Are there any interaction pairs? When identifying interaction pairs, keep in mind that they always occur in two different free-body diagrams, and they always will have the symmetry of subscripts noted on the previous page. In this case, the interaction pair is $F_{\text{book on hand}}$ and $F_{\text{hand on book}}$.

The ball in **Figure 15** interacts with the table and with Earth. First, analyze the forces acting on one system, the ball. The table exerts an upward force on the ball, and the mass of Earth exerts a downward gravitational force on the ball. Even though these forces are in opposite directions, they are not an interaction pair because they act on the same object. Now consider the ball and the table together. In addition to the upward force exerted by the table on the ball, the ball exerts a downward force on the table. This is an interaction pair. Notice also that the ball has a weight. If the ball experiences a force due to Earth's mass, then there must be a force on Earth's mass due to the ball. In other words, they are an interaction pair.

$$F_{\text{Earth's mass on ball}} = -F_{\text{ball on Earth's mass}}$$

An unbalanced force on Earth would cause Earth to accelerate. But acceleration is inversely proportional to mass. Because Earth's mass is so huge in comparison to the masses of other objects that we normally consider, Earth's acceleration is so small that it can be neglected. In other words, Earth can be often treated as part of the external world rather than as a second system. The problem-solving strategies below summarize how to deal with interaction pairs.

Get It?

Explain why Earth's acceleration is usually very small compared to the acceleration of the object that Earth interacts with.

PROBLEM-SOLVING STRATEGY

Interaction Pairs

Use these strategies to solve problems in which there is an interaction between objects in two different systems.

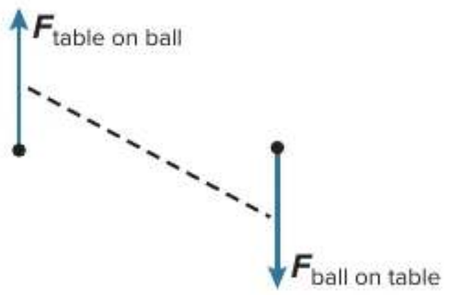
1. Separate the system or systems from the external world.
2. Draw a pictorial model with coordinate systems for each system.
3. Draw a physical model that includes free-body diagrams for each system.
4. Connect interaction pairs by dashed lines.
5. To calculate your answer, use Newton's second law to relate the net force and acceleration for each system.
6. Use Newton's third law to equate the magnitudes of the interaction pairs and give the relative direction of each force.
7. Solve the problem and check the reasonableness of the answers' units, signs, and magnitudes.

Newton's Third Law

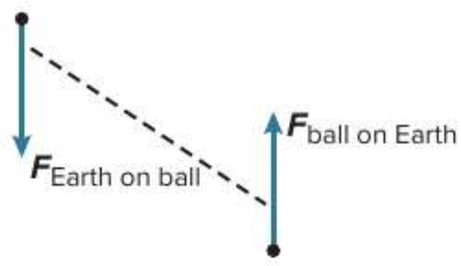




The two forces acting on the ball are $F_{\text{table on ball}}$ and $F_{\text{Earth's mass on ball}}$. These forces are not an interaction pair.



Force interaction pair between ball and table.



Force interaction pair between ball and Earth.

Figure 15 A ball resting on a table is part of two interaction pairs.

EXAMPLE Problem 4

EARTH'S ACCELERATION A softball has a mass of 0.18 kg. What is the gravitational force on Earth due to the ball, and what is Earth's resulting acceleration? Earth's mass is 6.0×10^{24} kg.

1 ANALYZE AND SKETCH THE PROBLEM

- Draw free-body diagrams for the two systems: the ball and Earth.
- Connect the interaction pair by a dashed line.



KNOWN

$m_{ball} = 0.18$ kg

$m_{Earth} = 6.0\times10^{24}$ kg $\quad g = 9.8$ N/kg

UNKNOWN

$F_{Earth\ on\ ball} = ?$

$a_{Earth} = ?$

2 SOLVE FOR THE UNKNOWN

Use Newton's second law to find the weight of the ball.

$F_{Earth\ on\ ball} = m_{ball}g = (0.18\text{ kg})(-9.0\text{ N/kg}) = -1.8\text{ N}$ Substitute $m_{ball} = 0.18$ kg.g = –9.8 N/kg.

Use Newton's third law to find $F_{ball\ on\ Earth}$.

$F_{Ball\ on\ Earth} = -F_{Earth\ on\ ball} = -(-1.8\text{ N}) = +1.8\text{ N}$ Substitute $F_{Earth\ on\ ball} = -1.8$ N.

Use Newton's second law to find a_{Earth}.

$$a_{Earth} = \frac{F_{net}}{m_{Earth}} = \frac{1.8\text{ N}}{6.0\times10^{24}\text{ kg}}$$ Substitute $F_{net} = 1.8$ N, $m_{Earth} = 6.0\times10^{24}$ kg.

$= 2.9\times10^{-25}$ m/s^2 toward the softball

3 EVALUATE THE ANSWER

- **Are the units correct?** Force is in N and acceleration is in m/s^2.
- **Do the signs make sense?** Force and acceleration should be positive.
- **Is the magnitude realistic?** It makes sense that Earth's acceleration is small; Earth's mass is large.

PRACTICE Problems

ADDITIONAL PRACTICE

28. You lift a relatively light bowling ball with your hand, accelerating it upward. What are the forces on the ball? What forces does the ball exert? What objects are these forces exerted on?

29. A brick falls from a construction scaffold. Identify any forces acting on the brick. Also identify any forces the brick exerts and the objects on which these forces are exerted. (Air resistance may be ignored.)

30. A suitcase sits on a stationary airport luggage cart, as in **Figure 16.** Draw a free-body diagram for each object and specifically indicate any interaction pairs between the two.

31. CHALLENGE You toss a ball up in the air. Draw a free-body diagram for the ball after it has lost contact with your hand but while it is still moving upward. Identify any forces acting on the ball. Also identify any forces that the ball exerts and the objects on which these forces are exerted. Assume that air resistance is negligible.



Figure 16

Tension

Tension is simply a specific name for the force that a string or rope exerts. A simplification within this textbook is the assumption that all strings and ropes are massless. In **Figure 17**, the rope is about to break in the middle. If the rope breaks, the bucket will fall; before it breaks, there must be forces holding the rope together. The force that the top part of the rope exerts on the bottom part is $F_{\text{top on bottom}}$. Newton's third law states that this force must be part of an interaction pair. The other member of the pair is the force that the bottom part of the rope exerts on the top, $F_{\text{bottom on top}}$. These forces, equal in magnitude but opposite in direction, also are shown in **Figure 17**.

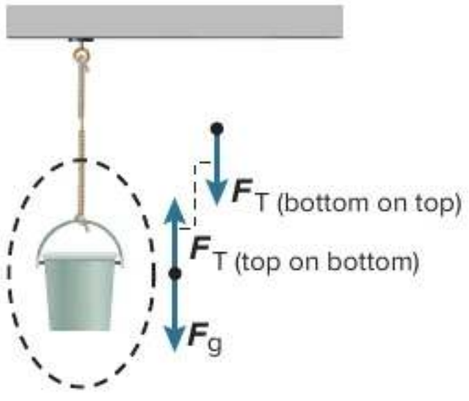


Figure 17 The tension in the rope is equal to the weight of all the objects hanging from it.

Think about this situation in another way. Before the rope breaks, the bucket is in equilibrium. This means that the force of its weight downward must be equal in magnitude but opposite in direction to the tension in the rope upward. Similarly, if you look at the point in the rope just above the bucket, it also is in equilibrium. Therefore, the tension of the rope below it pulling down must be equal to the tension of the rope above it pulling up. You can move up the rope, considering any point in the rope, and see that the tension forces at any point in the rope are pulling equally in both directions. Thus, the tension in the rope equals the weight of all objects below it.

Examine the tension forces shown in **Figure 18**. If team A is exerting a 500-N force and the rope does not accelerate, then team B also must be pulling with a force of 500 N. What is the tension in the rope? If each team pulls with 500 N of force, is the tension 1000 N? To decide, think of the rope as divided into two halves. The left side is not accelerating, so the net force on it is zero. Thus, $F_{\text{A on left side}} = F_{\text{right side on left side}} = 500$ N. Similarly, $F_{\text{B on right side}} = F_{\text{left side on right side}} = 500$ N. But the two tensions, $F_{\text{right side on left side}}$ and $F_{\text{left side on right side,}}$ are an interaction pair, so they are equal and opposite. Thus, the tension in the rope equals the force with which each team pulls, or 500 N. To verify this, you could cut the rope in half and tie the ends to a spring scale. The scale would read 500 N.

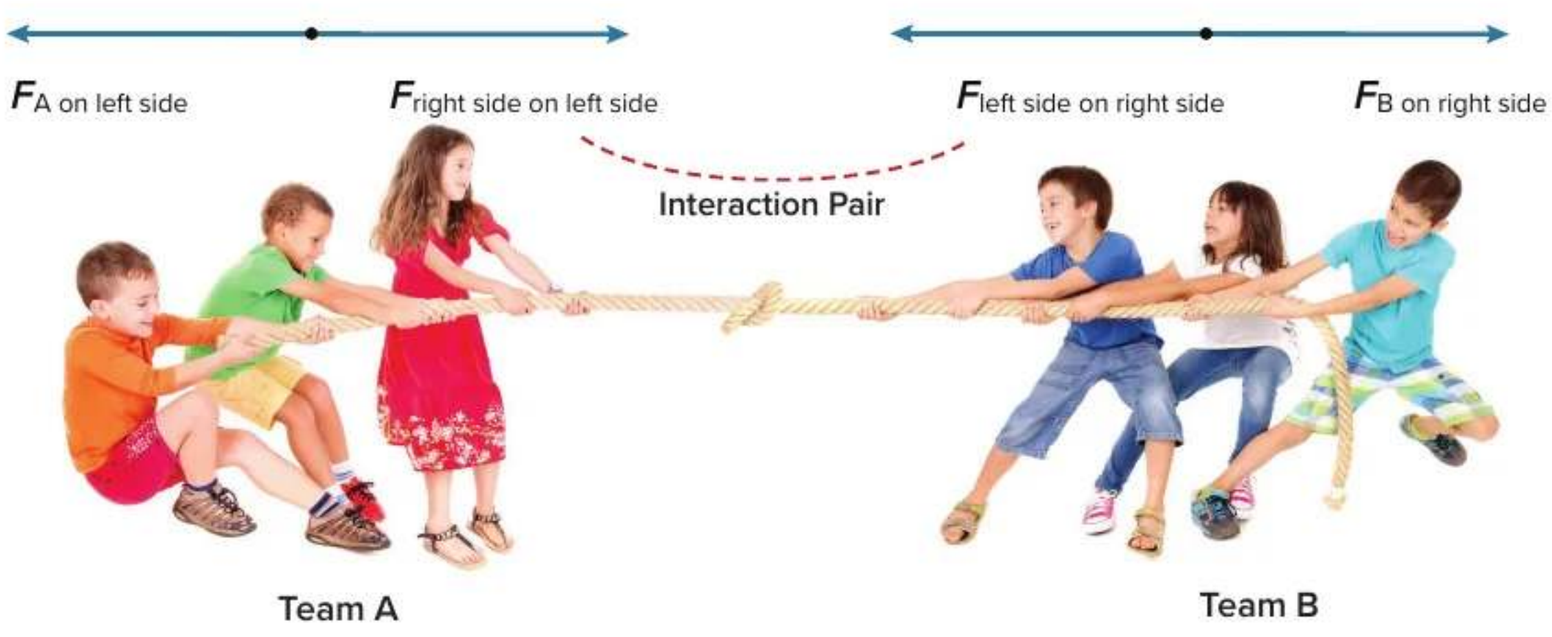


Figure 18 The rope is not accelerating, so the tension in the rope equals the force with which each team pulls.

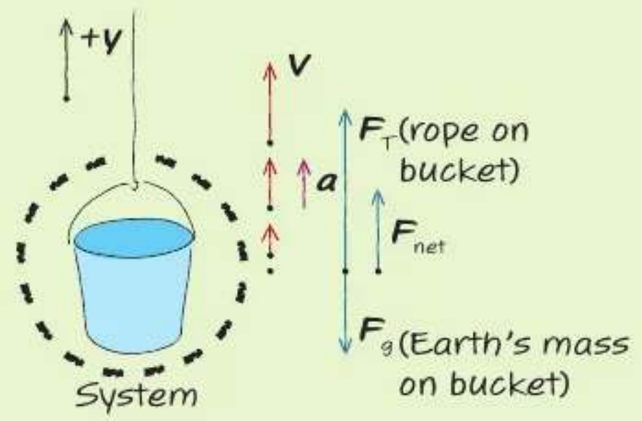
Veronica Louro/Shutterstock

EXAMPLE Problem 5

LIFTING A BUCKET A 50.0-kg bucket is being lifted by a rope. The rope will not break if the tension is 525 N or less. The bucket started at rest, and after being lifted 3.0 m, it moves at 3.0 m/s. If the acceleration is constant, is the rope in danger of breaking?

1 ANALYZE AND SKETCH THE PROBLEM

- Draw the situation, and identify the forces on the system.
- Establish a coordinate system with the positive axis upward.
- Draw a motion diagram; include ***v*** and ***a***.
- Draw the free-body diagram, and label for forces.



KNOWN

$m = 50.0$ kg $v_t = 3.0$ m/s

$v_i = 0.0$ m/s $d = 3.0$ m

UNKNOWN

$F_T = ?$

2 SOLVE FOR THE UNKNOWN

F_{net} is the sum of the positive force of the rope pulling up (T_F) and the negative weight force ($-F_g$) pulling down as defined by the coordinate system.

$$F_{net} = F_T + (-F_g)$$

$$F_T = F_{net} + F_g = ma + mg \qquad \text{Substitue } F_{net} = ma,\ F_g = mg$$

v_i, v_f, and d are known.

$$v_f^2 = v_i + 2ad$$

$$a = \frac{v_f^2 - v_i^2}{2d} = \frac{v_f^2}{2d} \qquad \text{Substitute } v_i = 0.0 \text{ m/s}$$

$$F_T = ma + mg \qquad \text{Substitute } a = v_f^2 / (2d).$$

$$= m\left(\frac{v_f^2}{2d}\right) + mg$$

$$= (50.0 \text{ kg})\left(\frac{(3.0 \text{ m/s})^2}{2(3.0 \text{ m})}\right) + (50.0 \text{ kg})(9.8 \text{ N/kg}) \qquad \text{Substitute } m = 50.0 \text{ kg},\ v_f = 3.0 \text{ m/s},\ d = 3.0 \text{ m},\ g = 9.8 \text{ N/Kg}.$$

$$= 560 \text{ N}$$

The rope is in danger of breaking because the tension exceeds 525 N.

3 EVALUATE THE ANSWER

- **Are the units correct?** dimensional analysis verifies kg•m/s^2, which is N
- **Does the sign make sense?** The upward force should be positive.
- **Is the magnitude realistic?** The magnitude is a little larger than 490 N, which is the weight of the bucket. $F_g = mg = (50.0 \text{ kg})(9.8 \text{ N/kg}) = 490 \text{ N}$

PRACTICE Problems

ADDITIONAL PRACTICE

32. Diego and Mika are trying to remove a tire from Diego's car. When they pull together in the same direction, Mika with a force of 23 N and Diego with a force of 31 N, they just barely get the tire to move off the wheel. What is the magnitude of the force between the tire and the wheel?

33. CHALLENGE You are loading equipment into a bucket that roofers will hoist to a rooftop. If the rope will not break as long as the tension does not exceed 450 N and you fill the bucket until it has a mass of 42 kg, what is the greatest acceleration the workers can give the bucket as they hoist it?

The Normal Force

Any time two objects are in contact, they exert a force on each other. Consider a box sitting on a table. There is a downward force on the box due to gravity. There also is an upward force that the table exerts on the box. This force must exist because the box is in equilibrium. The **normal force** is the perpendicular contact force that a surface exerts on another surface.

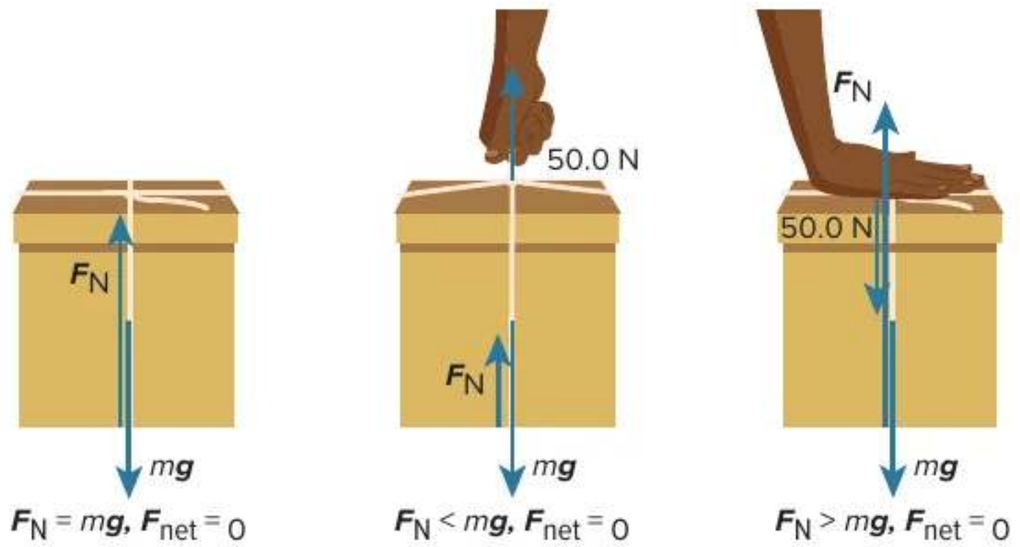


Figure 19 The normal force is not always equal to the object's weight.

The normal force always is perpendicular to the plane of contact between two objects, but is it always equal to the weight of an object? **Figure 19** shows three situations involving a box with the same weight. What if you tied a string to the box and pulled up on it a little bit, but not enough to accelerate the box, as shown in the middle panel in **Figure 19**? When you apply Newton's second law to the box and the forces acting on the box, you see $F_N + F_{\text{string on box}} - F_g = ma = 0$ N, which can be rearranged to show $F_N = F_g - F$ string on box.

You can see that in this case the normal force that the table exerts on the box is less than the box's weight (F_g). Similarly, if you pushed down on the box on the table as shown in the final panel in **Figure 19**, the normal force would be more than the box's weight. Finding the normal force will be important when you study friction in detail.

Check Your Progress

34. **Interaction Pair** Identify each force acting on the ball and its interaction pair in **Figure 20.**



Figure 20

35. **Force** Imagine lowering the ball in **Figure 20** at increasing speed. Draw separate free-body diagrams for the forces acting on the ball and for each set of interaction pairs.

36. **Tension** A block hangs from the ceiling by a massless rope. A second block is attached to the first block and hangs below it on another piece of massless rope. If each of the two blocks has a mass of 5.0 kg, what is the tension in the rope?

37. **Tension** A block hangs from the ceiling by a massless rope. A 3.0-kg block is attached to the first block and hangs below it on another piece of massless rope. The tension in the top rope is 63.0 N. Find the tension in the bottom rope and the mass of the top block.

38. **Critical Thinking** A curtain prevents two tug-of-war teams from seeing each other. One team ties its end of the rope to a tree. If the other team pulls with a 500-N force, what is the tension in the rope? Explain.

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NATURE OF SCIENCE

Finding the Source of the Force

Racetrack Playa in Death Valley, CA has a flat surface dotted with rocks and boulders of varying sizes. The rocks occasionally move, leaving trails in the mud that show their path of motion. For a long time, no one knew what caused the rocks to move. This scientific mystery was finally solved during an investigation that gathered data using advanced technology.



For many years, scientists tried to determine the source of the forces that caused these rocks to move.

Wandering Stones

The rocks on Racetrack Playa have earned many nicknames over the years. They have been called wandering stones, sliding rocks, slithering stones, and sailing stones. The rocks vary greatly in size—some of the largest have a mass of more than 300 kg. The motion of the rocks had never been observed first-hand.

Hypotheses

Many scientists developed hypotheses to explain the motion of the rocks. Some thought that the force of gravity was pulling the rocks slowly down a very slight slope. Others thought that high winds occasionally pushed the rocks with enough force to cause their motion. However, evidence did not support these hypotheses. Gravity was ruled out by data showing the rocks were moving up a slight slope, not downhill. High winds were also ruled out, because data showed that more massive rocks often moved farther than smaller rocks.

Solving the Mystery

To solve the mystery of the rocks' movement, researchers embedded GPS in fifteen rocks to track their motion. They set up a weather station to keep track of the wind speed and direction. They also set up a camera to record video of the rocks' motion.

After analyzing the data, the researchers concluded that rock movement occurred when a shallow pond formed and froze on the Playa's surface. When the ice sheet on the top of the pond started to melt, very thin floating sheets of ice were blown by light breezes. The floating ice sheets pushed against rocks, making the rocks slide, similar to the way a sail makes a boat move across the water. Data providing evidence that thin ice and light breezes could generate enough force to cause the motion of the rocks was an unexpected answer to this scientific mystery.

Panther Media GmbH/Alamy Stock Photo



DEVELOP A MODEL TO ILLUSTRATE

Work with a team to draw a free-body diagram of the forces acting on a sliding rock in Racetrack Playa. Indicate the direction of the acceleration and of the net force. Next to the free-body diagram, make an illustration that uses a circle to designate the system.

MODULE 4
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 FORCE AND MOTION

- A force is a push or a pull. Forces have both direction and magnitude. A force might be either a contact force or a field force.
- Newton's second law states that the acceleration of a system equals the net force acting on it divided by its mass.

$$a = \frac{F_{net}}{m}$$

- Newton's first law states that an object that is at rest will remain at rest and an object that is moving will continue to move in a straight line with constant speed, if and only if the net force acting on that object is zero. An object with zero net force acting on it is in equilibrium.

- force
- system
- free-body diagram
- net force
- Newton's second law
- Newton's first law
- inertia
- equilibrium

Lesson 2 WEIGHT AND DRAG FORCE

- The object's weight (F_g) depends on the object's mass and the gravitational field at the object's location.

$$F_g = mg$$

- An object's apparent weight is the magnitude of the support force exerted on it. An object with no apparent weight experiences weightlessness.
- A falling object reaches a constant velocity when the drag force is equal to the object's weight. The constant velocity is called the terminal velocity. The drag force on an object is determined by the object's weight, size, and shape as well as the fluid through which it moves.

- weight
- gravitational field
- apparent weight
- weightlessness
- drag force
- terminal velocity

Lesson 3 NEWTON'S THIRD LAW

- Newton's third law states that the two forces that make up an interaction pair of forces are equal in magnitude, but opposite in direction and act on different objects. In an interaction pair, $F_{A\text{ on }B}$ does not cause $F_{B\text{ on }A}$. The two forces either exist together or not at all.

$$F_{A\text{ on }B} = -F_{B\text{ on }A}$$

- The normal force is a support force resulting from the contact between two objects. It is always perpendicular to the plane of contact between the two objects.

- interaction pair
- Newton's third law
- tension
- normal force

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

How do wing suits help BASE jumpers control their velocity?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will apply your evidence from this module and complete your project.

GO FURTHER

SEP Data Analysis Lab

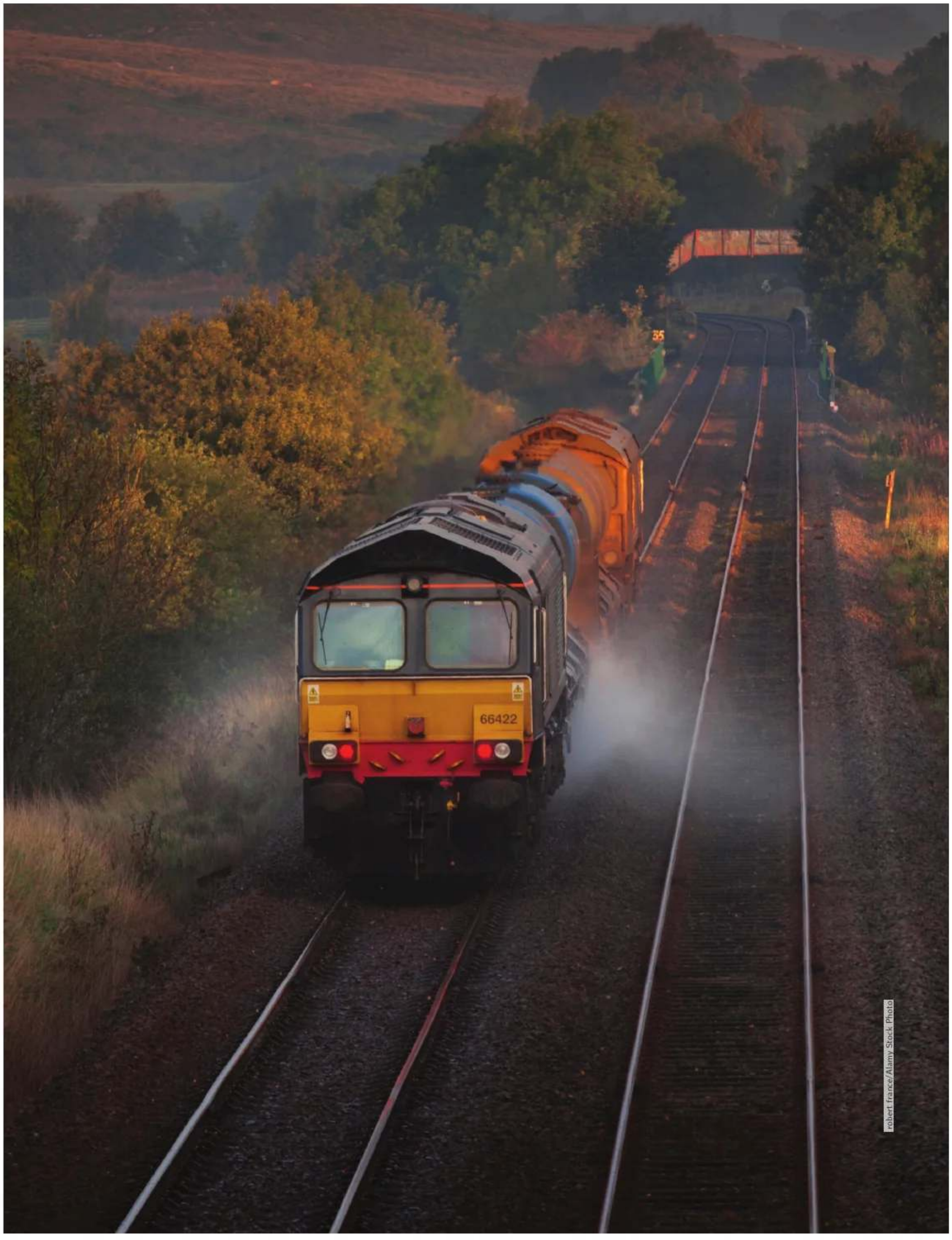
How does weight change during a rocket launch?

A rocket is launched vertically. When the rocket reaches its maximum height, a parachute is deployed, and the rocket descends to the ground.

CER Analyze and Interpret Data

1. **Draw and label** a free-body diagram for the rocket during each of the following intervals:
   a. While the engine is firing
   b. After the engine shuts down but before the parachute is deployed
   c. The moment the parachute is deployed
2. **Claim** Imagine the rocket is full-sized and a person stands on a bathroom scale inside. Is the scale reading less than, equal to, or greater that the scale reading when the rocket is at rest for the following intervals?
   a. While the engine is firing
   b. After the engine shuts down but before the parachute is deployed
   c. The moment the parachute is deployed
3. **Evidence and Reasoning** Justify your answers by using your diagrams and explaining your reasoning.

Oliver Furrer/Cultura/Getty Images


66422
robert france/Alamy Stock Photo

MODULE 5

DISPLACEMENT AND FORCE IN TWO DIMENSIONS

ENCOUNTER THE PHENOMENON

Why is this specialized train washing the train tracks?



GO ONLINE to play a video about the importance of cleaning train tracks.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

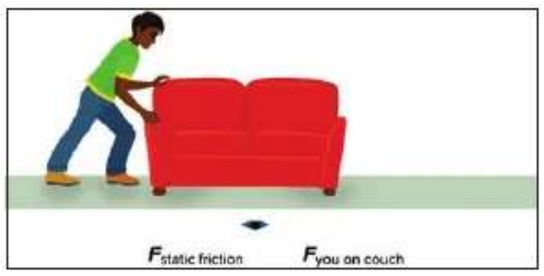
CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about why the train is washing the tracks. Explain your reasoning.

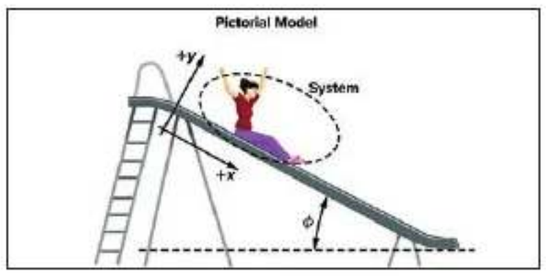
Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 2: Explore & Explain: Kinetic and Static Friction



LESSON 3: Explore & Explain: Inclined Planes

Smithsonian

Additional Resources

Video Supplied by BBC Worldwide Learning

LESSON 1

VECTORS

FOCUS QUESTION

How can you add forces in two dimensions?

Vectors in Two Dimensions

How do climbers cling to a rock wall? Often the climber has more than one support point. This means there are multiple forces acting on him. Because he grips crevices in the rock, the rock pulls back on him. Also the rope secures him to the rock, so there are two contact forces acting on him. Gravity pulls on him as well, so there are a total of three forces acting on the climber.

One aspect of this situation that is different from the ones you might have studied earlier is that the forces exerted by the rock face on the climber do not push or pull only in the horizontal or vertical direction. You know from previous study that you can pick your coordinate system and orient it in the way that is most useful to analyze a situation. But how can you set up a coordinate system for a net force when you are dealing with more than one dimension? And what happens when the forces are not at right angles to each other?

Vectors revisited Let's begin by reviewing force vectors in one dimension. Consider the case in **Figure 1** in which you and a friend both push on a table together. Suppose that you each exert a 40-N force to the right. The sum of the forces is 80 N to the right, which is what you probably expected. But how do you find the sum of these force vectors?

In earlier discussions of adding vectors along a straight line, you learned that the resultant vector always points from the tail of the first vector to the tip of the final vector. Does this rule still apply when the vectors do not lie along a straight line?

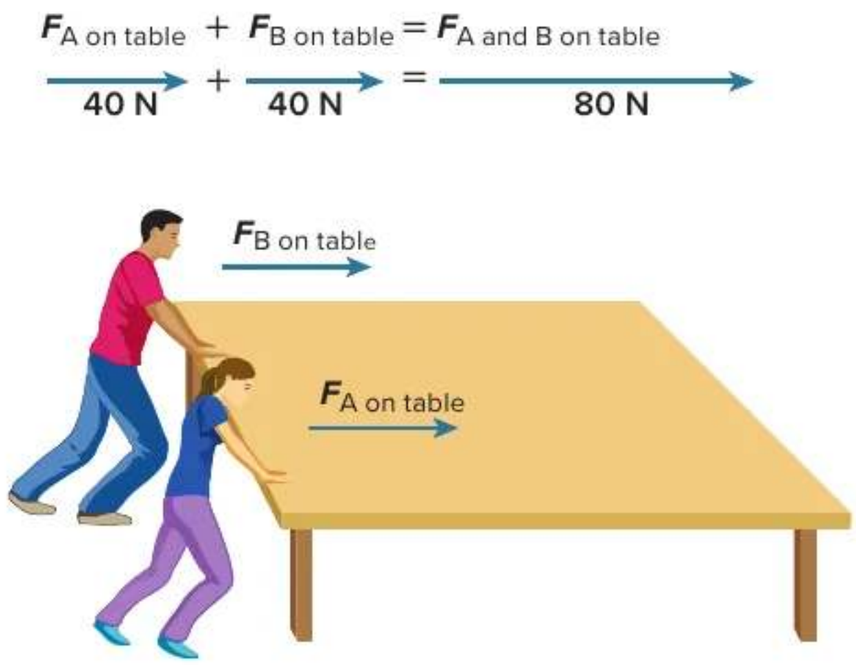


Figure 1 The sum of the two applied forces is 80 N to the right.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Launch Lab: Adding Vectors

Carry out an investigation to determine the net force on a system when forces are exerted in two dimensions.

CCC Identify Crosscutting Concepts

Create a table of the crosscutting concepts and fill in examples you find as you read.

Adding vectors in two dimensions

Even when vectors do not lie on a straight line, the resultant vector always points from the tail of the first vector to the tip of the final vector. You can use a protractor and a ruler both to draw the vectors at the correct angles and to measure the magnitude and the direction of the resultant vector.

Figure 2 illustrates how to add vectors graphically in two dimensions. Notice that when vector ***B*** is moved, its magnitude and direction are unchanged. This is always the case—you do not change a vector's length or direction when you move that vector.

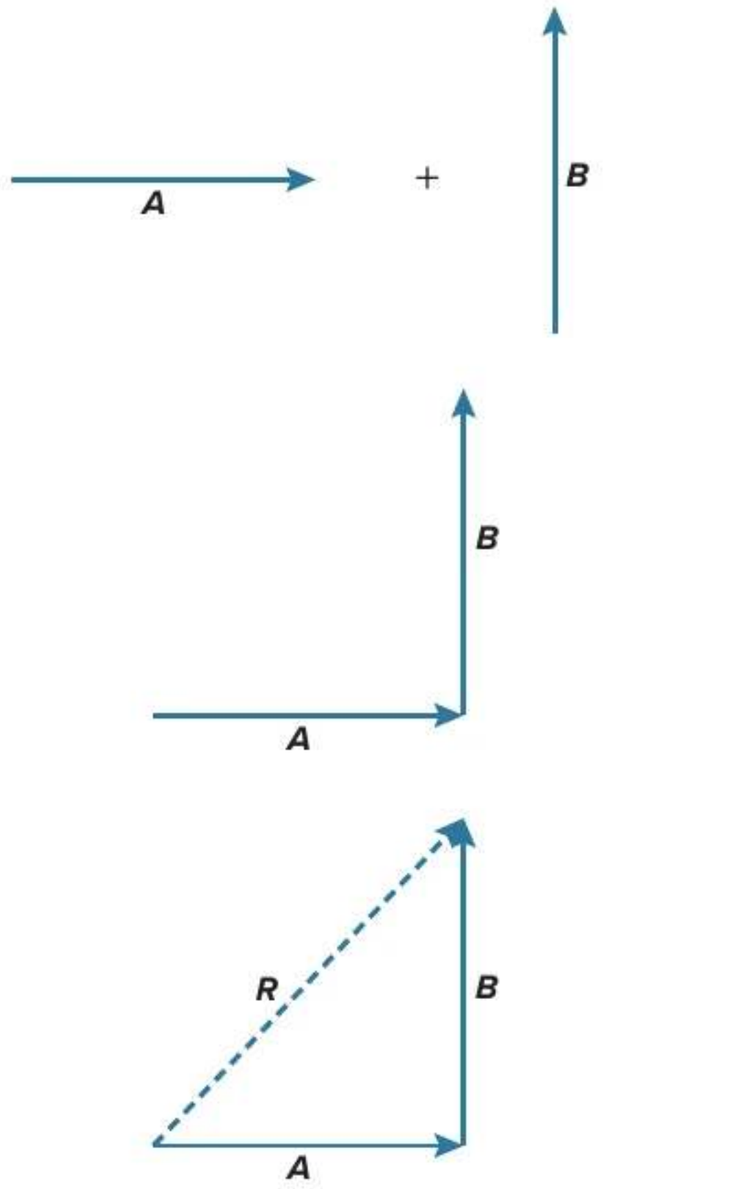


Figure 2 When adding vectors in two dimensions, the process is the same as adding vectors in one dimension: place vectors tip to tail, then connect the tail of the first vector to the tip of the second vector (or final vector if more than two) to find the resultant.

Perpendicular vectors

If you are adding together two vectors at right angles, such as vector ***A*** pointing east and vector ***B*** pointing north in **Figure 2,** you could use the Pythagorean theorem to find the magnitude (R) of the resultant vector.

$$R^2 = A^2 + B^2$$

Angles other than 90°

If you are adding two vectors that are at an angle other than 90°, as in **Figure 3,** then you can use trigonometry, such as the law of sines or the law of cosines. It is best to use the law of sines when you are given two angle measurements and only one vector magnitude. The law of cosines is particularly useful when given two vectors and the angle between them.

Law of Sines: $\frac{R}{\sin\theta} = \frac{A}{\sin a} = \frac{B}{\sin b}$

Law of Cosines: $R^2 = A^2 + B^2 - 2AB\cos\theta$

What happens if you apply the law of cosines to a triangle in which $\theta = 90^\circ$? Notice that the first three terms in the law of cosines are the same three terms found in the Pythagorean theorem. The final term in the law of cosines, $-2AB\cos\theta$, equals zero if $\theta = 90^\circ$ because $\cos 90^\circ = 0$.

$$R^2 = A^2 + B^2 - 2AB\cos 90^\circ$$

$$R^2 = A^2 + B^2 - 2AB\,(0)$$

$$R^2 = A^2 + B^2$$

If $\theta = 90^\circ$, the triangle is a right triangle and the law of cosines reduces to the Pythagorean theorem.

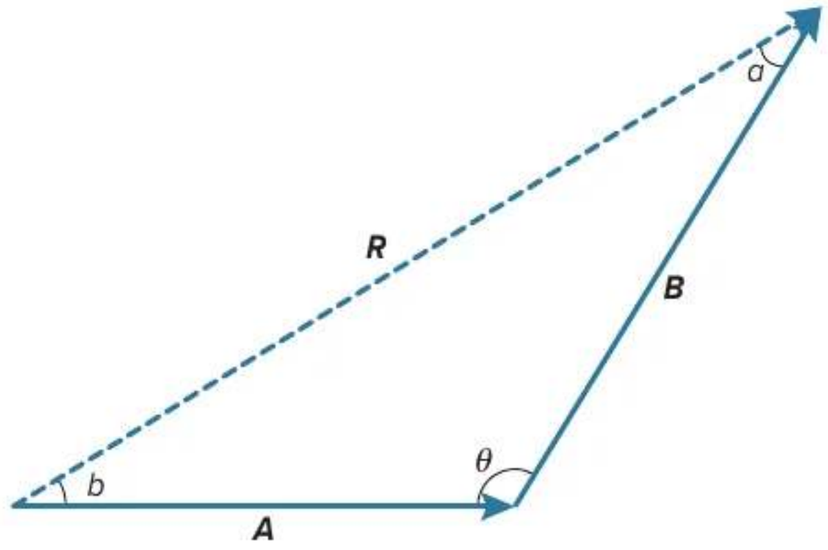


Figure 3 If the vectors are not at right angles, the Pythagorean theorem does not apply. Instead, use the law of cosines or the law of sines with the variables as shown here.

EXAMPLE Problem 1

FINDING THE MAGNITUDE OF THE SUM OF TWO VECTORS Find the magnitude of the sum of a 15-km displacement and a 25-km displacement when the angle θ between them is 90° and when the angle θ between them is 135°.

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the two displacement vectors, A and B, and the angle between them.

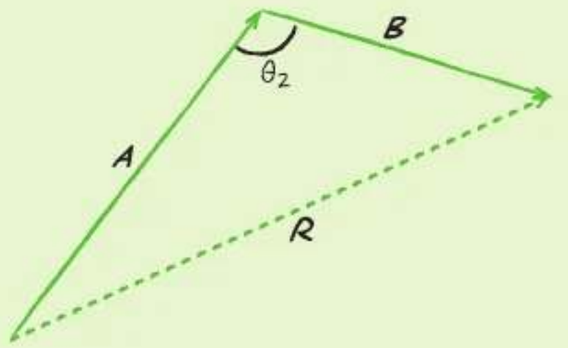
Known

$A = 25$ km $\quad \theta = 90°$ or $\theta_2 = 135°$

$B = 15$ km

Unknown

$R = ?$



2 SOLVE FOR THE UNKNOWN

When θ is 90°, use the Pythagorean theorem to find the magnitude of the resultant vector.

$$R^2 = A^2 + B^2$$

$$R = \sqrt{A^2 + B^2}$$

$$= \sqrt{(25\text{ km})^2 + (15\text{ km})^2} \qquad \text{Substitute } A = 25\text{ km}, B = 15\text{ km}$$

$$= 29\text{ km}$$

When θ does not equal 90°, use the law of cosines to find the magnitude of the resultant vector.

$$R^2 = A^2 + B^2 - 2AB(\cos\theta_2)$$

$$R = \sqrt{A^2 + B^2 - 2AB(\cos\theta_2)}$$

$$= \sqrt{(25\text{ km})^2 + (15\text{ km})^2 - 2(25\text{ km})(15\text{ km})(\cos 135°)} \qquad \text{Substitute } A = 25\text{ km}, B = 15\text{ km}, \theta_2 = 135°$$

$$= 37\text{ km}$$

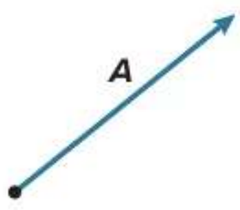
3 EVALUATE THE ANSWER

- **Are the units correct?** Each answer is a length measured in kilometers.
- **Do the signs make sense?** The sums are positive.
- **Are the magnitudes realistic?** From the sketch, you can see that the resultant should be longer than either vector.

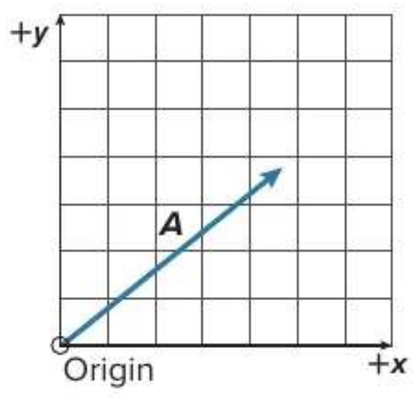
PRACTICE Problems

ADDITIONAL PRACTICE

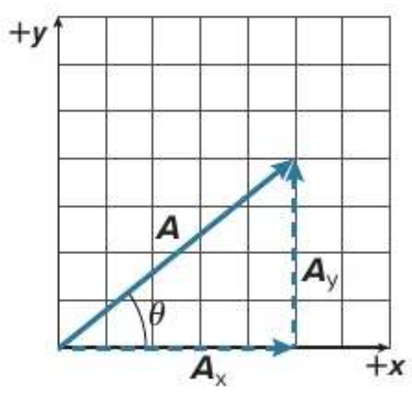
1. You and your family are out for a drive. You drive 125.0 km due west, then turn due south and drive for another 65.0 km. What is the magnitude of your displacement? Solve this problem both graphically and mathematically, and check your answers against each other.
2. On a fine, sunny day, you and your siblings decide to go for a nearby hike. You walk 4.5 km in one direction, then make a 45° turn to the right and walk another 6.4 km. What is the magnitude of your displacement?
3. After picking up school supplies, you and your caregiver walk from the door of the mall to the car. You first walk 250.0 m down a lane of cars, and then turn 90° to the right and walk an additional 60.0 m. How far is the car from the door to the mall? Solve this problem both graphically and mathematically, and check your answers against each other.
4. **CHALLENGE** An ant crawls on the sidewalk. It first moves south a distance of 5.0 cm. It then turns southwest and crawls 4.0 cm. What is the magnitude of the ant's displacement?



You may place the vector on any coordinate system as long as the vector's direction and magnitude remain unchanged.



The coordinate system can be oriented to make the problem easier to solve.



If the components of a vector are added together, they result in the original vector.

Figure 4 Vector ***A*** is placed on a coordinate system. Notice in the right panel that ***A***'s direction is measured counterclockwise from the positive *x*-axis. $\boldsymbol{A}_x$ is parallel to the *x*-axis, and $\boldsymbol{A}_y$ is parallel to the *y*-axis.

Describe *a vector whose y-component is zero.*

Vector Components

What is the direction of the vector shown on the left of **Figure 4?** To determine the exact direction, we need to choose a coordinate system. This coordinate system, such as the one in the center of **Figure 4,** is similar to laying down a grid drawn on transparent plastic on top of a vector problem. You choose where to put the center of the grid (the origin) and establish the directions in which the axes point. Notice that the *x*-axis is drawn through the origin with an arrow pointing in the positive direction. The positive *y*-axis is located 90° from the positive *x*-axis and crosses the *x*-axis at the origin.

How do you choose the direction of the *x*-axis? There is never a single correct answer, but some choices make the problem easier to solve than others. When the motion you are describing is confined to the surface of Earth, have the *x*-axis point east and the *y*-axis point north. If the motion is on an incline, place the positive *x*-axis in the direction of the motion parallel to the surface of the incline

Component vectors Now you can describe a vector in a useful way. Vector ***A,*** shown on the right in **Figure 4,** can be described as going 5 units in the positive *x*-direction ($\boldsymbol{A}_x$) and 4 units in the positive *y*-direction ($\boldsymbol{A}_y$). If you add $\boldsymbol{A}_x$ and $\boldsymbol{A}_y$, the resultant is the original vector, ***A***. A vector can be broken into its **components,** which are a vector parallel to the *x*-axis and another vector parallel to the *y*-axis: $\boldsymbol{A} = \boldsymbol{A}_x + \boldsymbol{A}_y$.

The process of breaking a vector into its components is sometimes called **vector resolution.** The original vector is the hypotenuse of a right triangle. This means that the magnitude of the original vector will always be greater than or equal to the magnitude of either component vector.

SCIENCE USAGE v. COMMON USAGE

Components

Science usage: projections of a vector parallel to the *x*- and *y*-axes
A vector is the algebraic sum of its components.

Common usage: constituent parts or ingredients
The computer's electronic components malfunctioned during the power surge.

STEM CAREER Connection

Civil Engineer

When in a moving car on a wet surface, do you notice the difference in friction between the tires and the wet road? One task for a civil engineer is to design roads and pavements to account for the friction between the tires and the road in different driving conditions.

Direction in coordinate systems In a two-dimensional force problem, you should carefully select your coordinate system because the direction of any vector will be specified relative to those coordinates. We define the direction of a vector as the angle that the vector makes with the x-axis, measured counterclockwise from the positive x-axis. In **Figure 5,** the angle θ represents the direction of the vector $\boldsymbol{A}$**.** All algebraic calculations involve only the components of vectors, not the vectors themselves.

In addition to measuring the lengths of the component vectors graphically, you can find the components by using trigonometry as shown below:

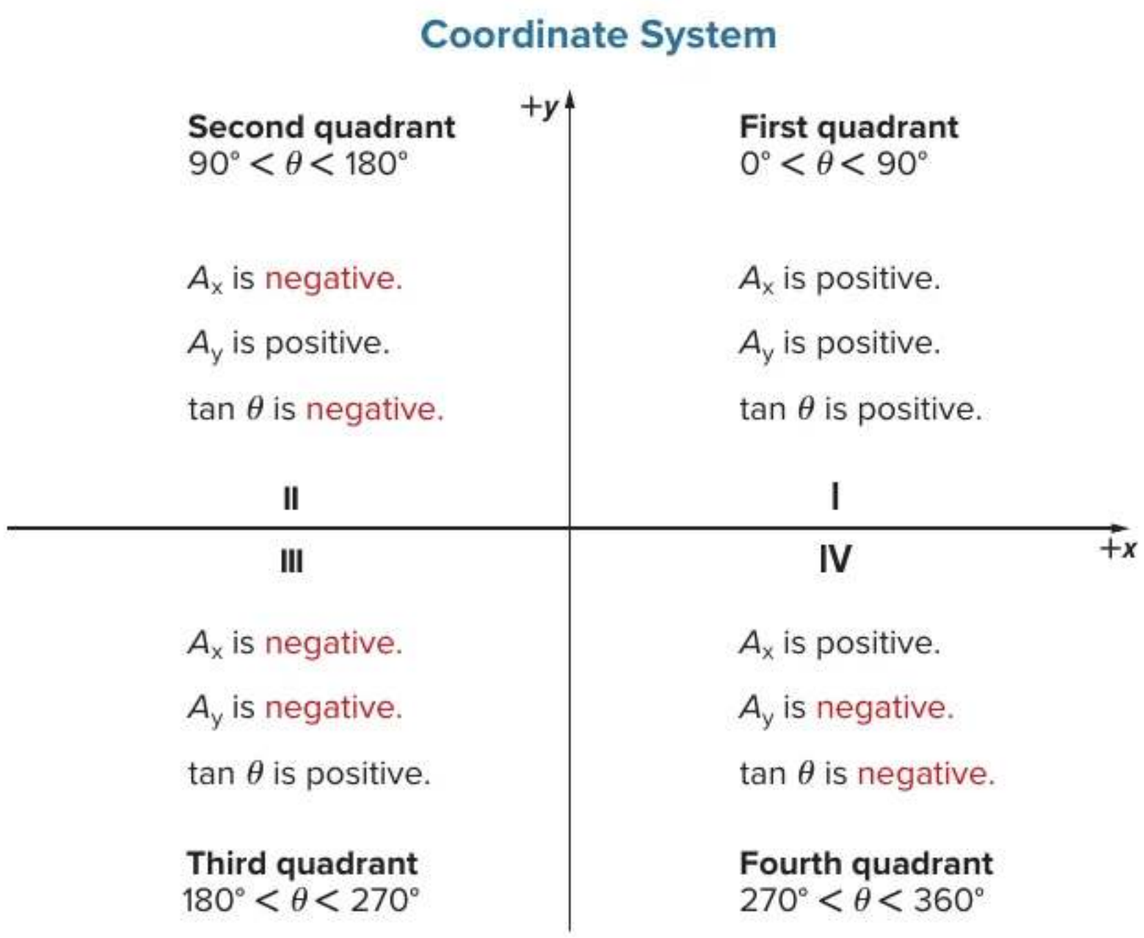
$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{A_x}{A};\ \text{therefore, } A_x = A\cos\theta$$

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{A_y}{A};\ \text{therefore, } A_y = A\sin\theta$$

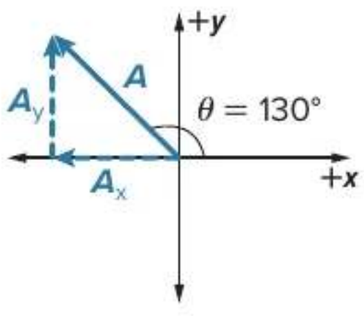
When the angle that a vector makes with the x-axis is larger than 90°, the sign of one or more components is negative, as shown in the top coordinate system of **Figure 5.**

Get It?

Explain how you should measure the direction of a vector.



Example:



- ***A*** is in 2nd quadrant.
- Expect A_x to be negative:

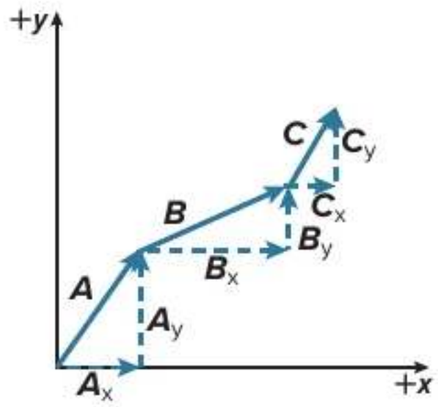
$$A_x = A\cos\theta = (5.0\text{ N})\cos 130° = -3.2\text{ N}$$

- Expect A_y to be positive:

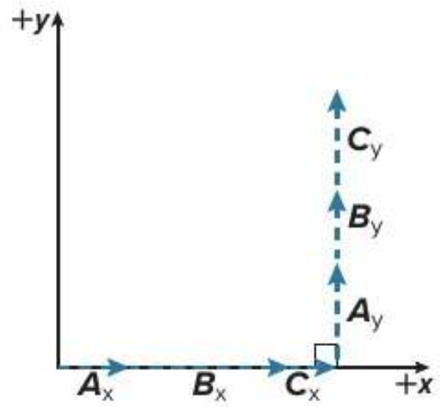
$$A_y = A\sin\theta = (5.0\text{ N})\sin 130° = 3.8\text{ N}$$

Figure 5 The coordinate plane is divided into four quadrants. A vector's components will be positive or negative depending on the vector's quadrant.

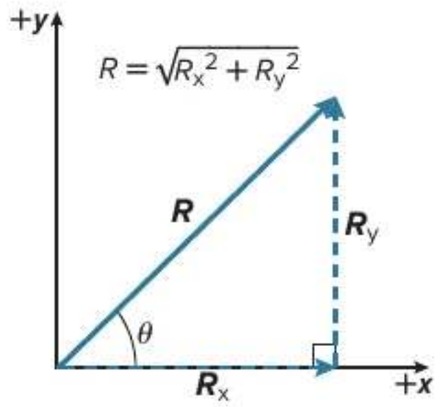
Classify *as positive or negative the components of a vector whose angle is 280°. In which quadrant does the vector lie?*



Add the vectors graphically by placing them tip to tail.



Add the *x*-components together and the *y*-components together.



The magnitude of ***R*** can be calculated using the Pythagorean theorem.

Figure 6 The vector sum of ***A***, ***B***, and ***C*** is the same as the vector sum of $\boldsymbol{R}_x$ and $\boldsymbol{R}_y$.

Algebraic Addition of Vectors

Figure 6 shows how resolving vectors into components makes adding vectors together easier. First, resolve each vector into its *x*- and *y*-components. Add the *x*-components to form the *x*-component of the resultant:

$$\boldsymbol{R}_x = \boldsymbol{A}_x + \boldsymbol{B}_x + \boldsymbol{C}_x.$$

Similarly, add the *y*-components to form the *y*-component of the resultant:

$$\boldsymbol{R}_y = \boldsymbol{A}_y + \boldsymbol{B}_y + \boldsymbol{C}_y.$$

Because $\boldsymbol{R}_x$ and $\boldsymbol{R}_y$ are at a right angle (90°), you can calculate the magnitude of the resultant vector using the Pythagorean theorem:

$$R^2 = R_x^{\,2} + R_y^{\,2}$$

Recall that the angle the vector makes with the *x*-axis is given by the following equation.

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

You can find the angle, or direction of the vector, using the $\tan^{-1}$ key on your calculator. When $\tan\theta > 0$, most calculators give the angle between 0° and 90°. When $\tan\theta < 0$, the angle will be between 0° and −90°.

PROBLEM-SOLVING STRATEGIES

Vector Addition

Use the following technique to solve problems for which you need to add or subtract vectors.

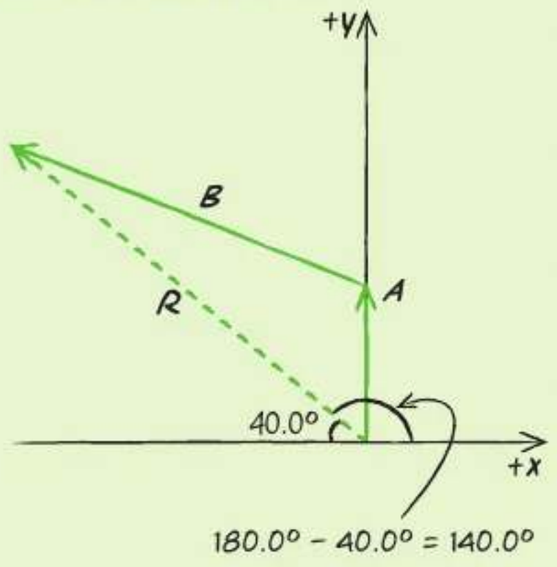
1. Choose a coordinate system.
2. Resolve the vectors into their *x*-components using $A_x = A\cos\theta$ and their *y*-components using $A_y = A\sin\theta$, where θ is the angle measured counterclockwise from the positive *x*-axis.
3. Add or subtract the component vectors in the *x*-direction.
4. Add or subtract the component vectors in the *y*-direction.
5. Use the Pythagorean theorem, $R = \sqrt{R_x^{\,2} + R_y^{\,2}}$, to find the magnitude of the resultant vector.
6. Use $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$ to find the angle of the resultant vector.

EXAMPLE Problem 2

FINDING YOUR WAY HOME You are on a hike. Your camp is 15.0 km away, in the direction 40.0° north of west. The only path through the woods leads directly north. If you follow the path 5.0 km before it opens into a field, how far, and in what direction, would you have to walk to reach your camp?

1 ANALYZE AND SKETCH THE PROBLEM

- Draw the resultant vector, ***R***, from your original location to your camp.
- Draw ***A***, the known vector, and draw ***B***, the unknown vector.



Known

$\boldsymbol{A} = 5.0$ km, due north

$\boldsymbol{R} = 15.0$ km, 40.0° north of west

$\theta = 140.0°$

Unknown

$\boldsymbol{B} = ?$

2 SOLVE FOR THE UNKNOWN

Find the components of ***R***.

$$R_x = R\cos\theta$$
$$= (15.0\text{ km})\cos 140.0° \qquad \text{Substitute } R = 15.0\text{ km}, \theta = 140.0°.$$
$$= 11.5\text{ km}$$
$$R_y = R\sin\theta$$
$$= (15.0\text{ km})\sin 140.0° \qquad \text{Substitute } R = 15.0\text{ km}, \theta = 140.0°.$$
$$= 9.64\text{ km}$$

Because ***A*** is due north, $A_x = 0.0$ km and $A_y = 5.0$ km.
Use the components of ***R*** and ***A*** to find the components of ***B***.

$$B_x = R_x - A_x$$
$$= -11.5\text{ km} - 0.0\text{ km} \qquad \text{Substitute } R_x = -11.5\text{ km}, A_x = 0.0\text{ km}.$$
$$= -11.5\text{ km} \qquad \text{The negative sign means that this component points west.}$$
$$B_y = R_y - A_y$$
$$= 9.64\text{ km} - 5.0\text{ km} \qquad \text{Substitute } R_y = 9.64\text{ km}, A_y = 5.0\text{ km}.$$
$$= 4.6\text{ km} \qquad \text{This component points north.}$$

Use the components of vector ***B*** to find the magnitude of vector ***B***.

$$B = \sqrt{B_x^2 + B_y^2}$$
$$= \sqrt{(-11.5\text{ km})^2 + (4.6\text{ km})^2} \qquad \text{Substitute } B_x = -11.5\text{ km}, B_y = 4.6\text{ km}.$$
$$= 12.4\text{ km}$$

Locate the tail of vector ***B*** at the origin of a coordinate system, and draw the components B_x and B_y. The vector ***B*** is in the second quadrant. Use the tangent to find the direction of vector ***B***.

$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right)$$
$$\theta = \tan^{-1}\left(\frac{4.6\text{ km}}{-11.5\text{ km}}\right) \qquad \text{Substitute } B_y = 4.6\text{ km}, B_x = -11.5\text{ km}.$$
$$= -22° \text{ or } 158° \qquad \text{The tangent of an angle is negative in quadrants II and IV, so two answers are possible.}$$

Since ***B*** is in the second quadrant, θ, measured from the positive x-axis, must be 158°. This direction can also be given as 22° north of west. Thus, $\boldsymbol{B} = 12.4$ km at 22° north of west.

3 EVALUATE THE ANSWER

- **Are the units correct?** Kilometers and degrees are correct.
- **Do the signs make sense?** They agree with the diagram.
- **Is the magnitude realistic?** The length of ***B*** should be longer than R_x because the angle between ***A*** and ***B*** is greater than 90°.

PRACTICE Problems

ADDITIONAL PRACTICE

Solve problems 5–10 algebraically. You may also solve some of them graphically to check your answers.

5. Sudhir walks 0.40 km in a direction 60.0° west of north, then goes 0.50 km due west. What is his displacement?
6. You first walk 8.0 km north from home then walk east until your displacement from home is 10.0 km. How far east did you walk?
7. In a coordinate system in which the positive *x*-axis is east, for what range of angles is the *x*-component positive? For what range is it negative?
8. Could a vector ever be shorter than one of its components? Could a vector ever be equal in length to one of its components? Explain.
9. Two ropes tied to a tree branch hold up a child's swing as shown in **Figure 7.** The tension in each rope is 2.28 N. What is the combined force (magnitude and direction) of the two ropes on the swing?
10. **CHALLENGE** Afua and Chrissy are going to sleep overnight in their tree house and are using some ropes to pull up a 3.20-kg box containing their pillows and blankets. The girls stand on different branches, as shown in **Figure 8,** and pull at the angles with the forces indicated. Find the *x*- and *y*-components of the initial net force on the box. *Hint: Draw a free-body diagram so you do not leave out a force.*

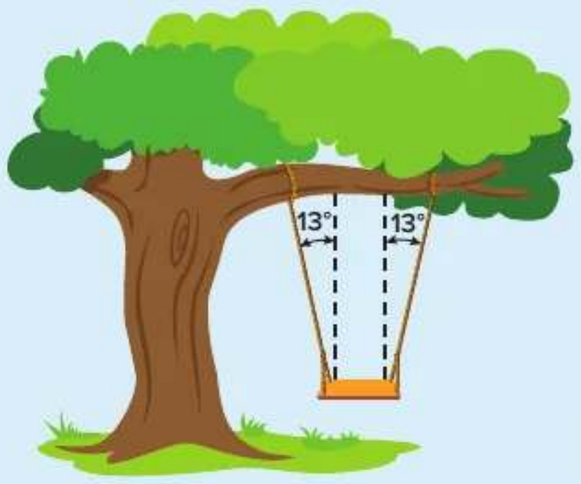


Figure 7



Figure 8

Check Your Progress

11. **Vectors** Use **Figure 9** for these questions.
    a. Find the components of vectors ***K***, ***L***, and ***M***.
    b. Find the sum of the three vectors.
    c. Subtract vector ***K*** from vector ***L***.

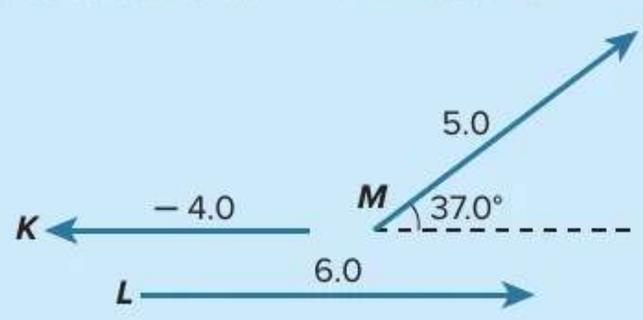


Figure 9

12. **Distance v. Displacement** Are distance and displacement always the same? Give an example that supports your conclusion.
13. **Commutative Operations** Vector addition is commutative because the order in which vectors are added does not matter.
    a. Use the vectors from **Figure 9** to show graphically that $M + L = L + M$.
    b. Are addition, subtraction, multiplication, and division commutative? Give an example of each operation to support your conclusion.
14. **Critical Thinking** You move a box through two unequal displacements. Could the displacements have directions such that the resultant displacement is zero? Suppose you now move the box through three displacements of unequal magnitude. Could the resultant displacement be zero? Support your conclusion with a diagram.

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LESSON 2

FRICTION

FOCUS QUESTION

What force causes a train to stop?

Kinetic and Static Friction

You push a book across your desk. When you stop pushing, the book stops due to friction.

Types of friction When you pushed your book across the desk, the book experienced a type of friction that acts on moving objects—**kinetic friction** is exerted on one surface by another when the two surfaces rub against each other because one or both surfaces are moving.

Now imagine trying to push a couch across the floor, as shown on the left in **Figure 10.** You push on it with a small force, but it does not move. Newton's laws tell you that the net force on the couch must be zero. There must be a second horizontal force acting on the couch that opposes your force and is equal in size. This force is **static friction,** which is the force exerted on one surface by another when there is no motion between the two surfaces.

You push harder, and the couch still does not move. The static friction force is increasing in response to your applied force. When you push hard enough, the couch begins to move as in the right side of **Figure 10.** There is a limit to how large the static friction force can be. Once your force is greater than this maximum static friction, the couch moves and kinetic friction acts on it.



Figure 10 An applied force is balanced by static friction up to a maximum limit. When this limit is exceeded, the object begins to move.

Identify *the type of friction force acting on the couch when it begins to move.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

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INVESTIGATE

GO ONLINE to find these activities and more resources.

Forensics Lab: Hit-and-Run Driver

Analyze data about friction forces to determine cause and effect at a crime scene.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Mathematical models for friction forces

The materials that the surfaces are made of affect the force of friction. For example, there is more friction between your shoes and concrete than there is between your socks and a polished wood floor. It might seem reasonable that the force of friction depends on either the surface area in contact or the speed at which the surfaces move past each other. Experiments have shown that this is not true. The normal force between the two objects does matter, however. The harder one object is pushed against the other, the greater the force of friction that results.

Get It?

Identify the two factors that affect friction forces.

Kinetic friction Imagine pulling a block along a surface at a constant velocity, as shown in **Figure 11.** Since the block is not accelerating, according to Newton's laws, the friction force must be equal and opposite to the force with which you pull. The spring scale in **Figure 11** indicates the force exerted on the block. You can then stack additional blocks on top of the first block to increase the normal force and repeat the measurement. **Table 1** shows the results of such an experiment.

Table 1 Kinetic Friction v. Normal Force (sandpaper)

Number of blocks	Normal force (N)	Kinetic friction (N)
1	0.98	0.53
2	1.96	0.95
3	2.94	1.4
4	3.92	1.8
5	4.90	2.3
6	5.88	3.1
7	6.86	3.3
8	7.84	4.0

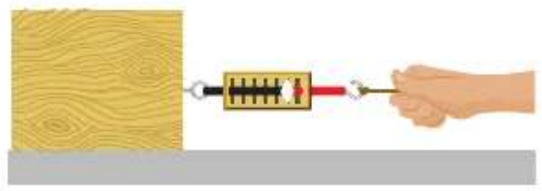


Figure 11 The spring scale applies a constant force on the block.

Plotting the data will yield a graph like the one in **Figure 12.** There is a linear relationship between the kinetic friction force and the normal force. The different lines on the graph correspond to dragging the block along different surfaces. Notice that the line for the sandpaper surface has a steeper slope than the line for the highly polished table. You would expect it to be much more difficult to pull the block along sandpaper than along a polished table, so the slope must be related to the magnitude of the resulting friction force. The slope of the line on a kinetic friction force v. normal force graph, designated μ_k, is called the **coefficient of kinetic friction** and relates the friction force to the normal force.

Kinetic Friction Force

The kinetic friction force equals the product of the coefficient of kinetic friction and the normal force.

$$F_{f,\,kinetic} = \mu_k F_N$$

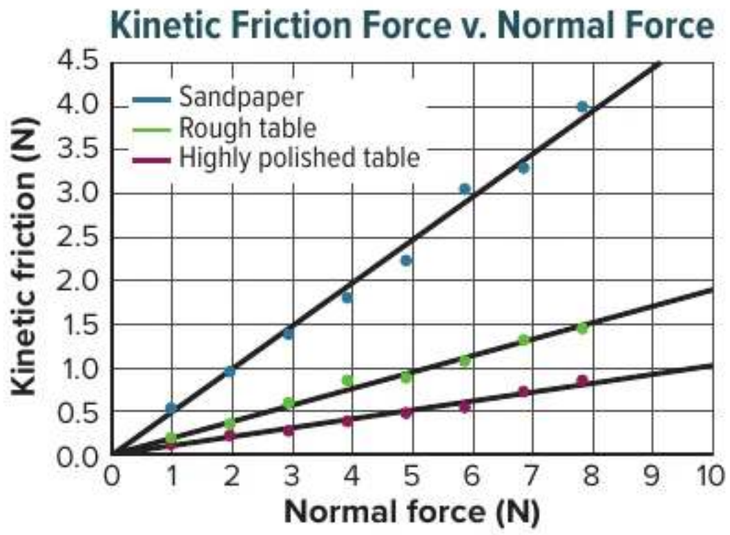


Figure 12 A plot of kinetic friction v. normal force for a block pulled along different surfaces shows a linear relationship between the two forces for each surface. The slope of the line is μ_k.

Compare *the coefficient of kinetic friction for the three surfaces shown on the graph.*

Get It?

Describe, in terms of friction, why the railway track maintenance mentioned at the beginning of this module should include wheel cleaning as well.

Static friction The maximum static friction force relates to the normal force in a similar way as the kinetic friction force. Remember that the force of static friction acts in response to a force trying to cause a stationary object to start moving. If there is no such force acting on an object, the force of static friction is zero. If there is a force trying to cause motion, the force of static friction will increase up to a maximum value before it is overcome and motion starts.

Static Friction Force

The static friction force is less than or equal to the product of the coefficient of static friction and the normal force.

$$F_{\text{f, static}} = \mu_s F_N$$

In the equation for the maximum static friction force, μ_s is the **coefficient of static friction** between the two surfaces. The maximum static friction force that must be overcome before motion can begin is $\mu_s F_N$. In the example of pushing the couch in **Figure 10,** the maximum force of static friction balances the force of the person pushing on the couch the instant before the couch begins to move.

Real-World Physics

ALL SURFACES, even surfaces that appear to be smooth, are rough at a microscopic level. If you look at a photograph of a graphite crystal magnified by a scanning tunneling microscope, the atomic level surface irregularities of the crystal are revealed. When two surfaces touch, the high points on each are in contact and temporarily bond. This is the origin of both static and kinetic friction. The details of this process are still unknown and are the subject of research in both physics and engineering.

Note that the equations for the kinetic and maximum static friction forces involve only the magnitudes of the forces. Also, the friction forces (F_f) are always perpendicular to the normal force (F_N).

Measuring coefficients of friction **Table 2** shows coefficients of friction between various surfaces. These coefficients are estimates for each combination of surfaces. Measurements of coefficients of friction are quite sensitive to the conditions of the surfaces. Surface impurities, such as dust or small amounts of oil from experimenters' hands, can significantly affect the results of the measurements.

Table 2 Typical Coefficients of Friction*

Surfaces	Coefficient of static friction (μ_s)	Coefficient of kinetic friction (μ_k)
Cast iron on cast iron	1.1	0.15
Glass on glass	0.94	0.4
Leather on oak	0.61	0.52
Nonstick coating on steel	0.04	0.04
Oak on oak	0.62	0.48
Steel on steel	0.78	0.42
Steel on steel (with castor oil)	0.15	0.08

*All measurements are for dry surfaces unless otherwise stated. All problems in this text assume dry surfaces.

CCC CROSSCUTTING CONCEPTS

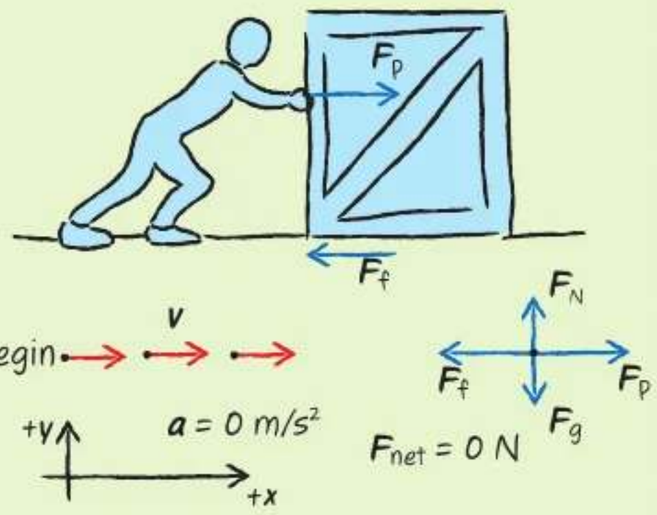
Cause and Effect Systems can be designed to cause a desired effect. When designing and building vehicles such as cars and airplanes, manufacturers use steel for many of the components. Do you think that any of the steel components are in direct contact with each other in a vehicle? What would be the effect if they were? Explain your answer, using evidence from **Table 2.**

EXAMPLE Problem 3

BALANCED FRICTION FORCES You push a 25.0-kg wooden box across a wooden floor at a constant speed of 1.0 m/s. The coefficient of kinetic friction is 0.20. How large is the force that you exert on the box?

1 ANALYZE AND SKETCH THE PROBLEM

- Identify the forces, and establish a coordinate system.
- Draw a motion diagram indicating constant v and $a = 0$.
- Draw the free-body diagram.



Known

$m = 25.0$ kg $\quad v = 1.0$ m/s

$a = 0.0$ m/s^2 $\quad \mu_k = 0.20$

Unknown

$F_{\text{person on box}} = ?$

2 SOLVE FOR THE UNKNOWN

The normal force is in the y-direction, and the box does not accelerate in that direction.

$$F_N = -F_g$$

$$= -mg \qquad \text{Substitute } F_g = -mg$$

$$= -(25.0 \text{ kg})(-9.8 \text{ N/kg}) \qquad \text{Substitute } m = 25.0 \text{ kg}, g = -9.8 \text{ N/kg}$$

$$= +245 \text{ N}$$

The pushing force is in the x-direction; v is constant; thus the box does not accelerate.

$$F_{\text{person on box}} = \mu_k F_N$$

$$= (0.20)(245 \text{ N}) \qquad \text{Substitute } \mu_k = 0.20, F_N = 245 \text{ N}$$

$$= 49 \text{ N}$$

$F_{\text{person on box}} = 49$ N, to the right

3 EVALUATE THE ANSWER

- **Are the units correct?** Force is measured in newtons.
- **Do the signs make sense?** The positive sign agrees with the sketch.
- **Is the magnitude realistic?** The pushing force is $\frac{1}{5}$ the weight of the box. This corresponds with $\mu_k = 0.20 = \frac{1}{5}$.

PRACTICE Problems

ADDITIONAL PRACTICE

15. Gwen exerts a 36-N horizontal force as she pulls a 52-N sled across a cement sidewalk at constant speed. What is the coefficient of kinetic friction between the sidewalk and the metal sled runners? Ignore air resistance.

16. Mr. Ames is dragging a box full of books from his office to his car. The box and books together have a combined weight of 134 N. If the coefficient of static friction between the pavement and the box is 0.55, how hard must Mr. Ames push horizontally on the box in order to start it moving?

17. Thomas sits on a small rug on a polished wooden floor. The coefficient of kinetic friction between the rug and the slippery wooden floor is only 0.12. If Thomas weighs 650 N, what horizontal force is needed to pull the rug and Thomas across the floor at a constant speed?

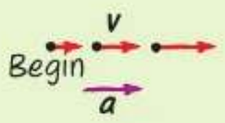
18. CHALLENGE You need to move a 105-kg sofa to a different location in the room. It takes a 403-N force to start the sofa moving. What is the coefficient of static friction between the sofa and the carpet?

EXAMPLE Problem 4

UNBALANCED FRICTION FORCES Imagine that the force you exert on the 25.0-kg box in Example Problem 3 is doubled.

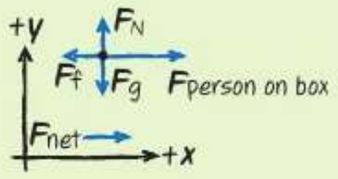
a. What is the resulting acceleration of the box?

b. How far will you push the box if you push it for 3 s?



1 ANALYZE AND SKETCH THE PROBLEM

- Draw a motion diagram showing v and a.
- Draw the free-body diagram with a doubled $F_{\text{person on box}}$.



Known

$m = 25.0\ \text{kg}$ $\quad \mu_k = 0.20$

$v = 1.0\ \text{m/s}$ $\quad F_{\text{person on box}} = 2(49\ \text{N}) = 98\ \text{N}$

$t = 3.0\ \text{s}$

Unknown

$a = ?$

2 SOLVE FOR THE UNKNOWN

a. The normal force is in the y-direction, and the box does not accelerate in that direction.

$$\boldsymbol{F}_N = -\boldsymbol{F}_g$$
$$= -mg$$

The box does accelerate in the x-direction. So the forces must be unequal.

$$F_{net} = F_{\text{person on box}} - F_f$$
$$ma = F_{\text{person on box}} - F_f \qquad \text{Substitute } F_{net} = ma.$$
$$a = \frac{F_{\text{person on box}} - F_f}{m}$$

Find F_f and substitute it into the expression for a.

$$F_f = \mu_k F_N$$
$$= \mu_k mg \qquad \text{Substitute } F_N = mg.$$
$$a = \frac{F_{\text{person on box}} - \mu_k mg}{m} \qquad \text{Substitute } F_f = \mu_k mg.$$
$$= \frac{98\ \text{N} - (0.20)(25.0\ \text{kg})(9.8\ \text{N/kg})}{25.0\ \text{kg}} \qquad \text{Substitute } F_{\text{person on box}} = 98\ \text{N}, \mu_k = 0.20, m = 25.0\ \text{kg}, g = 9.8\ \text{N/kg}.$$
$$= 2.0\ \text{m/s}^2$$

b. Use the relationship between distance, time, and constant acceleration.

$$x_f = x_i + v_i t_f + \left(\frac{1}{2}\right) a t_f^2$$
$$x_f = 0\ \text{m} + v_i t + \left(\frac{1}{2}\right) a t^2 \qquad \text{Substitute } x_i = 0\ \text{m}.$$
$$= (1.0\ \text{m/s})(3.0\ \text{s}) + \left(\frac{1}{2}\right)(2.0\ \text{m/s}^2)(3.0\ \text{s})^2 \qquad \text{Substitute } v_i = 1.0\ \text{m/s}, t = 3.0\ \text{s}, a = 2.0\ \text{m/s}^2.$$
$$= 12\ \text{m}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** a is measured in m/s^2, and x_f is measured in meters.
- **Does the sign make sense?** In this coordinate system, the sign should be positive.
- **Is the magnitude realistic?** 12 m is about the length of two full-sized cars. It is realistic to push a box this distance in 3 s.

PRACTICE Problems

ADDITIONAL PRACTICE

19. You want to move a 41-kg bookcase to a different place in the living room. If you push with a force of 65 N and the bookcase accelerates at 0.12 m/s^2, what is the coefficient of kinetic friction between the bookcase and the carpet?

20. Consider the force pushing the box in Example Problem 4. How long would it take for the velocity of the box to double to 2.0 m/s?

21. Ke Min is driving at 23 m/s. He sees a tree branch lying across the road. He slams on the brakes when the branch is 60.0 m in front of him. If the coefficient of kinetic friction between the car's locked tires and the road is 0.41, will the car stop before hitting the branch? The car has a mass of 1200 kg.

22. CHALLENGE Isabel pushes a shuffleboard disk, accelerating it to a speed of 6.5 m/s before releasing it as indicated in **Figure 13.** If the coefficient of kinetic friction between the disk and the concrete court is 0.31, how far does the disk travel before it comes to a stop? Will Isabel's shot stop in the 10-point section of the board?

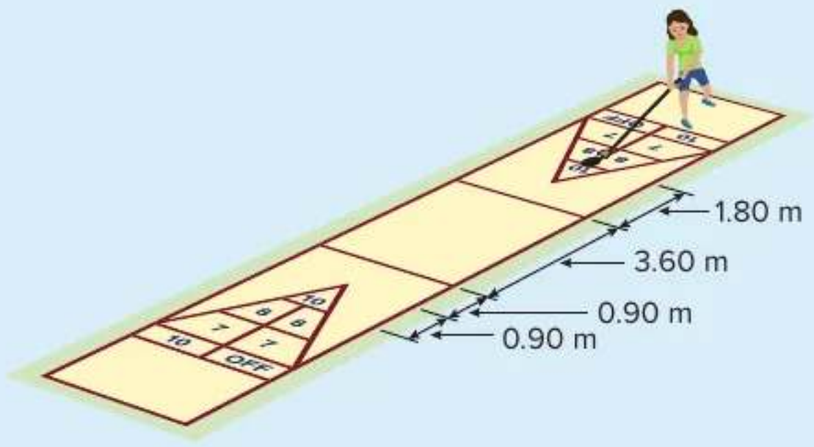


Figure 13

Check Your Progress

23. **Analyze** Compare static friction and kinetic friction. How are the frictional forces similar, and how do they differ?

24. **Friction** At a wedding reception, you notice a boy who looks like his mass is about 25 kg running across the dance floor, then sliding on his knees until he stops. If the coefficient of kinetic friction between the boy's pants and the floor is 0.15, what is the friction force acting on him as he slides?

25. **Velocity** Dinah is playing cards with her friends, and it is her turn to deal. A card with a mass of 2.3 g slides 0.35 m along the table before it stops. If the coefficient of kinetic friction between the card and the table is 0.24, what was the initial speed of the card as it left Dinah's hand?

26. **Force** The coefficient of static friction between a 40.0-kg picnic table and the ground is 0.43. Find the greatest horizontal force that could be exerted on the table without moving the table.

27. **Acceleration** You push a 13-kg table in the cafeteria with a horizontal force of 20 N, but the table does not move. You then push the table with a horizontal force of 25 N, and it accelerates at 0.26 m/s^2. What, if anything, can you conclude about the coefficients of static and kinetic friction?

28. **Critical Thinking** Rachel puts a dresser in the back of her pickup truck. When the truck accelerates forward, what force accelerates the dresser? Under what circumstances could the dresser slide? In which direction?

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LESSON 3

FORCES IN TWO DIMENSIONS

FOCUS QUESTION

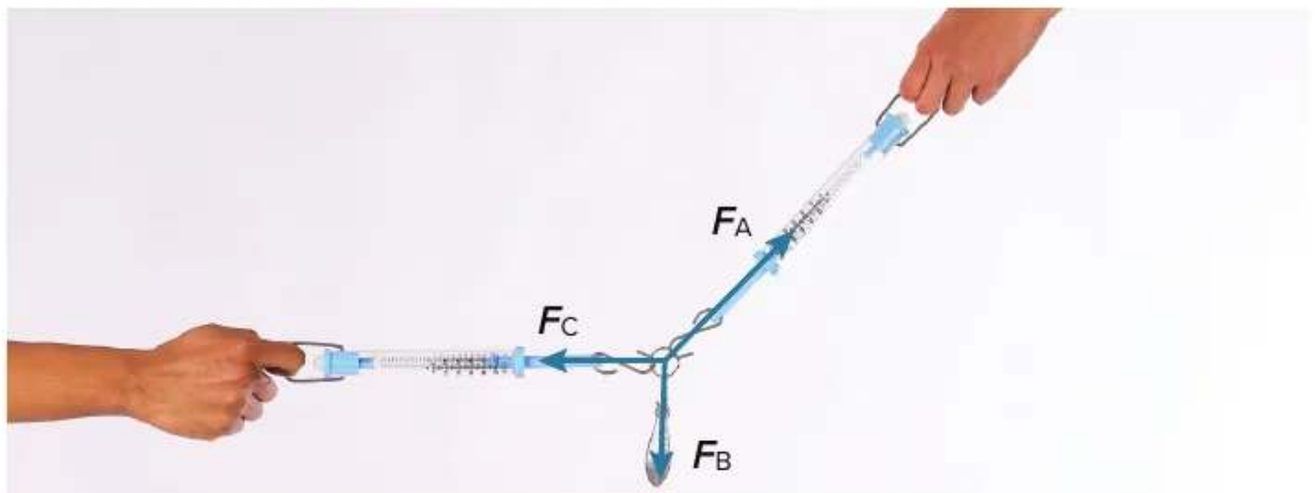
How can you analyze forces on a train as it climbs a hill?

Equilibrium Revisited

So far, you have considered motion along a horizontal surface. Now you will analyze situations in which the forces acting on an object are at angles other than 90°.

When the net force on an object is zero, the object is in equilibrium. According to Newton's laws, the object will not accelerate because there is no net force acting on it; an object in equilibrium moves with constant velocity. (Remember that staying at rest is a state of constant velocity.) Equilibrium can also occur if more than two forces act on an object. As long as the net force on the object is zero, the object is in equilibrium.

What is the net force acting on the ring in **Figure 14?** The free-body diagram in the figure shows the three forces acting on the ring. The ring is not accelerating, so the net force is zero. The free-body diagram, however, does not immediately indicate that the net force is zero. To find the net force, you must add all the vectors together. **Figure 15** on the next page shows the process of adding the force vectors to discover the net force.



Richard Hutchings/Digital Light Source

Figure 14 The ring does not accelerate, so the net force acting on it must be zero.

Compare *the vertical component of the force pulling up and to the right to the weight of the mass hanging from the ring.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Sliding Down a Slope

Use mathematics to apply Newton's second law of motion to a system on an inclined plane.

Quick Investigation: Equilibrium

Carry out an investigation of the forces on a system in equilibrium.



Figure 15 The forces acting on the ring sum to a zero net force.

Equilibrants **Figure 15** shows the addition of the three forces, F_A, F_B, and F_C, acting on the ring. Note that the three vectors form a closed triangle. The sum $F_A + F_B + F_C$ is zero. Now suppose two forces are exerted on an object and the sum is not zero, as shown in the free-body diagram in **Figure 16.** We want to find a third force that, when added to the other two, would add up to zero and put the object in equilibrium. First, add the forces already being exerted on the object. This single force that produces the same effect as the two or more individual forces together is called the resultant force. To put the object in equilibrium, add another force, called the **equilibrant,** that has the same magnitude as the resultant force but is in the opposite direction. Notice that in **Figure 15** the force F_C is the equilibrant of $F_A + F_B$. **Figure 16** illustrates the procedure for finding the equilibrant for two vectors.

Get It?

Identify the relationship between the equilibrant and the resultant vector.

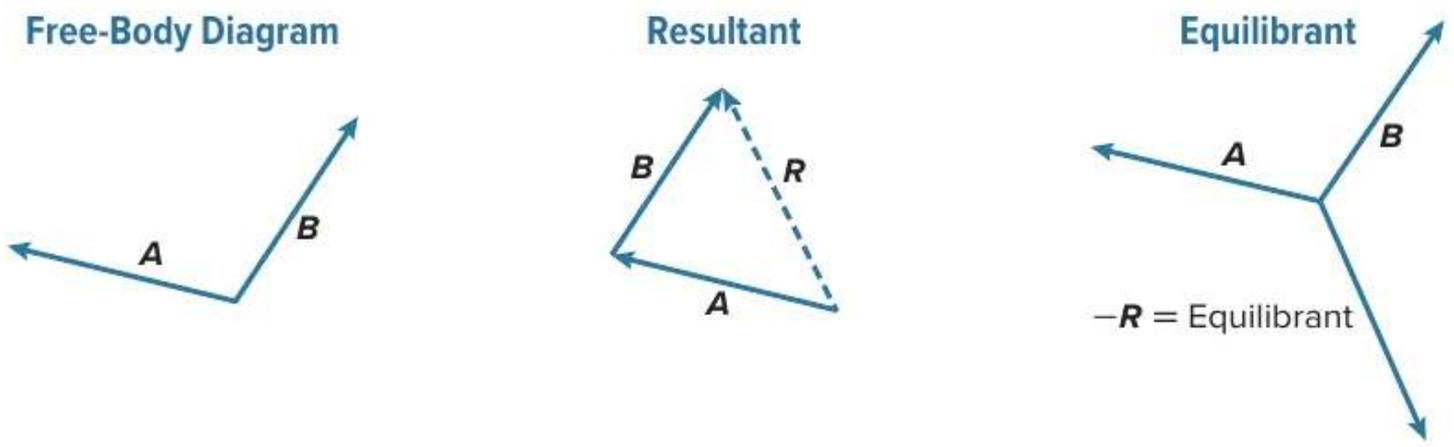


Figure 16 The equilibrant is the force required to put an object in equilibrium. This general procedure for finding the equilibrant works for any number of vectors.

PHYSICS Challenge

Find the equilibrant for the entire set of forces listed below, also shown in the figure.

F_1 = 61.0 N at 17.0° north of east

F_2 = 38.0 N at 64.0° north of east

F_3 = 54.0 N at 8.0° west of north

F_4 = 93.0 N at 53.0° west of north

F_5 = 65.0 N at 21.0° south of west

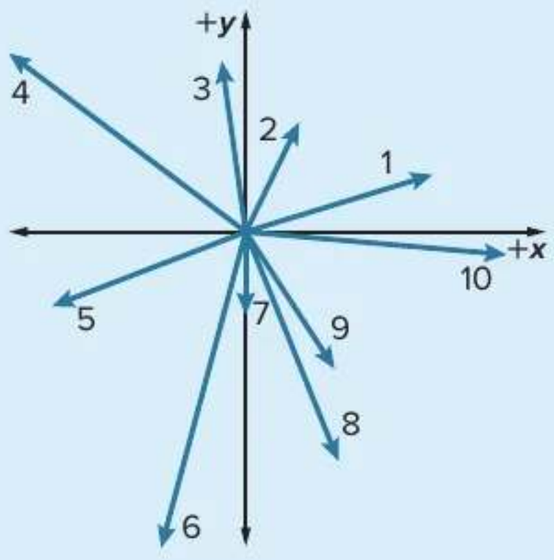
F_6 = 102.0 N at 15.0° west of south

F_7 = 26.0 N south

F_8 = 77.0 N at 22.0° east of south

F_9 = 51.0 N at 33.0° east of south

F_{10} = 82.0 N at 5.0° south of east



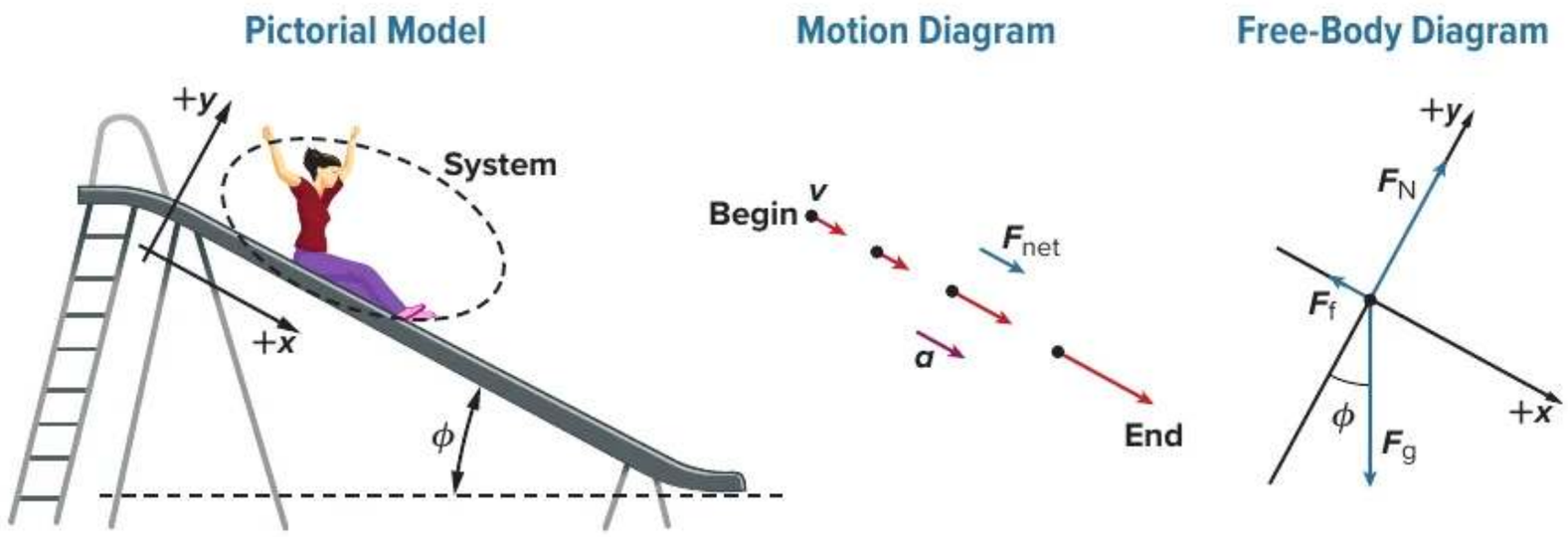


Figure 17 The girl's weight causes her to accelerate down the slide.

Describe *how the component of the girl's weight parallel to the incline changes when the angle between the slide and the horizontal increases.*

Inclined Planes

You have applied Newton's laws to a variety of situations but only to motions that were either horizontal or vertical. How would you apply Newton's laws in a situation such as the one in **Figure 17,** in which you want to find the net force on the girl?

First, identify the forces acting on the system. In this case, the girl is the system. The gravitational force on the girl is downward, toward the center of Earth. The normal force acts perpendicular to the slide, and the kinetic friction force acts parallel to the slide. You can see the resulting free-body diagram on the right in **Figure 17.** You know from experience that the girl's acceleration will be along the slide.

Choosing a coordinate system Second, you should carefully choose your coordinate system. Because the girl's acceleration is parallel to the incline, one axis, the x-axis, should be in that direction. The y-axis is perpendicular to the x-axis and perpendicular to the incline's surface. With this coordinate system, you now have two forces, the normal force and the friction force, in the directions of the coordinate axes. The weight, however, has components in both the x- and y-directions. This means that when you place an object on an inclined plane, the magnitude of the normal force between the object and the plane usually will not equal the object's weight.

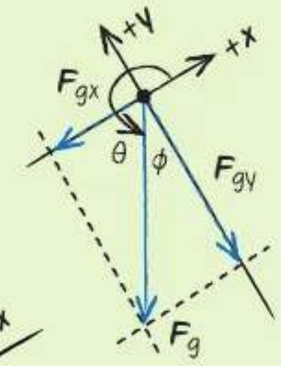
Get It?

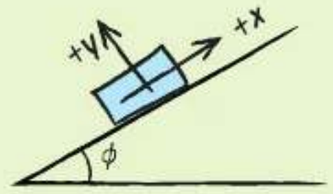
Explain why you would choose the x-direction to be parallel to the slope of an inclined plane.

You will need to apply Newton's laws once in the x-direction and once in the y-direction. Because the weight does not point in either of these directions, you will need to break this vector into its x- and y-components before you can sum the forces in these two directions. Example Problem 5 and Example Problem 6 both show this procedure.

EXAMPLE Problem 5

COMPONENTS OF WEIGHT FOR AN OBJECT ON AN INCLINE A 562-N crate is resting on a plane inclined 30.0° above the horizontal. Find the components of the crate's weight that are parallel and perpendicular to the plane.





1 ANALYZE AND SKETCH THE PROBLEM

- Include a coordinate system with the positive *x*-axis pointing uphill.
- Draw the free-body diagram showing $\boldsymbol{F}_g$, the components F_{gx} and F_{gy}, and the angles θ and ϕ.

Known
$\boldsymbol{F}_g = 562$ N down
$\phi = 30.0°$

Unknown
$F_{gx} = ?$
$F_{gy} = ?$
$\theta = ?$

2 SOLVE FOR THE UNKNOWN

$\theta + \phi = 270°$ The angle from the positive *x*-axis to the negative *y*-axis is 270°.

$\theta = 270° - 30°$ Rearrange, substitute $\phi = 30°$, and solve for θ.

$= 240°$

$F_{gx} = F_g(\cos\theta)$

$= (562\text{ N})(\cos 240.0°)$ Substitute $F_g = 562$ N, $\theta = 240.0°$.

$= -281$ N

$F_{gy} = F_g(\sin\theta)$

$= (562\text{ N})(\sin 240.0°)$ Substitute $F_g = 562$ N, $\theta = 240.0°$.

$= -487$ N

3 EVALUATE THE ANSWER

- **Are the units correct?** Force is measured in newtons.
- **Do the signs make sense?** The components point in directions opposite to the positive axes.
- **Are the magnitudes realistic?** The values are less than F_g, as expected.

PRACTICE Problems

ADDITIONAL PRACTICE

29. An ant climbs at a steady speed up the side of its anthill, which is inclined 30.0° from the vertical. Sketch a free-body diagram for the ant.

30. Ryan and Becca are moving a folding table out of the sunlight. A cup of lemonade, with a mass of 0.44 kg, is on the table. Becca lifts her end of the table before Ryan does, and as a result, the table makes an angle of 15.0° with the horizontal. Find the components of the cup's weight that are parallel and perpendicular to the plane of the table.

31. Fernando, who has a mass of 43.0 kg, slides down the banister at his grandparents' house. If the banister makes an angle of 35.0° with the horizontal, what is the normal force between Fernando and the banister?

32. CHALLENGE A suitcase is on an inclined plane as shown in **Figure 18.** At what angle θ will the component of the suitcase's weight parallel to the plane be equal to half the component of its weight perpendicular to the plane?



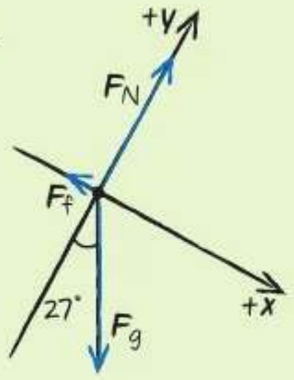
Figure 18

EXAMPLE Problem 6

SLIDE Ichiko, who has a mass of 45 kg, is going down a slide sloped at 27°. The coefficient of kinetic friction is 0.23. How fast does she slide 1.0 s after starting from rest?

1 ANALYZE AND SKETCH THE PROBLEM

- Establish a coordinate system.
- Draw a free-body diagram that shows the forces acting on the girl as she travels down the slide.
- The three forces acting on the girl are the normal force, kinetic friction, and the girl's weight.



Known

$m = 45$ kg $\quad \phi = 27°$

$\mu_k = 0.23 \quad v_i = 0.0$ m/s

$t = 1.0$ s

Unknown

$a = ?$

$v_f = ?$

$\theta = ?$

2 SOLVE FOR THE UNKNOWN

$\theta = 270° + \phi$ The angle from the positive x-axis to the negative y-axis is 270°.

$= 270° + 27°$ Substitute $\phi = 27°$ and solve for θ.

$= 297°$

y-direction:

$F_{net,y} = ma_y$ There is no acceleration in the y-direction, so $a_y = 0.0$ m/s^2.

$= 0.0$ N

Add forces in the y-direction to find F_N.

$F_N + F_{gy} = F_{net,y}$

$F_N = -F_{gy}$ Substitute $F_{net,y} = 0.0$ N, rearrange, and solve for F_N.

$= -mg(\sin\theta)$ Substitute $F_{gy} = mg\ (\sin\theta)$.

x-direction:

Use the net force in the x-direction and Newton's second law to solve for a.

$F_{net,x} = F_{gx} - F_f$ F_f is negative because it is in the negative x-direction.

$ma_x = mg(\cos\theta) - \mu_k F_N$ Substitute $F_{net,x} = ma_x$, $F_{gx} = mg\ (\cos\theta)$, $F_f = \mu_k F_N$.

$ma = mg(\cos\theta) + \mu_k mg(\sin\theta)$ Substitute $a_x = a$ (all acceleration is in the x-direction), $F_N = -mg\ (\sin\theta)$.

$a = g(\cos\theta + \mu_k \sin\theta)$

$= (9.8\text{ m/s}^2)(\cos 297° + (0.23)(\sin 297°))$ Substitute $g = 9.8$ m/s^2, $\theta = 297°$, $\mu_k = 0.23$.

$= 2.4\text{ m/s}^2$

Because v_i, a, and t are all known, use the relationship between velocity, acceleration, and time.

$v_f = v_i + at$

$= 0.0\text{ m/s} + (2.4\text{ m/s}^2)(1.0\text{ s})$ Substitute $v_i = 0.0$ m/s, $a = 2.4$ m/s^2, $t = 1.0$ s.

$= 2.4$ m/s

3 EVALUATE THE ANSWER

- **Are the units correct?** Performing dimensional analysis on the units verifies that v_f is in m/s and a is in m/s^2.
- **Do the signs make sense?** Because v_f and a are both in the $+x$-direction, the signs make sense.
- **Are the magnitudes realistic?** The velocity is similar to a person's running speed, which is realistic for a steep slide and a low coefficient of kinetic friction.

PRACTICE Problems

ADDITIONAL PRACTICE

33. Consider the crate on the incline in Example Problem 5. Calculate the magnitude of the acceleration. After 4.00 s, how fast will the crate be moving?

34. Jorge decides to try the slide discussed in Example Problem 6. Jorge's trip down the slide is quite different from Ichiko's. After giving himself a push to get started, Jorge slides at a constant speed. What is the coefficient of kinetic friction between Jorge's pants and the slide?

35. Stacie, who has a mass of 45 kg, starts down a slide that is inclined at an angle of 45° with the horizontal. If the coefficient of kinetic friction between Stacie's shorts and the slide is 0.25, what is her acceleration?

36. CHALLENGE You stack two physics books on top of each other as shown in **Figure 19.** You tilt the bottom book until the top book just begins to slide. You perform five trials and measure the angles given in **Table 3.**

a. What is the average ϕ measured during the five trials?

b. What is the coefficient of static friction between the covers of the two books? Use the average ϕ found in part **a.**

c. You measure the top book's acceleration down the incline to be 1.3 m/s^2. What is the coefficient of kinetic friction? Assume ϕ is the average value found in part **a.**



Figure 19

Table 3 Trial Number and Angle of Tilt

Trial	ϕ
1	21°
2	17°
3	21°
4	18°
5	19°

Check Your Progress

37. **Acceleration** A rope pulls a 63-kg water skier up a 14.0° incline with a tension of 512 N. The coefficient of kinetic friction between the skier and the ramp is 0.27. What are the magnitude and direction of the skier's acceleration?

38. **Forces** One way to get a car unstuck is to tie one end of a strong rope to the car and the other end to a tree, then pull the rope at its midpoint at right angles to the rope. Draw a free-body diagram. Explain how even a small force on the rope can exert a large force on the car.

39. **Mass** A large scoreboard is suspended from the ceiling of a sports arena by ten strong cables. Six of the cables make an angle of 8.0° with the verticals and the other four make an angle of 10.0°. If the tension in each cable is 1300 N, what is the scoreboard's mass?

40. **Equilibrium** You are hanging a painting using two lengths of wire. The wires will break if the force is too great. Should you hang the painting as shown on the left or the right of **Figure 20?** Explain.

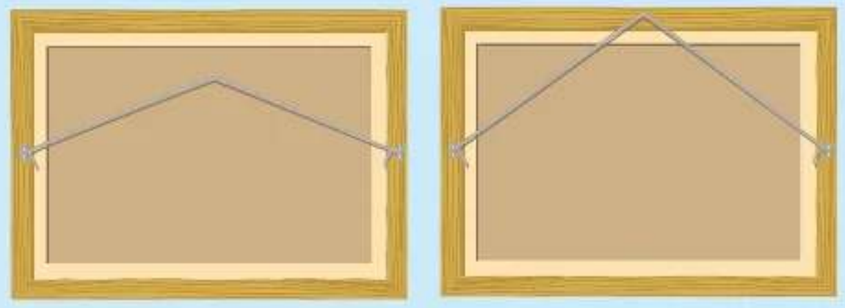


Figure 20

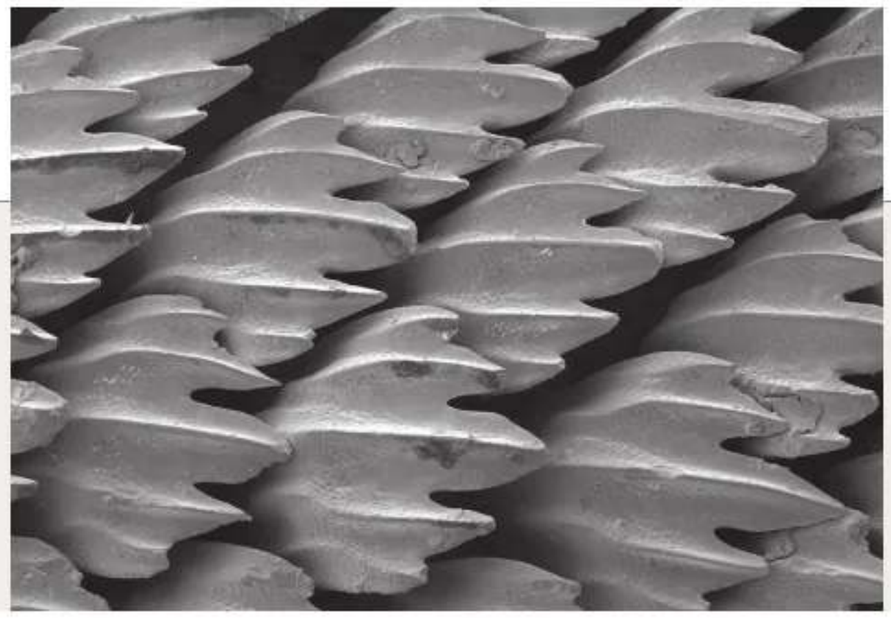
41. **Critical Thinking** Can the coefficient of friction ever have a value such that a child would be able to slide *up* a slide at a constant velocity? Explain. Assume that no one pushes or pulls on the child.

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ENGINEERING & TECHNOLOGY

More or Less—Using Biomimetics to Control Friction

As solutions are designed to meet engineering challenges, the force of friction needs to be considered. In some applications, friction between surfaces needs to be maximized. In other applications, friction between surfaces needs to be minimized. Structures of living things have provided insight into how these goals can be accomplished.



This SEM image shows the structure of shark skin. Scientists and engineers apply properties of this structure to reduce friction on everything from boats to airplanes.

Biomimetics

Biomimetics is the application of ideas from nature and living things to solve design problems. For example, burdock seeds stick to living things to move from place to place. Scientists imitated the structure of burdock seeds to develop hook-and-loop tape.

Using Biomimetics to Increase Friction

Clingfish are a species that live near the coast, so are subject to tides. These fish have a suction-cup-like disc with an astonishing ability to hold the fish in place in their ebbing and flowing environment. Friction is one of the forces that make this clingfish structure so effective at sticking to surfaces, even when they are wet. Scientists have found that the outer edge of the disc is made up of hair-like structures so tiny they require a microscope to see. Recall that when two surfaces touch, high points on each surface temporarily bond. The tiny hair-like structures of the clingfish's disc vastly increase the number of points of contact between the fish and the surface to which they cling. Engineers can apply information about the structure of the clingfish disc when they need to maximize friction between two surfaces, such as in the development of adhesives.

Using Biomimetics to Decrease Friction

Friction can negatively affect the motion of vehicles and the function of machines, so engineers look for ways to reduce the friction between surfaces. Shark skin is made up of structures called denticles. Denticles have several properties that interest engineers, including the reduction of friction between the shark and the water it moves through. Applying this technology to vehicles and other machines has the potential to decrease friction, reducing fuel usage and energy consumption.



USE A MODEL TO DESCRIBE

Draw a labeled diagram to describe how friction can be increased or decreased using biomimetics. You might want to carry out additional research to gather details to add to your model.

hooky13/Getty Images

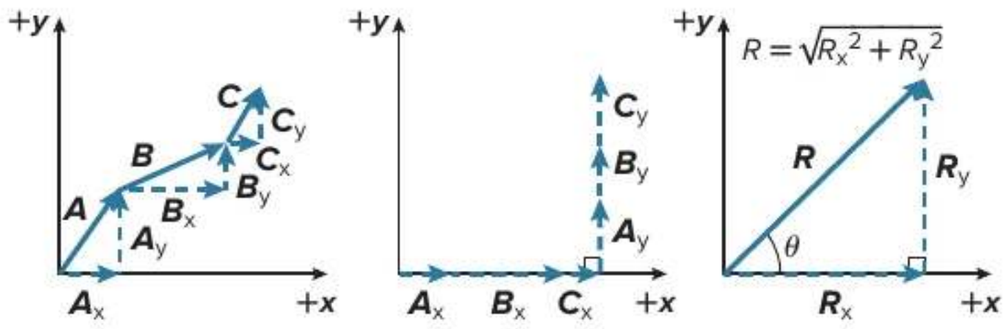
MODULE 5
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 VECTORS

- components
- vector resolution

- Vectors are added graphically by placing the tail of the second vector on the tip of the first vector. The resultant vector points from the tail of the first vector to the tip of the final vector.
- The components of a vector are projections of the component vectors onto axes. Vectors can be summed by separately adding the x- and y-components.



- When two vectors are at right angles, you can use the Pythagorean theorem to determine the magnitude of the resultant vector. The law of sines and the law of cosines can be used to find the resultant of any two vectors.

$$R^2 = A^2 + B^2$$

$$\text{Law of Sines: } \frac{R}{\sin\theta} = \frac{A}{\sin a} = \frac{B}{\sin b}$$

$$\text{Law of Cosines: } R^2 = A^2 + B^2 - 2AB\cos\theta$$

Lesson 2 FRICTION

- kinetic friction
- static friction
- coefficient of kinetic friction
- coefficient of static friction

- Friction is a force that acts parallel to the surfaces when two surfaces touch.
- The kinetic friction force equals the coefficient of kinetic friction times the normal force.

$$F_{f,\,\text{kinetic}} = \mu_k F_N$$

- The static friction force is less than or equal to the coefficient of static friction times the normal force.

$$F_{f,\,\text{static}} \leq \mu_s F_N$$

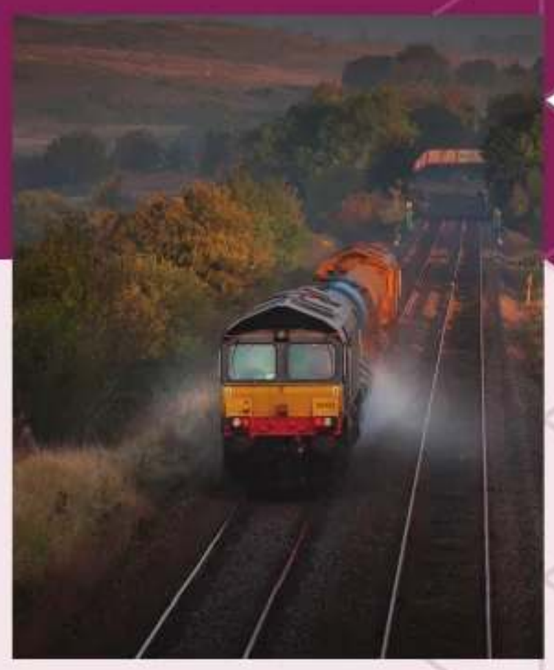
Lesson 3 FORCES IN TWO DIMENSIONS

- equilibrant

- The equilibrant is a force of the same magnitude but opposite direction as the resultant force, which is the sum of all the other forces acting on an object.
- Friction forces are parallel to an inclined plane but point up the plane if the motion of the object is down the plane. An object on an inclined plane has a component of the force of gravity parallel to the plane; this component can accelerate the object down the plane.

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

Why is this specialized train washing the train tracks?

CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

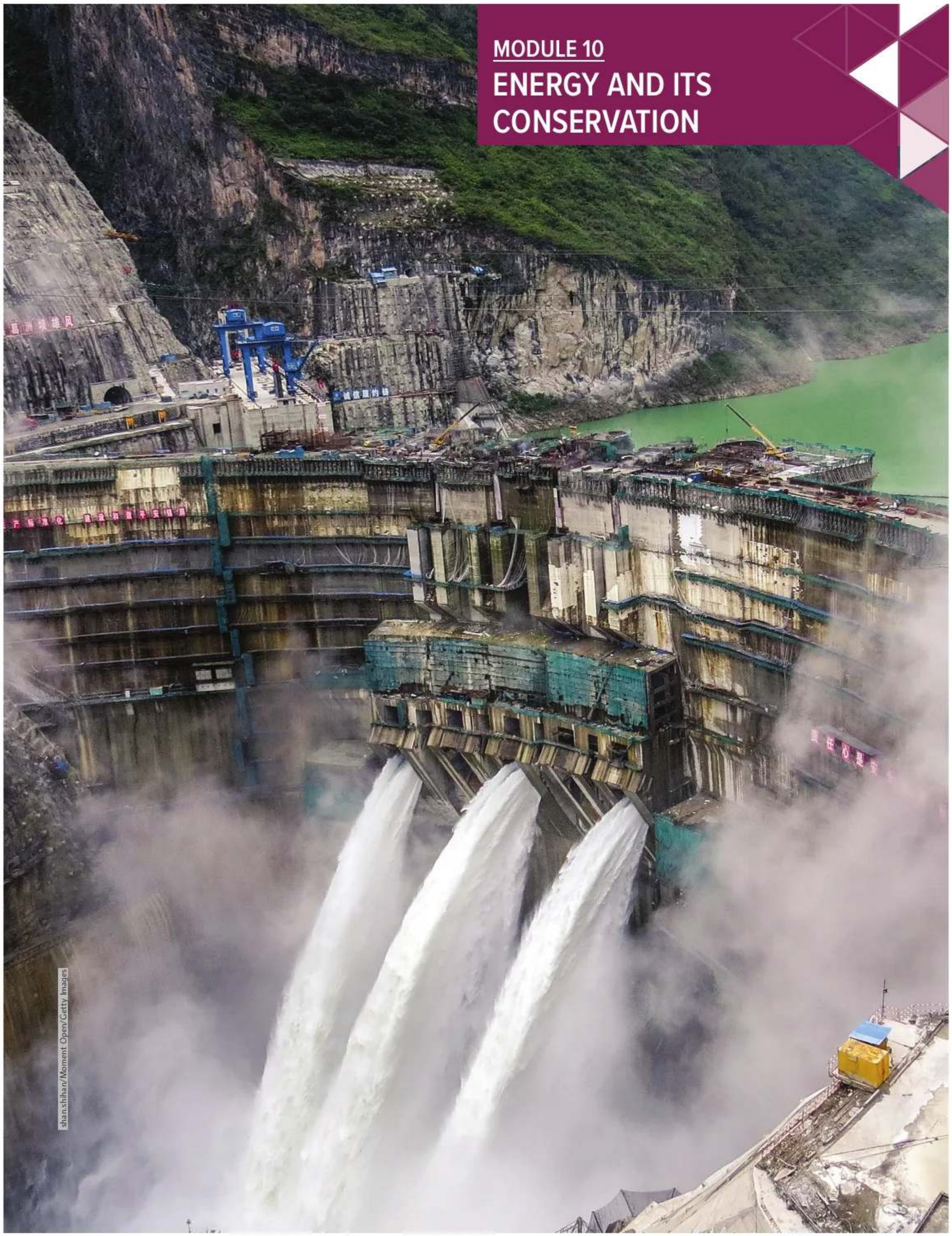
Who goes down the slide fastest?

Margaret Mary, Doug, and Kako are at a local amusement park. They see an attraction called the Giant Slide, which is simply a very long and high inclined plane. Visitors at the amusement park climb a long flight of stairs to the top of the 27° inclined plane and are given canvas sacks. They sit on the sacks and slide down the plane 70 m long. At the time when the three friends walked past the slide, a 135-kg man and a 20-kg boy are each at the top preparing to slide down. "I wonder how much less time it will take the man to slide down than it will take the boy," said Margaret Mary. "I think the boy will take less time," said Doug. "You're both wrong," said Kako. "They will reach the bottom at the same time."

CER Analyze and Interpret Data

1. **Sketch the situation.**
2. **Claim** Who is correct?
3. **Evidence and Reasoning** Justify your claim.

robert france/Alamy Stock Photo



MODULE 10

ENERGY AND ITS CONSERVATION

shan.shihan/Moment Open/Getty Images

MODULE 10

ENERGY AND ITS CONSERVATION

ENCOUNTER THE PHENOMENON

How can energy from power plants be stored in the power grid for later use?



GO ONLINE to play a video about energy can be stored for later use.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about how energy from power plants can be stored in the power grid for later use. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 2: Explore & Explain: Gravitational Potential Energy



LESSON 3: Explore & Explain: Analyzing Collisions

Smithsonian

Additional Resources

(t)Video Supplied by BBC Worldwide Learning; (bl)Sviat Studio/Shutterstock; (br)A_Lesik/Shutterstock

LESSON 1

WORK AND ENERGY

FOCUS QUESTION

What is energy?

Work

Consider a force exerted on an object while the object moves a certain distance, such as the book bag in **Figure 1.** There is a net force, so the object is accelerated, $a = \frac{F}{m}$, and its velocity changes. Recall from your study of motion that acceleration, velocity, and distance are related by the equation $v_f^2 = v_i^2 + 2ad$. This can be rewritten as $2ad = v_f^2 - v_i^2$. Replace a with the term $\frac{F}{m}$ to get $2\left(\frac{F}{m}\right)d = v_f^2 - v_i^2$. Multiplying both sides by $\frac{m}{2}$ gives $Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.

The left side of the equation describes an action that was done to the system by the external world. Recall that a system is the object or objects of interest and the external world is everything else. A force (F) was exerted on a system while the point of contact moved. When a force is applied through a displacement, **work** (W) is done on the system.

The SI unit of work is called a **joule** (J). One joule is equal to 1 N·m. One joule of work is done when a force of 1 N acts on a system over a displacement of 1 m. An apple weighs about 1 N, so it takes roughly 1 N of force to lift the apple at a constant velocity. Thus, when you lift an apple 1 m at a constant velocity, you do 1 J of work on it.

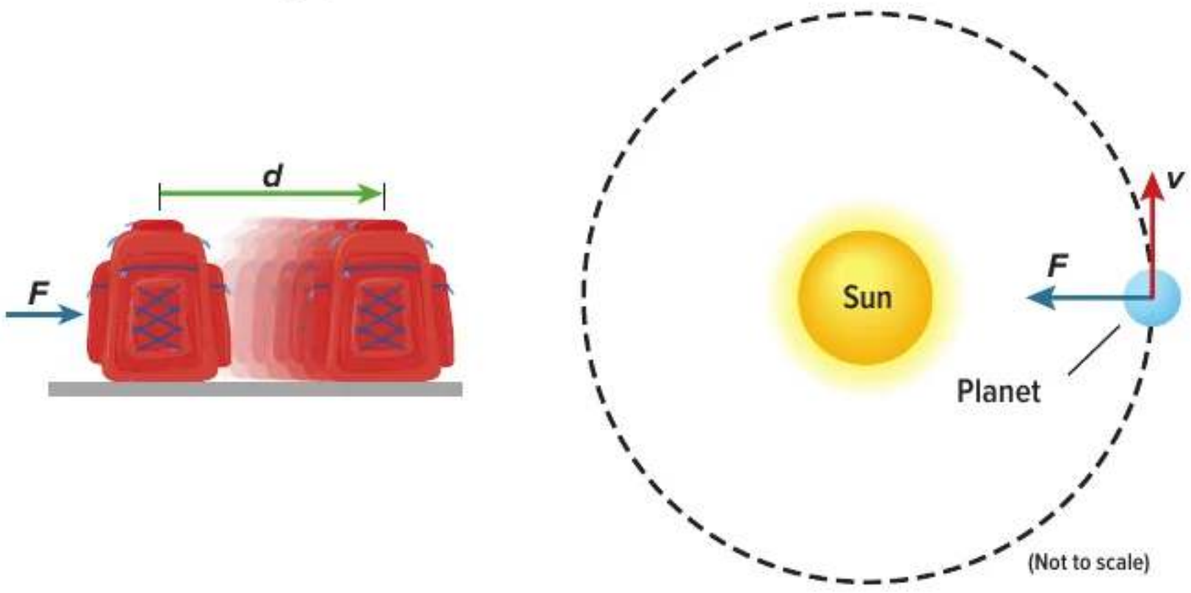


Figure 1 Work is done when a force is applied through a displacement.

Identify *another example of when a force does work on an object.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Stair Climbing and Power

Analyze and interpret data to determine the relationships among force, power, and time.

Quick Investigation: Force Applied at an Angle

Carry out an investigation to determine how the angle at which you apply force affects the amount work done.

Work done by a constant force In the book bag example, F is a constant force exerted in the direction in which the object is moving. In this case, work (W) is the product of the force and the system's displacement. That is,

$$W = Fd.$$

What happens if the exerted force is perpendicular to the direction of motion? For example, for a planet in a circular orbit, the force is always perpendicular to the direction of motion, as shown in **Figure 1.** Recall that a perpendicular force only changes the direction. The speed of the planet doesn't change, so the right side of the equation, $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, is zero. Therefore, the work done is also zero.

Constant force exerted at an angle What work does a force exerted at an angle do? For example, what work does the person pushing the car in **Figure 2** do? Recall that any force can be replaced by its components. If you use the coordinate system shown in **Figure 2,** the 125-N force (F) exerted in the direction of the person's arm has two components.

The magnitude of the horizontal component (F_x) is related to the magnitude of the applied force (F) by $\cos 25.0^\circ = \frac{F_x}{F}$. By solving for F_x, you obtain

$$F_x = F \cos 25.0^\circ = (125 \text{ N})(\cos 25.0^\circ) = 113 \text{ N}.$$

Using the same method, the vertical component is

$$F_y = -F \sin 25.0^\circ$$

$$F_y = -(125 \text{ N})(\sin 25.0^\circ) = -52.8 \text{ N}.$$

The negative sign shows that the force is downward. Because the displacement is in the x direction, only the x-component does work. The y-component does no work. The work you do when you exert a force on a system at an angle to the direction of motion is equal to the component of the force in the direction of the displacement multiplied by the displacement.

The magnitude of the component (F_x) force acting in the direction of displacement is found by multiplying the magnitude of force (F) by the cosine of the angle (θ) between the force and the direction of the displacement:



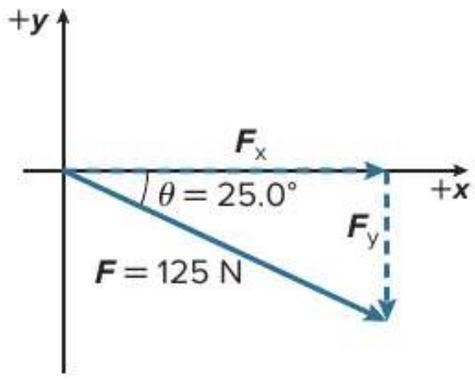


Figure 2 Only the horizontal component of the force that the man exerts on the car does work because the car's displacement is horizontal.

$F_x = F \cos \theta$. Thus, the work done is represented by the following equation.

Work

Work is equal to the product of the magnitude of the force and magnitude of displacement times the cosine of the angle between them.

$$W = Fd \cos \theta$$

Get It?

Determine the work you do when you exert a force of 3 N at an angle of 45° from the direction of motion for 1 m.

The equation above agrees with our expectations for constant forces exerted in the direction of displacement and for constant forces perpendicular to the displacement. In the book bag example, $\theta = 0^\circ$ and $\cos 0^\circ = 1$. Thus, $W = Fd(1) = Fd$, just as we found before. In the case of the orbiting planet, $\theta = 90^\circ$ and $\cos 90^\circ = 0$. Thus, $W = Fd(0) = 0$. This agrees with our previous conclusions.

AW/Alamy Stock Photo

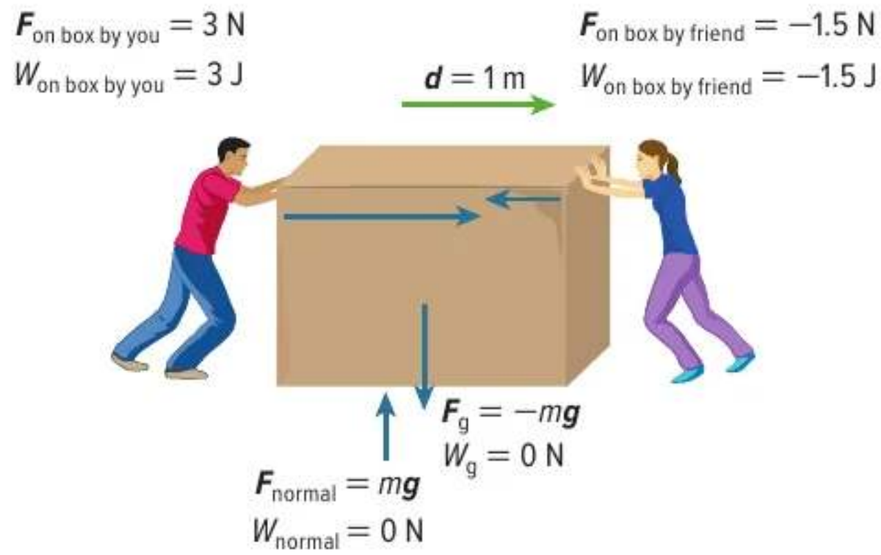


Figure 3 The total work done on a system is the sum of the work done by each agent that exerts a force on the system.

Describe *the total work done on the box if your friend exerted a greater force than you did. Be sure to consider the direction of the displacement.*

Work done by many forces

Suppose you are pushing a box on a frictionless surface while your friend is trying to prevent you from moving it, as shown in **Figure 3.** What forces are acting on the box? You are exerting a force to the right and your friend is exerting a force to the left. Earth's gravity exerts a downward force, and the ground exerts an upward normal force. How much work is done on the box?

When several forces are exerted on a system, calculate the work done by each force, and then add the results. For the box in **Figure 3,** the upward and downward forces (gravity and the normal force) are perpendicular ($\theta = 90^\circ$) to the direction of motion and do no work. For these forces, $\theta = 90^\circ$, which makes $\cos\theta = 0$, and thus, $W = 0$.

The force you exert ($F_{\text{on box by you}}$) is in the direction of the displacement, so the work is

$$W = F_{\text{on box by you}}d.$$

Your friend exerts a force (F_{friend}) in the direction opposite the displacement ($\theta = 180^\circ$). Because $\cos 180^\circ = -1$, your friend does negative work:

$$W = -F_{\text{on box by friend}}d$$

The total work done on the box is

$$W = F_{\text{on box by you}}d - F_{\text{on box by friend}}d.$$

Get It?

Explain why you do positive work on the box and your friend does negative work on the box.

It is also important to consider each distance separately. Say you push a box 1 m with a force of 3 N. You then pull the box back 1 m to the starting point with a force of 3 N. Is $W = 0$ because the total displacement is zero?

This would be true if the force was constant, but your force changed direction. Your push was in the box's direction of motion for the first part, so you did 3 J of work. In the second part, both the force you exerted and the direction of motion reversed. Your pull and the box's motion were in the same direction, and you did 3 J of work on the box. Therefore, you did a total of 6 J of work on the box.

Get It?

Describe another scenario in which you do work on a system, and explain how much work is done on the system.

SCIENCE USAGE v. COMMON USAGE

Work

Science usage: a force applied through a displacement
The student did work on the car, pushing it out of the mud.

Common usage: physical or mental effort
Studying physics can require a lot of work.

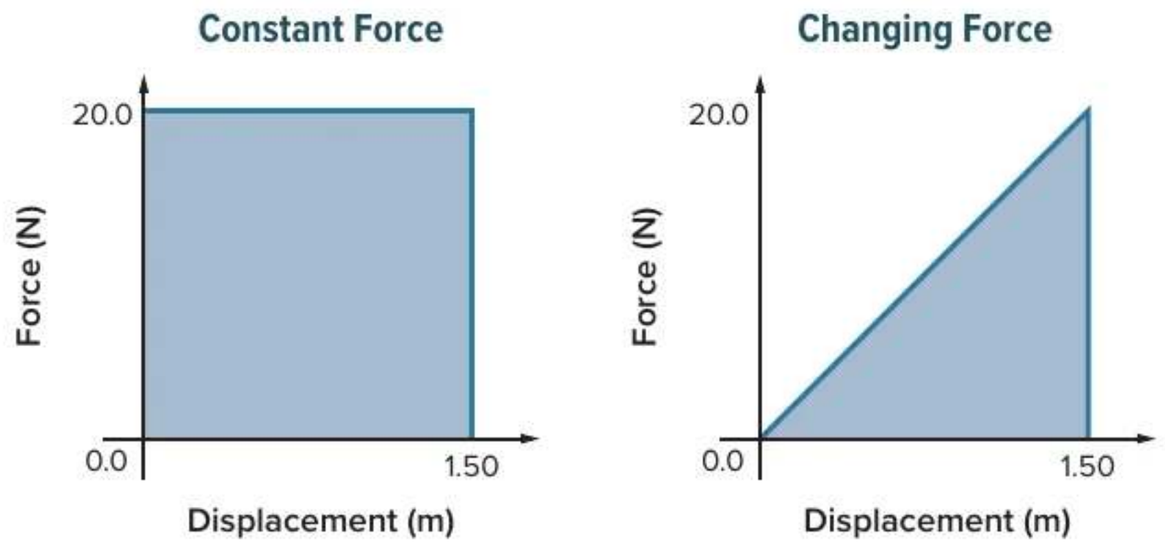


Figure 4 The area under a force-displacement graph is equal to the work.

Finding work done when forces change In the last example, the force changed but we could determine the work done in each segment. But what if the force changes in a more complicated way?

A graph of force versus displacement lets you determine the work done by a force. This graphical method can be used to solve problems in which the force is changing. The left graph in **Figure 4** shows the work done by a constant force of 20.0 N that is exerted to lift an object 1.50 m. The work done by this force is represented by

$$W = Fd = (20.0\ \text{N})(1.50\ \text{m}) = 30.0\ \text{J}.$$

Note that the shaded area under the left graph is also equal to $(20.0\ \text{N})(1.50\ \text{m})$, or 30.0 J. The area under a force-displacement graph is equal to the work done by that force.

This is true even if the force changes. The right graph in **Figure 4** shows the force exerted by a spring that varies linearly from 0.0 to 20.0 N as it is compressed 1.50 m. The work done by the force that compressed the spring is the area under the graph, which is the area of a triangle, $\left(\frac{1}{2}\right)$(base)(altitude), or

$$W = \left(\frac{1}{2}\right)(20.0\ \text{N})(1.50\ \text{m}) = 15.0\ \text{J}.$$

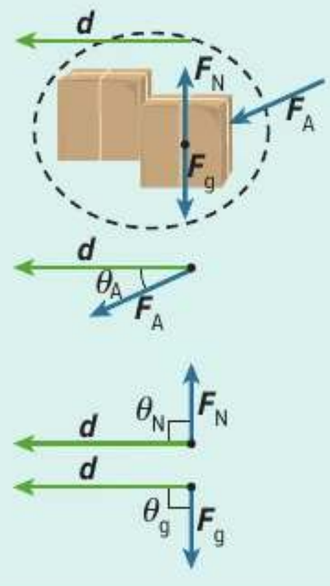
Use the problem-solving strategy below when you solve problems related to work.

PROBLEM-SOLVING STRATEGY

Work

When solving problems related to work, use the following strategy:

1. Identify and sketch the system. Show any forces doing work on the system.
2. Establish a coordinate system. Draw the displacement vectors of the system and each force vector doing work on the system.
3. Find the angle (θ) between each force and displacement.
4. Calculate the work done by each force using $W = Fd\cos\theta$.
5. Calculate the net work done.

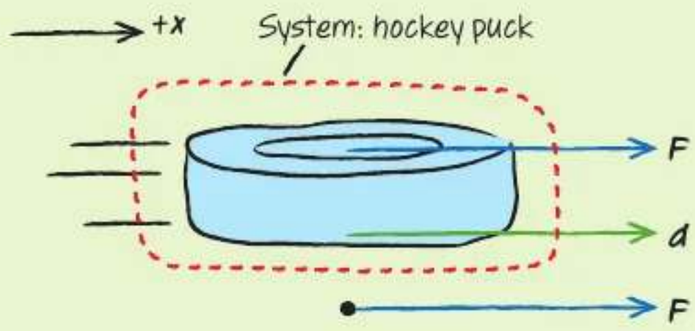


EXAMPLE Problem 1

WORK A hockey player uses a stick to apply a constant 4.50-N force forward to a 105-g puck sliding on ice over a displacement of 0.150 m forward. How much work does the stick do on the puck? Assume friction is negligible.

1 ANALYZE AND SKETCH THE PROBLEM

- Identify the system and the force doing work on it.
- Sketch the situation showing initial conditions.
- Establish a coordinate system with $+x$ to the right.
- Draw a vector diagram.



Player does work on hockey puck.

Known

$m = 105$ g

$F = 4.50$ N

$d = 0.150$ m

$\theta = 0°$

Unknown

$W = ?$

2 SOLVE FOR THE UNKNOWN

Use the definition for work.

$W = Fd \cos \theta$

$= (4.50 \text{ N})(0.150 \text{ m})(\cos \theta)$ Substitute $F = 4.50$ N, $d = 0.150$ m, $\cos \theta = \cos 0° = 1$.

$= 0.675 \text{ N·m}$

$= 0.675 \text{ J}$ $1 \text{ J} = 1 \text{ N·m}$

3 EVALUATE THE ANSWER

- **Are the units correct?** Work is measured in joules.
- **Does the sign make sense?** The stick (external world) does work on the puck (the system), so the sign of work should be positive.

PRACTICE Problems

ADDITIONAL PRACTICE

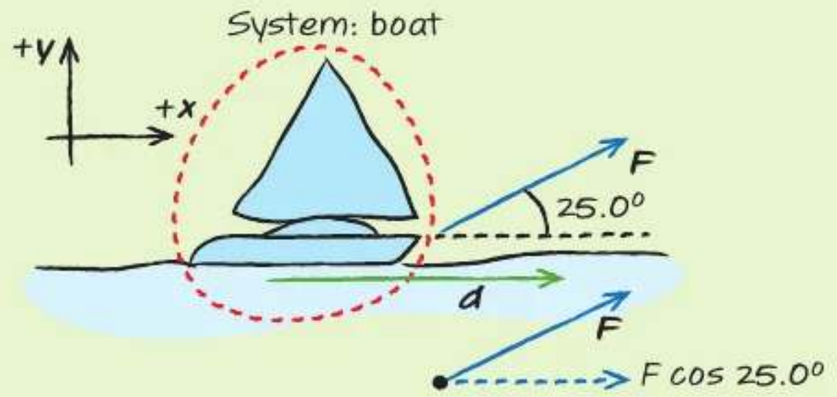
1. Refer to Example Problem 1 to solve the following problem.
   a. If the hockey player exerted twice as much force (9.00 N) on the puck over the same distance, how would the amount of work the stick did on the puck be affected?
   b. If the player exerted a 9.00-N force, but the stick was in contact with the puck for only half the distance (0.075 m), how much work does the stick do on the puck?
2. Together, two students exert a force of 825 N in pushing a car a distance of 35 m.
   a. How much work do the students do on the car?
   b. If their force is doubled, how much work must they do on the car to push it the same distance?
3. A rock climber wears a 7.5-kg backpack while scaling a cliff. After 30.0 min, the climber is 8.2 m above the starting point.
   a. How much work does the climber do on the backpack?
   b. If the climber weighs 645 N, how much work does she do lifting herself and the backpack?
4. **CHALLENGE** Marisol pushes a 3.0-kg box 7.0 m across the floor with a force of 12 N. She then lifts the box to a shelf 1 m above the ground. How much work does Marisol do on the box?

EXAMPLE Problem 2

FORCE AND DISPLACEMENT AT AN ANGLE A sailor pulls a boat a distance of 30.0 m along a dock using a rope that makes a 25.0° angle with the horizontal. How much work does the rope do on the boat if its tension is 255 N?

1 ANALYZE AND SKETCH THE PROBLEM

- Identify the system and the force doing work on it.
- Establish coordinate axes.
- Sketch the situation showing the boat with initial conditions.
- Draw vectors showing the displacement, the force, and its component in the direction of the displacement.



Sailor does work on the boat.

Known

$F = 255\ \text{N}$ $\theta = 25.0°$

$d = 30.0\ \text{m}$

Unknown

$W = ?$

2 SOLVE FOR THE UNKNOWN

Use the definition of work.

$$W = Fd\cos\theta$$

$$= (255\ \text{N})(30.0\ \text{m})(\cos 25.0°)$$

Substitute $F = 255$ N, $d = 30.0$ m, $\theta = 25.0°$.

$$= 6.93\times10^{3}\ \text{J}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Work is measured in joules.
- **Does the sign make sense?** The rope does work on the boat, which agrees with a positive sign for work.

PRACTICE Problems

ADDITIONAL PRACTICE

5. If the sailor in Example Problem 2 pulls with the same force through the same displacement but at an angle of 50.0°, how much work is done on the boat by the rope?

6. Two people lift a heavy box a distance of 15 m. They use ropes, each of which makes an angle of 15° with the vertical. Each person exerts a force of 225 N. Calculate the work done by the ropes.

7. An airplane passenger carries a 215-N suitcase up the stairs, a displacement of 4.20 m vertically and 4.60 m horizontally.

a. How much work does the passenger do on the suitcase?

b. The same passenger carries the same suitcase back down the same set of stairs. How much work does the passenger do on the suitcase to carry it down the stairs?

8. A rope is used to pull a metal box a distance of 15.0 m across the floor. The rope is held at an angle of 46.0° with the floor, and a force of 628 N is applied to the rope. How much work does the rope do on the box?

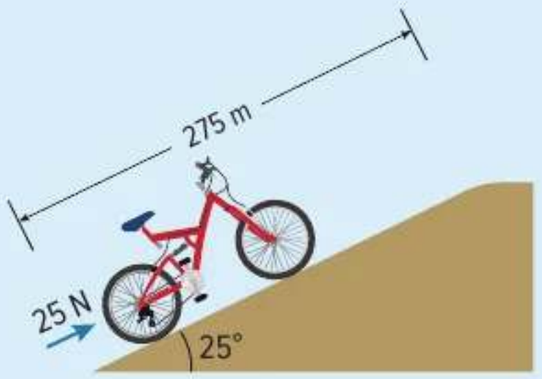


Figure 5

9. CHALLENGE A bicycle rider pushes a 13-kg bicycle up a steep hill. The incline is 25° and the hill is 275 m long, as shown in **Figure 5.** The rider pushes the bike parallel to the road with a force of 25 N.

a. How much work does the rider do on the bike?

b. How much work is done by the force of gravity on the bike?

Energy

Look again at the equation $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. What property of a system does $\frac{1}{2}mv^2$ describe? A massive, fast-moving vehicle can do damage to objects around it, and a baseball hit at high speed can rise high into the air. That is, a system with this property can produce a change in itself or the world around it. This ability of a system to produce a change in itself or the world around it is called **energy** and is represented by the symbol E.

The right side of the equation, $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, indicates a change in a specific kind of energy. That is, work causes a change in energy. This is the **work-energy theorem,** which states that when work is done on a system, the result is a change in the system's energy. The work-energy theorem can be represented by the following equation.

Work-Energy Theorem

Work done on a system is equal to the change in the system's energy.

$$W = \Delta E$$

Since work is measured in joules, energy must also be measured in joules. In fact, the unit gets its name from the nineteenth-century physicist James Prescott Joule, who established the relationship between work done and the change in energy. Recall that 1 joule equals 1 N·m and that 1 N equals 1 kg·m/s^2. Therefore, 1 joule equals 1 kg·m^2/s^2. These are the same units that $\frac{1}{2}mv^2$ has.

Through the process of doing work, energy can move between the external world and the system. The direction of energy transfer can be either way. If the external world does work on a system, then W is positive and the energy of the system increases. If a system does work on the external world, then W is negative and the energy of the system decreases. In summary, work is the transfer of energy that occurs when a force is applied through a displacement.

Changing kinetic energy

So far, we have discussed the energy associated with a system's motion. For example, the bobsledders in **Figure 6** do work on their sled to get it moving at the beginning of a race. The energy associated with motion is called **kinetic energy** (KE).

In the case of the bobsled, work resulted in a change in the object's kinetic energy. There are, however, many other forms of energy.

Work can cause a change in these other forms, as well. Some of these forms, such as potential energy and thermal energy, will be explored later in this module and in subsequent modules.



PCN Photography/Alamy Stock Photo

Figure 6 The bobsledders do work on the bobsled when they push it. The result is a change in the bobsled's kinetic energy.

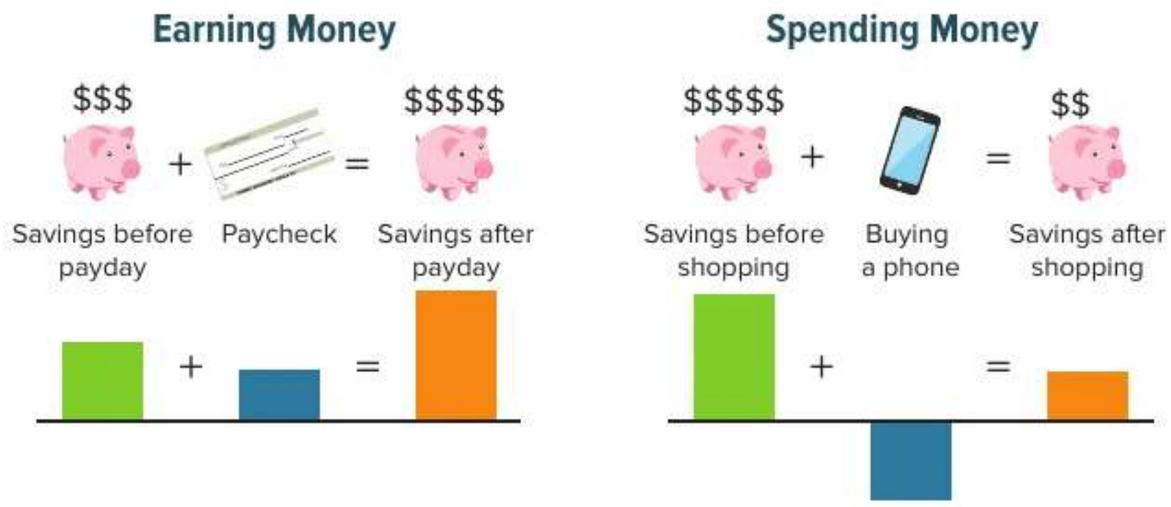


Figure 7 Energy is similar to a savings account. The amount can increase or decrease.

A Model of the Work-Energy Theorem

Keeping track of work and energy changes is like keeping track of your savings account. The amount of money you have changes when you earn more or spend it. Similarly, energy can be stored or it can be used. In each case, the behavior of a system is affected. We can use bar diagrams, such as those shown in **Figure 7,** to track money or energy.

Throwing and catching a ball Suppose you exert a force to throw a stationary ball. Consequently, the ball gains kinetic energy, as shown on the left in **Figure 8.** In the energy bar diagram, the height of the bar represents the amount of work (in joules).

The kinetic energy after the work is done equals the initial kinetic energy plus the work done on the ball.

Before the moving ball is caught, it has kinetic energy. In catching it, you exert a force on the ball in the direction *opposite* to its motion. Therefore, you do negative work on it, causing it to stop. Now that the ball is not moving, it has no kinetic energy, as represented in the energy bar diagram on the right in **Figure 8.** Kinetic energy is always positive, so KE_{before} is positive. The work done on the ball is negative and KE_{after} is zero. Again,

$$KE_{after} = KE_{before} + W.$$

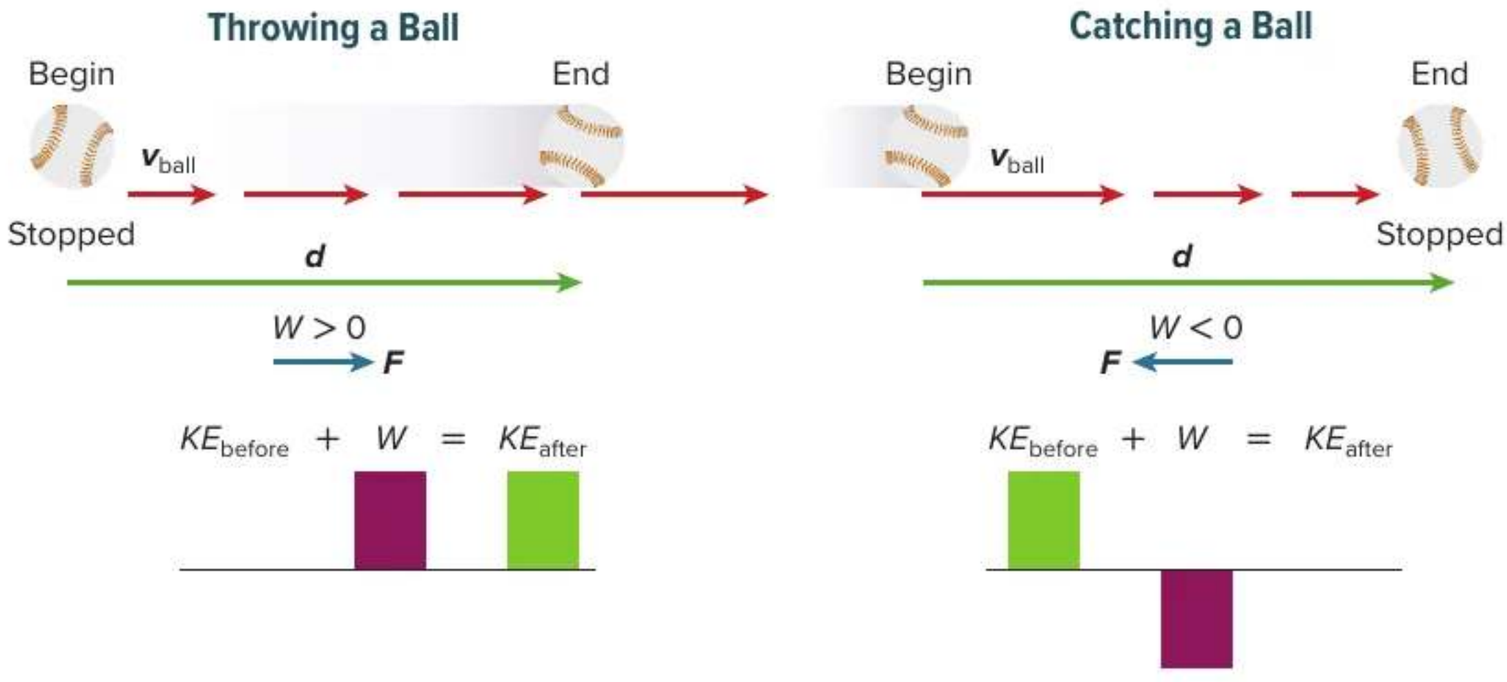


Figure 8 When you do work on an object, such as a ball, you change its energy.
Identify *two other events in which work causes a change in energy.*

PRACTICE Problems

ADDITIONAL PRACTICE

10. A catamaran with a mass of 5.44×10^3 kg is moving at 12 knots. How much work is required to increase the speed to 16 knots? (One knot = 0.51 m/s.)

11. A 52.0-kg skater moves at 2.5 m/s and stops over a distance of 24.0 m. Find the skater's initial kinetic energy. How much of her kinetic energy is transformed into other forms of energy by friction as she stops? How much work must she do to speed up to 2.5 m/s again?

12. An 875.0-kg car speeds up from 22.0 m/s to 44.0 m/s. What are the initial and final kinetic energies of the car? How much work is done on the car to increase its speed?

13. CHALLENGE A comet with a mass of 7.85×10^{11} kg strikes Earth at a speed of 25.0 km/s. Find the kinetic energy of the comet in joules, and compare the work that is done by Earth in stopping the comet to the 4.2×10^{15} J of energy that was released by the largest nuclear weapon ever exploded.

Power

Suppose you had a stack of books to move to a shelf. You could lift the stack at once, or you could move the books one at a time. Since the total force applied and the displacement are the same in both cases, the work is the same. However, the time needed is different. Recall that work causes a change in energy. The rate at which energy is transformed is **power.**

Power

Power is equal to the change in energy divided by the time required for the change.

$$P = \frac{\Delta E}{t}$$

When work causes the change in energy, power is equal to the work done divided by the time taken to do the work: $P = \frac{W}{t}$.

Consider the two forklifts in **Figure 9.** The left forklift raises its load in 5 seconds, and the right forklift raises the same load in 10 seconds. The left forklift is more powerful than the right forklift. Even though the same work is accomplished by both forklifts, the left forklift accomplishes the work in less time and thus develops more power.

Power is measured in watts (W). One **watt** is 1 J of energy transformed in 1 s. That is, 1 W = 1 J/s.

A watt is a relatively small unit of power. For example, a glass of water weighs about 2 N. If you lift it 0.5 m to your mouth at a constant speed, you do 1 J of work. If you lift it in 1 s, you are doing work at the rate of 1 W. Because a watt is such a small unit, power often is measured in kilowatts (kW). One kilowatt is equal to 1000 W.

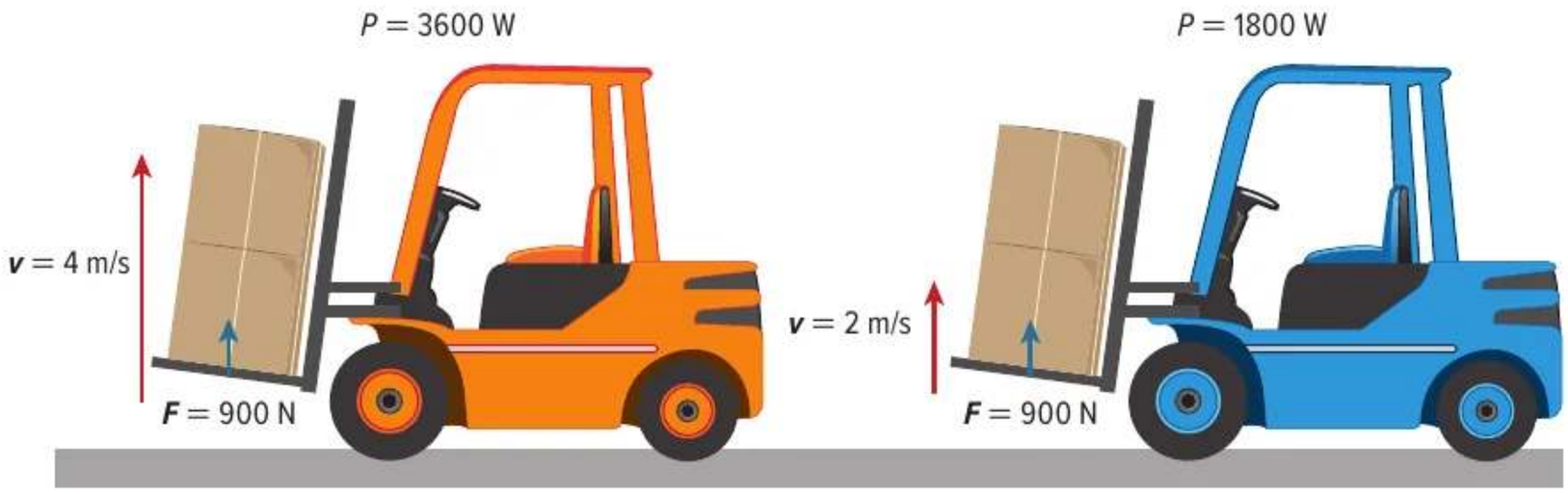


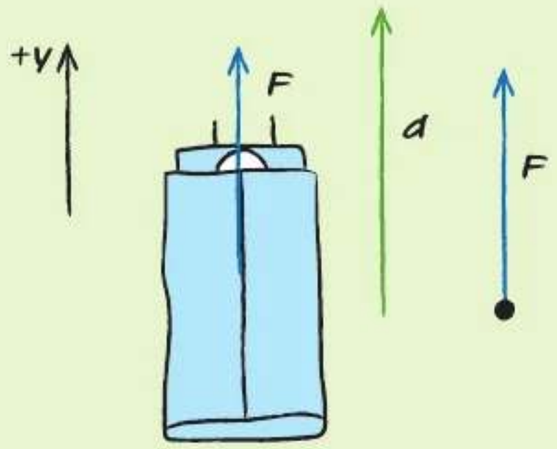
Figure 9 The forklift on the left develops more power than the forklift on the right because it lifts the load at a faster rate.

EXAMPLE Problem 3

POWER An electric motor lifts an elevator 9.00 m in 15.0 s by exerting an upward force of 1.20×10^4 N. What power does the motor produce in kW?

1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation showing the system as the elevator with its initial conditions.
- Establish a coordinate system with up as positive.
- Draw a vector diagram for the force and displacement.



Known
$d = 9.00$ m
$t = 15.0$ s
$F = 1.20\times10^4$ N

Unknown
$P = ?$

2 SOLVE FOR THE UNKNOWN

Use the definition of power.

$$P = \frac{W}{t}$$

$$= \frac{Fd}{t}$$ Substitute $W = Fd\cos 0° = Fd$.

$$= \frac{(1.20\times10^4\text{ N})(9.00\text{ m})}{(15.0\text{ s})}$$ Substitute $F = 1.20\times10^4$ N, $d = 9.00$ m, $t = 15.0$ s.

$$= 7.20\text{ kW}$$

3 EVALUATE THE ANSWER

- **Are the units correct?** Power is measured in joules per second, or watts.
- **Does the sign make sense?** The positive sign agrees with the upward direction of the force.

PRACTICE Problems

ADDITIONAL PRACTICE

14. A cable attached to a motor lifts a 575-N box up a distance of 20.0 m. The box moves with a constant velocity and the job is done in 10.0 s. What power is developed by the motor in W and kW?

15. You push a wheelbarrow a distance of 60.0 m at a constant speed for 25.0 s by exerting a 145-N force horizontally.

a. What power do you develop?

b. If you move the wheelbarrow twice as fast, how much power is developed?

16. What power does a pump develop to lift 35 L of water per minute from a depth of 110 m? (One liter of water has a mass of 1.00 kg.)

17. An electric motor develops 65 kW of power as it lifts a loaded elevator 17.5 m in 35 s. How much force does the motor exert?

18. CHALLENGE A winch designed to be mounted on a truck, as shown in **Figure 10,** is advertised as being able to exert a 6.8×10^3-N force and to develop a power of 0.30 kW. How long would it take the truck and the winch to pull an object 15 m?



Figure 10

McGraw-Hill Education

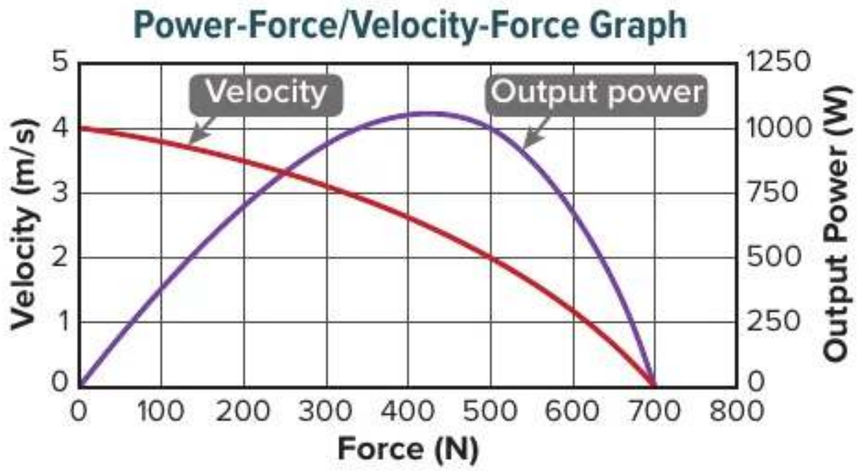


Figure 11 When riding a multispeed bicycle, the output power depends on the force you exert and your speed.

Power and speed When the force has a component (F_x) in the same direction as the displacement, $P = \frac{F_x d}{t}$. However, $v = \frac{d}{t}$ so power can be calculated using $P = F_x v$. From the equation $P = Fv$, you can see that either zero force or zero speed results in zero power. Our muscles cannot exert large forces nor can they move very fast. Some combination of moderate force and moderate speed will produce the largest amount of power when cycling, for example. **Figure 11** shows that for a certain cyclist, the maximum power output is over 1000 W when the force is about 400 N and speed is about 2.6 m/s.

Real-World Physics

A BICYCLIST in the Tour de France rides at an average speed of 8.94 m/s for more than 6 h a day. The power output of the racer is about 1 kW. One-fourth of that power goes into moving the bike against the resistance of the air, gears, and tires. Three-fourths of the power is used to cool the racer's body.

Check Your Progress

19. **Work-Energy Theorem** How can you apply the work-energy theorem to lifting a bowling ball from a storage rack to your shoulder?
20. **Work and Energy** If the work done on an object doubles its kinetic energy, does it double its speed? If not, by what ratio does it change the speed?
21. **Work** Murimi pushes a 20-kg mass 10 m across a floor with a horizontal force of 80 N. Calculate the amount of work done by Murimi on the mass.
22. **Work** Suppose you are pushing a stalled car. As the car gets going, you need less and less force to keep it going. For the first 15 m, your force decreases at a constant rate from 210.0 N to 40.0 N. How much work did you do on the car? Draw a force-displacement graph to represent the work done during this period.
23. **Work** A mover loads a 185-kg refrigerator into a moving van by pushing it at a constant speed up a 10.0-m, friction-free ramp at an angle of inclination of 11°. How much work is done by the mover on the refrigerator?
24. **Work** A 0.180-kg ball falls 2.5 m. How much work does the force of gravity do on the ball?
25. **Work and Power** Does the work required to lift a book to a shelf depend on how fast you raise it? Does the power required to lift it depend on how fast you raise it? Explain.
26. **Power** An elevator lifts a total mass of 1.1×10^3 kg a distance of 40.0 m in 12.5 s. How much power does the elevator deliver?
27. **Mass** A forklift raises a box 1.2 m and does 7.0 kJ of work on it. What is the mass of the box?
28. **Work** You and a friend carry identical boxes from the first floor of a building to a room on the second floor, farther down the hall. You carry the box first up the stairs, and then down the hall to the room. Your friend carries it down the hall on the first floor, then up a different stairwell to the second floor. How do the amounts of work done by the two of you on your boxes compare?
29. **Critical Thinking** Explain how to find the change in energy of a system if three agents exert forces on the system at once.

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LESSON 2

THE MANY FORMS OF ENERGY

FOCUS QUESTION

What are some forms of energy?

Changing Forms of Energy

Recall that the work-energy theorem states that doing work on a system causes a change in the energy of that system. That is, work is the process that transfers energy between a system and the external world. When an agent performs work on a system, the system's energy increases. When the system does work on its surroundings, the system's energy decreases.

In some ways, energy is like ice cream—it comes in different varieties. You can have vanilla, chocolate, or peach ice cream. These are different varieties, but they are all ice cream. However, unlike ice cream, energy can be changed from one form to another. A system of objects can possess energy in a variety of forms.



LHB Photo/Alamy Stock Photo

Figure 12 Energy can manifest itself in a variety of ways, such as light and sound in a July 4 fireworks display.

At the macroscopic scale, energy manifests itself in multiple ways, such as in motion, sound, light, and thermal energy. For example, when you experience a fireworks display, such as the one shown in **Figure 12,** you hear and see the results of the energy released in the chemical reactions in the fireworks in the form of moving, colorful lights and loud pops. You may even feel the warmth given off during the explosion if you are close enough (but not too close).

In this lesson, you will learn how energy is transformed from one variety (or form) to another and how to keep track of the changes.

Get It?

Describe an example of an event in which more than one form of energy is evident.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Review the News

Obtain information from a current news story about energy or energy resources. Evaluate your source and communicate your findings to your class.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Translational and Rotational Kinetic Energy

In the examples we have considered so far, moving objects that were changing position had kinetic energy $\left(\frac{1}{2}mv^2\right)$ due to their motion. What about energy due to an object's changing position?

Translational kinetic energy

Energy due to changing position is called **translational kinetic energy** and can be represented by the following equation.

Translational Kinetic Energy

A system's translational kinetic energy is equal to one-half times the system's mass multiplied by the system's speed squared.

$$KE_{trans} = \frac{1}{2}mv^2$$

Translational kinetic energy is proportional to the object's mass. For example, a 7.26-kg bowling ball thrown through the air has more translational kinetic energy than a 0.148-kg baseball, like the one shown in **Figure 13,** moving with the same speed.

An object's translational kinetic energy is also proportional to the square of the object's speed. A car moving at 20 m/s has four times the translational kinetic energy of the same car moving at 10 m/s.



Thearon W. Henderson/Getty Images

Figure 13 A baseball has less translational kinetic energy than a bowling ball moving with the same speed.

Get It?

Explain Using the equation for translational kinetic energy, show why a car moving at 20 m/s has four times the translational kinetic energy of the same car moving at 10 m/s.

SCIENCE USAGE v. COMMON USAGE

Energy

Science usage: the ability of a system to produce a change in itself or the world around it

The kinetic energy of the soccer ball decreased as it slowed down.

Common usage: the capacity of acting or being active

The young students had a lot of energy during recess.

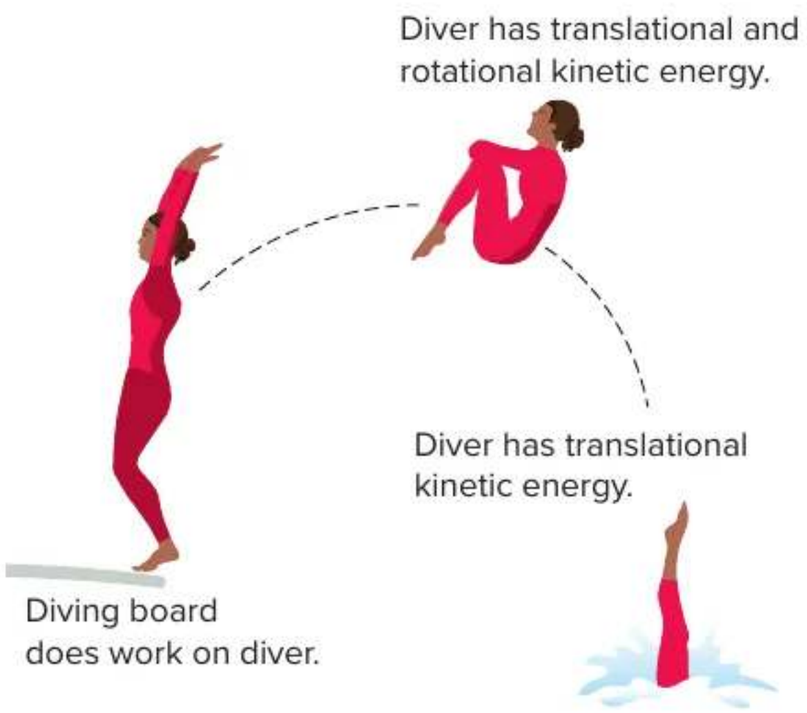


Figure 14 The diving board does work on the diver. This work increases the diver's kinetic energy.

Rotational kinetic energy If you spin a toy top in one spot, you might say that it does not have kinetic energy because the top does not change its position. However, to make the top rotate, you had to do work on it. The top has **rotational kinetic energy,** which is energy due to rotational motion. Rotational kinetic energy can be calculated using $KE_{rot} = \frac{1}{2} I\omega^2$, where I is the object's moment of inertia and ω is the object's angular velocity.

In **Figure 14,** the diving board does work on a diver, transferring translational and rotational kinetic energy to the diver. Her center of mass moves as she leaps, so she has translational kinetic energy. She rotates about her center of mass, so she has rotational kinetic energy. When she slices into the water, she has mostly translational kinetic energy.

Potential Energy

Let's return to the money and energy model from Lesson 1. Money and energy can be in different forms. You can have one five-dollar bill, 500 pennies, or 20 quarters—in all forms, you still have five dollars. In the same way, energy can be kinetic energy, stored energy, or another form. Consider some boulders on a mountain. They have been moved from Earth's center by geological processes against gravity. Thus, the Earth-boulders system has stored energy. Energy that is stored due to interactions between objects in a system is called **potential energy.** But not all potential energy is due to gravity. A spring-loaded toy is a system that has potential energy, but the energy is stored due to a compressed spring, not gravity.

CCC CROSSCUTTING CONCEPTS

Systems and System Models Mathematical models can be used to predict the behavior of a system, but models can be limited. Choose an example of a mathematical model used to make an energy calculation based on a realistic situation, such as ice-skating or pushing different materials across different surfaces. What evidence do you have that the model is limited? Explain your model as well as its limitations to a peer.



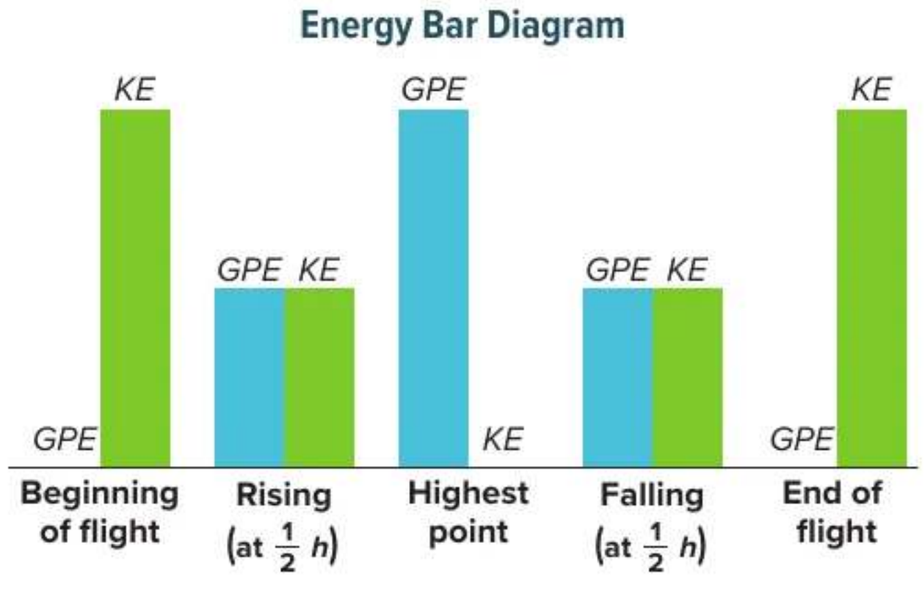


Figure 15 As the ball rises, its kinetic energy transforms into potential energy in the ball-Earth system. As it falls, the potential energy is transformed into kinetic energy. In this process, Earth also gains and loses very small amounts of kinetic energy. The energy bar diagrams show how the ball's energy changes.

Gravitational Potential Energy

Look at the balls being juggled in **Figure 15.** If you consider the system to be only one ball, then it has several external forces acting on it. The force of the juggler's hand does work, giving the ball its initial kinetic energy. After the ball leaves the juggler's hand, only the gravitational force acts on it, assuming that air resistance is negligible. How much work does gravity do on the ball as its height changes?

Work done by gravity

Let h represent the ball's height measured from the juggler's hand. On the way up, its displacement is upward but the gravitational force on the ball (F_g) is downward, so the work done by gravity is negative: $W_g = -mgh$. On the way back down, the force and displacement are in the same direction, so the work done by gravity is positive: $W_g = mgh$. Thus, while the ball is moving upward, gravity does negative work, slowing the ball to a stop. On the way back down, gravity does positive work, increasing the ball's speed and thereby increasing its kinetic energy. The ball recovers all of its original kinetic energy when it returns to the same height as the juggler's hand, as shown in the energy bar diagrams in **Figure 15.** Notice in the diagrams that the ball has equal amounts of gravitational potential energy and kinetic energy at $\frac{1}{2}h$.

Get It?

Describe the energy transformations in the dam at the beginning of the module.

According to Newton's law of universal gravitation, there is a gravitational attraction between any pair of objects. This gravitational attraction between the objects is a force that performs work when one of the objects moves. Usually, we are most concerned with the gravitational potential energy between an object and Earth. For this particular example, we choose to discuss the object and Earth together as a system. If the object moves away from Earth, the system stores energy due to the gravitational force between the object and Earth. The stored energy due to gravity is called **gravitational potential energy,** or *GPE*. The height to which the object has risen is determined by using a **reference level,** the position where *GPE* is defined to be zero. In the example of the juggler, the reference level is the juggler's hand.

gjohnstonphoto/123RF

Calculating gravitational potential energy

For an object with mass m at height h above the reference level, the *GPE* of the object-Earth system is represented by the following equation.

Gravitational Potential Energy

The *GPE* of an object-Earth system is equal to the product of the object's mass, the gravitational field strength, and the object's distance above the reference level.

$$GPE = mgh$$

In this equation, g is the gravitational field strength. *GPE*, like kinetic energy, is measured in joules.

Energy transformations Consider the energy of a system consisting of Earth and a ball being juggled. The energy of the system is in two forms: kinetic energy and gravitational potential energy. At the beginning of the ball's flight, all the energy is in the form of kinetic energy, as shown on the left in **Figure 16.** As the ball rises, it slows down and the system's energy changes from kinetic energy to gravitational potential energy. At the highest point of the ball's flight, the velocity is zero and all the energy is in the form of gravitational potential energy.

Changes of energy in a system can be described in terms of energy flowing into, out of, and within that system. As the ball falls, gravitational potential energy changes back into kinetic energy, or flows within that system as it transforms back into kinetic energy. The sum of kinetic energy and gravitational potential energy is constant at all times because no work is done on the system by forces outside of the system.

Reference levels So far, we have set the reference level to be the juggler's hand. That is, the height of the ball is measured from where it left contact with the juggler's hand. Thus, at the juggler's hand, $h = 0$ m and $GPE = 0$ J. However, you can set the reference level at any height that is convenient for solving a given problem.

Suppose the reference level is set at the highest point of the ball's flight. Then, $h = 0$ m and the system's $GPE = 0$ J at that point, as illustrated on the right side of **Figure 16.** The system's potential energy is negative at the beginning of the ball's flight, zero at the highest point, and negative at the end of the ball's flight. If you were to calculate the total energy of the system represented on the left in **Figure 16,** it would be different from the total energy of the system represented on the right in **Figure 16.** This is because the reference levels are different in each case. However, the total energy of the system in each situation is constant at all times during the flight of the ball.

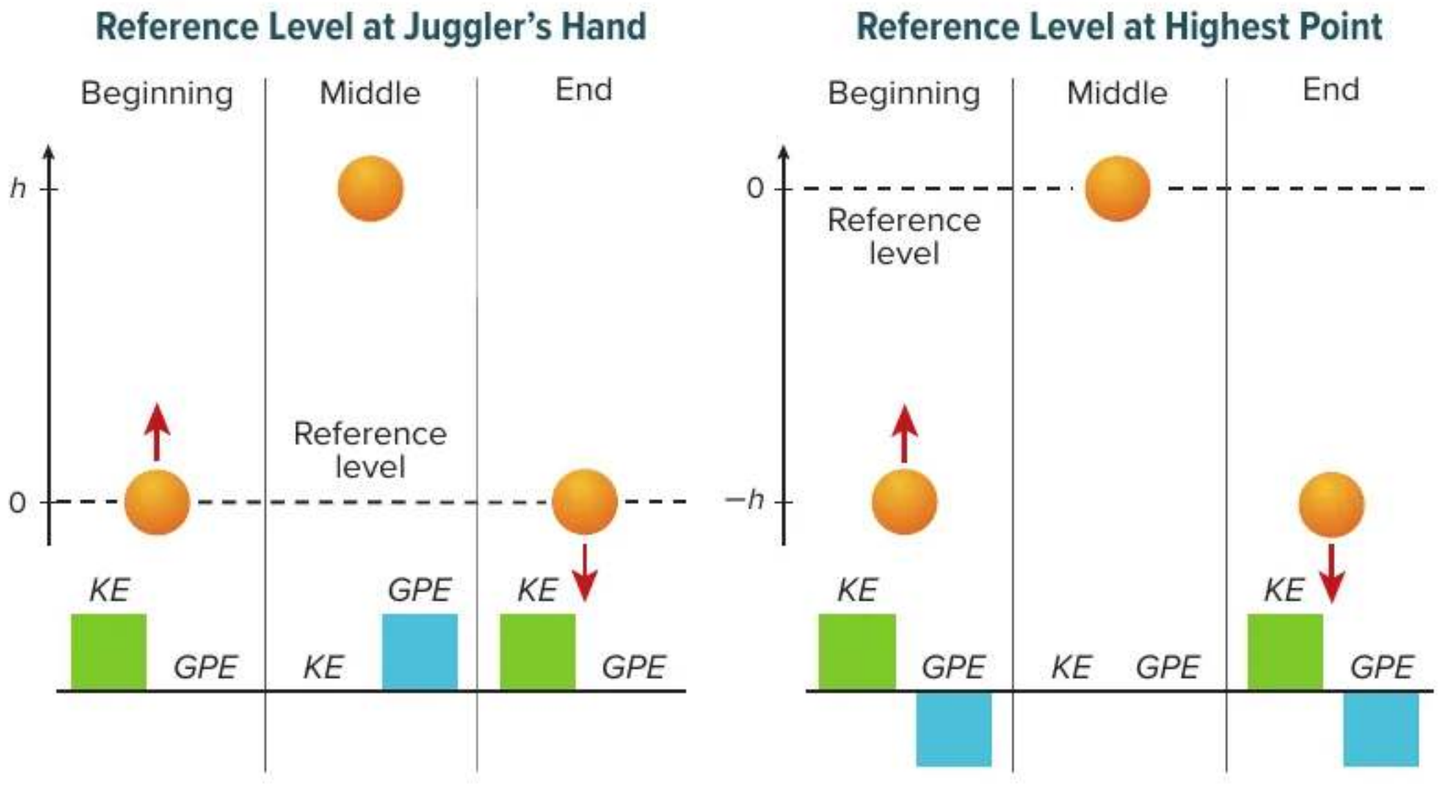


Figure 16 Where you put the reference level affects the system's *GPE*. Where you put the reference level does not affect how the system's *GPE* changes over time, however.

Analyze *Where else might you place the reference level?*

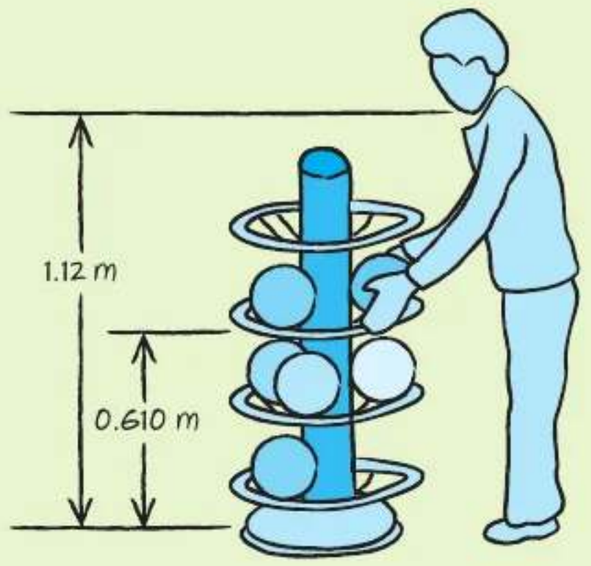
EXAMPLE Problem 4

GRAVITATIONAL POTENTIAL ENERGY You lift a 7.30-kg bowling ball from the storage rack and hold it up to your shoulder. The storage rack is 0.610 m above the floor and your shoulder is 1.12 m above the floor.

a. When the bowling ball is at your shoulder, what is the ball-Earth system's gravitational potential energy relative to the floor?

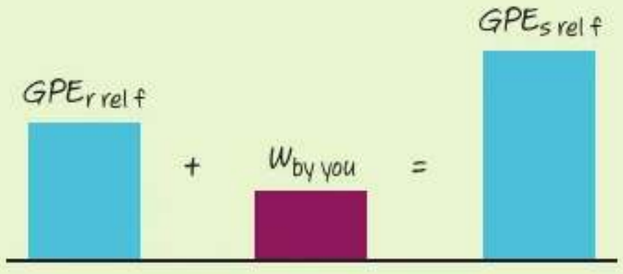
b. When the bowling ball is at your shoulder, what is the ball-Earth system's gravitational potential energy relative to the rack?

c. How much work was done by gravity as you lifted the ball from the rack to shoulder level?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Choose a reference level.
- Draw an energy bar diagram showing the gravitational potential energy with the floor as the reference level.



Known

$m = 7.30$ kg $\quad g = 9.8$ N/kg

$h_r = 0.610$ m (rack relative to the floor)

$h_s = 1.12$ m (shoulder relative to the floor)

Unknown

$GPE_{s\ rel\ f} = ?$

$GPE_{s\ rel\ r} = ?$

$W = ?$

2 SOLVE FOR THE UNKNOWN

a. Set the reference level to be at the floor.

Determine the gravitational potential energy of the system when the ball is at shoulder level.

$GPE_{s\ rel\ f} = mgh_s = (7.30\ \text{kg})(9.8\ \text{N/kg})(1.12\ \text{m})$ Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m

$= 8.0 \times 10^1\ \text{J}$

b. Set the reference level to be at the rack height.

Determine the height of your shoulder relative to the rack.

$h = h_s - h_r$

Determine the gravitational potential energy of the system when the ball is at shoulder level.

$GPE_{s\ rel\ r} = mgh = mg(h_s - h_r)$ Substitute $h = h_s - h_r$

$= (7.30\ \text{kg})(9.8\ \text{N/kg})(1.12\ \text{m} - 0.610\ \text{m})$ Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m, $h_r = 0.610$ m

$= 36\ \text{J}$ This also is equal to the work done by you as you lifted the ball.

c. The work done by gravity is the weight of the ball times the distance the ball was lifted.

$W = Fd = -(mg)h = -(mg)(h_s - h_r)$ The weight opposes the motion of lifting, so the work is negative.

$= -(7.30\ \text{kg})(9.8\ \text{N/kg})(1.12\ \text{m} - 0.610\ \text{m})$ Substitute $m = 7.30$ kg, $g = 9.8$ N/kg, $h_s = 1.12$ m, $h_r = 0.610$ m

$= -36\ \text{J}$

3 EVALUATE THE ANSWER

- **Are the units correct?** Both potential energy and work are measured in joules.
- **Does the answer make sense?** The system should have a greater *GPE* measured relative to the floor than relative to the rack because the ball's distance above the floor level is greater than the ball's distance above the rack.

PRACTICE Problems

ADDITIONAL PRACTICE

30. In Example Problem 4, what is the potential energy of the ball-Earth system when the bowling ball is on the floor? Use the rack as your reference level.

31. If you slowly lower a 20.0-kg bag of sand 1.20 m from the trunk of a car to the driveway, how much work do you do?

32. A boy lifts a 2.2-kg book from his desk, which is 0.80 m high, to a bookshelf that is 2.10 m high. What is the potential energy of the book-Earth system relative to the desk when the book is on the shelf?

33. You are walking around an old building and notice that it is falling apart. If a 1.8-kg brick falls to the ground from the building's chimney, which is 6.7 m high, what is the change in the potential energy of the brick-Earth system?

34. **CHALLENGE** A worker picks up a 10.1-kg box from the floor and sets it on a table that is 1.1 m high. He slides the box 5.0 m along the table and then lowers it back to the floor. What were the changes in the box-Earth system's energy, and how did the system's total energy change? (Ignore friction.)

Elastic Potential Energy

When the bow string shown in **Figure 17** is pulled, work is done on the bow string and energy is transferred to the bow. If you identify the system as the bow, the arrow, and Earth, the energy of the system increases. When the string and the arrow are released, the stored energy is changed into kinetic energy.

The stored energy due to the pulled string and bent bow is **elastic potential energy,** which is stored energy due to an object's change in shape. Systems that include springs, rubber bands, and trampolines have elastic potential energy.

Get It?

Define the term *elastic potential energy*.

You can also store elastic potential energy in a system when you bend an elastic object in that system. For example, when stiff metal or bamboo poles were used in pole-vaulting, the poles did not bend easily. Little work was done on the poles, and the poles did not store much potential energy. Since flexible fiberglass poles were introduced, however, record pole-vaulting heights have soared.



Figure 17 The archer-bow-arrow system has maximum elastic potential energy before the string is released, as shown on the left. When the arrow and string disengage, the elastic potential energy is completely transformed into kinetic energy, as shown on the right.

(bl)Tetra Images - Erik Isakson/Brand X Pictures/Getty Images; (br)Steve Allen/Brand X Pictures

A pole-vaulter runs with a flexible pole and plants its end into a socket in the ground. When the pole-vaulter bends the pole, as shown in **Figure 18,** some of the pole-vaulter's kinetic energy is transformed to elastic potential energy. When the pole straightens, the elastic potential energy is transformed to gravitational potential energy and kinetic energy as the pole-vaulter is lifted as high as 6 m above the ground.



Figure 18 The pole-vaulter stores elastic potential energy in the pole-vaulter-track system. The elastic potential energy of this system then changes into kinetic energy and gravitational potential energy.

Other Forms of Energy

Think about all the forms and sources of energy you encounter every day. Gasoline provides energy to run your car. You eat food to obtain energy. Power plants harness the energy of wind, water, fossil fuels (**Figure 19**), and atoms and transform it into electrical energy.

Some of these forms of energy are best understood if we think of energy as a quantitative property of a system that depends on the motion and interactions of matter and radiation within that system. (Radiation, including light, is the transfer of energy by electromagnetic waves, which you will encounter in a later module.) For example, nuclear energy is the energy stored in the bonds inside the nucleus of an atom. Without understanding the structure and chemical subprocesses within an atom, we would not be able to develop mathematical models to predict the amount of energy that could be released.



Figure 19 Coal-burning plants harness energy and transform it into something more useful, such as electrical energy.

Get It?

Explain In your own words, explain why we need to understand the interactions in a process or subprocess in order to understand a system.

Chemical and nuclear energy When you eat, your body uses the energy from the chemical bonds in your food as an energy source. The bonds inside an atom's nucleus also store energy. This energy is called nuclear energy and is released when the structure of an atom's nucleus changes.

(t)moodboard/Alamy Stock Photo; (b) Bildagentur Zoonar GmbH/Shutterstock.com

CCC CROSSCUTTING CONCEPTS

Energy and Matter Changes of energy and matter in a system can be described in terms of energy and matter flow within that system. In a labeled poster, show the changes in energy flow in a pole-vaulter-track system from the moment the vaulter places the pole in the socket in the ground to the moment the vaulter lands. What evidence is there of a change in energy?

Thermal energy

Thermal energy, which is associated with temperature, is the sum of the kinetic energy and potential energy of the particles in a system. When you rub your hands together, you transform kinetic energy into thermal energy. Thermal energy can also be transferred, such as a stove like the one in **Figure 20** transferring thermal energy to a pan.

Electrical and radiant energy

Power plants transform various types of energy into electrical energy, which is energy associated with charged particles. Electrical energy can be transmitted through wires like those in **Figure 21** to your home, where appliances transform it into other forms of energy, such as radiant energy in lightbulbs.

Mass

Albert Einstein demonstrated that mass represents a form of energy, called rest energy (E_0): $E_0 = mc^2$. Mass is a form of energy that can be transformed into other forms of energy. Mass-energy equivalence is only apparent when large amounts of energy are involved, such as during nuclear explosions and particle physics experiments.



Figure 20 In your daily activities, you encounter many types of energy, such as thermal energy when cooking your food.



Figure 21 Transmitting electrical energy is critical for our day-to-day needs.

Check Your Progress

35. **Elastic Potential Energy** You compress the spring in a jumping toy, then release it. The toy then flies straight up. Draw bar graphs that show the forms of energy present in the following instances. Assume the system includes the spring toy and Earth.
    a. The toy is pushed down thereby compressing the spring.
    b. The spring expands and the toy jumps.
    c. The toy reaches the top of its flight.
    d. Describe how energy manifests itself in different ways in this system.
36. **Potential Energy** A 25.0-kg shell is shot from a cannon at Earth's surface. The reference level is Earth's surface.
    a. What is the shell-Earth system's gravitational potential energy when the shell's height is 425 m?
    b. What is the change in the system's potential energy when the shell falls to a height of 225 m?
    c. What motions and interactions does this system depend on?
37. **Rotational Kinetic Energy** On a playground, some children push a merry-go-round so that it turns twice as fast as it did before they pushed it. What are the relative changes in angular momentum and rotational kinetic energy of the merry-go-round?
38. **Critical Thinking** In the school lab, Karl uses an air hose to exert a constant horizontal force to move a puck a fixed distance on a frictionless air table.
    a. Explain what happens in terms of work and energy. Draw bar graphs.
    b. Karl uses a different puck with half the first one's mass. All other conditions remain the same. How do the kinetic energy and work done differ from the kinetic energy and work done in part a?
    c. Describe what happened in parts a and b in terms of impulse and momentum.

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LESSON 3

CONSERVATION OF ENERGY

FOCUS QUESTION

Can energy be lost?

The Law of Conservation of Energy

The **law of conservation of energy** states that in a closed, isolated system, energy can neither be created nor destroyed. So, the total change of energy in any system is always equal to the total energy transferred into or out of the system. In other words, energy can be transported from one place to another, transferred between systems, and converted to different forms, such as thermal energy released to the surrounding environment from a running car. However, the system's total energy in all of its forms remains constant.

Within a system, energy is continually transferred from one object to another and between its various possible forms. Therefore, energy is considered a single quantity due to the law of conservation of energy.

Get It?

Explain whether the light from a battery-powered lamp can have more energy than the battery's energy.

Mechanical energy You often focus on the energy that comes from the motions of and interactions between objects. The sum of the *KE* and *PE* of the objects in a system is the system's **mechanical energy** (*ME*). The *KE* includes both the translational and rotational kinetic energies of the objects in the system. Potential energy includes the gravitational and elastic potential energies of the system. In any system, mechanical energy is represented as follows.

Mechanical Energy of a System

The mechanical energy of a system is equal to the sum of the *KE* and *PE* of the system's objects.

$$ME = KE + PE$$

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Applying Practices: Earth Power

HS-PS3-3. Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

Applying Practices: Modeling Changes in Energy

HS-PS3-1. Create a computational model to calculate the change in the energy of...

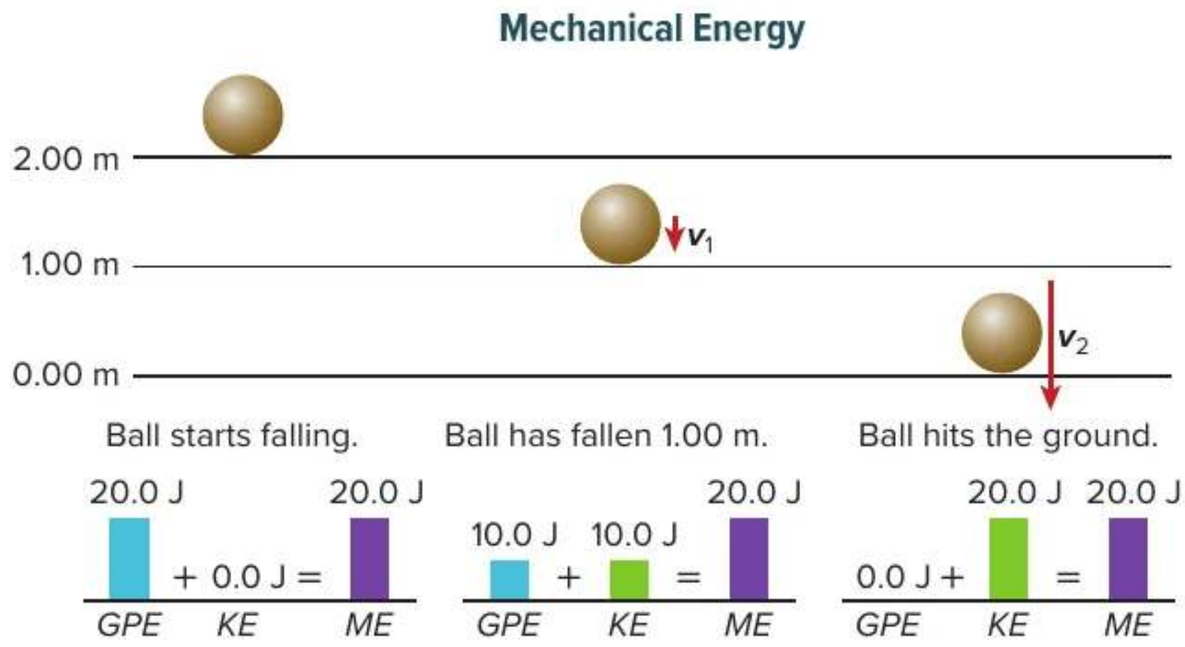


Figure 22 When a bowling ball is dropped, mechanical energy is conserved.

Predict *Suppose the ball started with an upward velocity. What would its energy graph look like?*

Imagine a system consisting of a 10.0-N bowling ball falling to Earth, as shown in **Figure 22.** For this case, the system's mechanical energy is 20.0 J.

Conservation of mechanical energy

Suppose that you let go of the bowling ball. As it falls, the *GPE* of the ball-Earth system is transformed into kinetic energy. When the ball is 1.00 m above Earth's surface, $GPE = mgh = F_g h = (10.0\ \text{N})(1.00\ \text{m}) = 10.0\ \text{J}$. What is the system's kinetic energy when the ball is at a height of 1.00 m? The system consisting of the ball and Earth is closed and isolated because no external forces are doing work on it. Therefore, the total mechanical energy of the system remains constant at 20.0 J.

$$ME = KE + PE$$

$$KE = ME - PE$$

$$KE = 20.0\ \text{J} - 10.0\ \text{J} = 10.0\ \text{J}$$

When the ball reaches ground level, the system's potential energy is zero, and the system's kinetic energy is 20.0 J. The availability of energy limits what can occur in this and any system; for example, at a height of 1.00 m, the *GPE* of this 10-N bowling ball cannot exceed 10.0 J.

Conservation of Mechanical Energy

When mechanical energy is conserved, the sum of the system's *KE* and *PE* before an event equals the sum of the system's *KE* and *PE* after the event.

$$KE_i + PE_i = KE_f + PE_f$$

CCC CROSSCUTTING CONCEPTS

Systems and System Models Models can be used to predict the behavior of a system, but these predictions have limited precision and liability. Develop an argument that explains why the bowling ball model discussed in the text is limited. What evidence from your life supports your argument? How would you present your case to your peers?

What happens if the ball does not fall straight down but rolls down a ramp, as shown in **Figure 23?** If friction does not affect the system, then the system of the ball and Earth remains closed and isolated. The ball still moves down a vertical distance of 2.00 m, so the decrease in potential energy is 20.0 J. Therefore, the increase in kinetic energy is 20.0 J.

As long as friction doesn't affect the system's energy, the path the ball takes does not matter. However, in the case with the ramp, the kinetic energy is split between translational kinetic energy (it is moving forward) and rotational kinetic energy (it is rolling). Three situations involving conservation of mechanical energy are shown in **Figure 24** on the next page.

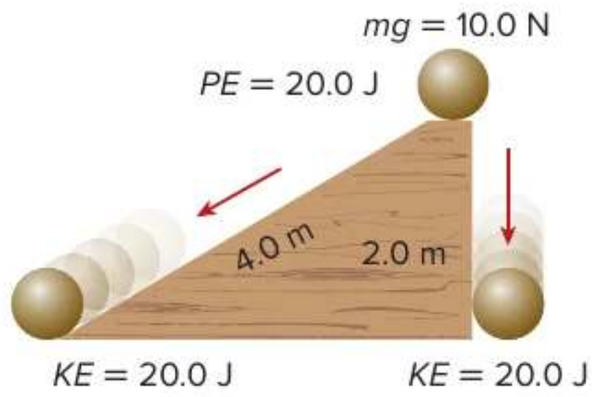


Figure 23 If friction does no work on the ball, the ball's final kinetic energy is equal to its initial gravitational potential energy regardless of which path it follows.

Get It?

Analyze how the ball's final kinetic energy in **Figure 23** would be different if friction transformed some of the system's energy.

Conservation and other forms of energy

A swinging pendulum eventually stops, a bouncing ball comes to rest, and the heights of roller-coaster hills get lower and lower as the ride progresses. Where does the mechanical energy in such systems go? Objects moving through the air experience the forces of air resistance. In a roller coaster, there are also frictional forces between the wheels and the tracks. Each force transforms mechanical energy into other forms of energy.

When a ball bounces from a surface, it compresses. Most of the kinetic energy transforms into elastic potential energy. Some, but not all, of this elastic potential energy transforms back into kinetic energy during the bounce. But as in the cases of the pendulum and the roller coaster, some of the original mechanical energy in the system transforms into another form of energy within the system or transmits to energy outside the system, as in air resistance.

Get It?

Analyze If most of the kinetic energy of a bouncing ball transforms into elastic potential energy, what happens to the rest of the kinetic energy?

Mathematical expressions, which quantify how kinetic energy depends on mass and speed, allow the concept of conservation of energy to be used to predict and describe system behavior. The Example Problems in this lesson use the concept of conservation of energy to solve problems, including collisions.

STEM CAREER Connection

Solar Photovoltaic Installer

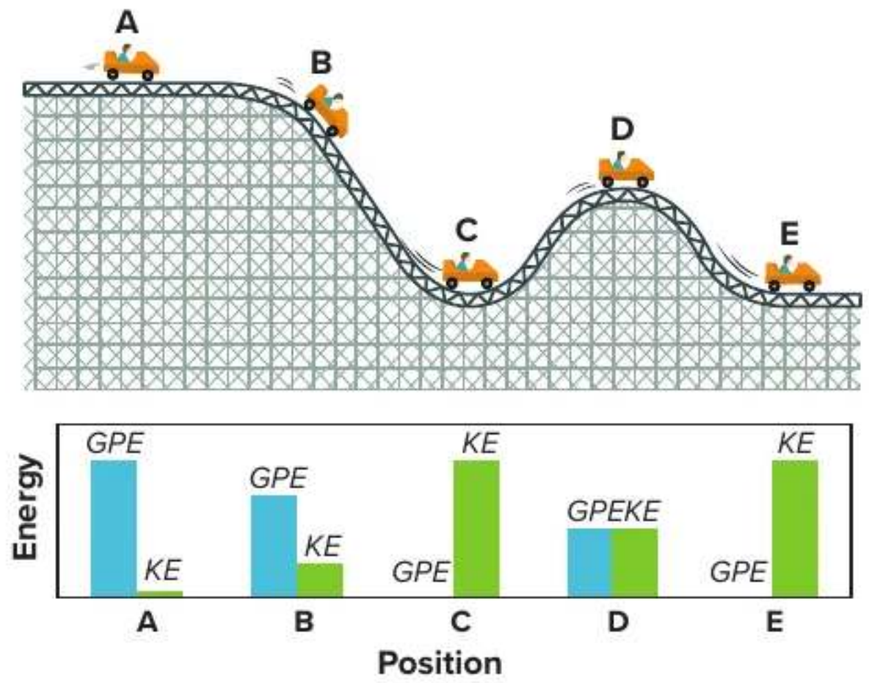
Alternative energy is an area of global importance today. Solar photovoltaic installers, or PV installers, install and maintain solar panels on homes and for industry. They may also assemble systems. Installers in this rapidly growing field work mostly outside, sometimes work inside as well. They also must travel to their job sites.

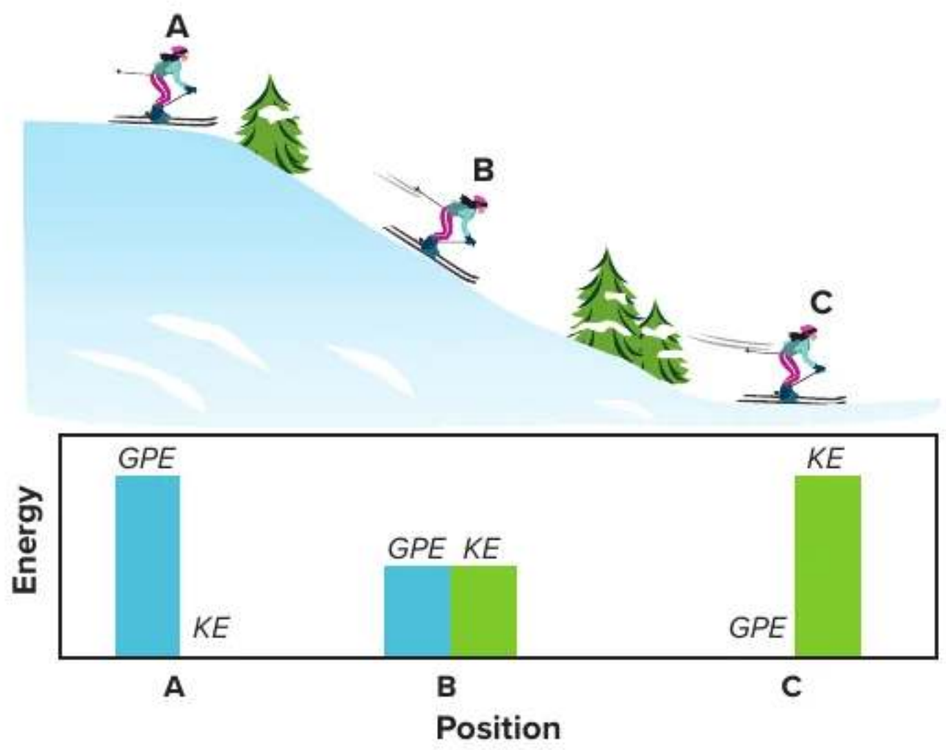
CCC CROSSCUTTING CONCEPTS

Energy and Matter Make a simple ramp with a book or a board. Roll a ball down the ramp. Explain how your ramp and ball provide evidence of the law of conservation of energy.

Figure 24 The conservation of mechanical energy is an important consideration in designing roller coasters, ski slopes, and the pendulums for grandfather clocks.

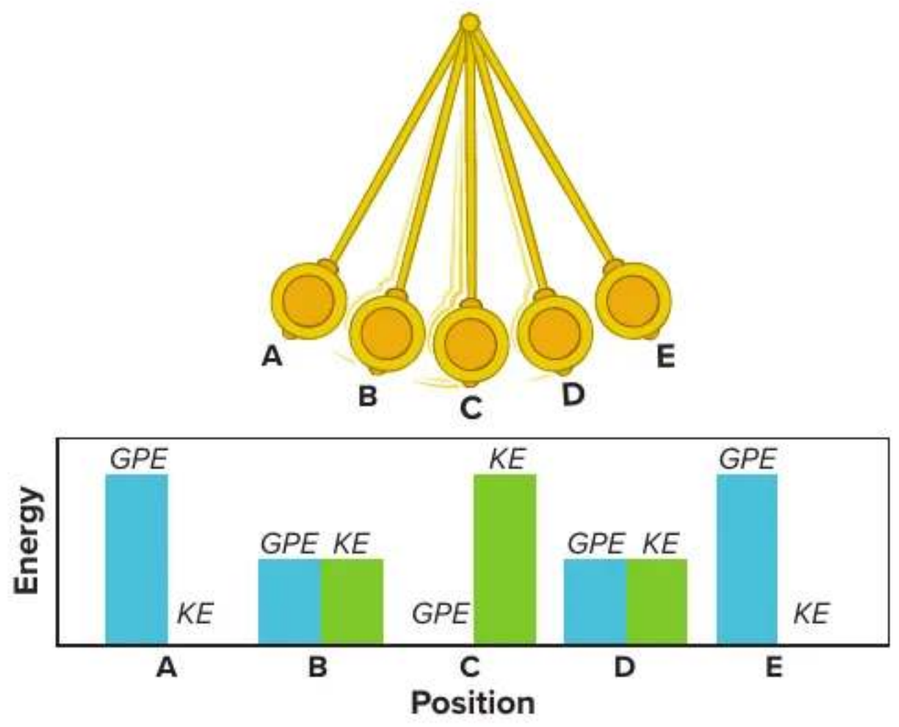
Roller Coasters The roller-coaster car is nearly at rest at the top of the first hill, and the total mechanical energy in the Earth-roller coaster car system is the system's gravitational potential energy at that point. If a hill farther along the track were higher than the first one, the roller-coaster car would not be able to climb the higher hill because the energy required to do so would be greater than the total mechanical energy of the system.





Skiing When you ski down a steep slope, you begin from rest at the top of the slope and the total mechanical energy is equal to the gravitational potential energy. Once you start skiing downhill, the gravitational potential energy is transformed to kinetic energy. As you ski down the slope, your speed increases as more potential energy is transformed to kinetic energy.

Pendulums The simple oscillation of a pendulum also demonstrates conservation of mechanical energy. The system is the pendulum bob and Earth. The reference level is chosen to be the height of the bob at the lowest point. When an external force pulls the bob to one side, the force does work that adds mechanical energy to the system. At the instant the bob is released, all the energy is in the form of potential energy, but as the bob swings downward, potential energy is transformed to kinetic energy. When the bob is at the lowest point, the gravitational potential energy is zero, and the kinetic energy is equal to the total mechanical energy.

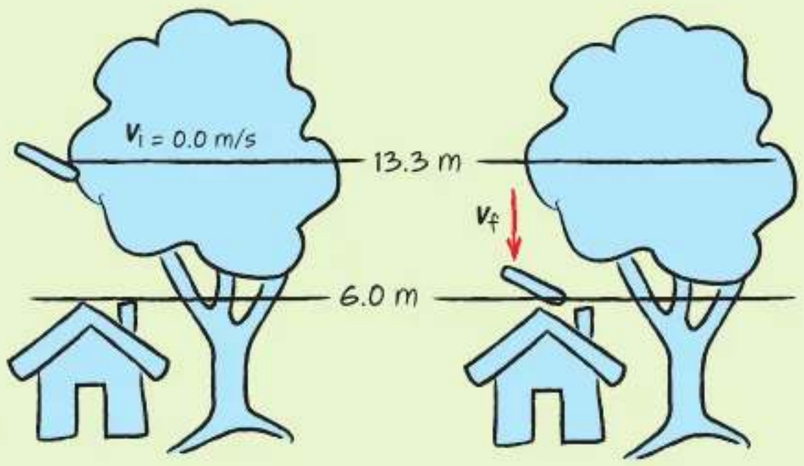


EXAMPLE Problem 5

CONSERVATION OF MECHANICAL ENERGY A 22.0-kg tree limb is 13.3 m above the ground. During a tropical storm, it falls on a roof that is 6.0 m above the ground.

a. Find the kinetic energy of the limb when it reaches the roof. Assume that the air does no work on the tree limb.

b. What is the limb's speed when it reaches the roof?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the initial and final conditions.
- Choose a reference level.
- Draw an energy bar diagram.

Known

$m = 22.0$ kg $\quad g = 9.8$ N/kg

$h_{limb} = 13.3$ m $\quad v_i = 0.0$ m/s

$h_{roof} = 6.0$ m $\quad KE_i = 0.0$ J

Unknown

$GPE_i = ?$ $\quad KE_f = ?$

$GPE_f = ?$ $\quad v_f = ?$

2 SOLVE FOR THE UNKNOWN

a. Set the reference level as the height of the roof.

Find the initial height of the limb relative to the roof.

$h = h_{limb} - h_{roof} = 13.3 \text{ m} - 6.0 \text{ m} = 7.3 \text{ m}$ Substitute $h_{limb} = 13.3$ m, $h_{roof} = 6.0$ m

Determine the initial gravitational potential energy of the limb-Earth system.

$GPE_i = mgh = (22.0 \text{ kg})(9.8 \text{ N/kg})(7.3 \text{ m})$ Substitute $m = 22.0$ kg, $g = 9.8$ N/kg, $h = 7.3$ m

$= 1.6 \times 10^3$ J

Identify the initial kinetic energy of the system.

$KE_i = 0.0$ J The tree limb is initially at rest.

Identify the final potential energy of the system.

$GPE_f = 0.0$ J $h = 0.0$ m at the roof.

Use the law of conservation of energy to find KE_f.

$KE_f + GPE_f = KE_i + GPE_i$

$KE_f = KE_i + GPE_i - GPE_f$

$= 0.0 \text{ J} + 1.6 \times 10^3 \text{ J} - 0.0 \text{ J}$ Substitute $KE_i = 0.0$ J, $GPE_i = 1.6 \times 10^3$ J, and $GPE_f = 0.0$ J.

$= 1.6 \times 10^3$ J

b. Determine the speed of the limb.

$KE_f = \frac{1}{2} m v_f^2$

$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2(1.6 \times 10^3 \text{ J})}{22.0 \text{ kg}}}$ Substitute $KE_f = 1.6 \times 10^3$ J, $m = 22.0$ kg

$= 12$ m/s

3 EVALUATE THE ANSWER

- **Are the units correct?** The magnitude of velocity is measured in m/s, and energy is measured in $kg \cdot m^2/s^2 = J$.
- **Do the signs make sense?** Kinetic energy and speed are both positive in this scenario.

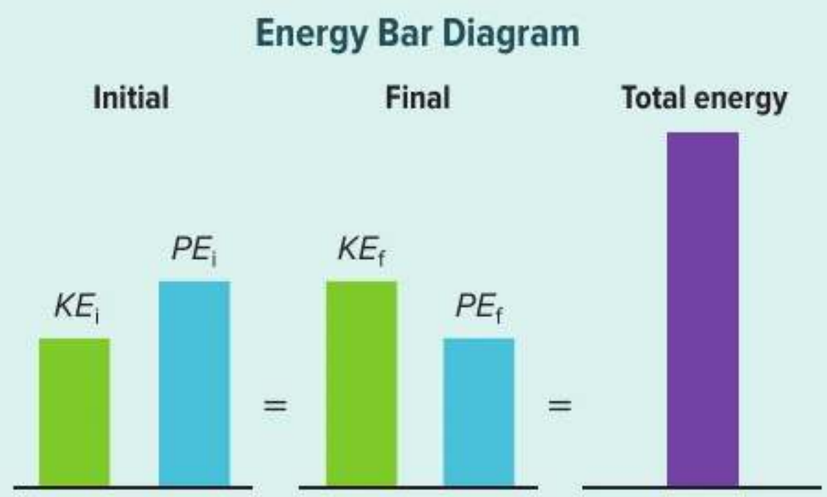
PROBLEM-SOLVING STRATEGY

CONSERVATION OF ENERGY

Use the following strategy when solving problems about the conservation of energy.

1. Identify the system. Determine whether the system is closed. No objects enter or leave a closed system.
2. Identify the forms of energy in the system. Identify which forms are part of the mechanical energy of the system.
3. Identify the system's initial and final states.
4. Is the system isolated?
   - **a.** If there are no external forces acting on the system, then the system is isolated and the total energy of the system is constant:
     $$E_{initial} = E_{final}$$
   - **b.** If there are external forces, then the final energy is the sum of the initial energy and the work done on the system.

   Remember that work can be negative:
   $$E_{final} = E_{initial} + W$$



5. For an isolated system, identify the types of energy in the system before and after. If the only forms of energy are potential and kinetic, mechanical energy is conserved.
   $$KE_{initial} + PE_{initial} = KE_{final} + PE_{final}$$
   Decide on the reference level for gravitational potential energy. Draw energy bar diagrams showing initial and final energies like the diagram above.

PRACTICE Problems

ADDITIONAL PRACTICE

39. A bike rider approaches a hill at a speed of 8.5 m/s. The combined mass of the bike and the rider is 85.0 kg. Choose a suitable system. Find the initial kinetic energy of the system. The rider coasts up the hill. Assuming friction is negligible, at what height will the bike come to rest?

40. Suppose that the bike rider in the previous problem pedaled up the hill and never came to a stop. In what system is energy conserved? From what form of energy did the bike gain mechanical energy?

41. A skier starts from rest at the top of a hill that is 45.0 m high, skis down a 30° incline into a valley, and continues up a hill that is 40.0 m high. The heights of both hills are measured from the valley floor. Assume that friction is negligible and ignore the effect of the ski poles.

- **a.** How fast is the skier moving at the bottom of the valley?
- **b.** What is the skier's speed at the top of the second hill?
- **c.** Do the angles of the hills affect your answers?

42. In a belly-flop diving contest, the winner is the diver who makes the biggest splash upon hitting the water. The size of the splash depends not only on the diver's style, but also on the amount of kinetic energy the diver has. Consider a contest in which each diver jumps from a 3.00-m platform. One diver has a mass of 136 kg and simply steps off the platform. Another diver has a mass of 100 kg and leaps upward from the platform. How high would the second diver have to leap to make a competitive splash?

43. CHALLENGE The spring in a pinball machine exerts an average force of 2 N on a 0.08-kg pinball over 5 cm. As a result, the ball has both translational and rotational kinetic energy. If the ball is a uniform sphere $\left(I = \frac{5}{2}mr^2\right)$, what is its linear speed after leaving the spring? (Ignore the table's tilt.)



Figure 25 In a car crash, the *KE* decreases.
Explain *What happened to the energy?*

Analyzing Collisions

A collision between any two objects, whether the objects include automobiles such as the ones shown in **Figure 25,** hockey players, or subatomic particles, is one of the most common situations analyzed in physics. The details of a collision can be complex during the collision itself, so the strategy is to find the motion of the objects just before and just after the collision. What conservation laws can be used to analyze such a system? If the system is closed and isolated, then momentum and energy are conserved. However, the potential or thermal energy in the system might decrease, remain the same, or increase. Therefore, you cannot predict whether kinetic energy is conserved. **Figure 26** on the next page shows four different kinds of collisions. Look at the calculations for kinetic energy for each of the cases. In all the cases except case 1, there is a change in the amount of kinetic energy.

In case 1, the momentum of the system before and after the collision is represented as

$$p_i = p_{Ai} + p_{Bi}$$
$$= (1.00\ \text{kg})(1.00\ \text{m/s}) + (1.00\ \text{kg})(0.00\ \text{m/s})$$
$$= 1.00\ \text{kg·m/s}$$
$$p_f = p_{Af} + p_{Bf} = (1.00\ \text{kg})(0.00\ \text{m/s}) +$$
$$(1.00\ \text{kg})(1.00\ \text{m/s})$$
$$= 1.00\ \text{kg·m/s}$$

Thus, in case 1, the momentum is conserved.

RobertCrum/iStock/Getty Images

Get It?
Calculate the initial and final momenta for the remaining cases in **Figure 26.** Verify that momentum is conserved for each case.

Elastic and inelastic collisions Next, consider the kinetic energy of the system in each case. In case 1, the kinetic energy of the system before and after the collision was the same. A collision in which the kinetic energy does not change is called an **elastic collision.** Collisions between hard objects, such as those made of steel or hard plastic, often are called nearly elastic collisions.

Kinetic energy decreased in cases 2 and 3. Some of it may have been transformed to thermal energy. A collision in which kinetic energy decreases is called an **inelastic collision.** Objects made of soft, sticky materials, such as clay, act in this way. In case 2, the objects stuck together after colliding. A collision in which colliding objects stick together is called a perfectly inelastic collision.

In case 4, the kinetic energy of the system increased. If energy in the system is conserved, then one or more other forms of energy must have decreased. Perhaps potential energy from a compressed spring was released during the collision, adding kinetic energy to the system. This kind of collision is called a superelastic or explosive collision.

The four kinds of collisions can be represented using energy bar diagrams, also shown in **Figure 26.** The kinetic energies before and after the collisions can be calculated, but we do not always have detailed information about the other forms of energy involved.

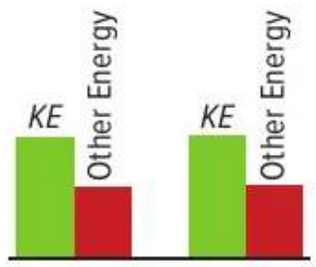
In automobile collisions, kinetic energy can be transformed into thermal energy and sound, but they are difficult to measure. We infer their amounts from the amount of kinetic energy that is transformed.

Get It?
Compare and contrast elastic collisions and inelastic collisions.

Figure 26 In a closed, isolated system, momentum is conserved when objects collide. Whether kinetic energy is conserved depends on whether the collision is elastic, inelastic, or superelastic.

Case 1: Elastic Collision

$KE_f = KE_i$



Before collision



$v_{Ai} = 1.00$ m/s $\quad v_{Bi} = 0.00$ m/s

$KE_i = KE_{Ai} + KE_{Bi}$

$= \frac{1}{2}(1.00\text{ kg})(1.00\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(0.00\text{ m/s})^2$

$= 0.50$ J

After collision



$v_{Af} = 0.00$ m/s $\quad v_{Bf} = 1.00$ m/s

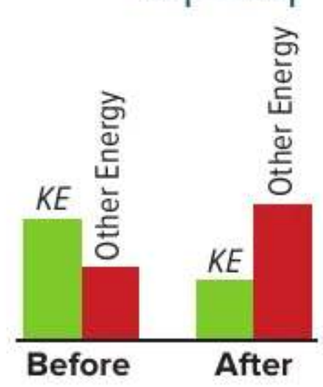
$KE_f = KE_{Af} + KE_{Bf}$

$= \frac{1}{2}(1.00\text{ kg})(0.00\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(1.00\text{ m/s})^2$

$= 0.50$ J

Case 2: Perfectly Inelastic Collision

$KE_f < KE_i$



Before collision



$v_{Ai} = 1.00$ m/s $\quad v_{Bi} = 0.00$ m/s

$KE_i = KE_{Ai} + KE_{Bi}$

$= \frac{1}{2}(1.00\text{ kg})(1.00\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(0.00\text{ m/s})^2$

$= 0.50$ J

After collision



$v_{Af} = v_{Bf} = 0.50$ m/s

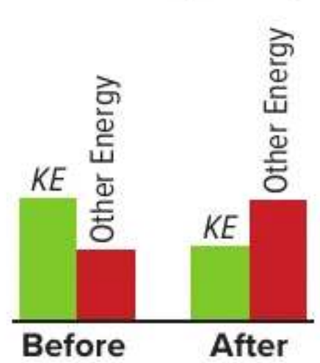
$KE_f = KE_{Af} + KE_{Bf}$

$= \frac{1}{2}(1.00\text{ kg})(0.50\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(0.50\text{ m/s})^2$

$= 0.25$ J

Case 3: Inelastic Collision

$KE_f < KE_i$



Before collision



$v_{Ai} = 1.00$ m/s $\quad v_{Bi} = 0.00$ m/s

$KE_i = KE_{Ai} + KE_{Bi}$

$= \frac{1}{2}(1.00\text{ kg})(1.00\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(0.00\text{ m/s})^2$

$= 0.50$ J

After collision



$v_{Af} = 0.25$ m/s $\quad v_{Bf} = 0.75$ m/s

$KE_f = KE_{Af} + KE_{Bf}$

$= \frac{1}{2}(1.00\text{ kg})(0.25\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(0.75\text{ m/s})^2$

$= 0.31$ J

Case 4: Superelastic Collision

$KE_f > KE_i$



Before collision



$v_{Ai} = 1.00$ m/s $\quad v_{Bi} = 0.00$ m/s

$KE_i = KE_{Ai} + KE_{Bi}$

$= \frac{1}{2}(1.00\text{ kg})(1.00\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(0.00\text{ m/s})^2$

$= 0.50$ J

After collision



$v_{Af} = 0.20$ m/s $\quad v_{Bf} = 1.20$ m/s

$KE_f = KE_{Af} + KE_{Bf}$

$= \frac{1}{2}(1.00\text{ kg})(-0.20\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(1.20\text{ m/s})^2$

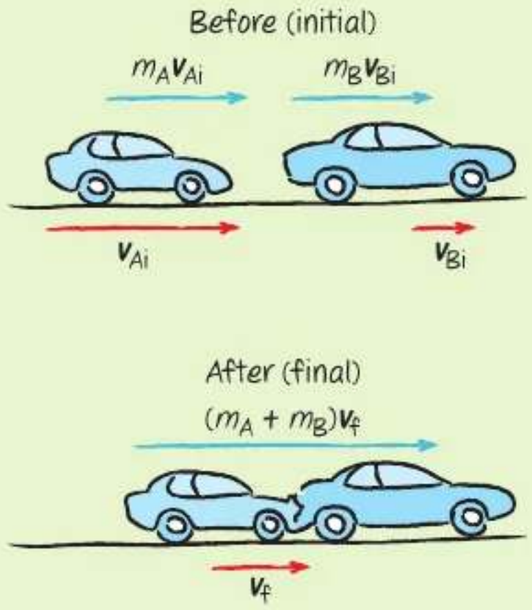
$= 0.74$ J

EXAMPLE Problem 6

INELASTIC COLLISION In an accident on a slippery road, a compact car with a mass of 1150 kg moving at 15.0 m/s smashes into the rear end of a car with a mass of 1575 kg moving at 5.0 m/s in the same direction.

a. What is the final velocity if the wrecked cars lock together?

b. How much is the kinetic energy decreased in the collision?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the initial and final conditions.
- Sketch the momentum diagram.

Known

$m_A = 1150$ kg

$m_B = 1575$ kg

$v_{Ai} = 15.0$ m/s

$v_{Bi} = 5.0$ m/s

$v_{Af} = v_{Bf} = v_f$

Unknown

$v_f = ?$

$\Delta KE = KE_f - KE_i = ?$

2 SOLVE FOR THE UNKNOWN

a. Use conservation of momentum to find the final velocity.

$$p_{Ai} + p_{Bi} = p_{Af} + p_{Bf}$$

$$m_A v_{Ai} + m_B v_{Bi} = (m_A + m_B)v_f$$

$$v_f = \frac{m_A v_{Ai} + m_B v_{Bi}}{m_A + m_B}$$

$$= \frac{(1150\text{ kg})(15.0\text{ m/s}) + (1575\text{ kg})(5.0\text{ m/s})}{(1150\text{ kg} + 1575\text{ kg})}$$

Substitute $m_A = 1150$ kg, $v_{Ai} = 15.0$ m/s, $m_B = 1575$ kg, $v_{Bi} = 5.0$ m/s

$= 9.2$ m/s, in the direction of the motion before the collision

b. To determine the change in kinetic energy of the system, KE_f and KE_i are needed.

$$KE_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(m_A + m_B)v_f^2$$

Substitute $m = m_A + m_B$

$$= \frac{1}{2}(1150\text{ kg} + 1575\text{ kg})(9.2\text{ m/s})^2$$

Substitute $m_A = 1150$ kg, $m_B = 1575$ kg, $v_f = 9.2$ m/s

$$= 1.2\times10^5\text{ J}$$

$$KE_i = KE_{Ai} + KE_{Bi}$$

$$= \frac{1}{2}(m_A v_{Ai}^2) + \frac{1}{2}(m_B v_{Bi}^2)$$

Substitute $KE_{Ai} = \frac{1}{2}(m_A v_{Ai}^2)$, $KE_{Bi} = \frac{1}{2}(m_B v_{Bi}^2)$

$$= \frac{1}{2}(1150\text{ kg})(15.0\text{ m/s})^2 + \frac{1}{2}(1575\text{ kg})(5.0\text{ m/s})^2$$

Substitute $m_A = 1150$ kg, $m_B = 1575$ kg, $v_{Ai} = 15.0$ m/s, $v_{Bi} = 5.0$ m/s

$$= 1.5\times10^5\text{ J}$$

Solve for the change in kinetic energy of the system.

$$\Delta KE = KE_f - KE_i$$

$$= 1.2\times10^5\text{ J} - 1.5\times10^5\text{ J} = -3\times10^4\text{ J}$$

Substitute $KE_f = 1.2\times10^5$ J, $KE_i = 1.5\times10^5$ J

3 EVALUATE THE ANSWER

- **Are the units correct?** Velocity is measured in meters per second; energy is measured in joules.
- **Does the sign make sense?** The change in kinetic energy is negative, meaning that kinetic energy decreased as expected in an inelastic collision.

PRACTICE Problems

ADDITIONAL PRACTICE

44. An 8.00-g bullet is fired horizontally into a 9.00-kg block of wood on an air table and is embedded in it. After the collision, the block and bullet slide along the frictionless surface together with a speed of 10.0 cm/s. Find the initial speed of the bullet.

45. A 91.0-kg hockey player is skating on ice at 5.50 m/s. Another hockey player of equal mass, moving at 8.1 m/s in the same direction, hits him from behind. They slide off together.

a. What are the total mechanical energy and momentum of the system before the collision?

b. What is the velocity of the two hockey players after the collision?

c. How much was the system's kinetic energy decreased in the collision?

46. CHALLENGE A 0.73-kg magnetic target is suspended on a string. A 0.025-kg magnetic dart, shot horizontally, strikes the target head-on. The dart and the target together act like a pendulum and swing 12.0 cm above the initial level before instantaneously coming to rest.

a. Sketch the situation and choose a system.

b. Decide what is conserved in each step of the process and explain why.

c. What was the initial velocity of the dart?

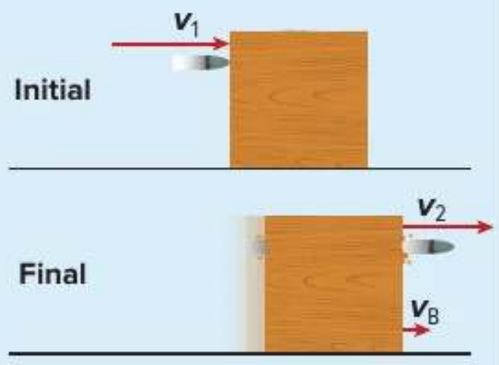
PHYSICS Challenge

A bullet of mass m, moving at speed v_1, goes through a motionless wooden block and exits with speed v_2. After the collision, the block, which has mass m_B, is moving.

1. What is the final speed (v_B) of the block?

2. What was the change in the bullet's mechanical energy?

3. How much energy was lost to friction inside the block?



Check Your Progress

47. **Energy Diagrams** A child jumps on a trampoline. Draw energy bar diagrams to show the forms of energy in the following situations.

a. The child is at the highest point.

b. The child is at the lowest point.

48. **Energy** Explain why energy is considered a single quantity.

49. **Kinetic Energy** Suppose a glob of chewing gum and a small, rubber ball collide head-on in midair and then rebound apart. Would you expect kinetic energy to be conserved? If not, what happens to the energy?

50. **Potential Energy** A rubber ball drops from a height of 8.0 m onto a concrete floor and bounces repeatedly. Each time it hits the floor, the ball-Earth system loses $\frac{1}{5}$ of its *ME*. How many times will the ball bounce before it bounces back up to a height of only 4 m?

51. **Energy** In **Figure 27,** a child slides down a playground slide. At the bottom, she is moving at 3.0 m/s. How much energy was transformed by friction as she slid down the slide?



Figure 27

52. **Conservation of Energy** Your friend wants to solve the world's energy problems by inventing a device that will deliver ten times more energy than put into the device. Can this device work? Explain.

53. **Critical Thinking** A ball drops 20 m. When it has fallen 10 m, half of the energy is potential energy and half is kinetic energy. When the ball has fallen for half the amount of time it takes to fall, will more, less, or exactly half of the energy be potential energy?

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LESSON 4

MACHINES

FOCUS QUESTION

How do machines make tasks easier?

Benefits of Machines

A **machine** is a device that makes tasks easier by changing either the magnitude or the direction of the applied force. When you lift the handle of the bottle opener in **Figure 28,** you transfer energy to the opener and do work on the opener. The opener lifts the cap, doing work on it. The work you do is called the input work (W_i). The work the machine does is called the output work (W_o).

The opener is not a source of energy. So, the cap cannot receive more energy than what you put into the opener. Thus, the output work can never be greater than the input work. The machine only helps transfer energy from you to the bottle cap.

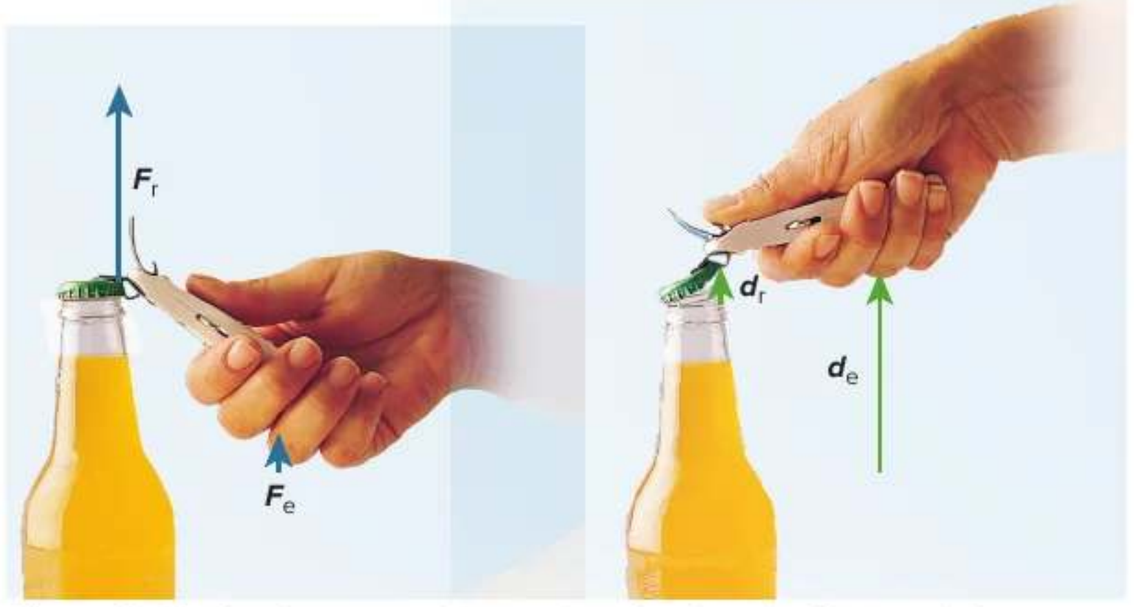


Figure 28 This bottle opener makes opening a bottle easier. However, it does not lessen the energy required to do so.

Mechanical advantage The force exerted by a user on a machine is called the **effort force** (F_e). The force exerted by the machine is called the **resistance force** (F_r). For the bottle opener in **Figure 28,** F_e is the upward force exerted by the person using the bottle opener and F_r is the upward force exerted by the bottle opener. The ratio of resistance force to effort force $\left(\frac{F_r}{F_e}\right)$ is called the **mechanical advantage** (MA) of the machine.

Mechanical Advantage

The mechanical advantage of a machine is equal to the resistance force divided by the effort force.

$$MA = \frac{F_r}{F_e}$$

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Lifting with Pulleys

Carry out an investigation into how the arrangement of a pulley system affects the force needed to lift a weight.

Quick Investigation: Wheel and Axle

Carry out an investigation into the effort and resistance forces on a wheel-and-axle system.

Hutchings Photography

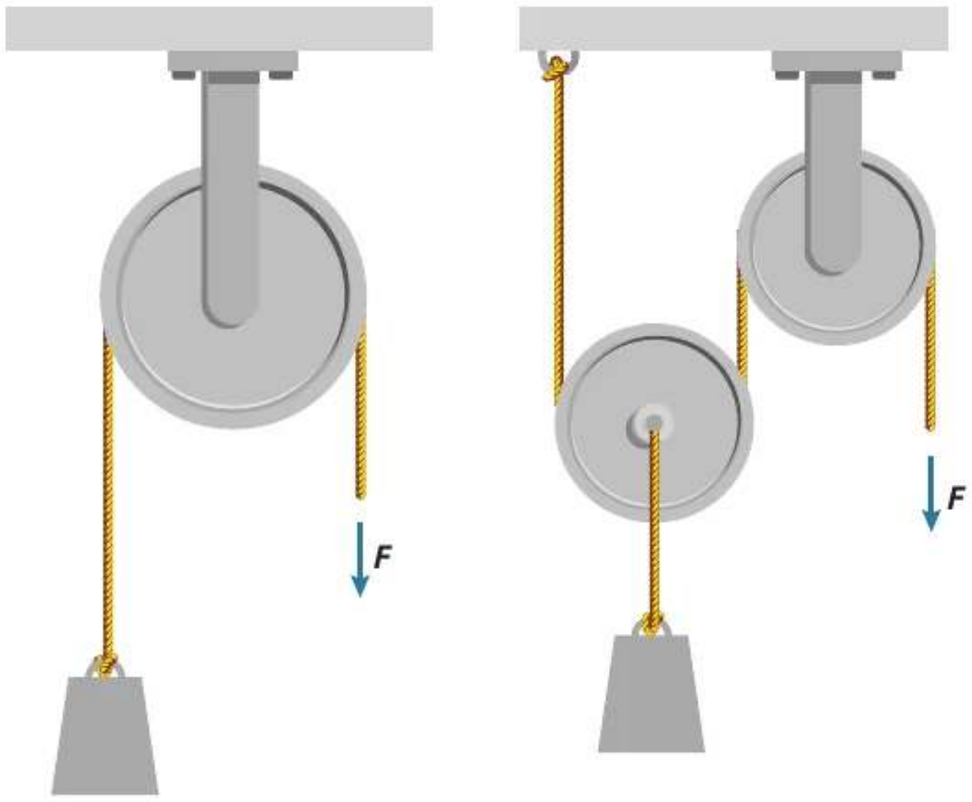


Figure 29 A fixed pulley has a mechanical advantage equal to 1. It is useful because it changes the direction of the force. A pulley system with a movable pulley has a mechanical advantage greater than 1, making the force applied to the object larger than the force originally exerted.

For a fixed pulley, such as the one shown on the left in **Figure 29,** the effort force (F_e) and the resistance force (F_r) are equal. Thus, the mechanical advantage is 1. What is the advantage of this machine? The fixed pulley is useful, not because the effort force is lessened, but because the direction of the effort force is changed. Many machines, such as the bottle opener shown in **Figure 28** and the pulley system shown on the right in **Figure 29,** have a mechanical advantage greater than 1. When the mechanical advantage is greater than 1, the machine increases the force applied by a person.

Get It?

Calculate A machine's *MA* is 3. If the input force is 2 N, what is the output force?

Ideal mechanical advantage A machine can increase force, but it cannot increase energy. An ideal machine transfers all the energy, so the output work equals the input work: $W_o = W_i$. The input work is the product of the effort force and the displacement the effort force acts through: $W_i = F_e d_e$. The output work is the product of the resistance force and the displacement the resistance force acts through: $W_o = F_r d_r$. Substituting these expressions into $W_o = W_i$ gives $F_r d_r = F_e d_e$. This equation can be rewritten $\frac{F_r}{F_e} = \frac{d_e}{d_r}$.

Recall that mechanical advantage is given by $MA = \frac{F_r}{F_e}$. Thus, for an ideal machine, **ideal mechanical advantage** (*IMA*) is equal to the displacement of the effort force divided by the displacement of the resistance force. *IMA* can be represented as follows.

Ideal Mechanical Advantage

The *IMA* of a machine is equal to the displacement of the effort force divided by the displacement of the resistance force.

$$IMA = \frac{d_e}{d_r}$$

Notice that you measure the displacements to calculate the ideal mechanical advantage, but you measure the forces exerted to find the actual mechanical advantage.

SCIENCE USAGE v. COMMON USAGE

Efficiency

Science usage: the ratio of output work to input work

The efficiency of the system of pulleys is 86 percent.

Common usage: production without waste

The new, high-efficiency washing machine uses less water and electricity than an older model of machine.

Efficiency In a real machine, not all of the input work is available as output work. Energy removed from the system through heat or sound means there is less output work from the machine. Consequently, the machine is less efficient at accomplishing the task. The **efficiency** of a machine (e) is defined as the ratio of output work to input work.

Efficiency

The efficiency of a machine (in %) is equal to the output work, divided by the input work, multiplied by 100.

$$e = \frac{W_o}{W_i} \times 100$$

An ideal machine has equal output and input work, $\frac{W_o}{W_i} = 1$, and its efficiency is 100 percent. All real machines have efficiencies that are less than 100 percent.

Efficiency can be expressed in terms of the mechanical advantage and ideal mechanical advantage. Efficiency, $e = \frac{W_o}{W_i}$, can be rewritten as follows:

$$e = \frac{W_o}{W_i} = \frac{F_r d_r}{F_e d_e}$$

Because $MA = \frac{F_r}{F_e}$ and $IMA = \frac{d_e}{d_r}$, the following expression can be written for efficiency.

Efficiency

The efficiency of a machine (in %) is equal to its mechanical advantage, divided by the ideal mechanical advantage, multiplied by 100.

$$e = \frac{MA}{IMA} \times 100$$

A machine's design determines its ideal mechanical advantage. An efficient machine has an *MA* almost equal to its *IMA*. A less-efficient machine has a small *MA* relative to its *IMA*. To obtain the same resistance force, a greater force must be exerted in a machine of lower efficiency than in a machine of higher efficiency.

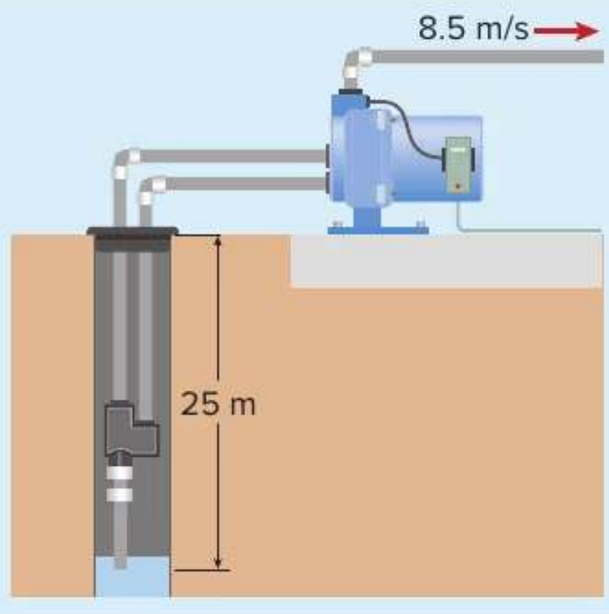
Get It?

Determine A machine has an efficiency of 50 percent and an *MA* of 3. What is its *IMA*?

PHYSICS Challenge

In many rural areas, residents do not get their water supply from a central water tower or other common source. Many residents need to use an electric pump to get their water from a well. A certain electric pump pulls water at a rate of 0.25 m^3/s from a well that is 25 m deep. The water leaves the pump at a speed of 8.5 m/s.

1. What power is needed to lift the water to the surface?
2. What power is needed to increase the water's kinetic energy?
3. If the pump's efficiency is 80 percent, how much power must be delivered to the pump?



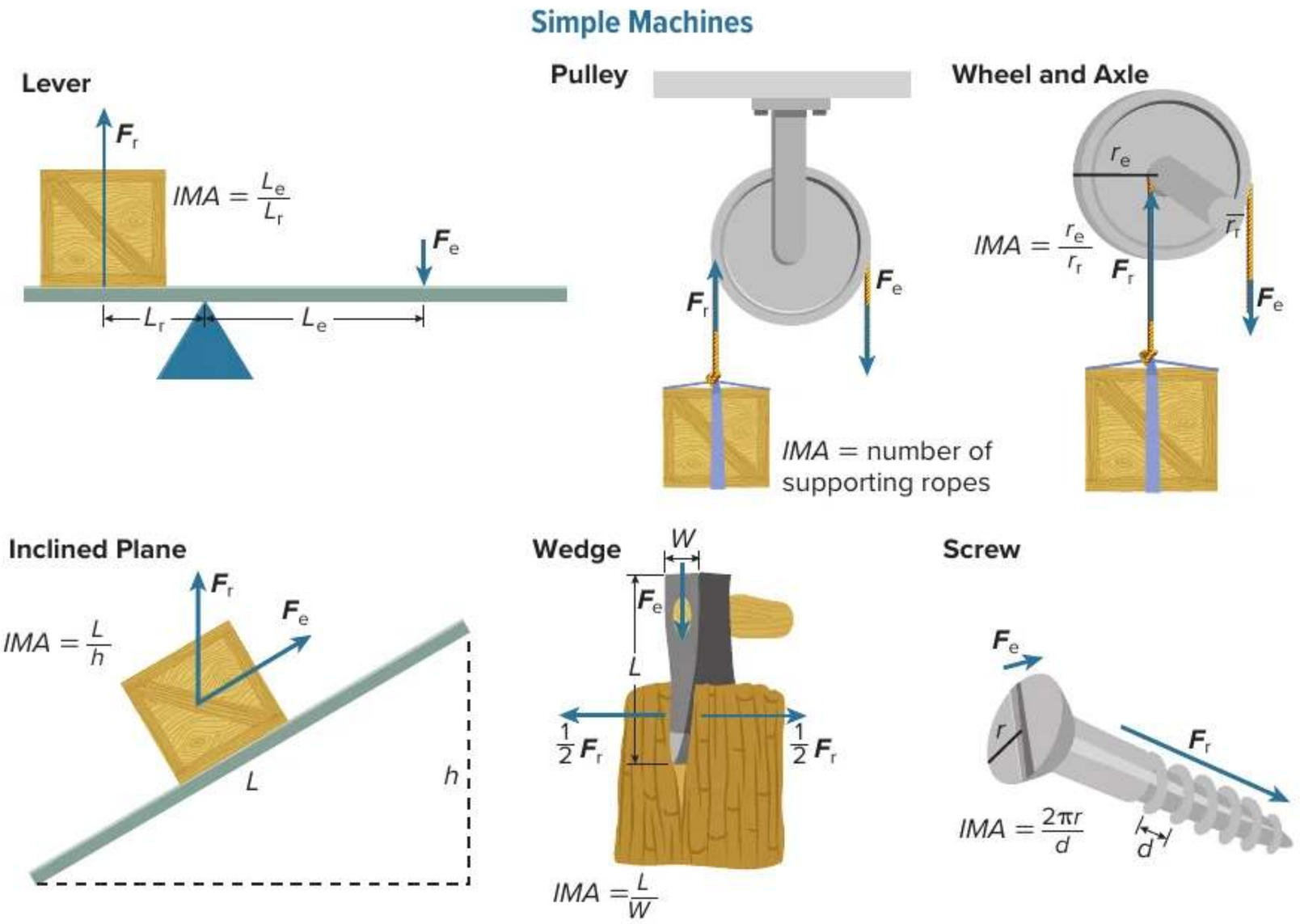


Figure 30 Each type of simple machine makes work easier by changing the direction or magnitude of the force. The *IMA* of each machine is related to the properties of the machine. For example, the *IMA* of a lever is equal to the length of the effort arm (L_e) divided by the length of the resistance arm (L_r). Similarly, the *IMA* of a wedge is equal to its length (L) divided by its width (W).

Identify *an everyday example of each simple machine.*

Compound Machines

Most machines are combinations of one or more of the six simple machines: the lever, pulley, wheel and axle, inclined plane, wedge, and screw, as shown in **Figure 30.**

The *IMA* of each machine in **Figure 30** is the ratio of the displacement of the effort force to the displacement of the resistance force. For machines such as the lever and the wheel and axle, this ratio can be replaced by the ratio of the displacements between the place where the force is applied and the pivot point. For example, a common version of the wheel and axle is a steering wheel, such as the one shown in **Figure 31.** The *IMA* is the ratio of the radii of the wheel and axle.

A machine consisting of two or more simple machines linked in such a way that the resistance force of one machine becomes the effort force of the second is called a **compound machine.** Some examples of compound machines are scissors (wedges and levers) and wheelbarrows (lever and wheel and axle).

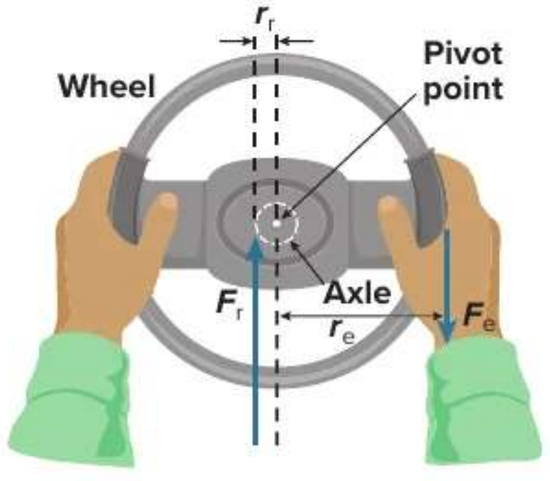


Figure 31 A steering wheel is an example of a wheel and axle. Its *IMA* is $\frac{r_e}{r_r}$.

Get It?

Compare and contrast simple machines and compound machines.



Figure 32 A bicycle is a compound machine. The rider exerts a force on the pedals, which exert a force on the chain. The chain then exerts a force on the rear gear.

Mechanical advantage and bicycles A bicycle, like the one shown in **Figure 32,** is a compound machine, consisting of two wheel-and-axle systems. The first is the pedal and front gear. Here, the effort force is the force that the rider exerts on the pedal, $F_{\text{rider on pedal}}$. The force of your foot is most effective when the force is exerted perpendicular to the arm of the pedal; that is, when the torque is largest. Therefore, we will assume that $F_{\text{rider on pedal}}$ is applied perpendicular to the pedal arm. The resistance is the force that the front gear exerts on the chain, $F_{\text{gear on chain}}$.

Get It?

Identify the effort force and the resistance force for the pedal and front gear.

The rear gear and the rear wheel act like another wheel and axle. The chain exerts an effort force on the rear gear, $F_{\text{chain on gear}}$. This force is equal to the force of the front gear on the chain. That is, $F_{\text{gear on chain}} = F_{\text{chain on gear}}$. The resistance force is the force that the wheel exerts on the road, $F_{\text{wheel on road}}$.

The mechanical advantage of a compound machine is the product of the mechanical advantages of the simple machines from which it is made. Therefore, the bicycle's mechanical advantage is given by the following equation:

$$MA_{\text{total}} = MA_{\text{pedal gear}} \times MA_{\text{rear wheel}}$$

$$MA_{\text{total}} = \left(\frac{F_{\text{gear on chain}}}{F_{\text{rider on pedal}}}\right)\left(\frac{F_{\text{wheel on road}}}{F_{\text{chain on gear}}}\right) = \frac{F_{\text{wheel on road}}}{F_{\text{rider on pedal}}}$$

Recall that $F_{\text{gear on chain}} = F_{\text{chain on gear}}$. Therefore, they cancel in the equation shown above.

Similarly, the IMA of the bicycle is $IMA = IMA_{\text{pedal gear}} \times IMA_{\text{rear wheel}}$. For each wheel and axle, the IMA is the ratio of the wheel radius to the axle radius.

For the pedal gear, $$IMA = \frac{\text{pedal radius}}{\text{front gear radius}}$$

For the rear wheel, $$IMA = \frac{\text{rear gear radius}}{\text{wheel radius}}$$

For the bicycle, then $$IMA = \left(\frac{\text{pedal radius}}{\text{front gear radius}}\right)\left(\frac{\text{rear gear radius}}{\text{wheel radius)}}\right)$$
$$= \left(\frac{\text{rear gear radius}}{\text{front gear radius}}\right)\left(\frac{\text{pedal radius}}{\text{wheel radius}}\right)$$

Laura Sifferlin

EXAMPLE Problem 7

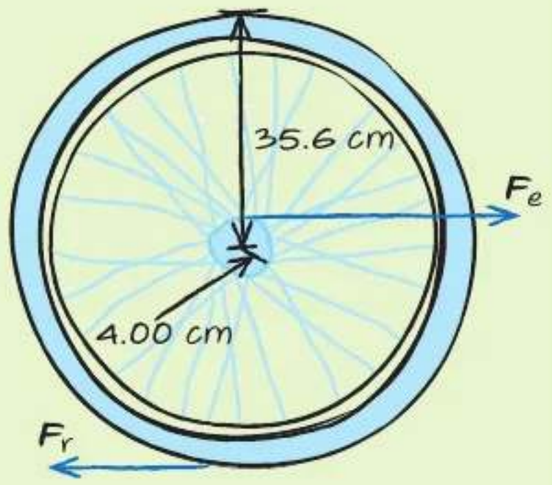
MECHANICAL ADVANTAGE You examine the rear wheel on your bicycle. It has a radius of 35.6 cm and has a gear with a radius of 4.00 cm. When the chain is pulled with a force of 155 N, the wheel rim moves 14.0 cm. The efficiency of this part of the bicycle is 95.0 percent.

a. What is the *IMA* of the wheel and gear?

b. What is the *MA* of the wheel and gear?

c. What is the resistance force?

d. How far was the chain pulled to move the rim 14.0 cm?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the wheel and axle.
- Sketch the force vectors.

Known

$r_e = 4.00$ cm $\quad e = 95.0\%$ $\quad F_e = 155$ N

$r_r = 35.6$ cm $\quad d_r = 14.0$ cm

Unknown

$IMA = ?$ $\quad F_r = ?$

$MA = ?$ $\quad d_e = ?$

2 SOLVE FOR THE UNKNOWN

a. Solve for *IMA*.

$$IMA = \frac{r_e}{r_r}$$

For a wheel-and-axle machine, *IMA* is equal to the ratio of radii.

$$= \frac{4.00 \text{ cm}}{35.6 \text{ cm}} = 0.112$$

Substitute $r_e = 4.00$ cm, $r_r = 35.6$ cm.

b. Solve for *MA*.

$$e = \frac{MA}{IMA} \times 100$$

$$MA = \left(\frac{e}{100}\right) \times IMA$$

$$= \left(\frac{95.0}{100}\right) \times 0.112 = 0.106$$

Substitute $e = 95.0\%$, $IMA = 0.112$.

c. Solve for force.

$$MA = \frac{F_r}{F_e}$$

$$F_r = (MA)(F_e)$$

$$F_r = (0.106)\ (155 \text{ N}) = 16.4 \text{ N}$$

Substitute $MA = 0.106$, $F_e = 155$ N.

d. Solve for distance.

$$IMA = \frac{d_e}{d_r}$$

$$d_e = (IMA)\ (d_r)$$

$$= (0.112)\ (14.0 \text{ cm}) = 1.57 \text{ cm}$$

Substitute $IMA = 0.112$, $d_r = 14.0$ cm.

3 EVALUATE THE ANSWER

- **Are the units correct?** Force is measured in newtons and distance in centimeters.
- **Is the magnitude realistic?** *IMA* is low for a bicycle because a greater F_e is traded for a greater d_r. *MA* is always smaller than *IMA*. Because *MA* is low, F_r also will be low. The small distance the axle moves results in a large distance covered by the wheel. Thus, d_e should be very small.

PRACTICE Problems

ADDITIONAL PRACTICE

54. If the gear radius of the bicycle in Example Problem 7 is doubled while the force exerted on the chain and the distance the wheel rim moves remain the same, what quantities change, and by how much?

55. A sledgehammer is used to drive a wedge into a log to split it. When the wedge is driven 0.20 m into the log, the log is separated a distance of 5.0 cm. A force of 1.7×10^4 N is needed to split the log, and the sledgehammer exerts a force of 1.1×10^4 N.

a. What is the *IMA* of the wedge?

b. What is the *MA* of the wedge?

c. Calculate the efficiency of the wedge as a machine.

56. A worker uses a pulley to raise a 24.0-kg carton 16.5 m, as shown in **Figure 33.** A force of 129 N is exerted, and the rope is pulled 33.0 m.

a. What is the *MA* of the pulley?

b. What is the efficiency of the pulley?

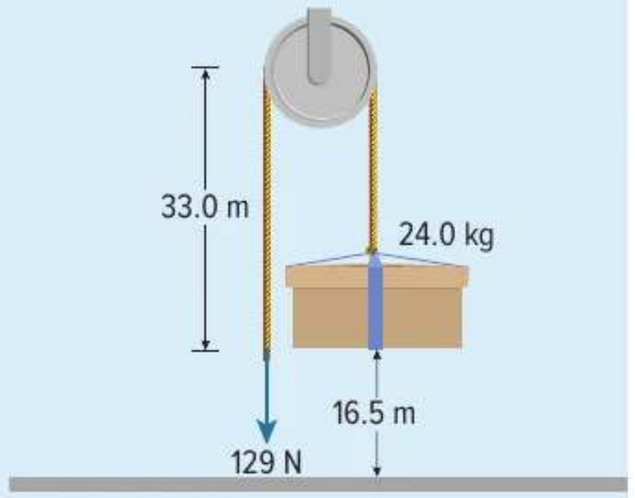


Figure 33

57. A winch has a crank with a 45-cm radius. A rope is wrapped around a drum with a 7.5-cm radius. One revolution of the crank turns the drum one revolution.

a. What is the ideal mechanical advantage of this machine?

b. If, due to friction, the machine is only 75 percent efficient, how much force would have to be exerted on the handle of the crank to exert 750 N of force on the rope?

58. CHALLENGE You exert a force of 225 N on a lever to raise a 1.25×10^3-N rock a distance of 13 cm. If the efficiency of the lever is 88.7 percent, how far did you move your end of the lever?

Multi-gear bicycle

Shifting gears on a bicycle is a way of adjusting the ratio of gear radii to obtain the desired *IMA*. On a multi-gear bicycle, the rider can change the *IMA* of the machine by choosing the size of one or both gears. **Figure 34** shows a rear gear with five different gear sizes. When accelerating or climbing a hill, the rider increases *IMA* to increase the force that the wheel exerts on the road.

To do this, the rider makes the rear gear radius large compared to the front gear radius. For the same force exerted by the rider, a larger force is exerted by the wheel on the road—the rider rotates the pedals through more turns for each revolution of the wheel.

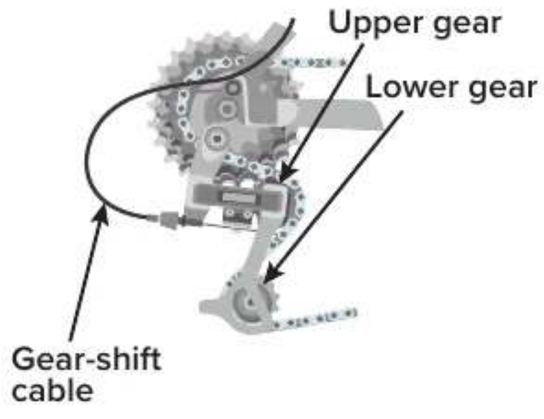


Figure 34 A rider can change the *IMA* of the bicycle by shifting gears.

However, less force is needed to ride the bicycle at high speed on a level road. The rider needs a small rear gear and a large front gear, resulting in a smaller *IMA*. Thus, for the same force exerted by the rider, a smaller force is exerted by the wheel on the road. In return, the rider does not have to move the pedals as far for each revolution of the wheel.

An automobile transmission works in the same way. To accelerate a car from rest, large forces are needed and the transmission increases the *IMA* by increasing the gear ratio.

At high speeds, however, the transmission reduces the gear ratio and the *IMA* because smaller forces are needed. Even though the speedometer shows a high speed, the tachometer indicates the engine's low angular speed.

Get It?

Explain why a car needs multiple gears.

The Human Walking Machine

BIOLOGY Connection Simple machines, in the form of levers, such as the lever system in **Figure 35,** give us the ability to walk, run, and do other activities. The complex lever systems of the human body have the following four basic parts:

1. a rigid bar (bone)
2. a source of force (muscle contraction)
3. a fulcrum or pivot (movable joints between bones)
4. a resistance (the weight of the body or an object being lifted or moved)

Lever systems of the body are not very efficient, and mechanical advantages are low. This is why walking and jogging require energy (burn calories) and help people lose weight.

When a person walks, the hip acts as a fulcrum and moves through the arc of a circle, centered on the foot. The center of mass of the body moves as a resistance around the fulcrum in the same arc. The length of the radius of the circle is the length of the lever formed by the bones of the leg. Athletes in walking races increase their velocity by swinging their hips upward to increase this radius.

Although tall people usually can walk faster than short people can, a tall person must apply a greater force to move the longer lever formed by the leg bones. Because of the inefficiency of their lever systems and the length of a 20- or 50-km walking race, very tall people rarely have the stamina to win.

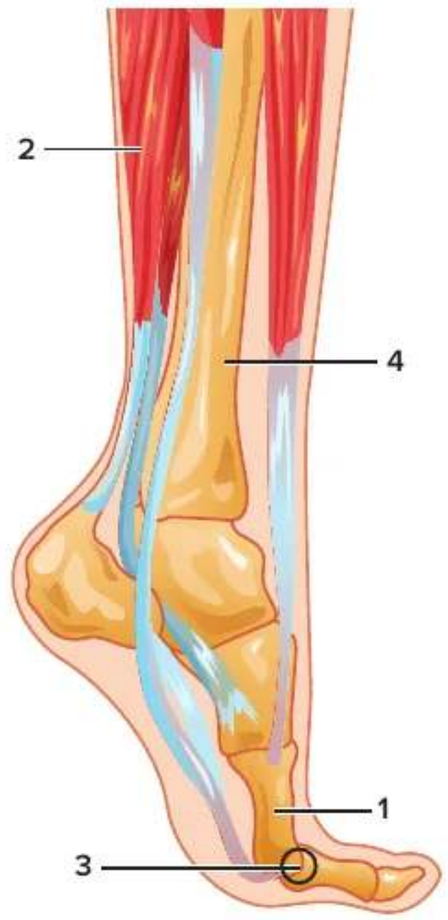


Figure 35 The human leg and foot function as a compound machine.

Check Your Progress

59. **Machines** Classify each tool below as a lever, a wheel and axle, an inclined plane, or a wedge. Describe how it changes the force to make the task easier.
    a. screwdriver
    b. pliers
    c. chisel
    d. nail puller
60. **IMA** A worker is testing a multiple pulley system to estimate the heaviest object that he could lift. The largest downward force he can exert is equal to his weight, 875 N. When the worker moves the rope 1.5 m, the object moves 0.25 m. What is the heaviest object that he could lift?
61. **Compound Machines** A winch has a crank on a 45-cm arm that turns a drum with a 7.5-cm radius through a set of gears. It takes three revolutions of the crank to rotate the drum through one revolution. What is the *IMA* of this compound machine?
62. **Efficiency** Suppose you increase the efficiency of a simple machine. Do the *MA* and *IMA* increase, decrease, or remain the same?
63. **Critical Thinking** The mechanical advantage of a multi-gear bicycle is changed by moving the chain to a suitable rear gear.
    a. To start out, you must accelerate the bicycle, so you want to have the bicycle exert the greatest possible force. Should you choose a small or large gear?
    b. As you reach your traveling speed, you want to rotate the pedals as few times as possible. Should you choose a small or large gear?
    c. Many bicycles also let you choose the size of the front gear. If you want even more force to accelerate while climbing a hill, would you move to a larger or smaller front gear? Explain.

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ENGINEERING & TECHNOLOGY

Greenhouse Gas Emission Efficiency: Excrement to Energy

Raising cattle is big business. It is also a big environmental headache. The waste produced by cows pollutes Earth's air and water. Cows release as much as 25% of global methane (CH_4) emissions, which is one of the main greenhouse gases contributing to climate change. Thanks to anaerobic digesters, methane gas from animal waste such as cow manure also has the potential to be harnessed to provide a source of renewable energy.



Anaerobic digesters on dairy farms help provide a source of renewable energy and eliminate harmful animal waste.

Rudmer Zwerver/Shutterstock

Anaerobic Digestion

There are over 90 million dairy and beef cattle in the United States. Each cow produces up to 40 kg of manure per day. How much manure is produced in the United States every year? A lot! In addition to contaminating runoff water with harmful bacteria and excess nitrogen, the methane gas emitted from decomposing manure and cow flatulence impacts Earth's atmosphere and climate change.

Anaerobic digestion is one process that is used to help mitigate this waste. Specific microorganisms are mixed with a watery manure slurry in a digestion tank. The conditions in the tank are kept oxygen-free and at a temperature optimal for anaerobic bacteria. The bacteria decompose the manure into smaller compounds, which are converted into biogas. Some impurities are filtered from the biogas as it rises in the fermenting tank. The result is a mixture of methane and carbon dioxide called *biogas*, which is fed into power plants through underground pipes. The gas is later burned in a power plant as a source of fuel in a process that generates heat and electricity.

The results of anaerobic digestion are overwhelmingly positive. The electricity and heat generated by biogas have been used to power farms and thousands of communities. And the remaining organic nutrient-rich materials left from production can be used to fertilize crops.

Some engineers are working on technology that can cheaply clean biogas to make it usable in converted car and airplane engines. Others are trying to make biogas feasible for home use. In the near future, manure may become one of the best renewable energy alternatives for our fast-depleting supply of fossil fuels, such as coal and oil.



OBTAIN, EVALUATE, AND COMMUNICATE INFORMATION

Using the internet or other appropriate resources, research other types of technology being used to generate biofuels. Write a brief summary of your findings.

MODULE 10
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 WORK AND ENERGY

- Work is done when a force is applied through a displacement. Work is the product of the force exerted on a system and the component of the distance through which the system moves:

$$W = Fd\cos\theta$$

- Energy is the ability of a system to produce a change in itself or its environment. A moving object has kinetic energy.
- The work done on a system is equal to the change in energy of the system. This is called the work-energy theorem:

$$W = \Delta E$$

- Power is the rate at which energy is transformed: $P = \frac{\Delta E}{t}$.

$$P = \frac{\Delta E}{t} = \frac{W}{t}$$

- work
- joule
- energy
- work-energy theorem
- kinetic energy
- power
- watt

Lesson 2 THE MANY FORMS OF ENERGY

- The translational kinetic energy of an object is proportional to its mass and the square of its velocity. The rotational kinetic energy of an object is proportional to the object's moment of inertia and the square of its angular velocity.
- The gravitational potential energy of an object-Earth system is the product of mass, gravitational field, and height above a reference level: $GPE = mgh$.
- Elastic potential energy is energy stored due to stretching, compressing, or bending an object. Other forms of energy are chemical, nuclear, thermal, electrical, radiant, and mass energy.

- translational kinetic energy
- rotational kinetic energy
- potential energy
- gravitational potential energy
- reference level
- elastic potential energy
- thermal energy

Lesson 3 CONSERVATION OF ENERGY

- The total energy of a closed, isolated system is constant. Within the system, energy can change form, but the total amount of energy does not change. Thus, energy is conserved.
- Mechanical energy is the sum of kinetic and potential energy:

$$ME = KE + PE$$

- Momentum is conserved in collisions if the external force is zero. The kinetic energy may be unchanged or decreased by the collision, depending on whether the collision is elastic or inelastic.

- law of conservation of energy
- mechanical energy
- elastic collision
- inelastic collision

Lesson 4 MACHINES

- Machines do not change the work done. They can make a task easier by changing the magnitude or direction of the effort force.
- Mechanical advantage is the ratio of resistance force to effort force.

$$MA = \frac{F_r}{F_e}$$

- Ideal mechanical advantage is the ratio of the distances moved.

$$IMA = \frac{d_e}{d_r}$$

- Efficiency is the ratio of output work to input work, in percent.

$$e = \frac{W_o}{W_i} \times 100$$

- machine
- effort force
- resistance force
- mechanical advantage
- ideal mechanical advantage
- efficiency
- compound machine

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

How can energy from power plants be stored in the power grid for later use?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

How much kinetic energy does a falling object have?

Five clay pots are dropped from the same rooftop. The results are summarized in the table.

Clay Pot	Mass (kg)	Initial Velocity (m/s)
A	1.0	0
B	1.0	2.0; downward
C	1.0	2.0; upward
D	1.0	2.0; horizontally
E	2.0	0

CER Analyze and Interpret Data

1. **Claim** Rank the clay pots according to speed when they strike the ground, from least to greatest. Specifically indicate any ties.
2. **Evidence and Reasoning** Explain how you determined your ranking.

shanshihan/Moment Open/Getty Images


Kayte Deioma/Alamy Stock Photo

MODULE 13

VIBRATIONS AND WAVES

ENCOUNTER THE PHENOMENON

How can this pendulum save a building from earthquake damage?



GO ONLINE to play a video about how dampers are used to prevent earthquake damage.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about how a pendulum could save a building from earthquake damage. Explain your reasoning.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

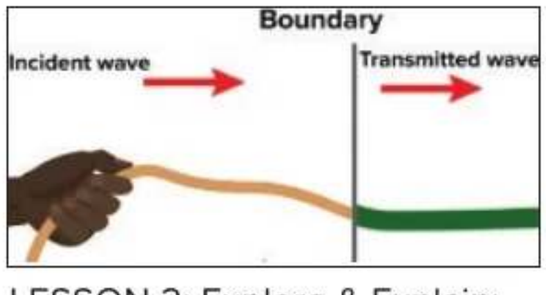
Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.

(t)Video Supplied by BBC Worldwide Learning, (b)Brian Jackson/123RF



LESSON 1: Explore & Explain: Pendulums



LESSON 2: Explore & Explain: Waves at Boundaries

Smithsonian

Additional Resources

LESSON 1

PERIODIC MOTION

FOCUS QUESTION

What are some types of repetitive motion?

Mass on a Spring

The bobbing of a mass on a spring or the swaying of a pendulum are examples of **periodic motion.** In each example, at one position the net force on the object is zero and the object is in equilibrium. When the object moves away from its equilibrium position, the net force on the system becomes nonzero. This net force acts to bring the object back toward equilibrium. The **period** (T) is the time needed for one full cycle of the motion. The **amplitude** of the motion is the maximum distance the object moves from the equilibrium position.

Simple harmonic motion In **Figure 1,** the force exerted by the spring is directly proportional to the distance it is stretched. When pulled down and released, the mass bobs up and down through equilibrium. Any system where the force acting to restore an object to its equilibrium position is directly proportional to the object's displacement shows **simple harmonic motion.**

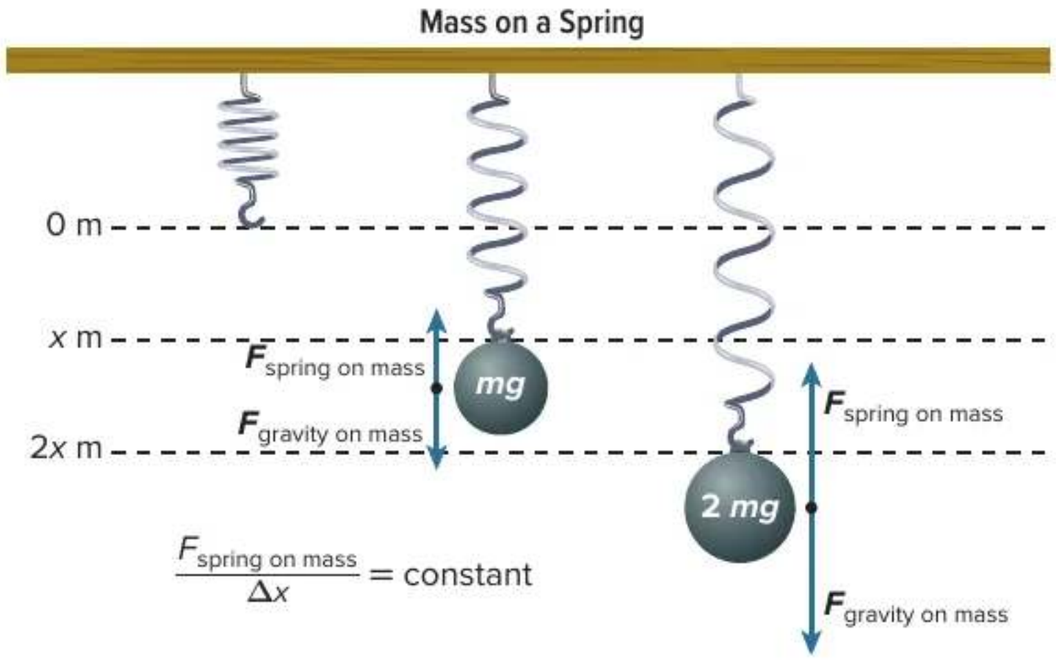


Figure 1 The force exerted on the mass by the spring is directly proportional to the mass's displacement.

Determine *the displacement if the mass is 0.5 mg.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Pendulum Vibrations

Plan and carry out an investigation to determine what variables affect a pendulum's period and use the resulting data to calculate the magnitude of the gravitation field.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Table 1 Force Magnitude-Stretch Distance in a Spring

Stretch Distance (m)	Magnitude of Force Exerted by Spring (N)
0.0	0.0
0.030	1.9
0.060	3.7
0.090	6.3
0.12	7.8

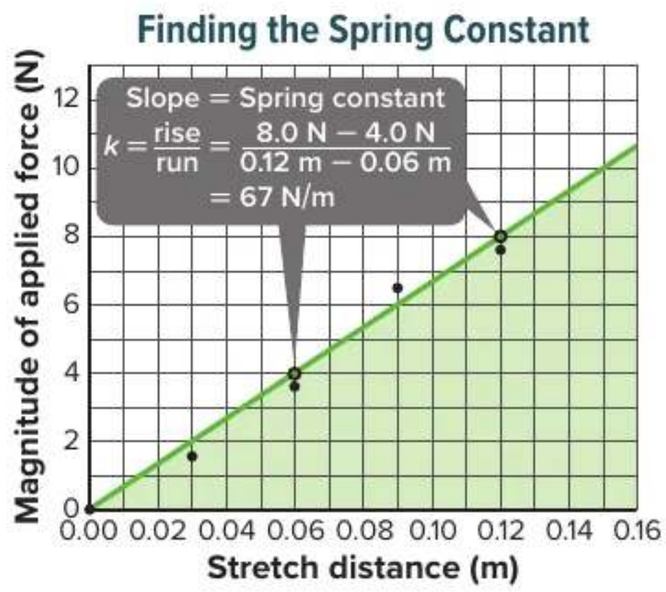


Figure 2 The spring constant can be determined from the slope of the force magnitude-stretch distance graph. The area under the curve is equal to the potential energy stored in the spring.

Hooke's Law

Table 1 shows the relationship between the magnitude of the force exerted by a spring and the distance the spring stretches. **Figure 2** is a graph of the data with the line of best fit. The linear relationship indicates that the magnitude of the force exerted by the spring is directly proportional to the amount the spring is stretched. A spring that exerts a force directly proportional to the distance stretched obeys **Hooke's law.**

Hooke's Law

The magnitude of the force exerted by a spring is equal to the spring constant times the distance the spring is stretched or compressed from its equilibrium position.

$$F = -kx$$

In this equation, k is the spring constant, which depends on the stiffness and other properties of the spring, and x is the distance the spring is stretched from its equilibrium position. Notice that k is the slope of the line in the magnitude of the force v. stretch distance graph. A steeper slope—a larger k—indicates that the spring is harder to stretch. The constant k has the same units as the slope, newtons/meter (N/m). The negative sign in Hooke's law indicates that the force is in the direction opposite the stretch or compression direction. The force exerted by the spring on the mass is always directed toward the spring's equilibrium position.

Hooke's law and real springs Not all springs obey Hooke's law. For example, rubber bands do not. Those that do obey Hooke's law are called elastic springs. Even for elastic springs, Hooke's law only applies over a limited range of distances. If a spring is stretched too far, it can become so deformed that the force is no longer proportional to the displacement.

Potential energy When you stretch a spring you transfer energy to the spring, giving it elastic potential energy. The work done by an applied force is equal to the area under a force v. distance graph like the one shown in **Figure 2.** This work is equal to the elastic potential energy stored in the spring. To calculate this stored energy, find the area of the triangle by multiplying one-half the base of the triangle, which is x, by the height of the triangle. According to Hooke's law, the height of the triangle—the magnitude of the force—is equal to kx.

Potential Energy in a Spring

The potential energy in a spring is equal to one-half times the product of the spring constant and the square of the displacement.

$$PE_{\text{spring}} = -\frac{1}{2}kx^2$$

This mathematical expression, which quantifies how the stored energy in a spring depends on its configuration, together with kinetic energy calculated from mass and speed data, allows the concept of the conservation of energy to be used to predict and describe system behavior. As shown in **Figure 3** on the next page, during horizontal simple harmonic motion the spring's elastic potential energy is converted to kinetic energy and then back to potential energy.

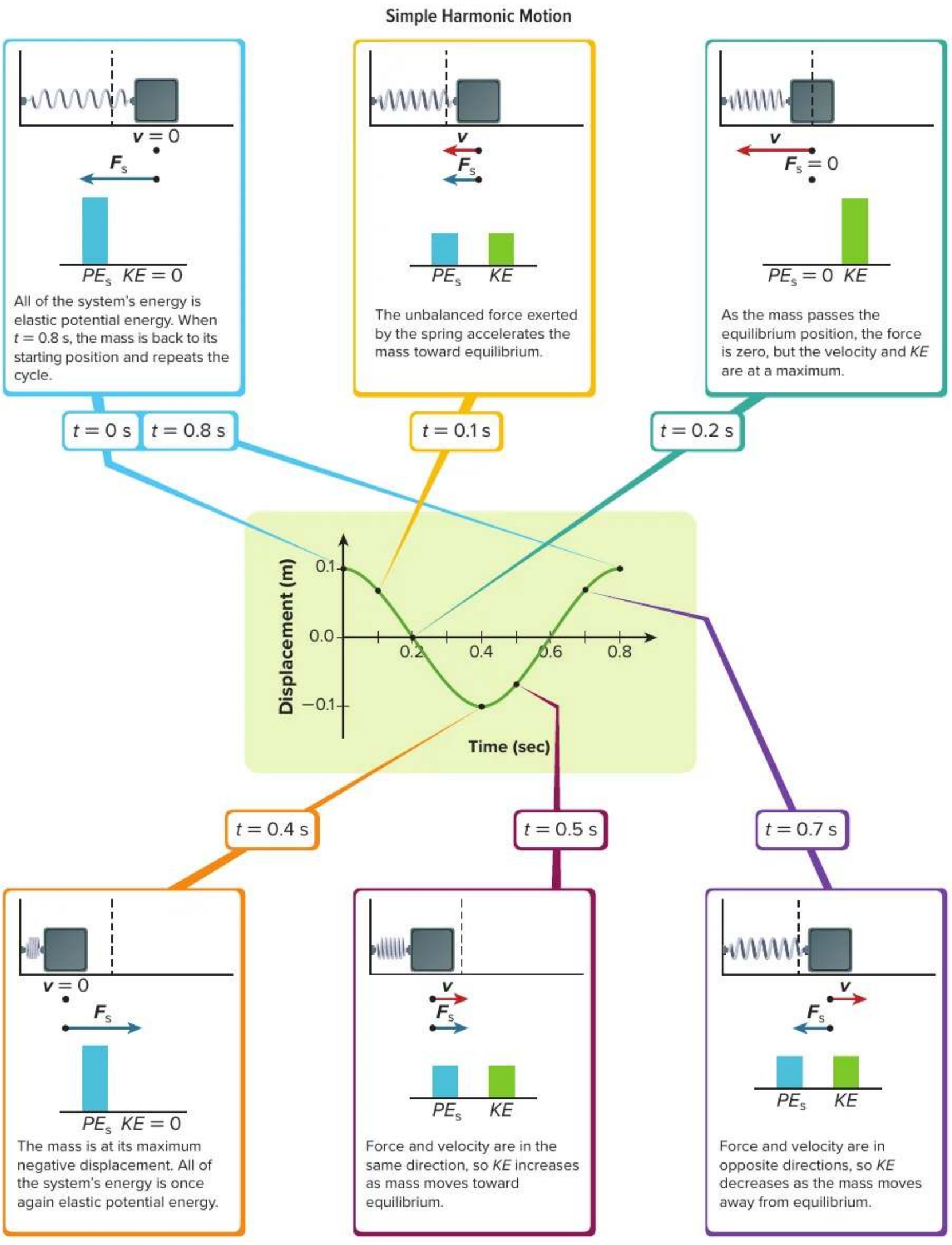


Figure 3 The total mechanical energy of the system is constant throughout the oscillation.

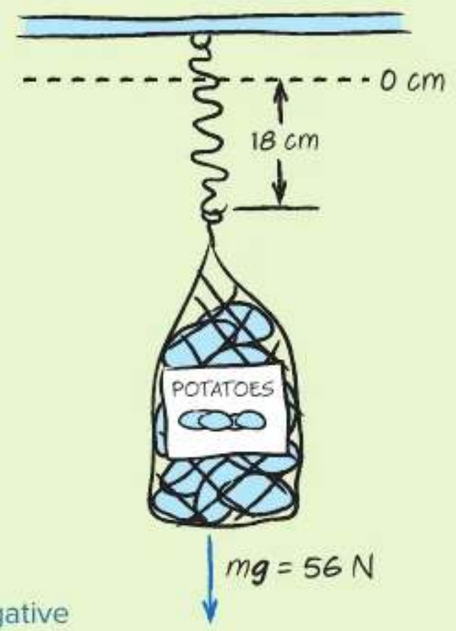
Explain *How can Hooke's law and the concept of conservation of energy be used to predict and describe the behavior of a system such as this?*

EXAMPLE Problem 1

THE SPRING CONSTANT AND THE ENERGY OF A SPRING A spring stretches by 18 cm when a bag of potatoes weighing 56 N is suspended from its end.

a. Determine the spring constant.

b. How much elastic potential energy does the stretched spring have?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Show and label the distance the spring has stretched and its equilibrium position.

Known	Unknown
$x = 18$ cm	$k = ?$
$F = -56$ N	$PE_s = ?$

2 SOLVE FOR THE UNKNOWN

a. Use Hooke's law and isolate k.

$$k = -\frac{F}{x}$$

$$= -\frac{-56\text{ N}}{0.18\text{ m}}$$

$$= 310\text{ N/m}$$

Substitute $F = -56$ N, $x = 0.18$ m. The force is negative because it is in the opposite direction of x.

b. $PE_s = \frac{1}{2}kx^2$

$$= \frac{1}{2}(310\text{ N/m})(0.18\text{ m})^2$$

$$= 5.0\text{ J}$$

Substitute $k = 310$ N/m, $x = 0.18$ m.

3 EVALUATE THE ANSWER

- **Are the magnitudes realistic?** N/m is the correct unit for the spring constant. $(\text{N/m})(\text{m}^2) = \text{N}\cdot\text{m} = \text{J}$, which is the correct unit for energy.
- **Is the magnitude realistic?** The average magnitude of the force the spring exerts is the average of 0 and 56 N. The work done is $W = Fx = (28\text{ N})(0.18\text{ m}) = 5.0$ J.

PRACTICE Problems

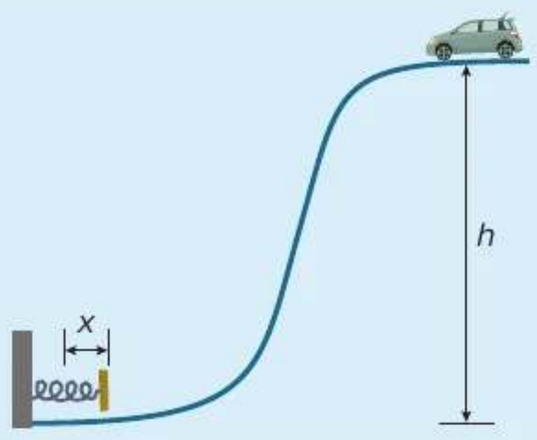
ADDITIONAL PRACTICE

1. What is the spring constant of a spring that stretches 12 cm when an object weighing 24 N is hung from it?
2. What is the elastic potential energy of a spring with $k = 144$ N/m that is compressed by 16.5 cm?
3. A spring has a spring constant of 56 N/m. How far will a block weighing 18 N stretch it?
4. **CHALLENGE** A spring has a spring constant of 256 N/m. How far must it be stretched to give it an elastic potential energy of 48 J?

PHYSICS Challenge

A car of mass m rests at the top of a hill of height h before rolling without friction into a crash barrier located at the bottom of the hill. The crash barrier contains a spring with a spring constant k, which is designed to bring the car to rest with minimum damage.

1. Determine, in terms of m, h, k, and g, the maximum distance (x) the spring will be compressed when the car hits it.
2. If the car rolls down a hill that is twice as high, by what factor will the spring compression increase?
3. What will happen after the car has been brought to rest?



Pendulums

Simple harmonic motion also occurs in the swing of a pendulum. A **simple pendulum** consists of a massive object, called the bob, suspended by a string or a light rod of length ℓ. The bob swings back and forth, as shown in **Figure 4.** The string or rod exerts a tension force (F_T), and gravity exerts a force (F_g) on the bob. Throughout the pendulum's path, the component of the gravitational force in the direction of the pendulum's circular path is a restoring force. At the left and right positions, the restoring force is at a maximum and the velocity is zero. At the equilibrium position, the restoring force is zero and the velocity is maximum.

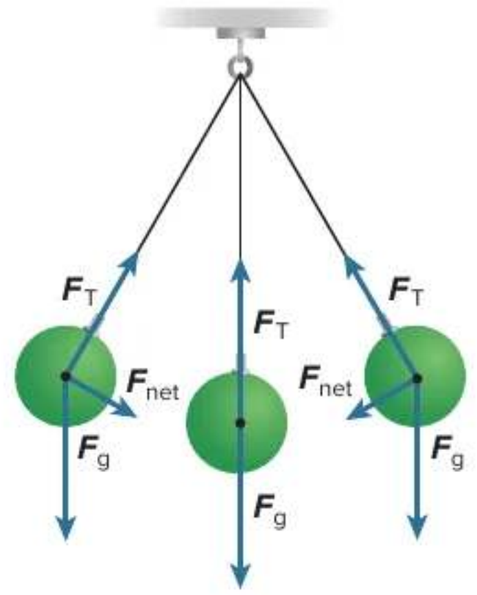


Figure 4 The pendulum's motion is an example of simple harmonic motion because the restoring force is directly proportional to the displacement from equilibrium.

For small angles (less than about 15°), the restoring force is proportional to the displacement from equilibrium. Similar to the motion of the mass on a spring discussed earlier, the motion of the pendulum is simple harmonic motion. The period of a pendulum is given by the following equation.

Period of a Pendulum

The period of a pendulum is equal to 2π times the square root of the length of the pendulum divided by the gravitational field.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Notice that the period depends only on the length of the pendulum and the gravitational field, not on the mass of the bob or the amplitude of oscillation. One practical use of the pendulum is to measure g, which can vary slightly at different locations on Earth.

Get It?

Compare the period of a very massive pendulum, like the one shown at the beginning of the module, with the period of a pendulum with the same length but a tiny mass.

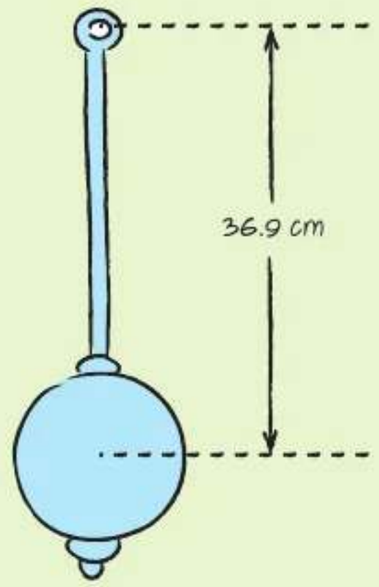
Resonance

To get a playground swing going, you can "pump" it by leaning back and pulling the chains at the same point in each swing. Another option is to have a friend give you repeated pushes at just the right times. **Resonance** occurs when forces are applied to a vibrating or oscillating object at time intervals equal to the period of oscillation. As a result, the amplitude of the vibration increases. Other familiar examples of resonance include rocking a car to free it from a snow bank and jumping rhythmically on a trampoline or a diving board to go higher.

Resonance in simple harmonic motion systems causes a larger and larger displacement as energy is added in small increments. As a child you may have been told to hold a seashell such as a conch up to your ear to "hear the sound of the ocean." The sound you hear when you hold a seashell or other similar-shaped object up to your ear actually comes from resonance. Sound waves resulting from background noise in the room interact with the seashell. Sounds with frequencies matching one of the natural frequencies at which the seashell vibrates result in resonance, and the sound becomes amplified and loud enough to hear. The large amplitude oscillations caused by resonance can also produce useful results. Resonance is used in musical instruments to amplify sounds and in clocks to increase accuracy.

EXAMPLE Problem 2

FINDING *g* USING A PENDULUM A pendulum with a length of 36.9 cm has a period of 1.22 s. What is the gravitational field at the pendulum's location?



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the situation.
- Label the length of the pendulum.

Known	Unknown
$\ell = 36.9$ cm	$g = ?$
$T = 1.22$ s	

2 SOLVE FOR THE UNKNOWN

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$ Solve for g.

$$g = (2\pi)\frac{2\ell}{T^2}$$

$$= \frac{4\pi^2(0.369\text{ m})}{(1.22\text{ s})^2}$$ Substitute $l = 0.369$ m, $T = 1.22$ s.

$$= 9.78\text{ m/s}^2 = 9.78\text{ N/kg}$$

3 EVALUATE THE ANSWER

- **Are the magnitudes realistic?** N/kg is the correct unit for gravitational field.
- **Is the magnitude realistic?** The calculated value of g is quite close to the accepted value of g, 9.8 N/kg. This pendulum could be at a higher elevation above sea level.

PRACTICE Problems

ADDITIONAL PRACTICE

5. What is the period on Earth of a pendulum with a length of 1.0 m?

6. How long must a pendulum be on the Moon, where $g = 1.6$ N/kg, to have a period of 2.0 s?

7. CHALLENGE On a certain planet, the period of a 0.75-m-long pendulum is 1.8 s. What is g for this planet?

Check Your Progress

8. **Periodic Motion** Explain why a pendulum is an example of periodic motion.

9. **Energy of a Spring** The springs shown in **Figure 5** are identical. Contrast the potential energies of the bottom two springs.

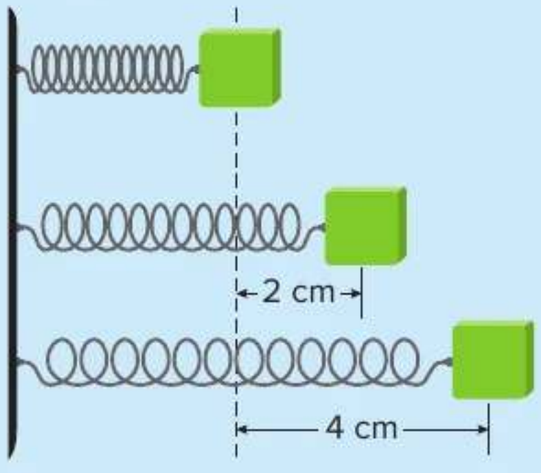


Figure 5

10. **Hooke's Law** Objects of various weights are hung from a rubber band that is suspended from a hook. The weights of the objects are plotted on a graph against the stretch of the rubber band. How can you tell from the graph whether the rubber band obeys Hooke's law?

11. **Pendulum** How must the length of a pendulum be changed to double its period? How must the length be changed to halve the period?

12. **Resonance** If a car's wheel is out of balance, the car will shake strongly at a specific speed but not at a higher or lower speed. Explain.

13. **Critical Thinking** How is uniform circular motion similar to simple harmonic motion? How are they different?

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LESSON 2

WAVE PROPERTIES

FOCUS QUESTION

What are some common types of waves?

Mechanical Waves

A **wave** is a disturbance that carries energy through matter or space without transferring matter. You have learned how Newton's laws of motion and the law of conservation of energy govern the behavior of particles. These laws also govern the behavior of waves. Water waves, sound waves, and the waves that travel along a rope or a spring are mechanical waves. Mechanical waves pass through a physical medium, such as water, air, or a rope.

Transverse waves A **wave pulse** is a single bump or disturbance that passes through a medium. In the left panel of **Figure 6,** the wave pulse disturbs the rope in the vertical direction, but the pulse travels horizontally. A wave that disturbs the particles in the medium perpendicular to the direction of the wave's travel is called a **transverse wave.** If the disturbances continue at a constant rate, a **periodic wave** is generated.

Longitudinal waves In a coiled spring toy, you can produce another type of wave. If you squeeze together several turns of the coiled spring toy and then suddenly release them, pulses will move away in both directions. The result is called a **longitudinal wave** because the disturbance is parallel to the direction of the wave's travel. Sound waves are longitudinal waves in which the molecules are alternately compressed or decompressed along the path of the wave.

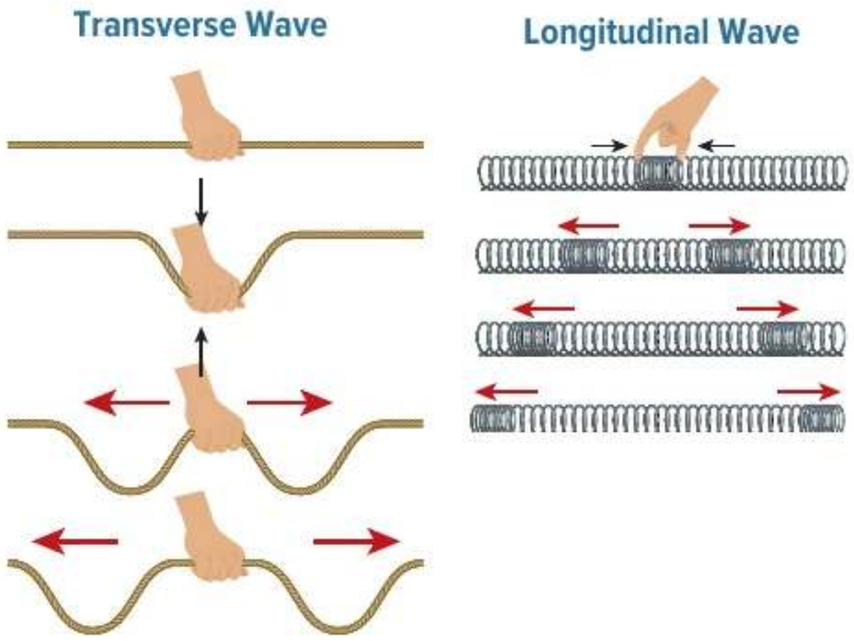


Figure 6 Shaking a rope up and down produces transverse wave pulses traveling in both directions. Squeezing and releasing the coils of a spring produces longitudinal wave pulses in both directions.

Explain *the difference between transverse and longitudinal waves.*

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Applying Practices: Wave Characteristics

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

CCC **Identify Crosscutting Concepts**

Create a table of the crosscutting concepts and fill in examples you find as you read.

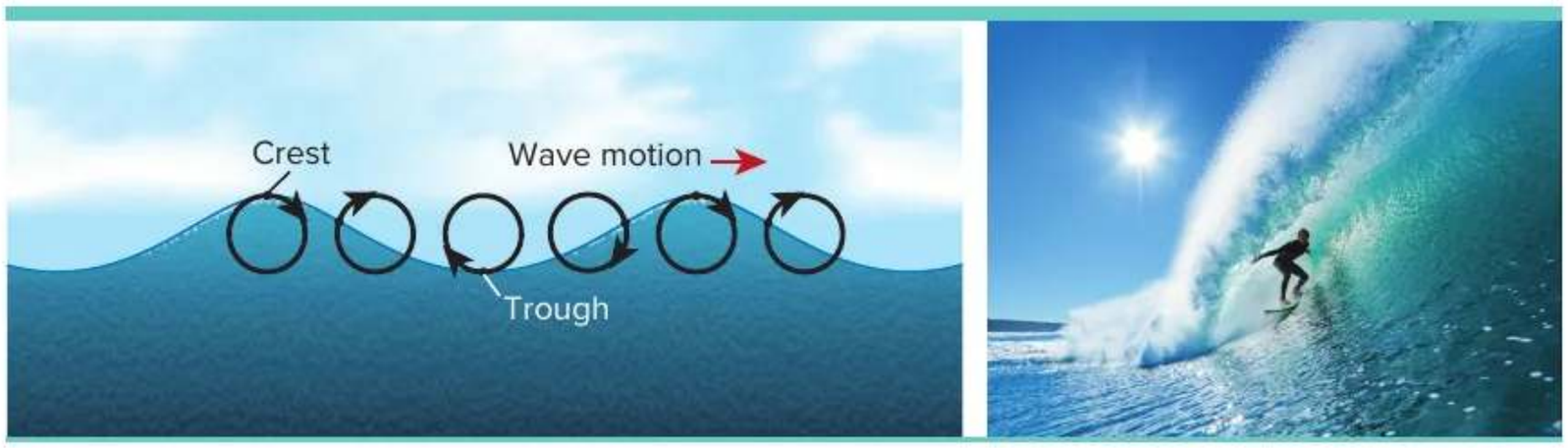


Figure 7 Surface waves in water cause movement both parallel and perpendicular to the direction of wave motion. When these waves interact with the shore, the regular, circular motion is disrupted and the waves break on the beach.

Surface waves

Waves that are deep in a lake or an ocean are longitudinal. In a **surface wave,** however, the medium's particles follow a circular path that is at times parallel to the direction of travel and at other times perpendicular to the direction of wave travel, as shown in **Figure 7.** Surface waves set particles in the medium, in this case water, moving in a circular pattern. At the top and bottom of the circular path, particles are moving parallel to the direction of the wave's travel. This is similar to a longitudinal wave. At the left and right sides of each circle, particles are moving up or down. This up-and-down motion is perpendicular to the wave's direction, similar to a transverse wave.

Real-World Physics

Tsunamis

On March 11, 2011, a wall of water estimated to be ten meters high hit areas on the east coast of Japan—tsunami! A tsunami is a series of ocean waves that can have wavelengths over 100 km, periods of one hour, and wave speeds of 500–1000 km/h.

Wave Properties

Many types of waves share a common set of wave properties. Some wave properties depend on how the wave is produced, whereas others depend on the medium through which the wave is passing.

Amplitude

How does the pulse generated by gently shaking a rope differ from the pulse produced by a violent shake? The difference is similar to the difference between a ripple in a pond and an ocean breaker—they have different amplitudes. You read earlier that the amplitude of periodic motion is the greatest distance from equilibrium. Similarly, as shown in **Figure 8,** a transverse wave's amplitude is the maximum distance of the wave from equilibrium. Since amplitude is a distance, it is always positive. You will learn more about measuring the amplitude of longitudinal waves when you study sound.

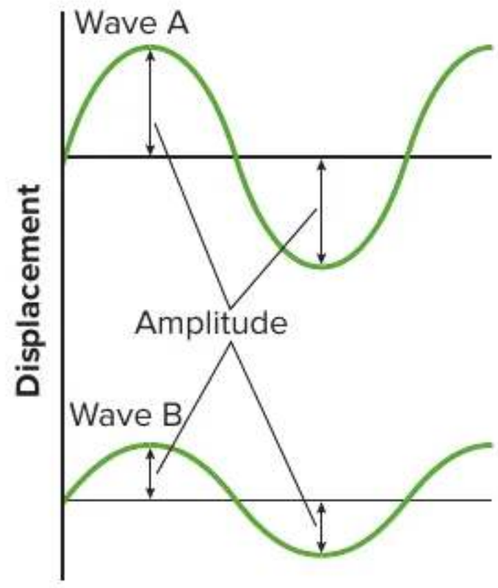


Figure 8 A wave's amplitude is measured from the equilibrium position to the highest or lowest point on the wave.

Energy of a wave

Waves, including water waves, are examples of the many ways that energy manifests at the macroscopic scale. The energy of a wave is related to its amplitude, and a wave's amplitude depends on how the wave is generated. More energy must be added to the system to generate a wave with a greater amplitude. For example, strong winds produce larger water waves than those formed by gentle breezes. Waves with greater amplitudes transfer more energy. Whereas a wave with a small amplitude might move sand on a beach a few centimeters, a giant wave can uproot and move a tree.

For waves that move at the same speed, the rate at which energy is transferred is proportional to the square of the amplitude. Thus, doubling the amplitude of a wave increases the amount of energy that wave transfers each second by a factor of four.

Epic Stock Media/Alamy Stock Photo

Wavelength Rather than focusing on one point on a wave, imagine taking a snapshot of the wave so you can see the whole wave at one instant in time. The top image in **Figure 9** shows each low point on a transverse wave, called a **trough,** and each high point on a transverse wave, called a **crest,** of a wave. The shortest distance between points where the wave pattern repeats itself is called the **wavelength.** Crests are spaced by one wavelength. Each trough also is one wavelength from the next. The Greek letter lambda (λ) represents wavelength.

Speed and velocity How fast does a wave travel? The speed of a wave pulse can be found in the same way the speed of a moving car is determined. First, measure the displacement of one of the wave's crests or compressions (Δd), then divide this by the time interval (Δt) to find the speed.

$$v = \frac{\Delta d}{\Delta t}$$

For most mechanical waves, transverse and longitudinal, except water surface waves, wave speed does not depend on amplitude, frequency, or wavelength. Speed depends only on the medium through which the waves are passing and on the type of wave. For example, sound waves have characteristic speeds in water and in air. The temperature of the medium also makes a difference. Sound waves travel faster in warm, dry air than in cool, dry air.

Get It?

Summarize how changing a wave's amplitude, frequency, or wavelength affects the wave's speed.

Phase Any two points on a wave that are one or more whole wavelengths apart are said to be in phase. Particles in the medium are in phase with one another when they have the same displacement from the equilibrium position and the same velocity. Particles with opposite displacements from the equilibrium position and opposite velocities are 180° out of phase. A crest and a trough, for example, are 180° out of phase with each other. Two particles in a wave medium can be anywhere from 0° to 360° out of phase with each other.

Period Although wave speed and amplitude can describe both wave pulses and periodic waves, period (T) applies only to periodic waves. You have learned that the period of simple harmonic motion, such as the motion of a simple pendulum, is the time it takes for the motion to complete one cycle. Such motion is usually the source, or cause, of a periodic wave. The period of a wave is equal to the period of the source. In **Figure 9** the period (T) equals 0.04 s, which is the time it takes the source to complete one cycle. The same time is taken by P, a point on the rope, to return to its initial position and velocity.

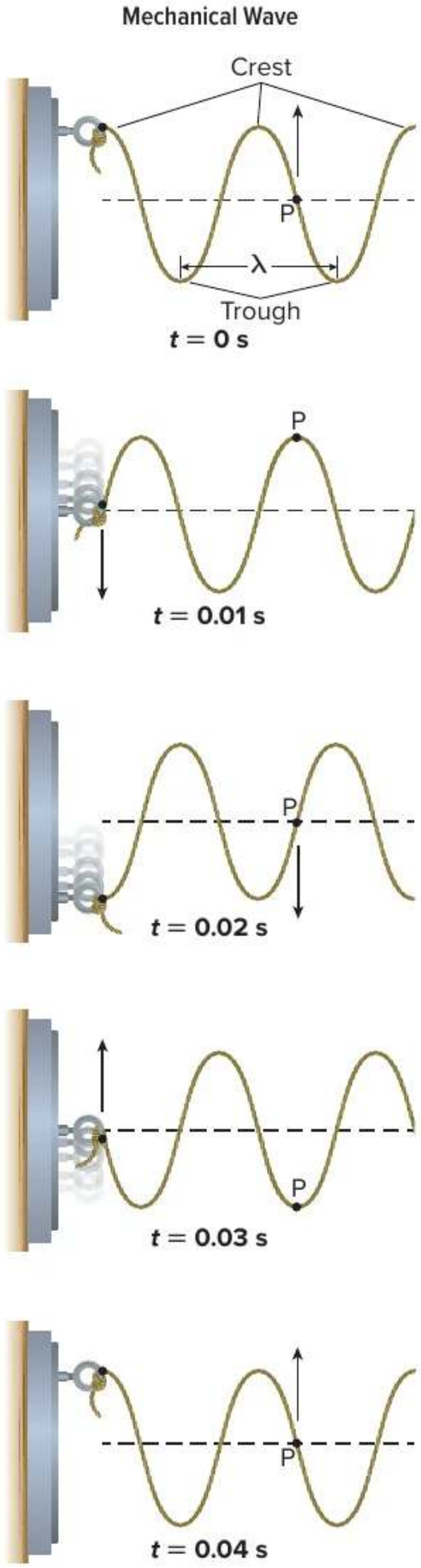


Figure 9 A mechanical oscillator moves the left end of the rope up and down, completing the cycle in 0.04 s.

Calculating frequency The **frequency** of a wave (f) is the number of complete oscillations a point on that wave makes each second. Frequency is measured in hertz (Hz). One hertz is one oscillation per second and is equal to 1/s or s^{-1}. The frequency and the period of a wave are related by the following equation.

Frequency of a Wave

The frequency of a wave is equal to the reciprocal of the period.

$$f = \frac{1}{T}$$

Both the period and the frequency of a wave depend only on the wave's source. They do not depend on the wave's speed or the medium.

Calculating wavelength You can directly measure a wave's wavelength by measuring the distance between adjacent crests or troughs. You can also calculate it because the wavelength and frequency of a wave are related to one another by the speed of travel of the wave, which depends on the type of wave and the medium through which it is passing. In the time interval of one period, a wave moves one wavelength. Therefore, the wavelength of a wave is the speed multiplied by the period, $\lambda = vT$. Using the relation that $f = \frac{1}{T}$, the wavelength equation is very often written in the following way.

Wavelength

The wavelength of a wave is equal to the velocity divided by the frequency.

$$\lambda = \frac{v}{f}$$

Graphing waves If you took a snapshot of a transverse wave on a coiled spring toy, it might look like one of the waves shown in **Figure 9.** This snapshot could be placed on a graph grid to show more information about the wave, as in the left panel of **Figure 10.** Measuring from peak to peak or trough to trough on such a snapshot provides the wavelength. Now consider recording the motion of a single particle, such as point P in **Figure 9.** That motion can be plotted on a displacement v. time graph, as in the graph on the right in **Figure 10.** Measuring from peak to peak or trough to trough in this graph provides the wave's period.

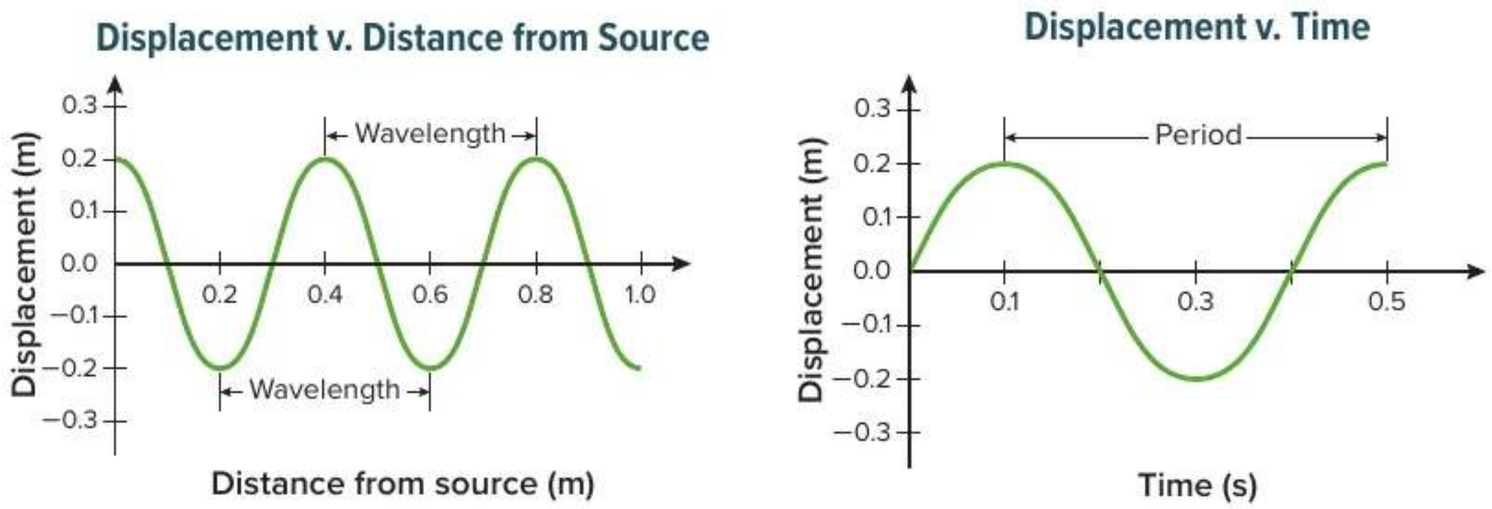


Figure 10 Graphing waves on different axes provides different kinds of information.

Determine *the period of the wave shown in the displacement v. time graph.*

EXAMPLE Problem 3

CHARACTERISTICS OF A WAVE A sound wave has a frequency of 192 Hz and travels the length of a football field, 91.4 m, in 0.271 s.

a. What is the speed of the wave?

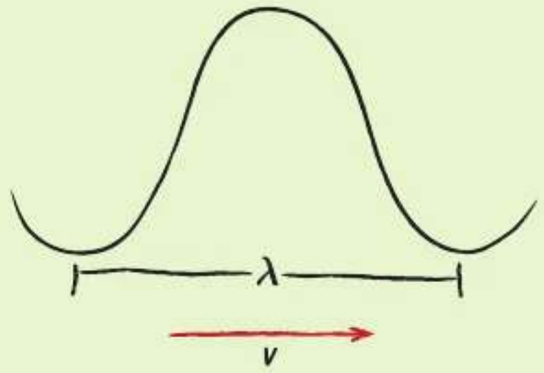
b. What is the wavelength of the wave?

c. What is the period of the wave?

d. If the frequency were changed to 442 Hz, what would be the new wavelength and period?

1 ANALYZE AND SKETCH THE PROBLEM

- Draw a diagram of the wave.
- Draw a velocity vector for the wave.



Known	Unknown
$f = 192$ Hz	$v = ?$
$\Delta d = 91.4$ m	$\lambda = ?$
$\Delta t = 0.271$ s	$T = ?$

2 SOLVE FOR THE UNKNOWN

a. Use the definition of velocity.

$$v = \frac{\Delta d}{\Delta t}$$
$$= \frac{91.4\text{ m}}{0.271\text{ s}} \qquad \text{Substitute } \Delta d = 91.4\text{ m}, \Delta t = 0.271\text{ s}$$
$$= 337\text{ m/s}$$

b. Use the relationship between wave velocity, wavelength, and frequency.

$$\lambda = \frac{v}{f}$$
$$= \frac{337\text{ m/s}}{192\text{ Hz}} \qquad \text{Substitute } v = 337\text{ m/s}, f = 192\text{ Hz}.$$
$$= 1.76\text{ m}$$

c. Use the relationship between period and frequency.

$$T = \frac{1}{f}$$
$$= \frac{1}{192}\text{ Hz} \qquad \text{Substitute } f = 192\text{ Hz}.$$
$$= 0.00521\text{ s}$$

d. $\lambda = \frac{v}{f}$

$$= \frac{337\text{ m/s}}{442\text{ Hz}} \qquad \text{Substitute } v = 337\text{ m/s}, f = 442\text{ Hz}.$$
$$= 0.762\text{ m}$$

$$T = \frac{1}{f}$$
$$= \frac{1}{442}\text{ Hz} \qquad \text{Substitute } f = 442\text{ Hz}.$$
$$= 0.00226\text{ s}$$

3 EVALUATE THE ANSWER

- **Are the magnitudes realistic?** Hz has the unit s^{-1}, so $\left(\frac{m}{s}\right)/\text{Hz} = \left(\frac{m}{s}\right)\cdot s = m$, which is correct for λ.
- **Are the magnitudes realistic?** A typical sound wave travels at approximately 340 m/s in air, so 337 m/s is reasonable. The frequencies and periods are reasonable for sound waves. The frequency of 442 Hz is close to a 440-Hz A, which is A above middle-C on a piano.

PRACTICE Problems

ADDITIONAL PRACTICE

14. A sound wave produced by a clock chime is heard 515 m away 1.50 s later.
- **a.** Based on these measurements, what is the speed of sound in air?
- **b.** The sound wave has a frequency of 436 Hz. What is the period of the wave?
- **c.** What is its wavelength?

15. How are the wavelength, frequency, and speed of a wave related? How do they depend on the medium through which the wave is passing and the type of wave?

16. What is the speed of a periodic wave disturbance that has a frequency of 3.50 Hz and a wavelength of 0.700 m?

17. How does increasing the wavelength by 50 percent affect the frequency of a wave on a rope?

18. The speed of a transverse wave in a string is 15.0 m/s. If a source produces a disturbance that has a frequency of 6.00 Hz, what is its wavelength?

19. Five wavelengths are generated every 0.100 s in a tank of water. What is the speed of the wave if the wavelength of the surface wave is 1.20 cm?

20. A periodic longitudinal wave that has a frequency of 20.0 Hz travels along a coiled spring toy. If the distance between successive compressions is 0.600 m, what is the speed of the wave?

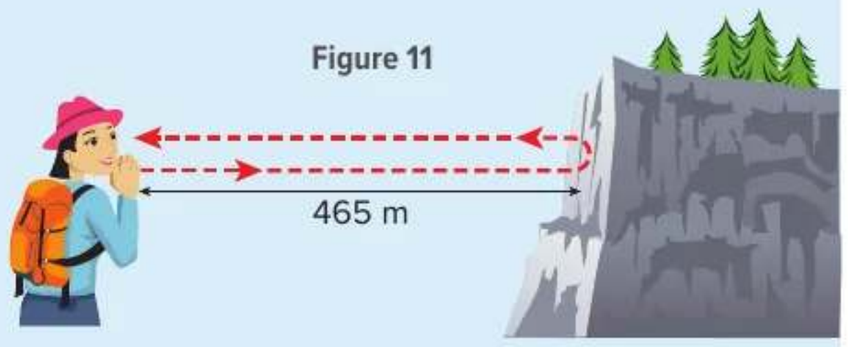
21. How does the frequency of a wave change when the period of the wave is doubled?

22. Describe the change in the wavelength of a wave when the period is reduced by one-half.

23. If the speed of a wave increases to 1.5 times its original speed while the frequency remains constant, how does the wavelength change?

24. CHALLENGE A hiker shouts toward a vertical cliff as shown in **Figure 11.** The echo is heard 2.75 s later.
- **a.** What is the speed of sound of the hiker's voice in air?
- **b.** The wavelength of the sound is 0.750 m. What is its frequency?
- **c.** What is the period of the wave?

Figure 11



Check Your Progress

25. **Transverse Waves** Suppose you and your lab partner are asked to demonstrate that a transverse wave transports energy without transferring matter. How could you do it?

26. **Wave Characteristics** You are creating transverse waves on a rope by shaking your hand from side to side. Without changing the distance your hand moves, you begin to shake it faster and faster. What happens to the amplitude, wavelength, frequency, period, and velocity of the wave?

27. **Longitudinal Waves** Describe longitudinal waves. What types of mediums transmit longitudinal waves?

28. **Speeds in Different Mediums** If you pull on one end of a coiled spring toy, does the pulse reach the other end instantaneously? What happens if you pull on a rope? What happens if you hit the end of a metal rod? Compare the pulses that are traveling through each of these three materials.

29. **Critical Thinking** If a raindrop falls into a pool, it produces waves with small amplitudes. If a swimmer jumps into a pool, he or she produces waves with large amplitudes. Why doesn't the heavy rain in a thunderstorm produce large waves?

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LESSON 3

WAVE BEHAVIOR

FOCUS QUESTION

What happens when two waves meet?

Waves at Boundaries

Recall from Lesson 2 that the speed of a mechanical wave depends only on the properties of the medium it passes through, not on the wave's amplitude or frequency. For example, for waves on a spring, the speed depends on the spring's tension and mass per unit length. Examine what happens when a wave travels from one medium to another. **Figure 12** shows a wave pulse traveling from a larger spring into a smaller one. The pulse that strikes the boundary is called the **incident wave.** One pulse from the larger spring continues in the smaller spring, but the speed is different in the smaller spring. Note that this transmitted wave pulse remains upward.

Some of the energy of the incident wave's pulse is reflected backward into the larger spring. This returning wave is called the reflected wave. Whether the **reflected wave** is upright or inverted depends on the characteristics of the two springs. For example, if the waves in the smaller spring have a greater speed because the spring is stiffer, then the reflected wave will be inverted.

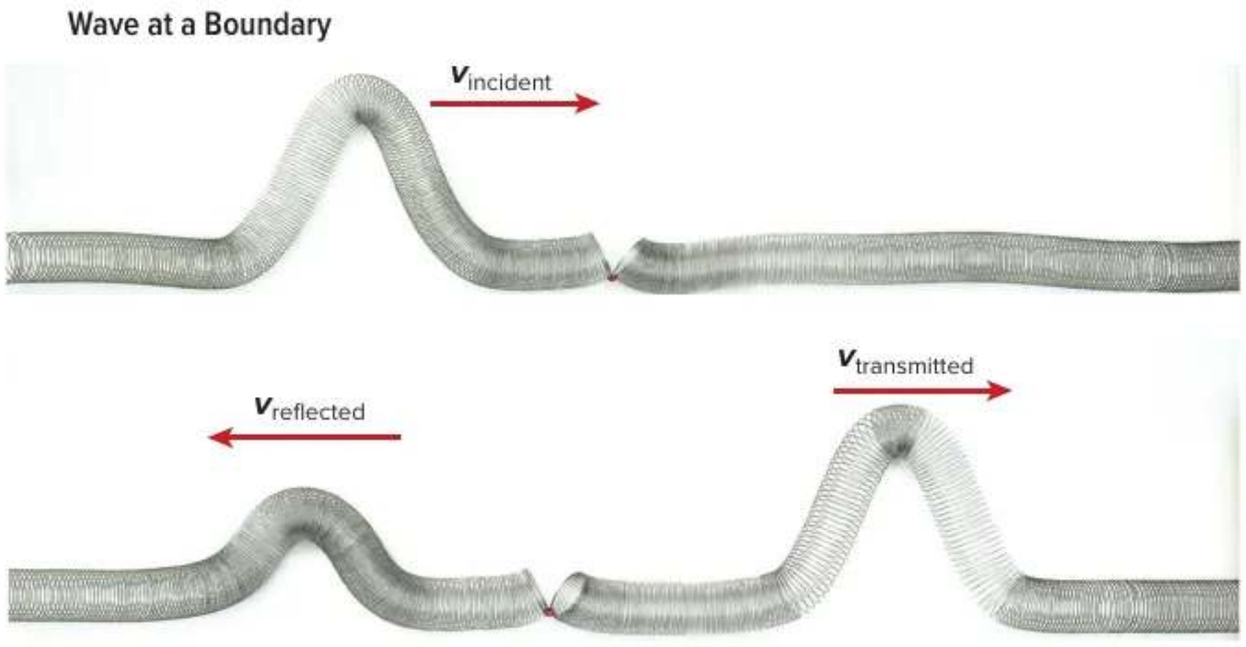


Figure 12 When the wave pulse meets the boundary between the two springs, a transmitted wave pulse and a reflected wave form.

Compare *the energy of the incident wave to the energy of the reflected wave.*

Matt Meadows/McGraw-Hill Education

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

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INVESTIGATE

GO ONLINE to find these activities and more resources.

PhysicsLAB: Reflection and Refraction

Analyze data from wave patterns in a ripple tank to predict the behavior of waves in water.

Review the News

Obtain information from a current news story about mechanical waves, such as seismic or water waves. Evaluate your source and communicate your findings to your class.

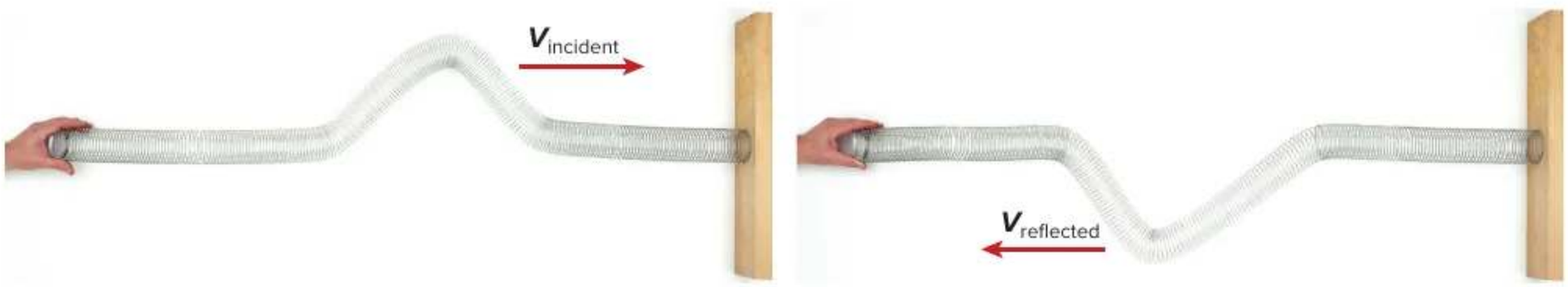


Figure 13 When a wave encounters a rigid boundary, the reflected wave is inverted. Note that the amplitude is not affected by the rigid boundary.

Rigid boundaries When a wave pulse hits a rigid boundary, the energy is reflected back, as shown in **Figure 13.** The wall is the boundary of a new medium through which the wave attempts to pass. Instead of passing through, the pulse is reflected from the wall with almost exactly the same amplitude as the pulse of the incident wave. Thus, almost all the wave's energy is reflected back. Very little energy is transmitted into the wall. Also note that the pulse is inverted.

Superposition of Waves

Suppose a pulse traveling along a spring meets a reflected pulse that is coming back from a boundary, as shown in **Figure 14.** In this case, two waves exist in the same place in the medium at the same time. Each wave affects the medium independently. The **principle of superposition** states that the displacement of a medium caused by two or more waves is the algebraic sum of the displacements caused by the individual waves. In other words, two or more waves can combine to form a new wave. If the waves move in the same medium, they can cancel or add or subtract to form a new wave of lesser or greater amplitude. They emerge unaffected by each other. The result of the superposition of two or more waves is called **interference.**

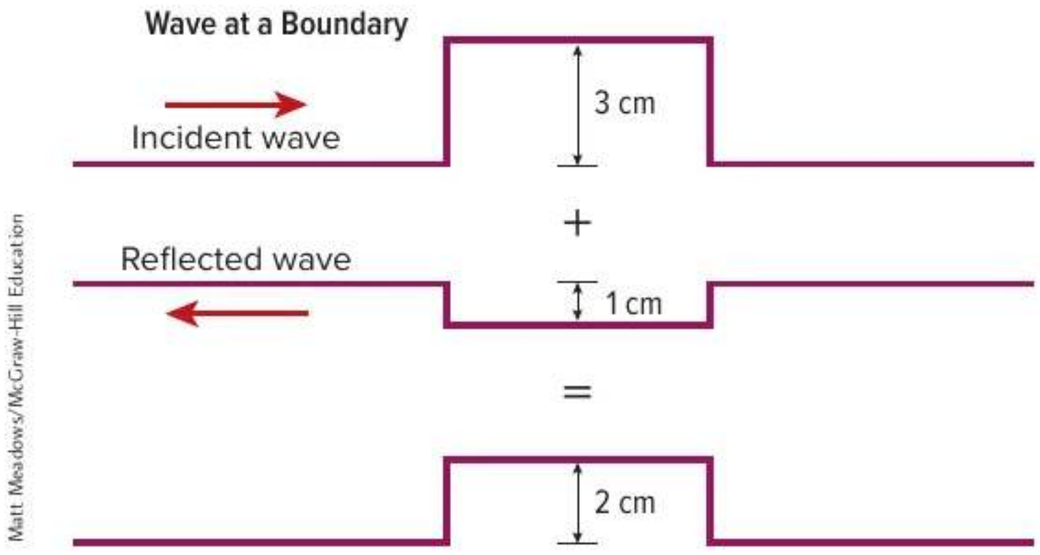


Figure 14 Waves add algebraically during superposition.

SCIENCE USAGE v. COMMON USAGE

Interference

Science usage: the result of superposition of two or more waves
The amplitude of the interference of several waves was much larger than the amplitude of the individual waves.

Common usage: the act of coming between in a way that hinders or impedes
Ehud was ejected from the game for an interference foul.

STEM CAREER Connection

Boat Designer

If you are drawn to the water, the career of boat designer might be for you. Whether designing ocean liners, freighters, yachts, or wakeboard boats, a boat designer must understand how a vessel will perform in various wave conditions.

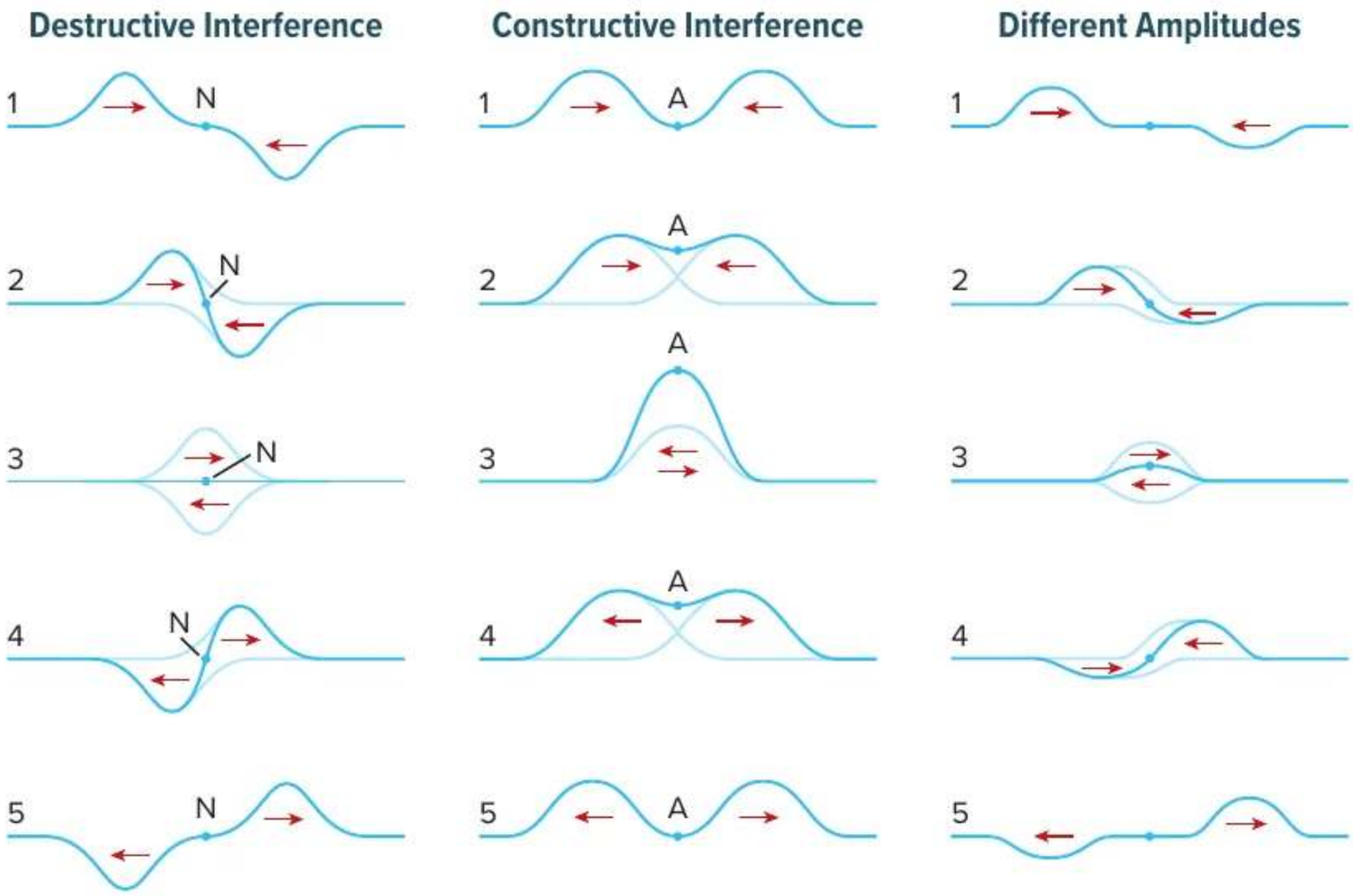


Figure 15 When waves add algebraically, the resulting combined waves can be quite different from the individual waves. **Summarize** *how waves behave during and after superposition.*

Wave interference

Wave interference can be either constructive or destructive. The first panel in **Figure 15** shows the superposition of waves with equal but opposite displacements, causing destructive interference. When the pulses meet and are in the same location, the displacement is zero. Point N, which does not move at all, is called a **node.** The pulses travel horizontally and eventually emerge unaffected by each other.

Constructive interference occurs when wave displacements are in the same direction. The result is a wave that has an amplitude greater than those of the individual waves. A larger pulse appears at point A when the two waves meet. Point A has the largest displacement and is called the **antinode.** The two pulses pass through each other without changing their shapes or sizes. Even if the pulses have unequal amplitudes, the resultant pulse at the overlap is still the algebraic sum of the two pulses, as shown in the final panel of **Figure 15.**

Get It?

Compare the wave medium's displacement at a node and at an antinode.

Two reflections

You can apply the concept of superimposed waves to the control of large-amplitude waves. Imagine attaching one end of a rope to a fixed point, such as a doorknob, a distance L away. When you vibrate the free end, the wave leaves your hand, travels along the rope toward the fixed end, is reflected and inverted at the fixed end, and returns to your hand. When it reaches your hand, the reflected wave is inverted and travels back down the rope. Thus, when the wave leaves your hand the second time, its displacement is in the same direction as it was when it left your hand the first time.

Standing waves

Suppose you adjust the motion of your hand so that the period of vibration equals the time needed for the wave to make one round-trip from your hand to the door and back. Then, the displacement given by your hand to the rope each time will add to the displacement of the reflected wave. As a result, the amplitude of oscillation of the rope will be much greater than the motion of your hand. This large-amplitude oscillation is an example of mechanical resonance.

The ends of the rope are nodes and an antinode is in the middle, as shown in the top photo in **Figure 16.** Thus, the wave appears to be standing still and is called a **standing wave.** Note, however, that the standing wave is the interference of waves traveling in opposite directions. If you double the frequency of vibration, you can produce one more node and one more antinode on the rope. Then it appears to vibrate in two segments. When you further increase the vibration frequency, it produces even more nodes and antinodes, as shown in the bottom photo in **Figure 16.**

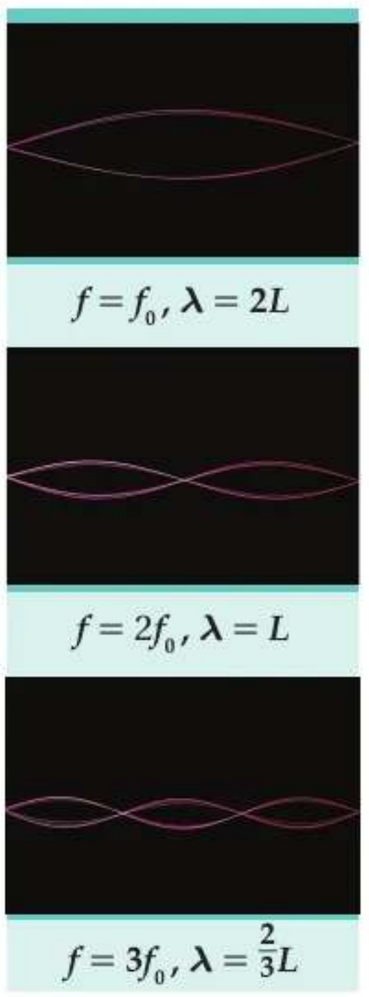


Figure 16 Interference produces standing waves only at certain frequencies.

Predict *the wavelength if the frequency is four times the lowest frequency.*

Waves in Two Dimensions

You have studied waves on a rope and on a spring reflecting from rigid supports. During some of these interactions, the amplitude of the waves is forced to be zero by destructive interference. These mechanical waves travel in only one dimension. Waves on the surface of water, however, travel in two dimensions, and sound waves and electromagnetic waves will later be shown to travel in three dimensions. How can two-dimensional waves be represented?

Picturing waves in two dimensions

When you throw a small stone into a calm pool of water, you see the circular crests and troughs of the resulting waves spreading out in all directions. You can sketch those waves by drawing circles to represent the wave crests. If you repeatedly dip your finger into water with a constant frequency, the resulting sketch would be a series of concentric circles, called wavefronts, centered on your finger. A **wavefront** is a line that represents the crest of a wave in two dimensions. Wavefronts can be used to show two-dimensional waves of any shape, including circular waves. The photo in **Figure 17** shows circular waves in water. The circles drawn on the diagram show the wavefronts that represent those water waves.

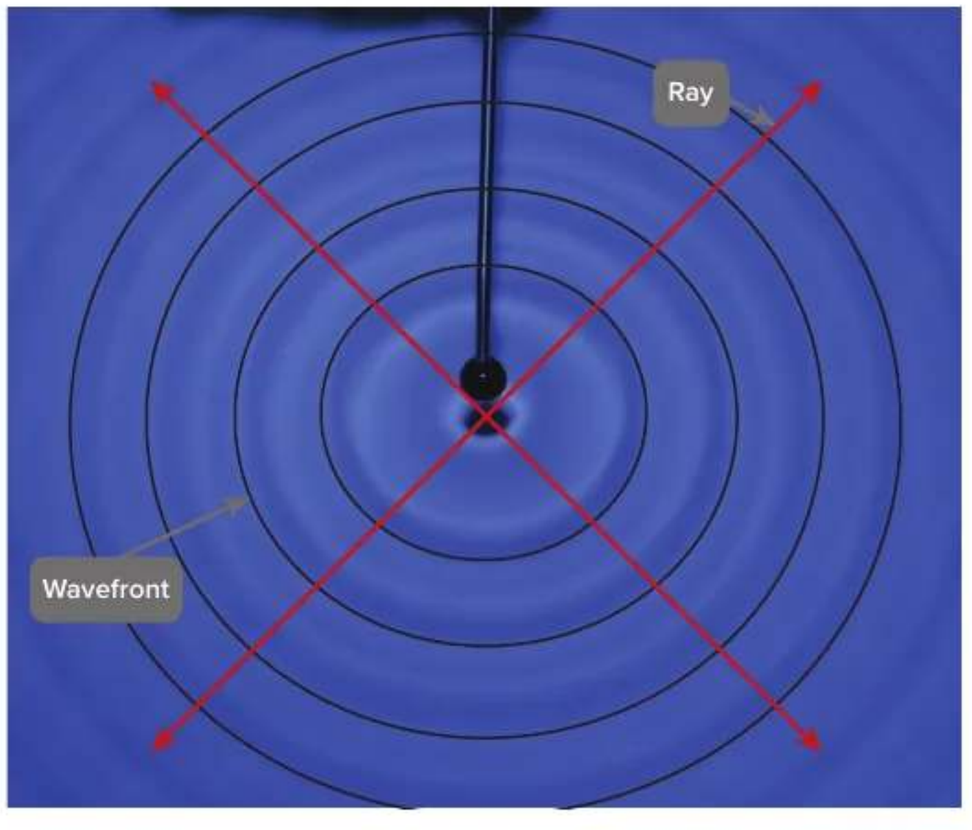


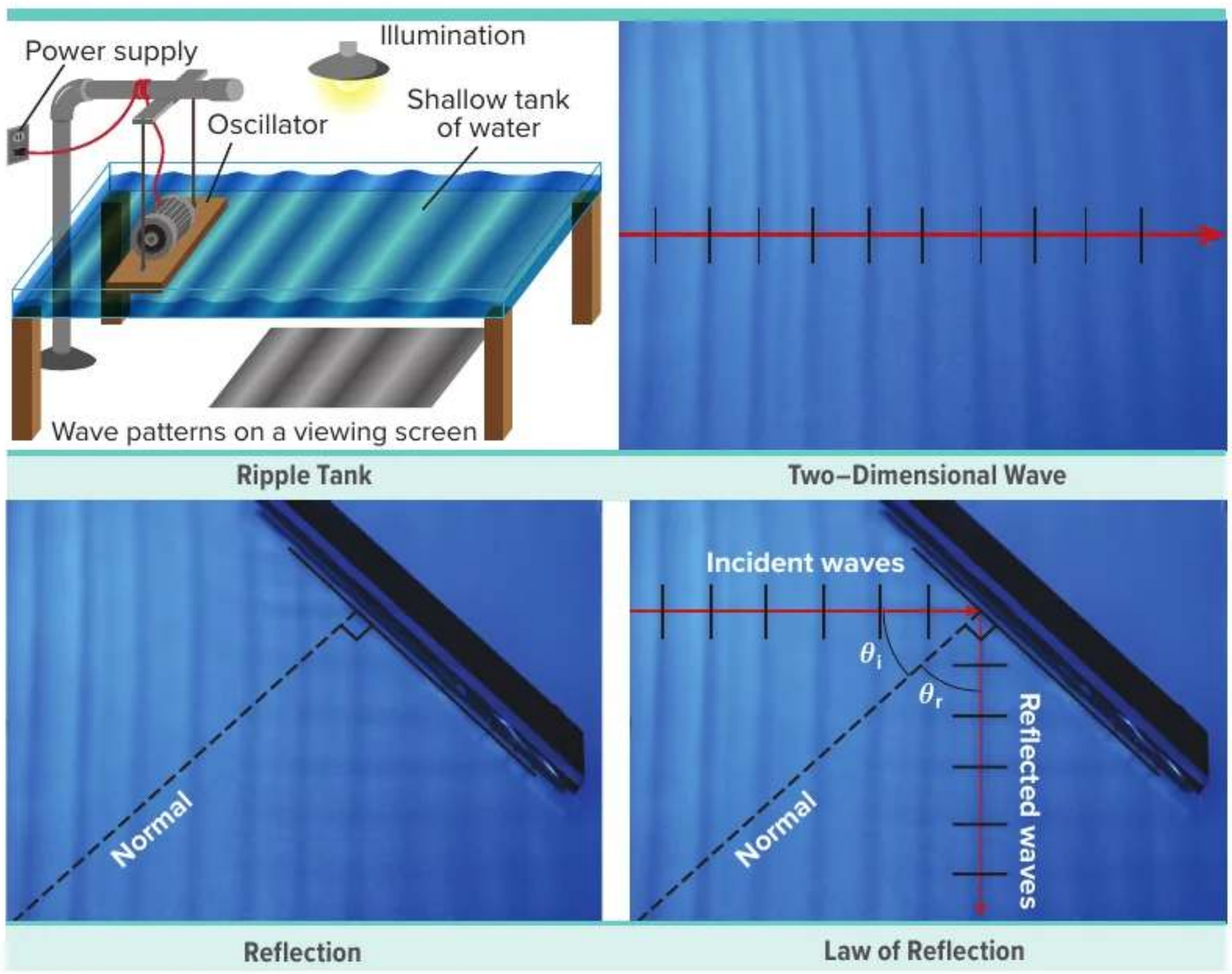
Figure 17 Waves spread out in a circular pattern from the oscillating source.

Whatever their shape, two-dimensional waves always travel in a direction that is perpendicular to their wavefronts. That direction can be represented by a **ray,** which is a line drawn at a right angle to the wavefront. When all you want to show is the direction in which a wave is traveling, it is convenient to draw rays instead of wavefronts. The red arrows in **Figure 17** are rays that show the water waves' direction of motion. One advantage of drawing wavefronts is when wavefronts are drawn to scale, they show the wave's wavelengths. In **Figure 17,** the wavelength equals the distance from one circle to the next.

(t)McGraw-Hill Education; (b)Matt Meadows/McGraw-Hill Education

Creating two-dimensional waves A ripple tank is a piece of laboratory equipment that is used to investigate the properties of two-dimensional waves. The main portion of the ripple tank shown in **Figure 18** is a shallow tank that contains a thin layer of water. A board attached to a mechanical oscillator produces waves with long, straight wavefronts. A lamp above the tank produces shadows below the tank that show the locations of the crests of the waves. The top photo in **Figure 18** shows a wave traveling through the ripple tank. The direction the wave travels is modeled by a ray diagram. For clarity, the wavefronts are not extended the entire length of the wave.

Reflection of two-dimensional waves The bottom row of pictures in **Figure 18** shows an incident ray encountering a rigid barrier placed at an angle to the ray's path. The orientation of the barrier is shown by a line, called the **normal,** drawn perpendicular to the barrier. The angle between the incident ray and the normal is called the angle of incidence and is labeled θ_i in the diagram. The angle between the normal and the reflected ray is called the angle of reflection and is labeled θ_r. The **law of reflection** states that the angle of incidence is equal to the angle of reflection. The law of reflection applies to many different kinds of waves, not just the waves in a ripple tank.



(t)Matt Meadows/McGraw-Hill Education; (b)McGraw-Hill Education

Figure 18 The ripple tank produces uniform waves that are useful for modeling wave behavior.

SCIENCE USAGE v. COMMON USAGE

Normal

Science usage: a line in a diagram that is drawn perpendicular to a surface
As measured from the normal, angles of incidence and reflection are equal.

Common usage: conforming to a type, standard, or regular pattern
Such cold temperatures in July are not normal.

CCC CROSSCUTTING CONCEPTS

Cause and Effect Write a procedure describing how you could use a ripple tank to gather empirical evidence to support the cause-and-effect relationship between the angle of a barrier and the angle of a reflected wave.

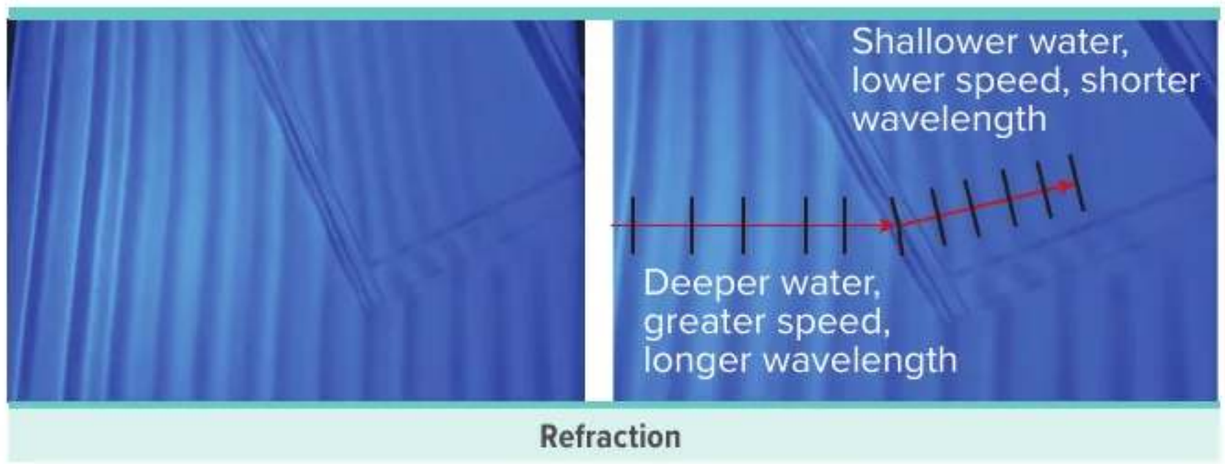


Figure 19 Waves in the ripple tank change direction as they enter shallower water.

Describe *how the wavelength changes as the wave travels into the shallow water.*

Refraction of waves in two dimensions A ripple tank can also model the behavior of waves as they travel from one medium into another. **Figure 19** shows a glass plate placed under the water in a ripple tank. The water above the plate is shallower than the water in the rest of the tank. As the waves travel from deep to shallow water, their speeds decrease and the direction of the waves changes. Such changes in speed are common when waves pass from one medium to another.

The waves in the shallow water are connected to the waves in the deep water. As a result, the frequency of the waves in the two mediums is the same. Based on the equation $\lambda = \frac{v}{f}$, the decrease in the speed of the waves means the wavelength is shorter in the shallower water. The change in the direction of waves at the boundary between two different mediums is known as **refraction.** **Figure 19** shows a wave front and ray model of refraction. Part of the wave will refract through the boundary, and part will be reflected from the boundary. Reflection and refraction occur for many different types of waves. Echoes are an example of reflection of sound waves by hard surfaces, such as the walls of a large gymnasium. Rainbows are the result of the reflection and refraction of light. As sunlight passes through a raindrop, reflection and refraction separate the light into its individual colors, producing a rainbow.

Check Your Progress

30. **Wave Characteristics** Which characteristics remain unchanged when a wave crosses a boundary into a different medium: frequency, amplitude, wavelength, velocity, direction?
31. **Superposition of Waves** Sketch two wave pulses whose interference produces a pulse with an amplitude greater than either of the individual waves.
32. **Refraction of Waves** In **Figure 19,** the wave changes direction as it passes from one medium to another. Can two-dimensional waves cross a boundary between two mediums without changing direction? Explain.
33. **Standing Waves** In a standing wave on a string fixed at both ends, how is the number of nodes related to the number of antinodes?
34. **Critical Thinking** As another way to understand wave reflection, cover the right-hand side of each drawing in the left panel in **Figure 15** with a piece of paper. The edge of the paper should be at point N, the node. Now, concentrate on the resultant wave, shown in darker blue. Note that it acts as a wave reflected from a boundary. Is the boundary a rigid wall? Repeat this exercise for the middle panel in **Figure 15.**

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Matt Meadows/McGraw-Hill Education

ENGINEERING & TECHNOLOGY

Harnessing the Motion of the Ocean

Ocean waves represent a huge, almost untapped potential source of energy. Estimates suggest that in the United States, wave power could produce one-quarter to one-third of the electricity used each year. Currently wave power technology lags behind wind and solar power, but around the world, engineers are working to change this and harness the power of ocean waves.



Wave power generators convert mechanical energy from ocean waves to electrical energy.

Wave Power Technologies

One wave power technology, sometimes called a point absorber, looks like a large floating buoy. The buoy has parts that are attached but are able to move independently. As waves rise and fall, the buoy's different parts move relative to each other. This motion drives an energy converter device such as a hydraulic pump, which transforms the mechanical energy of motion into electrical energy. Two different versions of these buoys are being tested off the coast of Hawaii.

Another technology that is designed for offshore use looks like a giant metal sea snake. Its segments can move independently but are attached to one another as it floats on the surface of the water. The whole device is anchored to the ocean floor. The motion of ocean waves causes the segments to move relative to one another. The mechanical energy of the segments is converted by a hydraulic pump or other device to electrical energy. The resulting electrical current is transported to shore via a cable.

A different style of generator, called an oscillating water column or a terminator, is usually designed for onshore use. Waves push water into a column-shaped chamber through an underwater opening. Air is trapped above. The captured water column moves up and down like a piston. Air is forced in and out of an opening at the top of the column. The movement of the air turns a turbine and this mechanical motion is converted to electricity.

Successful wave power devices need to do more than just convert mechanical energy from waves into electricity. They must also be extremely durable to last in harsh ocean conditions. Engineers must design devices that will withstand storms, constant impacts from waves, and corrosive seawater.

So far none of these technologies have been adopted for large-scale use. Engineers and scientists will continue to develop and test prototypes, working towards a goal of wave power devices that are effective, reliable, durable, and affordable.

DEVELOP AND USE MODEL TO ILLUSTRATE

Choose one type of wave power technology. Find out how it converts mechanical energy from waves to electricity. Develop and use a model to illustrate how the technology works.

Ashley Cooper/Alamy Stock Photo

MODULE 13
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 PERIODIC MOTION

- Simple harmonic motion results when the restoring force on an object is directly proportional to the object's displacement from equilibrium.
- The elastic potential energy of a spring that obeys Hooke's law is expressed by the following equation:

$$PE_s = \frac{1}{2}kx^2$$

- The period of a pendulum depends on the pendulum's length and the gravitational field strength at the pendulum's location. The period can be found using the following equation

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

- periodic motion
- period
- amplitude
- simple harmonic motion
- Hooke's law
- simple pendulum
- resonance

Lesson 2 PHOTOSYNTHESIS

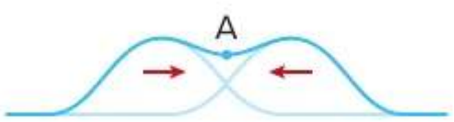
- Waves are disturbances that transfer energy without transferring matter.
- In transverse waves, the displacement of the medium is perpendicular to the direction the wave travels. In longitudinal waves, the displacement is parallel to the direction the wave travels.
- The velocity of a continuous wave is equal to the wave's frequency times its wavelength.

$$v = f\lambda$$

- wave
- wave pulse
- transverse wave
- periodic wave
- longitudinal wave
- surface wave
- trough
- crest
- wavelength
- frequency

Lesson 3 WAVE BEHAVIOR

- When two-dimensional waves are reflected from boundaries, the angles of incidence and reflection are equal. The change in direction of waves at the boundary between two different mediums is called refraction.
- Interference occurs when two or more waves travel through the same medium at the same time. The principle of superposition states that the displacement of a medium resulting from two or more waves is the algebraic sum of the displacements of the individual waves.



- incident wave
- reflected wave
- principle of superposition
- interference
- node
- antinode
- standing wave
- wavefront
- ray
- normal
- law of reflection
- refraction

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

How can this pendulum save a building from earthquake damage?

CER Claim, Evidence, Reasoning

Explain your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

How does the strength of gravity affect periodic motion?

An astronaut lands on an unknown planet and must determine the value of the gravitational acceleration *g*. He has the following instruments on hand.

Instrument A rests on the ground and shoots a ball vertically with a known speed. The astronaut can measure the time the ball takes to rise from its launch position and the time it takes to fall back to the launch position.

Instrument B is a simple unmarked weight suspended from a spring with constant $k = 22.5$ N/m.

Instrument C is a simple pendulum at the end of an arm that is 0.500 m long. The astronaut counts exactly 47 full swings in 1.00 min.

CER Analyze and Interpret Data

1. **Claim** Which instrument(s) can be used to determine *g*?
2. **Evidence and Reasoning** Explain how you made your decision.

Kayte Deioma/Alamy Stock Photo


©Charles D. Winters/Timeframe Photography/McGraw-Hill Education

MODULE 14

SOLIDS, LIQUIDS, AND GASES

ENCOUNTER THE PHENOMENON

Why does a balloon crumple and shrink when liquid nitrogen is poured on it?



GO ONLINE to play a video about changing the oxygen in air from a gas to a liquid.

(t)Video Supplied by BBC Worldwide Learning, (bl)Tetra Images/Getty Images, (br)KinoMasterskaya/Creatas Video+/Getty Images

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about what happens at the particle level as matter changes from a solid to a liquid to a gas.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain: Plasma State



LESSON 2: Explore & Explain: Viscosity

Smithsonian

Additional Resources

LESSON 1

MATTER AND THERMAL ENERGY

FOCUS QUESTION

How do changes in thermal energy affect the particles that make up matter?

Kinetic Theory

You encounter solids, liquids, and gases every day. Look at **Figure 1.** Can you identify the states of matter present? The tea is in the liquid state. The ice cubes dropped into the tea to cool it are in the solid state. Surrounding the glass, as part of the air, is water in the gas state. How do these states compare?

Gas state

To understand the states of matter, we must think about the particles that make up matter. Consider the air around you: it is composed of nitrogen, oxygen, and water, along with other gases. These atoms and molecules—the particles that make up the air—are constantly moving. The **kinetic theory,** also known as kinetic molecular theory, is an explanation of how the particles in gases behave. To explain the behavior of particles, it is necessary to make the following assumptions:

1. All matter is composed of tiny particles (atoms, molecules, and ions).
2. These particles are in constant, random motion.
3. The particles collide with each other and with the walls of any container in which they are held.
4. The amount of energy that the particles lose from these collisions is negligible.



Figure 1 Water is a substance that can exist in all three common states of matter at the same time.

Identify *the solid and liquid states of water in this photo.*

Figure 2, on the next page, illustrates the kinetic theory. Because the particles of a substance in the gas state are in constant motion, colliding with each other and with the walls of their container, gases do not have a fixed volume or shape. The particles that make up a gas spread out so that they fill whatever container they are in.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Lab: Phase Changes

Carry out an investigation into the energy absorbed or released from a system when matter changes state.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Module question?

Dana Hoff/Brand X Pictures/Getty Images

Liquid state

Although the kinetic theory explains the behaviors of gas particles, some of its assumptions also apply to liquids and solids. The particles of a substance in the liquid state, shown in **Figure 2,** are also constantly moving, although they are not moving as quickly as they would be if the substance were in the gas state. Particles that make up a substance have less kinetic energy when in its liquid state than when in its gas state.

Because they have less energy, the particles are less able to overcome their attractions to each other. They can slide past each other, allowing a liquid to flow and take the shape of its container. However, because the particles that make up a liquid have not completely overcome the attractive forces between them, the particles cling together, giving the liquid a definite volume. The structure of matter at the bulk scale is determined by forces within and between particles.

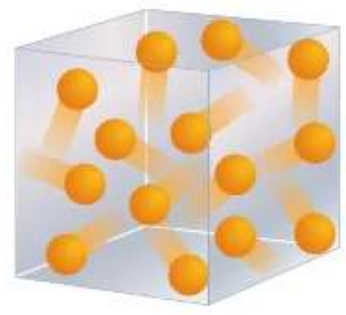
Solid state

A solid has a definite shape and volume. The particles that make up a solid are closely packed together, as shown in **Figure 2.** They are still in motion, but they have so little kinetic energy that the particles are unable to overcome their attractions to each other.

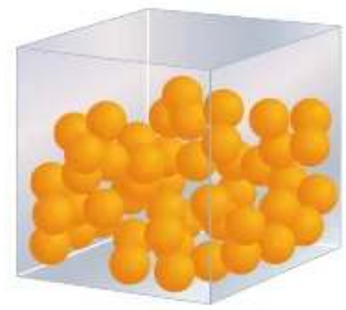
Many solids are crystalline, which means their particles have specific geometric arrangements. **Figure 3** shows the geometric arrangement of ice. The hydrogen and oxygen atoms alternate in the arrangement.

Thermal energy

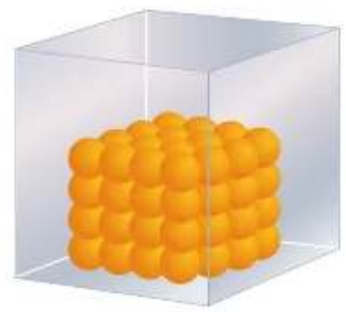
Think about the ice in **Figure 3.** How can frozen, solid ice have motion? The particles that make up solids are held tightly in place by the attractions between the particles. Those attractions give solids a definite shape and volume. However, the particles that make up a solid are still in constant motion. The particles' thermal energy causes them to vibrate.



Gas



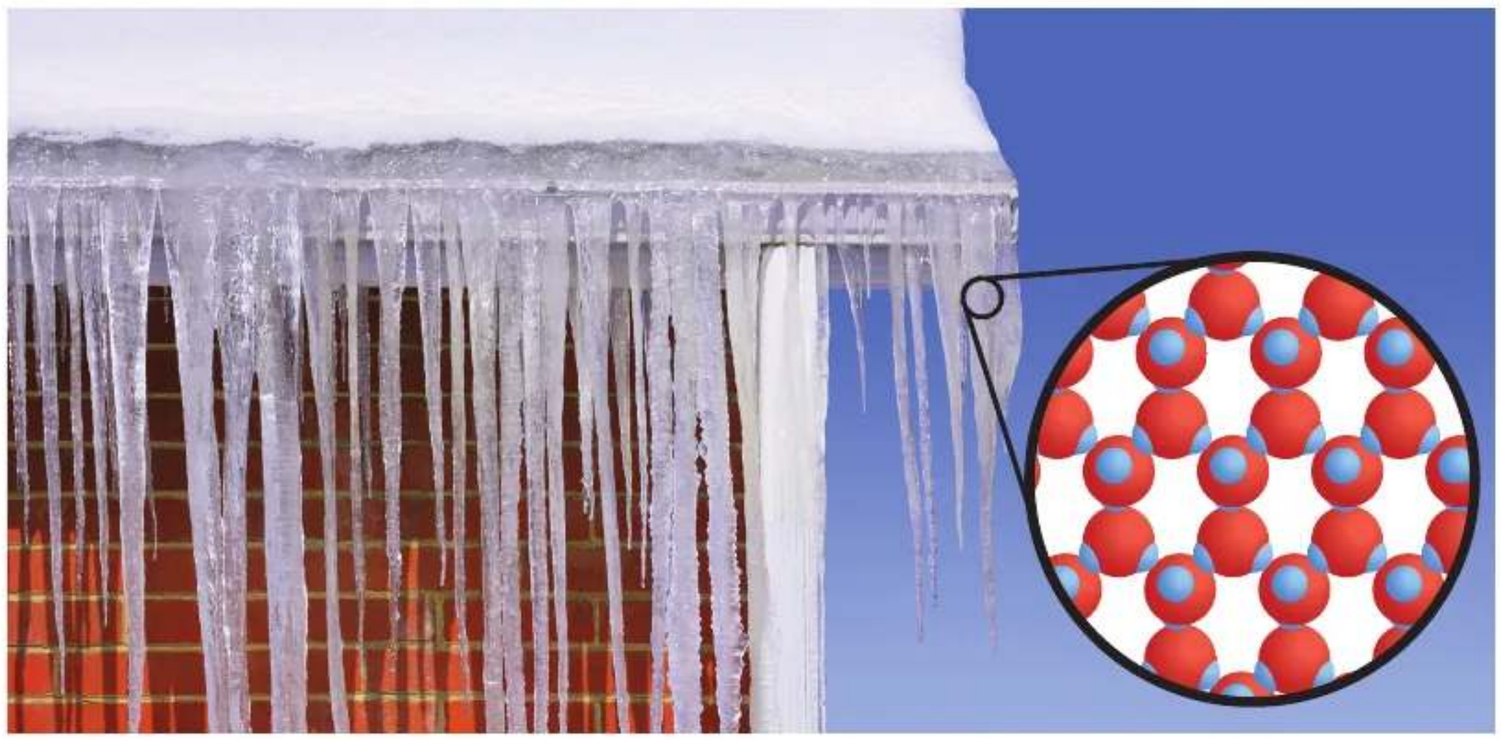
Liquid



Solid

Figure 2 Solids, liquids, and gases differ in how their particles move. These differences account for their physical properties.

Compare and contrast *the shape and volume of each state of matter.*



Design Pics Inc./Alamy Stock Photo

Figure 3 Ice is a crystalline solid—its particles have a specific geometric arrangement. Although ice does not look like it is moving, its molecules are vibrating in place.

Thermal energy is the total energy of a material's particles. Thermal energy is one of the ways energy manifests at the macroscopic scale. It includes both the kinetic energy of the particles as well as their potential energy. Energy from the motions of individual particles and energy from forces that act within or between particles are both forms of thermal energy. Energy from the motion of an object as a whole and energy from its interactions with its surroundings are not thermal energy.

Temperature

Temperature is the term used to explain how hot or cold an object is. Temperature represents the average kinetic energy of the particles that make up a substance. On average, molecules of water at 0°C have less kinetic energy than molecules of water at 100°C.

Changes of State

What happens to a solid when thermal energy is added to it? Think about the iced water in **Figure 4.** The particles that make up the water are moving fast and colliding with the particles that make up the ice cubes. Those collisions transfer energy from the water to the ice. The particles at the surface of an ice cube vibrate faster, transferring energy to other particles in the ice cube.



Figure 4 When ice is placed in water, energy from the particles of the liquid water is transferred to the particles of solid ice, melting the ice and cooling the water.

Melting and freezing

Soon, the particles that make up the ice have enough kinetic energy to overcome the attractive forces holding them in their crystalline structure. The ice melts. The **melting point** is the temperature at which a solid becomes a liquid. Energy is required for the particles to slip out of the ordered arrangement of a solid. The **heat of fusion** is the energy required to change a substance from solid to liquid at its melting point.

The transfer of energy between particles of liquid and particles of solid causes the ice to melt, but what happens to the particles of liquid after they collide with the solid? They slow down because they have less kinetic energy. As more of these collisions occur, the average kinetic energy of the particles of the liquid decreases, and the liquid cools.

Freezing is the reverse of melting. When a liquid's temperature is lowered, the average kinetic energy of the molecules decreases. When enough energy has been removed, the molecules become fixed into position. The freezing point is the temperature at which a liquid turns into a solid.

ACADEMIC VOCABULARY

definite
having distinct or certain limits
The teacher set definite standards for the students to meet.

CCC CROSSCUTTING CONCEPTS

Energy and Matter Energy cannot be created or destroyed. It moves between one place and another place, between objects and/or fields, or between systems. Create a graphic organizer that shows the movement of energy during different changes of state.

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Vaporization and condensation

How does a liquid become a gas? Remember that the particles that make up a liquid are constantly moving. When particles move fast enough to escape the attractive forces of other particles, they enter the gas state. This process is called vaporization. Vaporization can occur in two ways: evaporation and boiling. The process in which a gas becomes a liquid is called condensation. Condensation is the reverse of vaporization.

Evaporation Evaporation occurs at the surface of a liquid and can happen at nearly any temperature. To evaporate, particles must be at the liquid's surface and have enough kinetic energy to escape the attractive forces of the liquid.

Boiling Shown in **Figure 5,** boiling is the second way that a liquid can vaporize. Unlike evaporation, boiling occurs throughout a liquid at a specific temperature, depending on the pressure on the surface of the liquid.



Figure 5 As temperature increases, the particles that make up a substance in its liquid state move faster. When their energy creates sufficient pressure to surpass the air pressure above the liquid, the liquid boils.

Infer *What is inside the bubbles of the boiling liquid?*

The **boiling point** of a liquid is the temperature at which the pressure of the vapor in the liquid is equal to the external pressure acting on the surface of the liquid. This external pressure pushes down on the liquid, keeping particles from escaping. Particles require energy to overcome this pressure. The **heat of vaporization** is the amount of energy required for the liquid at its boiling point to become a gas.

Sublimation At certain pressures, some substances can change directly from solids into gases without going through the liquid phase. **Sublimation** is the process of a solid changing directly to a gas without forming a liquid. **Figure 6** shows frozen carbon dioxide, also known as dry ice, which is a common substance that undergoes sublimation.



(t)Richard Hutchings/Digital Light/McGraw-Hill Education, (b)IrinaK/Shutterstock

Figure 6 Carbon dioxide (CO_2) turns from a solid directly to a gas. Because this gas is very cold, it causes water in the air to condense, forming a white fog.

Explain, *at the molecular level, the behavior of CO_2 as it undergoes this phase transition.*

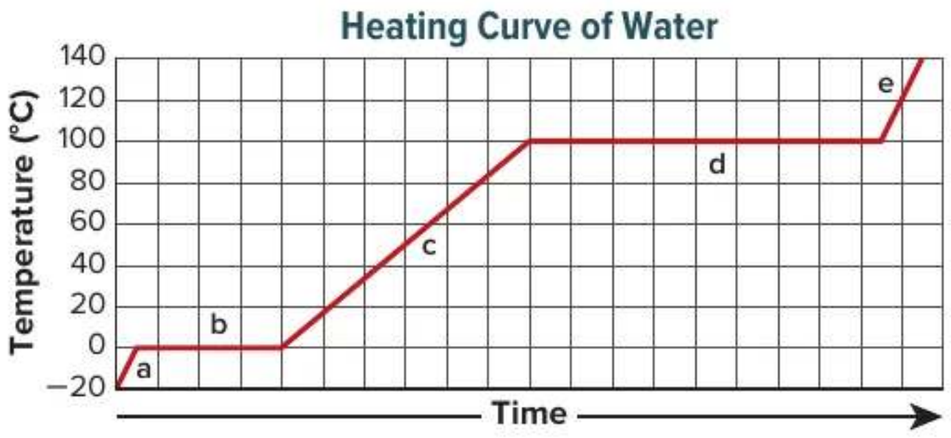


Figure 7 Although thermal energy is added at a constant rate, the temperature of the water increases only at *a*, *c*, and *e*. At *b* and *d*, the added energy is used to overcome the attractions between the particles.

Infer *how this graph would be different if 2.0 kg of water were being heated instead of 1.0 kg of water. How would this graph be different if 0.5 kg of water were being heated?*

Heating curves

A graph of temperature v. time for heating of 1.0 kg of water is shown in **Figure 7.** This type of graph is called a heating curve. It shows how temperature changes over time as thermal energy is continuously added. Notice the two areas on the graph where the temperature does not change. At 0°C, ice is melting. All of the energy put into the ice at this temperature is used to overcome the attractive forces between the particles. The flat line on the graph indicates that temperature remains constant during melting.

After the attractive forces between the particles of the solid are overcome, the particles move more freely, and their temperature increases. At 100°C, water is boiling, the temperature remains constant again, and the graph is flat. All of the energy that is put into the water goes to overcoming the remaining attractive forces between the particles. When all of the attractive forces between the particles of the liquid are overcome, the energy goes into increasing the temperature of the gas.

Get It?

Explain why the graph in Figure 7 is flat at b.

Plasma State

So far, you have learned about the three familiar states of matter—solids, liquids, and gases. However, there is a state of matter beyond the gas state. **Plasma** is matter that has enough energy to overcome not just the attractive forces between its particles but also the attractive forces within its atoms. The atoms that make up a plasma collide with such force that the electrons are stripped off the atoms. As a result, plasmas are made up of electrons and other charged particles.

You may be surprised to learn that most of the ordinary matter in the universe is in the plasma state. Every star that you can see in the sky, including the Sun, is composed of matter in the plasma state. Most of the matter between the stars and galaxies is also in the plasma state. The familiar states of matter—solid, liquid, and gas—are extremely rare in the universe.

Thermal Expansion

Have you ever wondered why a concrete sidewalk has seams? When thermal energy is transferred to a concrete sidewalk, the concrete expands. Without the seams, a concrete sidewalk would crack in hot weather. The kinetic theory can help to explain this behavior.

Recall that particles move faster and farther apart as the temperature rises. This separation of particles results in an expansion of the entire object, known as thermal expansion. **Thermal expansion** is an increase in the size of a substance when the temperature is increased. Substances also contract when they cool.

Thermometers

A common example of liquids undergoing thermal expansion occurs in thermometers, like the one shown in **Figure 8.** The addition of energy causes the particles that make up the liquid in the thermometer to move faster. As their motion increases, the particles that make up the liquid in the narrow thermometer tube start to move farther apart. This causes the liquid in a thermometer to expand and rise as the temperature increases.

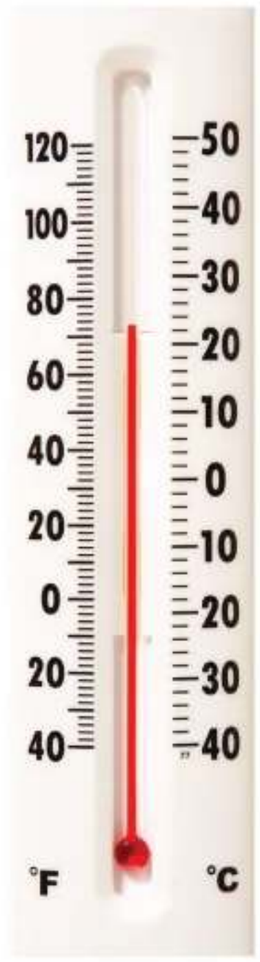


Figure 8 As the air temperature goes up, the liquid in the thermometer is heated and expands. As a result, the level of the liquid rises. The liquid in the thermometer contracts with a decrease in temperature.

Hot-air balloons

An application of gases undergoing thermal expansion is shown in **Figure 9.** Hot-air balloons are able to rise due to the thermal expansion of air. The air in the balloon is heated, causing the distance between the particles that make up the air to increase. As the hot-air balloon expands, the number of particles per cubic centimeter decreases. This expansion results in a decreased density of hot air. Because the density of the air in the hot-air balloon is lower than the density of the cooler air outside, the balloon will rise.



(t)Thomas Northcut/Photodisc/Getty Images, (b)John Foxx/Stockbyte/Getty Images

Figure 9 When the air inside a hot-air balloon is heated, its particles move farther apart. The balloon rises because the air inside it is less dense than the surrounding air.

Water's strange behavior

Ordinarily, substances contract as their temperatures decrease. However, an exception to this rule is water. Over a small range of temperatures, water expands as the temperature decreases. At first, it behaves like other substances. As the temperature begins to drop, the particles that make up water move closer together. This continues until the water reaches 4°C.

Water molecules are unusual in that they have highly positive and highly negative areas. These charged regions affect the behavior of water. As the temperature of water continues to drop under 4°C, the molecules line up so that only positive and negative areas are near each other, as shown in **Figure 10.** As a result, empty spaces occur in the structure. Water expands as it cools from about 4°C to 0°C and becomes less dense than liquid water. That is why ice floats in liquid water.

Get It?

Describe how forces between and within water molecules cause ice to float.

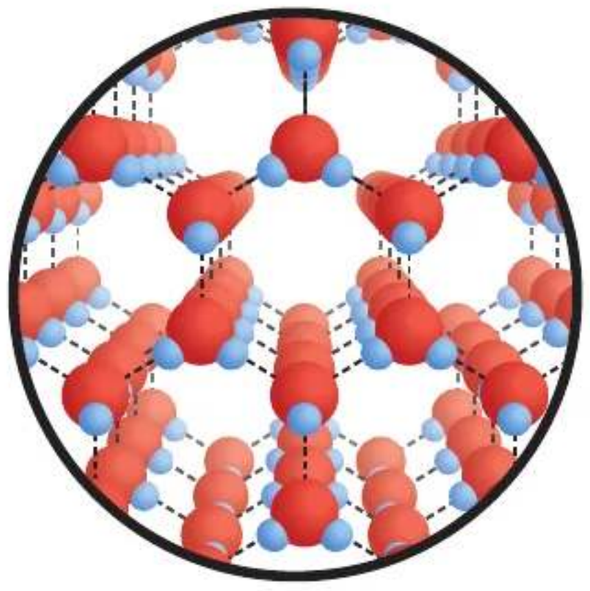


Figure 10 When water freezes, the positively and negatively charged ends of the water molecules interact, creating empty spaces in the crystal lattice.

Explain *why ice floats in water.*

Solid or Liquid?

Other substances also show unusual behaviors when changing states. Amorphous solids and liquid crystals are two classes of materials that do not react as you would expect when they are changing states.

Amorphous solids

Ice melts at 0°C, and lead melts at 327°C. But not all solids have a specific temperature at which they melt. Consider a stick of butter. Instead of having a specific melting point, butter softens and melts over a range of temperatures.

Other solids are similar to butter in this way. Instead of having a specific melting point, they soften and gradually turn into a liquid over a temperature range. These solids lack a crystalline structure and are called amorphous solids. One common amorphous solid is glass, shown in **Figure 11.**



Figure 11 Glass lacks the repeating crystal-line structure of solids like ice. Rather than melting at a specific temperature, glass becomes increasingly soft and malleable as temperature increases.

WORD ORIGINS

amorphous

comes from the Greek word *amorphos*, which means *without form, shapeless, deformed Clay, which can be shaped and formed easily, is amorphous.*

STEM CAREER Connection

Glass Artist

Glass artists take advantage of the amorphous properties of glass to create artistic pieces. They use different techniques to blow, shape, and join glass under high temperatures. These artists refine their skills through practice. Many study art at the postsecondary level. Others learn through workshops or privately, alongside an experienced artist.

Benoit Daoust/Shutterstock

Liquid crystals

Liquid crystals form another group of materials that do not change states in the usual manner. Normally, the ordered geometric arrangement of a solid is lost when the substance goes from the solid state to the liquid state. Liquid crystals start to flow during the melting phase, similar to a liquid. But they do not lose their ordered arrangement completely, as most substances do. Liquid crystals retain their geometric order in specific directions.

Liquid crystals are placed in classes, depending upon the type of order they maintain when they liquify. They are highly responsive to temperature changes and electric fields. Scientists use the unique properties of liquid crystals to make liquid crystal displays (LCDs) for cell phones, calculators, and netbooks, shown in **Figure 12.** LCD screens are composed of individual crystal picture elements, or "pixels" for short. Varying the amount of electricity that passes through the pixel determines how the crystals are aligned and whether light is able to pass through them.



Figure 12 Many small computing devices and electronics, such as cell phones, TVs, and tablets, utilize liquid crystal displays (LCDs).

Varin Rattanaburi/Blackzheep/123RF

Check Your Progress

Summary

- There are four main states of matter: solid, liquid, gas, and plasma.
- The kinetic theory is an explanation of how the particles that make up gases behave.
- Thermal energy is the total energy of the particles that make up a material, including kinetic and potential energy.
- Temperature is the average kinetic energy of a substance.

Demonstrate Understanding

1. **Describe** the movement of particles in solids, liquids, and gases.
2. **State** the basic assumptions of the kinetic theory.
3. **Describe,** in terms of kinetic theory, how the particles of a substance behave at its melting point.
4. **Describe,** in terms of kinetic theory, how the particles of a substance behave at its boiling point.

Explain Your Thinking

5. **Infer** How would the heating curve for glass be different from the heating curve for water?
6. **MATH Connection** Using the graph in **Figure 7,** describe the energy changes that occur when water goes from −15°C to 120°C.
7. **MATH Connection** The melting point of acetic acid is 17°C, and the boiling point is 118°C. Draw a graph similar to the graph in **Figure 7** showing the phase changes for acetic acid. Clearly mark the three phases, the boiling point, and the melting point.

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LESSON 2

PROPERTIES OF FLUIDS

FOCUS QUESTION

What principles describe the behavior of fluids?

Archimedes' Principle and Buoyancy

Some ships are like floating cities. For example, aircraft carriers are large enough to allow airplanes to take off and land on their decks. Despite their weights, these ships float. There is a force pushing up on the ship that opposes the gravitational force pulling the ship down.

What is the force pushing up on the ship? It is called the buoyant force. If the buoyant force is equal to an object's weight, the object will float. If the buoyant force is less than an object's weight, the object will sink. **Buoyancy** is the ability of a fluid—a liquid or a gas—to exert an upward force on an object immersed in it.

Archimedes' principle

In the third century B.C., a Greek mathematician named Archimedes made a discovery about buoyancy. Archimedes found that the buoyant force on an object is equal to the weight of the fluid displaced by the object. For example, if you place a block of wood in water, it will push water out of the way as it begins to sink—but only until the weight of the water displaced equals the block's weight.

When the weight of water displaced—the buoyant force—becomes equal to the weight of the block, the block floats. If the weight of the water displaced is less than that of the block, the block sinks. **Figure 13** shows the forces that affect objects in fluids.

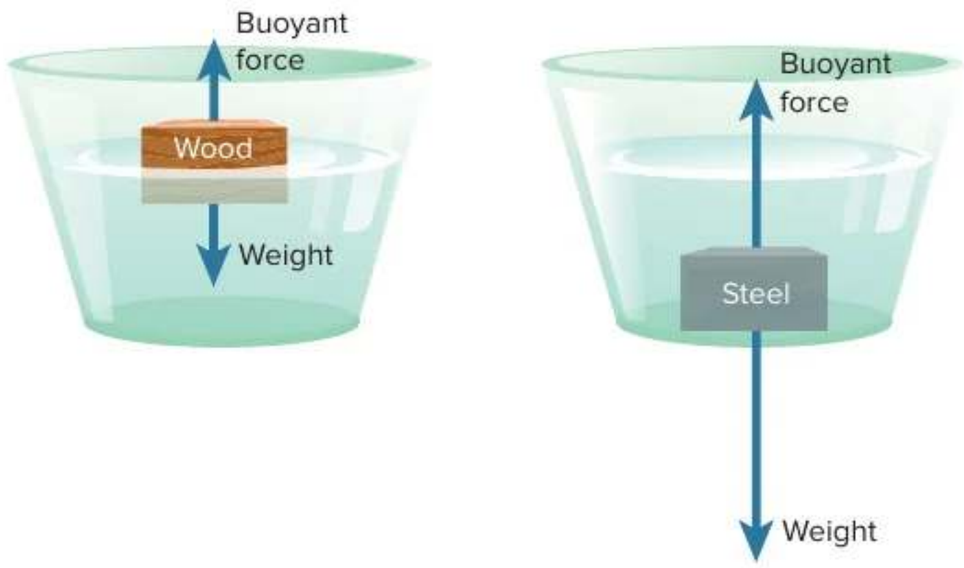


Figure 13 The steel block sinks because the buoyant force from the fluid is less than the gravitational force on the object. When the buoyant force is the same as or greater than the gravitational force—as with the wooden block—the object floats.

Compare *the volumes of the wood block and the steel block. How do the masses of the wood block and the steel block compare?*

Get It?

Infer why rocks sink and rubber balls float in water.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Laboratory: Density of a Liquid

Analyze and interpret data to calculate and compare the densities of several liquids.

CCC **Identify Crosscutting Concepts**

Create a table of the crosscutting concepts and fill in examples you find as you read.

Comparing buoyancy and weight

Look again at the wood and steel blocks in **Figure 13.** They both displace the same volume and weight of water when submerged. Therefore, the buoyant forces on the blocks are equal. Yet, the steel block sinks and the wood block floats. What is different?

The steel block weighs much more than the wood block. The gravitational force on the steel block is enough to make the steel block sink. The gravitational force on the wood block is not enough to make the wood block sink.

Density and buoyancy

One way to know whether an object will float or sink is to compare its density to the density of the fluid in which it is placed. An object floats if its density is less than that of the fluid. Remember that density is mass per unit volume. The density of the steel block is greater than the density of water. The wood block's density is less than that of water.

Suppose you formed the steel block into the shape of the hull of a ship filled with air, as shown in **Figure 14.** Now the same mass takes up a larger volume. The overall density of the steel boat and air is less than the density of water. The boat will now float.

Get It?

Explain why a steel block sinks but a steel ship floats.

Pascal's Principle and Pressure

Blaise Pascal (1623–1662), a French scientist, discovered that pressure applied to a fluid is transmitted throughout the fluid. This is why when you squeeze one end of a toothpaste tube, toothpaste emerges from the other end. The pressure has been transmitted through the toothpaste. In order to understand Pascal's principle, you must first understand pressure.

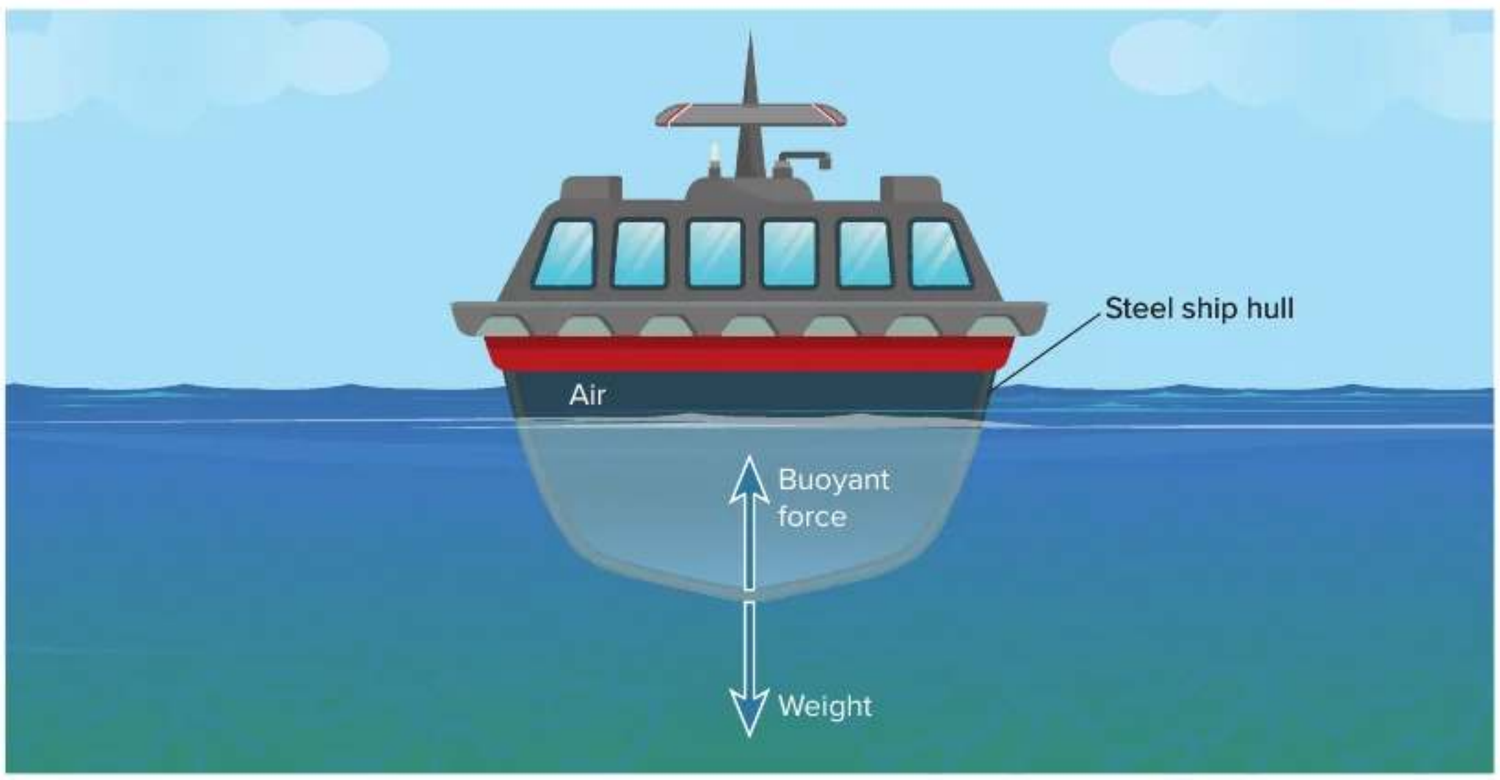


Figure 14 The overall density of a large ship is lower than the density of water because its empty hull contains mostly air.

Infer *why a boat cannot be made of solid steel.*

Pressure

Right now, pressure from the air is pushing on you from all sides, like the pressure you feel underwater in a swimming pool. **Pressure** is force exerted per unit area.

Pressure Equation

$$\text{Pressure (Pa)} = \frac{\text{force (N)}}{\text{area (m}^2\text{)}}$$

$$P = \frac{F}{A}$$

The SI unit of pressure is the pascal (Pa). Because pressure is the amount of force divided by area, one pascal is one newton per square meter (N/m^2). Most pressures are given in kilopascals (kPa) because 1 Pa is a very small amount of pressure.

EXAMPLE Problem 1

CALCULATE FORCE Atmospheric pressure at sea level is about 101 kPa. With how much total force does Earth's atmosphere push on an average human being at sea level? Assume that the surface area of an average human is 1.80 m^2.

List the Unknown: force: F

List the Knowns: pressure: $P = 101\ kPa = 101{,}000\ Pa$

area: $A = 1.80\ m^2$

Set Up the Problem: $P = \frac{F}{A}$

Solve the Problem: $101{,}000\ Pa = \frac{F}{1.80\ m^2}$

$F = 101{,}000\ Pa \times 1.80\ m^2$

$= 182{,}000\ Pa \cdot m^2 = 182{,}000\ \frac{N}{m^2} \cdot m^2 = 182{,}000\ N$

Check the Answer: You've set up your equation correctly if the units are the same on both sides: units of pressure = Pa = N/m^2, and (units of force)/(units of area) = N/m^2. The units on both sides of the equation match. Now, just double-check the calculation.

PRACTICE Problems

ADDITIONAL PRACTICE

8. A diver who is 10.0 m underwater experiences a pressure of 202 kPa. If the diver's surface area is 1.50 m^2, with how much total force does the water push on the diver?
9. A car weighs 15,000 N, and its tires are inflated to a pressure of 190 kPa. How large is the area of the car's tires that are in contact with the road?
10. **CHALLENGE** The atmospheric pressure at the surface of Venus is 91 times the pressure at sea level on Earth. With approximately how much total force would Venus's atmosphere push on an average human being at sea level? Assume that the surface area of an average human being is 1.8 m^2.

SCIENCE USAGE v. COMMON USAGE

pressure

Science usage: force per unit area
Increasing the pressure on a gas decreases its volume.

Common usage: the burden of physical or mental stress
Teachers often feel a lot of pressure to help their students do well in school.

Pascal's principle

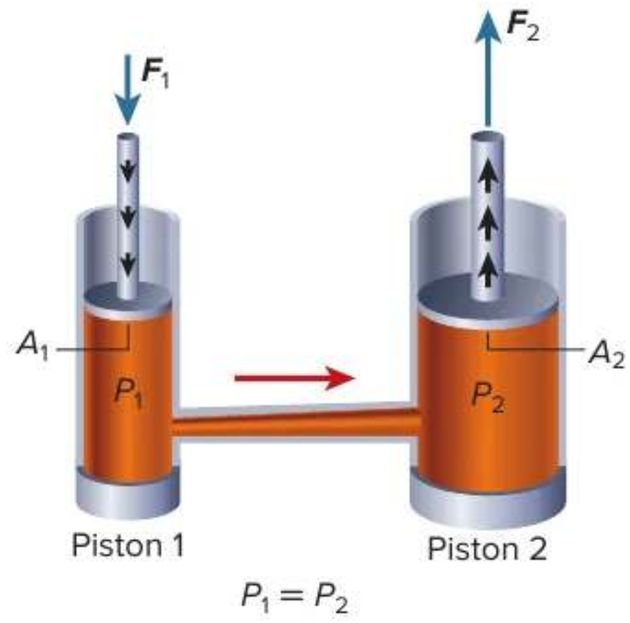
The idea that pressure is transferred through a fluid can be written as an equation: pressure in = pressure out. Since pressure is force over area, Pascal's principle can be written another way.

Pascal's Principle

$$\frac{\text{input force (N)}}{\text{input area (m}^2\text{)}} = \frac{\text{output force (N)}}{\text{output area (m}^2\text{)}}$$

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}$$

Hydraulic lifts Auto repair shops often make use of hydraulic lifts, which move heavy loads in accordance with Pascal's principle. A pipe that is filled with fluid connects small and large cylinders, as shown in **Figure 15.** Pressure applied to the small cylinder is transferred through the fluid and to the large cylinder. With a hydraulic lift, you could use your weight to lift something much heavier than you.



$$P_1 = P_2$$

Figure 15 The pressure of the fluid on one side of a hydraulic lift is equal to the pressure on the other side.

EXAMPLE Problem 2

CALCULATE FORCES A hydraulic lift is used to lift a heavy machine that is pushing down on a 2.8-m^2 platform with a force of 3700 N. What force must be exerted on a 0.072-m^2 piston to lift the heavy machine?

List the Unknowns: force on piston: F_{in}

List the Knowns: force on platform: $F_{out} = 3700\ \text{N}$
area of platform: $A_{out} = 2.8\ \text{m}^2$
area of piston: $A_{in} = 0.072\ \text{m}^2$

Set Up the Problem: $\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}$

Solve the Problem: $F_{in} = \left(\frac{F_{out}}{A_{out}}\right) A_{in} = \left(\frac{3700\ \text{N}}{2.8\ \text{m}^2}\right) 0.072\ \text{m}^2 = 95\ \text{N}$

Check the Answer: The ratio of the forces should be the same as the ratio of the areas. The area of the platform is about 40 times the area of the piston. Therefore, the force on the platform should be about 40 times the force on the piston. 3700 N is about 40 times greater than 95 N, so the answer is reasonable.

PRACTICE Problems

ADDITIONAL PRACTICE

11. A car weighing 15,000 N is on a hydraulic lift platform measuring 10 m^2. What is the area of the smaller piston if a force of 1100 N is used to lift the car?

12. CHALLENGE A heavy crate applies a force of 1500 N on a 25-m^2 piston. The smaller piston is 1/30 the size of the larger one. What force is needed to lift the crate?

Bernoulli's Principle

Daniel Bernoulli (1700–1782) was a Swiss scientist who studied the properties of moving fluids, such as water and air. Bernoulli found that fluid velocity increases when the flow of the fluid is restricted. Placing your thumb over a running garden hose, as shown in **Figure 16,** demonstrates this effect. When the opening of the hose is decreased in size, the water flows out more quickly.



Figure 16 Bernoulli's principle explains why covering the end of a hose causes the water to flow faster. When the flow of a fluid is restricted, its velocity increases.

Bernoulli examined the relationship between fluid flow and pressure. You might think that increasing the velocity of a fluid's flow would increase its pressure, but Bernoulli found the opposite to be true. According to Bernoulli's principle, as the velocity of a fluid increases, the pressure exerted by that fluid decreases. He published this discovery in 1738.

One application of Bernoulli's principle is the hose-end sprayer. This sprayer is used to apply fertilizers and pesticides to yards and gardens. To use this sprayer, a concentrated solution of the chemical that is to be applied is placed in the sprayer. The sprayer is attached to a garden hose, as shown in **Figure 17.** A strawlike tube is attached to the lid of the unit. The end of the tube is submerged into the concentrated chemical. The water to the garden hose is turned to a high flow rate.

When you are ready to apply the chemicals to the lawn or plant area, you must push a trigger on the sprayer attachment. This allows the water in the hose to flow at a high rate of speed, creating a low pressure area above the strawlike tube. The concentrated chemical solution is sucked up through the straw and into the stream of water. The concentrated solution mixes with water, reducing the concentration to the appropriate level and creating a spray that is easy to apply.

Get It?

Describe how pressure changes as the velocity of a fluid increases.

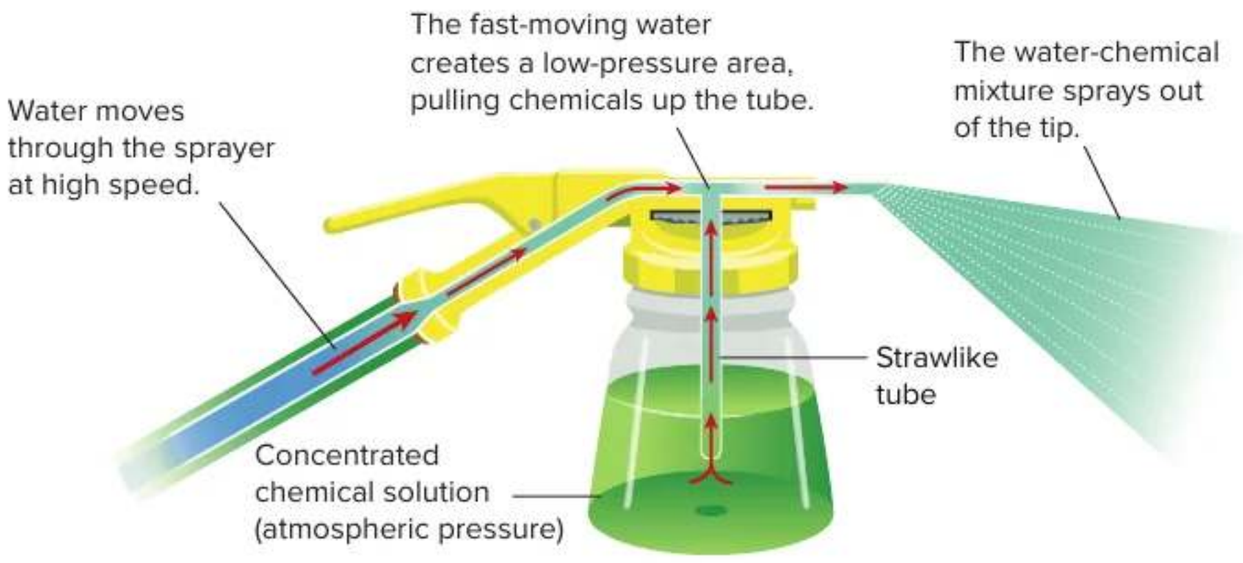


Figure 17 A hose-end sprayer makes use of Bernoulli's principle.

Val Loh/Photolibrary/Getty Images

Viscosity

Another property exhibited by a fluid is its tendency to flow. While all fluids flow, they vary in the rates at which they flow. **Viscosity** is the resistance of a fluid to flowing. For example, when you take syrup out of the refrigerator and pour it, as shown in **Figure 18,** the flow of syrup is slow. But if this syrup were heated, it would flow much faster. Water flows easily because it has low viscosity. Cold syrup flows slowly because it has high viscosity.

What causes viscosity? When a container of liquid is tilted to allow flow to begin, the flowing portion of the liquid transfers energy to the portion of the liquid that is stationary. In effect, the flowing portion of the liquid is pulling the stationary portion of the liquid, causing it to flow, too.

If the flowing portion of the liquid does not effectively pull the other portions of the liquid into motion, then the liquid has a high viscosity, which is a high resistance to flow. If the flowing portion of the liquid pulls the other portions of the liquid into motion easily, then the liquid has low viscosity, or a low resistance to flow.



Figure 18 This maple syrup flows slowly because it has high viscosity.

Identify *other examples of liquids with high viscosities.*

Check Your Progress

Summary

- If the buoyant force on an object is equal to or greater than the gravitational force on that object, the object will float. If the buoyant force on an object is less than the gravitational force on that object, the object will sink.
- Pascal's principle states that pressure applied to a fluid is transmitted throughout the fluid.
- Bernoulli's principle states that as the velocity of a fluid increases, the pressure exerted by the fluid decreases.
- The resistance to flow by a fluid is called viscosity.

Demonstrate Understanding

13. **Describe** how fluids exert forces on objects.
14. **Explain** why a steel boat floats on water, but a steel block does not.
15. **Explain** why squeezing a plastic mustard bottle forces mustard out the top.
16. **Describe,** using Bernoulli's principle, how tornadoes lift roofs off of buildings.

Explain Your Thinking

17. **Infer** If you blow up a balloon, tie it off, and release it, it will fall to the floor. Why does it fall instead of float? Explain what would happen if the balloon contained helium instead of air.
18. **MATH Connection** The density of water is 1.0 g/cm^3. How many kilograms of water does a submerged 120-cm^3 block displace? Recall that 1.0 kg weighs 9.8 N on Earth. What is the buoyant force on the block?
19. **MATH Connection** To lift an object weighing 21,000 N, how much force is needed on a piston with an area of 0.060 m^2 if the platform being lifted has an area of 3.0 m^2?

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Mahara/Shutterstock

LESSON 3

BEHAVIOR OF GASES

FOCUS QUESTION

How do gases respond to changes in pressure and temperature?

Boyle's Law–Volume and Pressure

Have you ever seen a weather balloon, like the one shown in **Figure 19?** They carry sensing instruments to very high altitudes to detect weather information. A weather balloon is inflated near Earth's surface with a low-density gas.

Recall that a gas completely fills its container. The balloon remains inflated because of collisions between the gas particles inside the balloon and the balloon itself. In other words, these collisions between gas particles and the container wall cause the gas to exert pressure on the container. As the balloon rises, the atmospheric pressure outside the balloon decreases. This decrease in pressure allows the balloon to expand, eventually reaching a volume between 30 and 200 times its original size. Boyle's law describes the relationship between gas pressure and volume that explains the behavior of weather balloons.

Get It?

Describe what happens to weather balloons as they rise.



Figure 19 A weather balloon expands as it rises due to decreased external pressure. Eventually, the balloon ruptures, and the instruments fall back to the ground.

Edward Haylan/Shutterstock

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Laboratory: The Behavior of Gases

Carry out an investigation into the effects of changes in pressure and temperature on a sample of a gas.

Revisit the Encounter the Phenomenon Question

What information from this lesson can help you answer the Unit and Module questions?

Volume and pressure

Because a balloon is flexible, its volume can change. In the case of the weather balloon, the volume increases as the external pressure decreases. The volume of the gas inside the weather balloon continues to increase until the balloon can no longer contain it. At this point, the balloon ruptures, and the sensing instruments it was carrying fall to the ground.

From the weather balloon, we know what happens to volume when you decrease pressure. What happens to the pressure from a gas if you decrease its volume—for example, by decreasing the size of the container in which the gas is held? Think about the kinetic theory of matter. The pressure from a gas depends on how often its particles strike the walls of the container. If you squeeze gas into a smaller space, its particles will strike the walls more often, causing increased pressure. The opposite is also true. If you give the particles that make up the gas more space, increasing the volume, they will hit the walls less often, and the pressure from the gas will be reduced.

Get It?

Explain the relationship between pressure and volume.

Robert Boyle (1627–1691), a British scientist, described this property of gases. According to **Boyle's law,** if you decrease the volume of a container of gas and hold the temperature constant, the pressure from the gas will increase. An increase in the volume of the container causes the pressure to drop if the temperature remains constant. **Figure 20** shows this relationship as the volume of a gas is decreased from 10 L to 5 L to 2.5 L. Note the points on the graph that correspond to each of these volumes.

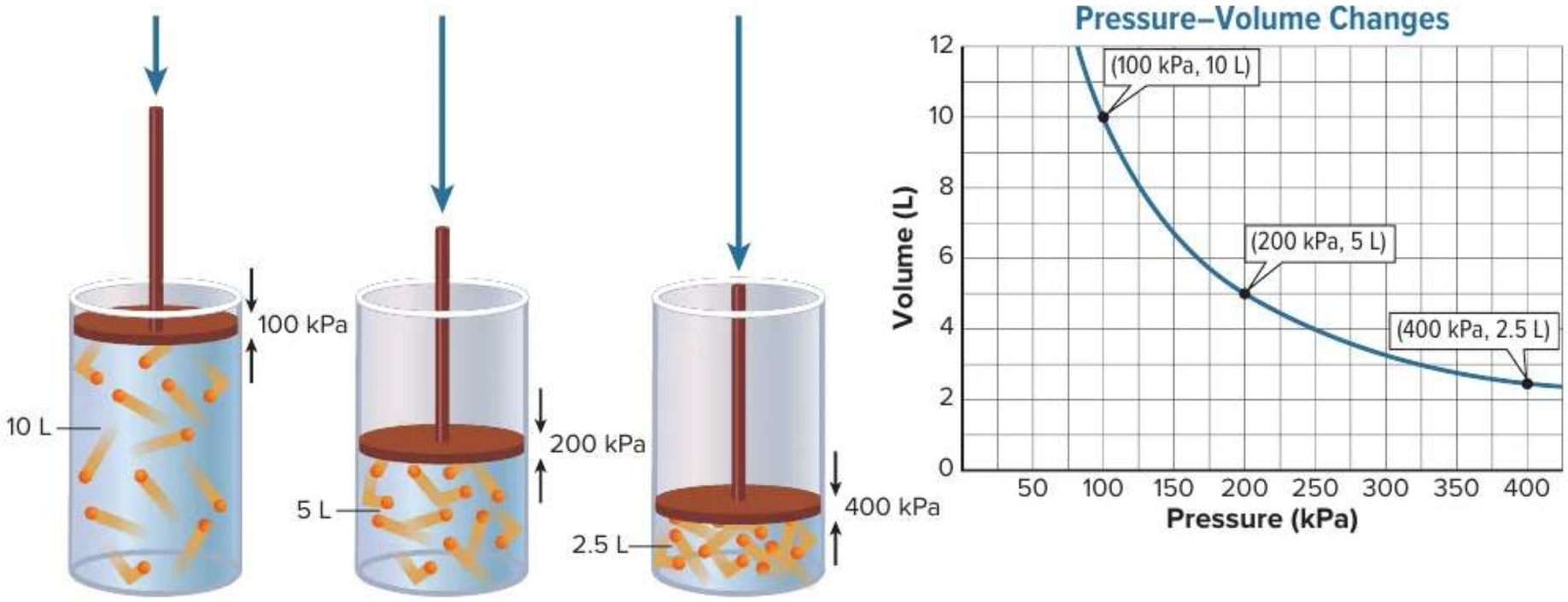


Figure 20 As volume is decreased, a gas exerts increased pressure on the walls of its container. The three canisters are depicted on the graph.

Interpret *What happens to the volume of a gas if the pressure on that gas is doubled?*

An equation for Boyle's law

Boyle's law can be expressed with a mathematical equation. When the temperature of a gas is constant, then the product of the pressure and volume of that gas does not change.

Boyle's Law Equation

initial pressure × **initial volume** = **final pressure** × **final volume**

$$P_i V_i = P_f V_f$$

The product of the initial pressure and volume—designated with the subscript *i*—is equal to the product of the final pressure and volume—designated with the subscript *f*. You can use this equation to find one unknown value when you have the other three. The equation will work with any units for either volume or pressure as long as you use the same pressure units for P_i and P_f and the same volume units for V_i and V_f.

Get It?

Show how to write the Boyle's law equation when it is solved for the final pressure of a gas.

EXAMPLE Problem 3

BOYLE'S LAW A weather balloon has a volume of 100.0 L when it is released from sea level, where the pressure is 101 kPa. What will be the balloon's volume when it reaches an altitude where the pressure is 43.0 kPa?

Identify the Unknown: final volume: V_f

List the Knowns: initial pressure: $P_i = 101\ \text{kPa}$
initial volume: $V_i = 100.0\ \text{L}$
final pressure: $P_f = 43.0\ \text{kPa}$

Set Up the Problem:

$$P_i V_i = P_f V_f$$

$$V_f = V_i \left(\frac{P_i}{P_f}\right)$$

Solve the Problem:

$$V_f = 100.0\ \text{L} \left(\frac{101\ \cancel{\text{kPa}}}{43.0\ \cancel{\text{kPa}}}\right)$$

$$= 235\ \text{L}$$

Check the Answer: You can do a quick estimate to check your answer. The pressure was slightly more than halved. Therefore, the volume should slightly more than double. The final volume of 235 L is slightly more than twice the initial volume of 100.0 L. Therefore, the answer seems reasonable.

PRACTICE Problems

ADDITIONAL PRACTICE

20. A volume of helium occupies 11.0 L at a pressure of 98.0 kPa. What is the new volume if the pressure drops to 86.2 kPa?

21. CHALLENGE A weather balloon has a volume of 90.0 L when it is released from sea level. What is the atmospheric pressure on the balloon when it has grown to a size of 175.0 L?

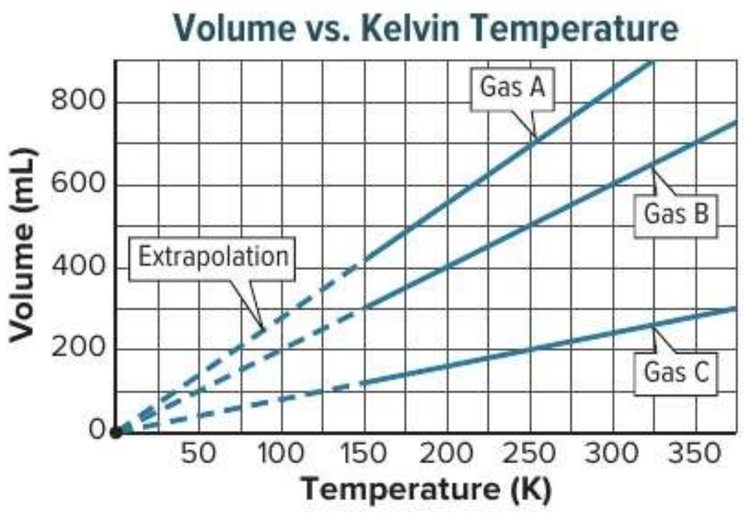


Figure 21 As the temperature of a sample of gas at constant pressure increases, the volume also increases. The dotted lines represent extrapolations of experimental data. Notice that all the extrapolated lines converge at 0 K.

Identify *which gas had the greatest volume change.*

Charles's Law—Temperature and Volume

If you've watched a hot-air balloon being inflated, you know that gases expand when they are heated. Jacques Charles (1746–1823), a French scientist, also noticed this. According to **Charles's law**, the volume of a gas increases with increasing temperature as long as the pressure on the gas does not change. As with Boyle's law, the reverse is also true. The volume of a gas shrinks with decreasing temperature, as shown in **Figure 21.**

Get It?
Explain the relationship between the temperature and volume of a gas.

The kinetic theory and Charles's law

Charles's law can be explained using the kinetic theory of matter. As a gas is heated, the particles that make up that gas move faster and faster. Because the particles that make up the gas move faster, they strike the walls of their container more often and with greater force. In the hot-air balloon, the walls have room to expand. So instead of pressure increasing, the volume increases.

Get It?
Describe How does the kinetic theory of matter explain Charles's law?

An equation for Charles's law

Like Boyle's law, Charles's law can be expressed mathematically. When the pressure on a gas is constant, then the ratio of the volume to the absolute temperature does not change. The absolute temperature is the temperature measured in kelvins.

Charles's Law Equation

$$\frac{\text{initial volume}}{\text{initial temperature (K)}} = \frac{\text{final volume}}{\text{final temperature (K)}}$$

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

This shows that the ratio of the initial volume to the initial temperature is equal to the ratio of the final volume to the final temperature. Remember that temperature must be in kelvins.

EXAMPLE Problem 4

USE CHARLES'S LAW A 2.0-L balloon at room temperature (20.0°C) is placed in a refrigerator at 3.0°C. What is the volume of the balloon after it cools in the refrigerator?

Identify the Unknown: final volume: V_f

List the Knowns: initial volume: $V_i = 2.0\ L$

initial temperature: $T_i = 20°C = 20.0°C + 273 = 293\ K$

final temperature: $T_f = 3.0°C = 3.0°C + 273 = 276\ K$

Set Up the Problem:

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

$$V_f = V_i\left(\frac{T_f}{T_i}\right)$$

Solve the Problem:

$$V_f = 2.0\ L\left(\frac{276\ \cancel{K}}{293\ \cancel{K}}\right) = 1.9\ L$$

Check the Answer: A good way to check your answer here is through experiment! If you place a balloon in a refrigerator, you will notice that the balloon shrinks, but not very much. This is consistent with our answer above.

PRACTICE Problems

ADDITIONAL PRACTICE

22. What would be the final size of the balloon in the example problem above if it were placed in a –18°C freezer?

23. CHALLENGE A gas is heated so that it expands from a volume of 1.0 L to a volume of 1.5 L. If the initial temperature of the gas was 5.0°C, then what is the final temperature of the gas?

Check Your Progress

Summary

- Boyle's law states that if the temperature is constant as the volume of a gas decreases, the pressure increases.
- Charles's law states that at constant pressure, the volume of a gas increases with increasing temperature.
- Both Boyle's law and Charles's law can be expressed as mathematical equations.

Demonstrate Understanding

24. **Describe** what would happen to the volume of a gas if the pressure on it were decreased and then the gas's temperature were increased.
25. **Predict,** using Boyle's law, what will happen to a balloon that an ocean diver takes to a pressure of 202 kPa.

Explain Your Thinking

26. **Predict** what would happen to the volume of a gas if the pressure on that gas were doubled and then the absolute temperature of the gas were doubled.
27. **MATH Connection** A helium balloon has a volume of 2.00 L at 101 kPa. As the balloon rises, the pressure drops to 97.0 kPa. What is the new volume?
28. **MATH Connection** If a 5.0-L balloon at 25°C were gently heated to 30°C, what would be the new volume?

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NATURE OF SCIENCE

Detecting Dark Matter

The Andromeda Galaxy, shown in the figure below, is a group of approximately one trillion stars held together by the force of gravity. Scientists estimate that the matter we can see, which makes up the stars, planets, and gas clouds in the galaxy, is about 15 percent of the galaxy's mass. The other 85 percent is in a form we do not currently understand.

"Missing" mass

Fritz Zwicky, working in the 1930s on gravity's effects on the motion of galaxies, proposed that much of the matter in the universe does not emit or absorb light. While Zwicky's data hinted at mysterious "dark matter," his idea was largely ignored because his measurements were not accurate enough to be conclusive.

In the 1970s, astronomer Vera Rubin used new technology to study light coming from the Andromeda Galaxy. Her accurate data supported the existence of dark matter and renewed interest in Zwicky's proposal. Today, scientists are investigating dark matter to study the 85 percent of the universe that is not explained by current scientific knowledge.



(t)National Aeronautics and Space Administration (NASA), (b)Viktar Malyshchyts/Shutterstock

Much of the visible matter is concentrated at the center of the Andromeda Galaxy. Dark matter is spread throughout.



These detectors attempt to measure rare interactions by searching for antimatter and dark matter and by measuring cosmic rays.

Undiscovered particles

Scientists are trying to detect massive particles that do not usually interact with other particles. This requires sensitive equipment, like that shown in the figure above. Attempts at detecting dark matter particles have been unsuccessful so far.

A new theory of gravity

Some scientists are trying to develop a gravitational theory that can explain the motion of distant galaxies without relying on mysterious undiscovered particles. To date, none of these proposals are able to explain the observed phenomena. Until a new particle is discovered or a new theory is written, dark matter will be a scientific mystery.

OBTAIN, EVALUATE, AND COMMUNICATE INFORMATION

The undiscovered particles of dark matter are commonly known by the descriptive acronym WIMP. Research what WIMP stands for and explain why it accurately describes the particles under consideration.

MODULE 14
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 MATTER AND THERMAL ENERGY

- There are four main states of matter: solid, liquid, gas, and plasma.
- The kinetic theory is an explanation of how the particles that make up gases behave.
- Thermal energy is the total energy of the particles that make up a material, including kinetic and potential energy.
- Temperature is the average kinetic energy of a substance.

- kinetic theory
- melting point
- heat of fusion
- boiling point
- heat of vaporization
- sublimation
- plasma
- thermal expansion

Lesson 2 PROPERTIES OF FLUIDS

- If the buoyant force on an object is equal to or greater than the gravitational force on that object, the object will float. If the buoyant force on an object is less than the gravitational force on that object, the object will sink.
- Pascal's principle states that pressure applied to a fluid is transmitted throughout the fluid.
- Bernoulli's principle states that as the velocity of a fluid increases, the pressure exerted by the fluid decreases.
- The resistance to flow by a fluid is called viscosity.

- buoyancy
- pressure
- viscosity

Lesson 3 BEHAVIOR OF GASES

- Boyle's law states that if the temperature is constant as the volume of a gas decreases, the pressure increases.
- Charles's law states that at constant pressure, the volume of a gas increases with increasing temperature.
- Both Boyle's law and Charles's law can be expressed as mathematical equations.

- Boyle's law
- Charles's law

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

Why does a balloon crumple and shrink when liquid nitrogen is poured on it?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

Carbonated Beverages

When you open a container of your favorite soft drink, what do you hear and see? Have you ever wondered why it went "pop" or why it fizzed? Carbon dioxide is added to the drink to give it a lively, tingling sensation. Inside that bottle or can is a lot of chemistry. Research to obtain the information needed to answer the following questions.

CER Analyze and Interpret Data

1. **Identify** What ingredients are in a soft drink?
2. **Identify** How is carbon dioxide added to the water?
3. **Claim** What happens in the space above the soft drink in a closed bottle or can as the soft drink warms to room temperature?
4. **Claim, Evidence, Reasoning** Why are bubbles released when the bottle or can is opened?
5. **Claim, Evidence, Reasoning** When a bottle or can of soft drink is opened, why does it "pop"?

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Sopon Chienwittayakun/Moment/Getty Images

MODULE 2

MATTER—PROPERTIES AND CHANGES

ENCOUNTER THE PHENOMENON

Why is this volcano spewing bright blue flames?



GO ONLINE to play a video about the mystery of the Kawah Ijen volcano.

SEP Ask Questions

Do you have other questions about the phenomenon? If so, add them to the driving question board.

CER Claim, Evidence, Reasoning

Make Your Claim Use your CER chart to make a claim about why this volcano is spewing bright blue flames.

Collect Evidence Use the lessons in this module to collect evidence to support your claim. Record your evidence as you move through the module.

Explain Your Reasoning You will revisit your claim and explain your reasoning at the end of the module.

GO ONLINE to access your CER chart and explore resources that can help you collect evidence.



LESSON 1: Explore & Explain: Physical Properties



LESSON 2: Explore & Explain: Chemical Changes

Smithsonian

Additional Resources

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LESSON 1

PROPERTIES OF MATTER

FOCUS QUESTION

What are the differences between physical and chemical properties?

Substances

As you know, matter is anything that has mass and takes up space. Everything around us is matter, including things that we cannot see, such as air and microbes. For example, table salt is a simple type of matter that you are probably familiar with. Table salt has a unique and unchanging chemical composition. Its chemical name is sodium chloride. It is always 100% sodium chloride, and its composition does not change from one sample to another. Salt harvested from the sea or extracted from a mine, as shown in **Figure 1**, always has the same composition and properties.



Salt from the sea | Salt from a mine

(l)Elad Sharon/Flickr RF/Getty Images, (r)Iakov Filimonov/Shutterstock.com

Figure 1 Harvested from the sea or extracted from a mine, salt always has the same composition.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Laboratory: The Density of Wood

Plan and carry out an investigation to determine the patterns in mass and volume of different types of wood.

Identify Crosscutting Concepts

Create a table of the crosscutting concepts and fill in examples you find as you read.

Recall that matter with a uniform and unchanging composition is called a substance, also known as a pure substance. Table salt is a pure substance. Another example of a pure substance is pure water. Water is always composed of hydrogen and oxygen. Seawater and tap water, on the other hand, are not pure substances because samples taken from different locations will often have different compositions. That is, the samples will contain different amounts of water, minerals, and other dissolved substances. Substances are important; much of your chemistry course will be focused on the composition of substances and how they interact with one another.

States of Matter

Imagine you are sitting on a sunny bench, breathing heavily and drinking water after playing a game of soccer. You are interacting with four different forms of matter—the bench is a solid, the water is a liquid, the air you breathe is a gas, and the Sun is a plasma. Although scientists recognize other forms, matter that exists naturally on Earth can be classified as one of these physical forms, which are called **states of matter.** Each state of matter can be distinguished by its properties.

Get It?
Name four states of matter.

Solids A **solid** is a form of matter that has its own definite shape and volume. Wood, iron, paper, and sugar are all examples of solids. The particles of matter in a solid are tightly packed. When heated, a solid expands, but only slightly. Because its shape is definite, a solid might not conform to the shape of the container in which it is placed. If you place a rock into a container, the rock will not take the shape of the container, as shown in **Figure 2**. The tight packing of particles in a solid makes it incompressible; that is, it cannot be pressed into a smaller volume. It is important to understand that a solid is not defined by its rigidity or hardness. For instance, although concrete is rigid and wax is soft, they are both solids.



Figure 2 A solid has a definite shape and does not take the shape of its container. Particles in a solid are tightly packed.

McGraw-Hill Education

Liquids A **liquid** is a form of matter that flows, has constant volume, and takes the shape of its container. Common examples of liquids include water, blood, and mercury. The particles in a liquid are not rigidly held in place and are less closely packed than the particles in a solid. Liquid particles are able to move past each other. This property allows a liquid to flow and take the shape of its container, as shown in **Figure 3**, although it might not completely fill the container. A liquid's volume is constant: regardless of the size and shape of the container in which the liquid is held, the volume of the liquid remains the same. Because of the way the particles of a liquid are packed, liquids are virtually incompressible. Like solids, however, liquids tend to expand when they are heated.

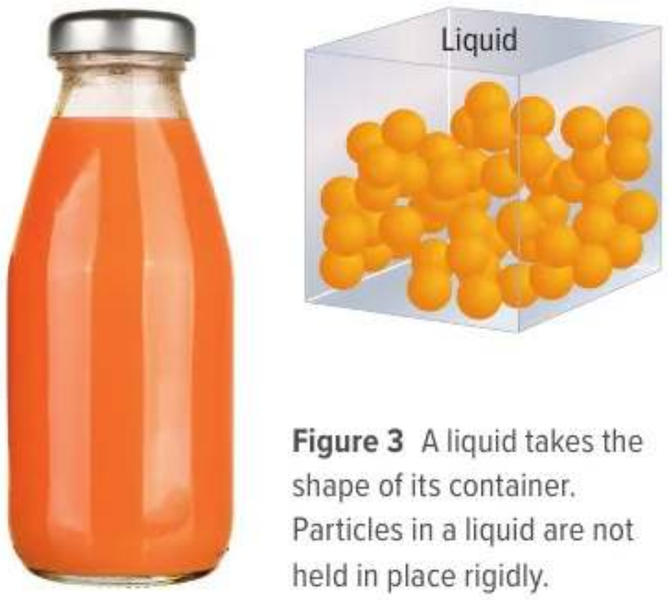


Figure 3 A liquid takes the shape of its container. Particles in a liquid are not held in place rigidly.

Get It?
Compare the properties of solids and liquids in terms of their particle arrangements.

Gases A **gas** is a form of matter that not only flows to conform to the shape of its container but also fills the entire volume of its container, as shown in **Figure 4**. If you flow gas into a container and close the container, the gas will expand to fill the container. Compared to solids and liquids, the particles of gases are far apart. Because of the significant amount of space between particles, gases are easily compressed.

You are probably familiar with the word *vapor* as it relates to the word *gas*. However, the words *gas* and *vapor*, while similar, do not mean the same thing, and should not be used interchangeably. The word *gas* refers to a substance that is naturally in the gaseous state at room temperature. The word **vapor** refers to the gaseous state of a substance that is a solid or a liquid at room temperature. For example, steam is a vapor because water exists as a liquid at room temperature.

Get It?
Differentiate between gas and vapor.



Figure 4 Gases take the shape and volume of their containers. Particles in a gas are very far ap

(t)Andrei Kuzmik/Shutterstock.com, (b)Nelstudio/Shutterstock.com



Figure 5 Plasmas are ionized gases.

Plasmas The fluorescent lights in a classroom or the bright, colorful lights in the sign in **Figure 5** involve matter as a plasma. A plasma is a form of matter that results when the particles of a gas become ionized and are broken apart into smaller charged particles. Ionization requires energy, such as that provided by an electric current in a neon or fluorescent bulb or the high temperatures of lightning or a star. Although plasmas can behave like gases in many ways, ionization causes plasmas to exhibit a strong response to electric and magnetic fields.

Physical Properties of Matter

You are probably used to identifying objects by their properties—their characteristics and behavior. For example, you can easily identify a pencil in your backpack because you recognize its shape, color, weight, or some other property. These characteristics are all physical properties of the pencil. A **physical property** is a characteristic of matter that can be observed or measured without changing the sample's composition. Physical properties also describe pure substances. Because substances have uniform and unchanging compositions, they also have consistent and unchanging physical properties. Density, color, odor, hardness, melting point, and boiling point are common physical properties that scientists record as identifying characteristics of a substance.

Get It?

Define *physical property* and provide examples.

Real-World Chemistry

Physical Properties



MINERALS Scientists use physical properties such as color and hardness to identify minerals. For instance, malachite is always green and relatively soft. Malachite was used as a pigment in paint and is now mainly used to make jewelry.

(t)Glow Images, (b)Albert Russ/Shutterstock.com

Table 1 Physical Properties of Common Substances

Substance	Color	State at 25°C	Melting Point (°C)	Boiling Point (°C)	Density (g/cm³) at 25°C
Oxygen	colorless	gas	–219	–183	0.0013
Mercury	silver	liquid	–39	357	13.5
Water	colorless	liquid	0	100	0.997
Sucrose	white	solid	186	decomposes	1.58
Sodium chloride	white	solid	801	1465	2.17

Extensive and intensive properties **Table 1** lists several common substances and their physical properties. Physical properties can be further described as being one of two types. **Extensive properties** are dependent on the amount of substance present. For example, mass is an extensive property. Length and volume are also extensive properties. Density, on the other hand, is an example of an intensive property of matter. **Intensive properties** are independent of the amount of substance present. For example, the density of a substance (at constant temperature and pressure) is the same no matter how much substance is present.

A substance can often be identified by its intensive properties. In some cases, a single intensive property is unique enough for identification. For instance, most of the spices shown in **Figure 6** can be identified by their scent.

Chemical Properties of Matter

Some properties of a substance are not obvious unless the substance has changed composition as a result of its contact with other substances or the application of thermal or electric energy. The ability or inability of a substance to combine with or change into one or more other substances is called a **chemical property.**

Iron forming rust when combined with the oxygen in air is an example of a chemical property of iron. Similarly, the inability of a substance to change into another substance is also a chemical property. For example, when iron is placed in nitrogen gas at room temperature, no chemical change occurs.

Get It?

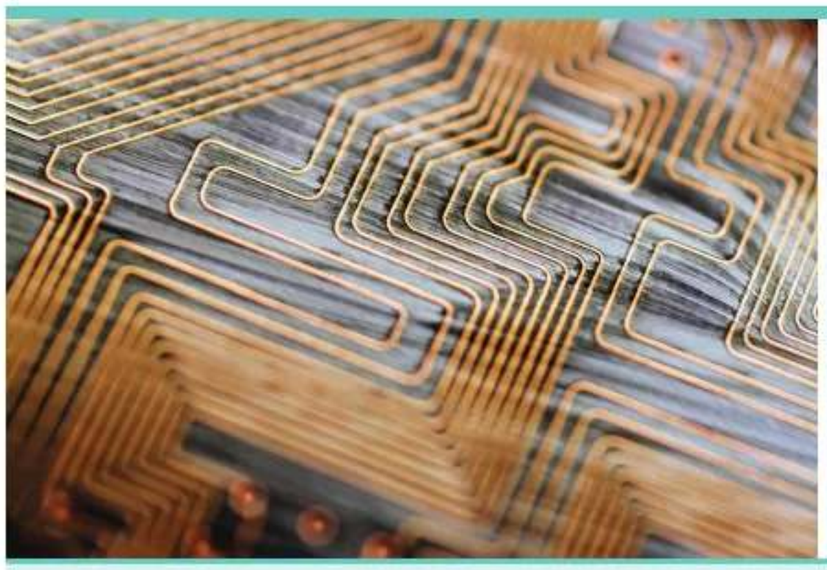
Compare physical and chemical properties.



Figure 6 Many spices can be identified by their scent, which is an intensive property.

Infer *Name an extensive property of one of the spices pictured in this figure.*

Erick Tseng (visiglow.com)/Moment/Getty Images



Copper wires



Copper roof

Figure 7 One of the physical properties of copper is that it can be shaped into different forms, such as the wires on circuit boards. The fact that copper turns from reddish to green when reacting with substances in the air is a chemical property.

Observing Properties of Matter

Every substance has its own unique set of physical and chemical properties. **Figure 7** shows physical and chemical properties of copper. Copper can be shaped into different forms, which is a physical property. When copper is in contact with air for a long time, it reacts with the substances in the air and turns green. This is a chemical property. **Table 2** lists several physical and chemical properties of copper.

Properties and states of matter

The properties of copper listed in **Table 2** might vary depending on the conditions under which they are observed. Because the particular form, or state, of a substance is a physical property, changing the state introduces or adds another physical property to its characteristics. It is important to state the specific conditions, such as temperature and pressure, under which observations are made because both physical and chemical properties depend on these conditions. Resources that provide tables of physical and chemical properties of substances, such as the *CRC Handbook of Chemistry and Physics*, generally include the physical properties of substances in all of the states in which they can exist.

Table 2 Properties of Copper

Physical Properties	Chemical Properties
• reddish brown, shiny • easily shaped into sheets (malleable) and drawn into wires (ductile) • a good conductor of heat and electricity • density = 8.96 g/cm^3 • melting point = 1085°C • boiling point = 2562°C	• forms green copper carbonate compound when in contact with moist air • reacts with nitric acid and sulfuric acid, forming new substances • one type of compound forms a deep-blue solution when in contact with ammonia

(l)Glow Images, (r)Wlad74/Shutterstock.com



Figure 8 Because the density of ice is lower than the density of water, icebergs float on the ocean.

Consider the properties of water, for example. You might think of water as a liquid (physical property) which is not particularly chemically reactive (chemical property). You might also find that water has a density of 1.00 g/cm^3 (physical property). These properties, however, apply only to water at standard temperature and pressure. At temperatures greater than 100°C, water is a gas (physical property) with a density of about 0.0006 g/cm^3 (physical property) that reacts rapidly with many different substances (chemical property). Below 0°C, water is a solid (physical property) with a density of about 0.92 g/cm^3 (physical property). The lower density of ice accounts for the fact that icebergs float on the ocean, as shown in **Figure 8**. Clearly, the properties of water are dramatically different under different conditions.

Check Your Progress

Summary

- Four states of matter are solid, liquid, gas, and plasma.
- Physical properties can be observed without altering a substance's composition.
- Chemical properties describe a substance's ability to combine with or change into one or more new substances.
- External conditions can affect both physical and chemical properties.

Demonstrate Understanding

1. **Create** a table that describes four states of matter in terms of their properties.
2. **Describe** the characteristics that identify a sample of matter as a substance.
3. **Classify** each of the following as a physical or a chemical property.
   a. Iron and oxygen form rust.
   b. Iron is more dense than aluminum.
   c. Magnesium burns brightly when ignited.
   d. Oil and water do not mix.
   e. Mercury melts at −39°C.
4. **Organize** Create a chart that compares physical and chemical properties. Give two examples for each type of property.

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LESSON 2

CHANGES IN MATTER

FOCUS QUESTION

How does a substance change when it burns?

Physical Changes

Imagine crumpling aluminum foil from a smooth, flat, mirrorlike sheet to a round, compact ball. The substance looks very different, but its composition is unchanged—it is still aluminum. A change such as this, which alters a substance without changing its composition, is a **physical change.**

Phase change

As with other physical properties, the state of matter depends on the temperature and pressure of the surroundings. As temperature and pressure change, most substances undergo a change from one state (or phase) to another. A **phase change** is a transition of matter from one state to another. **Figure 9** shows two common phase changes.



(l)Andrei Kuzmik/Shutterstock.com, (r)Stephen Bedtelyon/Shutterstock.com

Condensation Solidification

Figure 9 Condensation can occur when a gas is in contact with a cool surface, causing droplets to form. Solidification occurs when a liquid cools. Water dripping from the roof forms icicles as it cools.

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Laboratory: Properties of Water

Obtain, evaluate, and communicate information to determine the structure and function of water in Earth's surface processes.

CCC **Identify Crosscutting Concepts**

Create a table of the crosscutting concepts and fill in examples you find as you read.

EARTH SCIENCE Connection **The water cycle** Phase changes associated with water make up the water cycle, which allows life to exist on Earth. At atmospheric pressure and at temperatures of 0°C and below, water is in its solid state, which is known as ice. As heat is added to the ice, it melts and becomes liquid water. This change of state is a physical change because even though ice and water have different appearances, they have the same composition. If the temperature of the water increases to 100°C, the water begins to boil, and liquid water is converted to steam. Melting and formation of a gas are both physical changes and phase changes. Terms such as *boil, freeze, condense, vaporize,* or *melt* in chemistry generally refer to a phase change in matter.

The temperature and pressure at which a substance undergoes a phase change are important physical properties. These properties are called the melting and boiling points of the substance. Look again at **Table 1** to see this information for several common substances. Like density, the melting and boiling points are intensive physical properties. Tables of intensive properties, such as those given at the end of this textbook or in the *CRC Handbook of Chemistry and Physics,* are useful tools in identifying unknown substances from experimental data.

Chemical Changes

A process that involves one or more substances changing into new substances is called a **chemical change,** commonly referred to as a chemical reaction. The new substances formed in the reaction have different compositions and different properties from the substances present before the reaction occurred. For example, the formation of rust when iron reacts with oxygen in moist air is a chemical change. Rust, shown in **Figure 10**, is a chemical combination of iron and oxygen.

In chemical reactions, the starting substances are called reactants, and the new substances that are formed are called products. Terms such as *decompose, explode, rust, oxidize, corrode, tarnish, ferment, burn,* or *rot* generally refer to chemical reactions.

Get It?
Define *chemical change.*



Figure 10 When iron rusts, new substances are formed due to chemical change.
Identify *the reactants and products in the formation of rust.*

Anthony Grote/Gallo Images ROOTS RF/Getty Images



Figure 11 When food rots, new substances are formed due to chemical change.

Evidence of a chemical reaction

Take another look at **Figure 10.** As shown, rust is a brownish-orange powdery substance. It looks very different from the elements iron and oxygen that compose it. Rust is not attracted to a magnet, whereas iron is. The observation that the product (rust) has different properties than the reactants (iron and oxygen) is evidence that a chemical reaction has taken place. A chemical reaction always produces a change in properties.

Spoiled food, such as rotten fruit and bread, is another example of chemical reactions. The properties of spoiled food differ from the properties of fresh food. How the food tastes, how it smells, and its digestibility might all change as food rots. Examples of food that have undergone chemical reactions are shown in **Figure 11**.

Conservation of Mass

It was only in the late eighteenth century that scientists began to use quantitative tools to study and monitor chemical changes. The analytical balance, which was capable of measuring small changes in mass, was developed at that time. By carefully measuring mass before and after many chemical reactions, it was observed that, although chemical changes occurred, the total mass involved in the reaction remained constant. Assuming this was true for all reactions, chemists summarized this observation in a scientific law.

The **law of conservation of mass** states that mass is neither created nor destroyed during a chemical reaction—it is conserved. In other words, the mass of the reactants equals the mass of the products. The equation form of the law of conservation of mass is as follows:

The Law of Conservation of Mass

$$\text{mass}_{\text{reactants}} = \text{mass}_{\text{products}}$$

Mass is conserved in a chemical reaction; products have the same mass as reactants.

©j k/Alamy

EXAMPLE Problem 1

CONSERVATION OF MASS In an experiment, 10.00 g of red mercury(II) oxide powder is placed in an open flask and heated until it is converted to liquid mercury and oxygen gas. The liquid mercury has a mass of 9.26 g. What is the mass of oxygen formed in the reaction?

1 ANALYZE THE PROBLEM

You are given the mass of a reactant and the mass of one of the products in a chemical reaction. According to the law of conservation of mass, the total mass of the products must equal the total mass of the reactants.

Known

$m_{\text{mercury(II) oxide}} = 10.00\ \text{g}$

$m_{\text{mercury}} = 9.26\ \text{g}$

Unknown

$m_{\text{oxygen}} = ?\ \text{g}$

2 SOLVE FOR THE UNKNOWN

$\text{Mass}_{\text{reactants}} = \text{Mass}_{\text{products}}$ State the law of conservation of mass.

$m_{\text{mercury(II) oxide}} = m_{\text{mercury}} + m_{\text{oxygen}}$

$m_{\text{oxygen}} = m_{\text{mercury(II) oxide}} - m_{\text{mercury}}$ Solve for m_{oxygen}

$m_{\text{oxygen}} = 10.00\ \text{g} - 9.26\ \text{g}$ Substitute $m_{\text{mercury(II) oxide}} = 10.00\ \text{g}$ and $m_{\text{mercury}} = 9.26\ \text{g}$.

$m_{\text{oxygen}} = 0.74\ \text{g}$

3 EVALUATE THE ANSWER

The sum of the masses of the two products equals the mass of the reactant, verifying that mass has been conserved. The answer is correctly expressed to the hundredths place, making the number of significant digits correct.

PRACTICE Problems

ADDITIONAL PRACTICE

5. Use the data in the table to answer the following questions.

Aluminum and Liquid Bromine Reaction

Substance	Before Reaction	After Reaction
Aluminum	10.3 g	0.0 g
Liquid bromine	100.0 g	8.2 g
Compound	0.0 g	

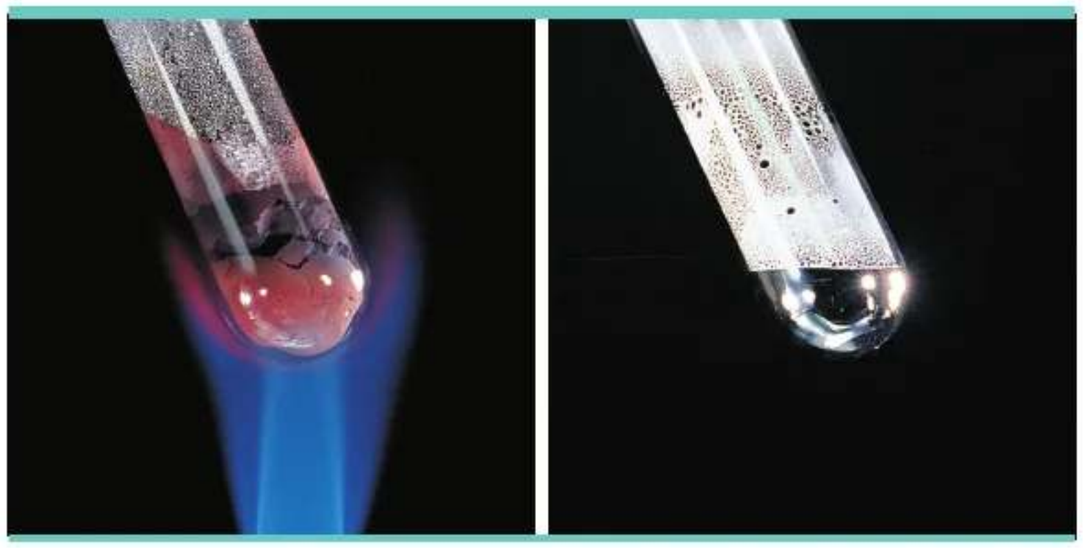
How many grams of bromine reacted? How many grams of compound were formed?

6. From a laboratory process designed to separate water into hydrogen and oxygen gas, a student collected 10.0 g of hydrogen and 79.4 g of oxygen. How much water was originally involved in the process?

7. A student carefully placed 15.6 g of sodium in a reactor supplied with an excess quantity of chlorine gas. When the reaction was complete, the student obtained 39.7 g of sodium chloride. Calculate how many grams of chlorine gas reacted. How many grams of sodium reacted?

8. A 10.0-g sample of magnesium reacts with oxygen to form 16.6 g of magnesium oxide. How many grams of oxygen reacted?

9. CHALLENGE 106.5 g of HCl(g) react with an unknown amount of NH_3(g) to produce 156.3 g of NH_4Cl(s). How many grams of NH_3(g) reacted? Is the law of conservation of mass observed in the reaction? Justify your answer.



Stephen Frisch/McGraw-Hill Education

Figure 12 When mercury(II) oxide is heated, the powdery red solid reacts to form silvery liquid mercury and colorless oxygen gas. The sum of the masses of mercury and oxygen produced during the reaction equals the mass of mercury(II) oxide.

French scientist Antoine Lavoisier (1743–1794) was one of the first to use an analytical balance to monitor chemical reactions. He studied the thermal decomposition of mercury(II) oxide, shown in **Figure 12.** The color change and production of a gas are indicators of a chemical reaction. In a closed container, the oxygen gas cannot escape and the mass before and after the reaction can be measured. The masses are the same. The law of conservation of mass is one of the most fundamental concepts in the study of chemistry and chemical reactions.

Check Your Progress

Summary

- A physical change alters the physical properties of a substance without changing its composition.
- A chemical change, also known as a chemical reaction, involves a change in a substance's composition.
- In a chemical reaction, reactants form products.
- The law of conservation of mass states that mass is neither created nor destroyed during a chemical reaction; it is conserved.

Demonstrate Understanding

10. **Classify** each change as physical or chemical.
    a. crushing an aluminum can
    b. recycling used aluminum cans to make new ones
    c. aluminum combining with oxygen to form aluminum oxide
11. **Describe** the results of a physical change and list three examples of physical change.
12. **Describe** the results of a chemical change. List four indicators of chemical change.
13. **Calculate** Solve each of the following.
    a. If 22.99 g of sodium and 35.45 g of chlorine fully react, how much sodium chloride forms?
    b. A 12.2-g sample of *X* reacts with a sample of *Y* to form 78.9 g of *XY*. What mass of *Y* reacted?
14. **Evaluate** A friend tells you, "Because composition does not change during a physical change, the appearance of a substance does not change." Is your friend correct? Explain.

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LESSON 3

ELEMENTS AND COMPOUNDS

FOCUS QUESTION

What are elements and compounds?

Elements

Earlier in this module, you considered the diversity of your surroundings in terms of matter. Although matter can take many different forms, all matter can be broken down into a relatively small number of basic building blocks called elements. An **element** is a pure substance that cannot be separated into simpler substances by physical or chemical means. On Earth, over 90 elements occur naturally. Copper, oxygen, and gold are examples of naturally-occurring elements. Elements are found in different physical states in normal conditions, as shown in **Figure 13**. There are also several elements that do not exist naturally but have been developed by scientists.

Each element has a unique chemical name and chemical symbol. The chemical symbol consists of one, two, or three letters; the first letter is always capitalized, and the remaining letter(s) are always lowercase. The names and symbols of the elements are universally accepted by scientists world-wide in order to make the communication of chemical information possible.



Copper pot—solid | Mercury switch—liquid | Helium balloon—gas

Figure 13 In normal conditions, elements exist in different states.

(l to r)Pabkov/Shutterstock.com, dcwcreations/Shutterstock.com, Tang2530/Shutterstock.com

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COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

ChemLAB: Identify the Products of a Chemical Reaction

Obtain, evaluate, and communicate information to determine the cause and effect of a chemical reaction.

Inquiry into Chemistry: Is it gold or fool's gold?

Construct an explanation for the importance of examining the structure of an unknown material.

The naturally-occurring elements are not equally abundant. For example, hydrogen is estimated to make up approximately 75% of the mass of the universe. Oxygen and silicon together comprise almost 75% of the mass of Earth's crust, while oxygen, carbon, and hydrogen account for more than 90% of the human body. Francium, on the other hand, is one of the least-abundant naturally-occurring elements. There is probably less than 20 g of francium dispersed throughout Earth's crust.

A first look at the periodic table As many new elements were being discovered in the early nineteenth century, chemists began to observe and study patterns of similarities in the chemical and physical properties of particular sets of elements. In 1869, Russian chemist Dmitri Mendeleev (1834–1907) devised a chart, shown in **Figure 14**, which organized all of the elements that were known at the time. His classification was based on the similarities and masses of the elements. Mendeleev's table was the first version of what has been further developed into the periodic table of the elements. The **periodic table** organizes the elements into a grid of horizontal rows called periods and vertical columns called groups or families. Elements in the same group have similar chemical and physical properties. The table is called periodic because the pattern of similar properties repeats from period to period.

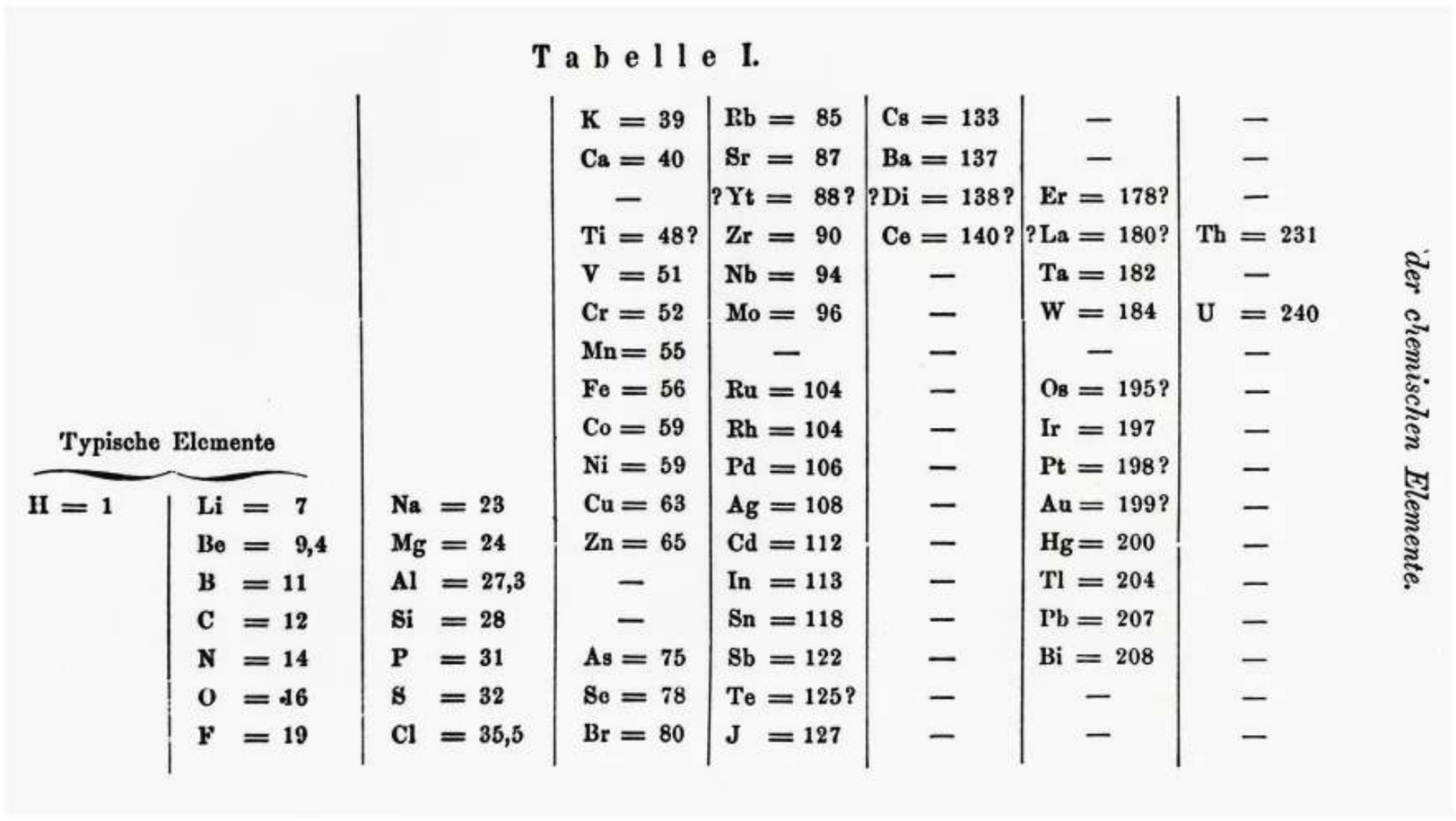


Tabelle I.

Typische Elemente

			K = 39	Rb = 85	Cs = 133	—	—
			Ca = 40	Sr = 87	Ba = 137	—	—
			—	?Yt = 88?	?Di = 138?	Er = 178?	—
			Ti = 48?	Zr = 90	Ce = 140?	?La = 180?	Th = 231
			V = 51	Nb = 94	—	Ta = 182	—
			Cr = 52	Mo = 96	—	W = 184	U = 240
			Mn = 55	—	—	—	—
			Fe = 56	Ru = 104	—	Os = 195?	—
			Co = 59	Rh = 104	—	Ir = 197	—
			Ni = 59	Pd = 106	—	Pt = 198?	—
H = 1	Li = 7	Na = 23	Cu = 63	Ag = 108	—	Au = 199?	—
	Be = 9,4	Mg = 24	Zn = 65	Cd = 112	—	Hg = 200	—
	B = 11	Al = 27,3	—	In = 113	—	Tl = 204	—
	C = 12	Si = 28	—	Sn = 118	—	Pb = 207	—
	N = 14	P = 31	As = 75	Sb = 122	—	Bi = 208	—
	O = 16	S = 32	Se = 78	Te = 125?	—	—	—
	F = 19	Cl = 35,5	Br = 80	J = 127	—	—	—

der chemischen Elemente.

Figure 14 Mendeleev was one of the first scientists to organize elements in a periodic manner, as shown in this chart, and to observe periodic patterns in the properties of the elements.

Science & Society Picture Library/Getty Images

SCIENCE USAGE V. COMMON USAGE

element

Science usage: a pure substance that cannot be separated into simpler substances by ordinary chemical means

Lead is one of the heaviest elements.

Common usage: the state or sphere that is natural or suited to any person or thing

In snow, huskies are in their element.

Compounds

Many pure substances can be classified as compounds. A **compound** is made up of two or more different elements that are combined chemically in a fixed ratio. Most matter in the universe exists in the form of compounds. Today, there are more than 50 million known compounds, and new compounds continue to be developed and discovered at the rate of about 100,000 per year. There appears to be no limit to the number of compounds that can be made or that will be discovered. Considering this virtually limitless potential, several organizations have assumed the task of collecting data and indexing the known chemical compounds. The information is stored in databases.

Get It?
Define *element* and *compound*.

Separating compounds into components As you have read earlier in this module, elements can never be separated into simpler substances. However, compounds can be broken down into simpler substances by chemical means. In general, compounds that occur naturally are more stable than the individual component elements. Separating a compound into its elements often requires external energy, such as heat or electricity. **Figure 15** shows the setup used to produce the chemical change of water into its component elements—hydrogen and oxygen—through a process called electrolysis. During electrolysis, one end of a long platinum electrode is exposed to the water in a tube and the other end is attached to a power source. An electric current splits water into hydrogen gas in the compartment on the right and oxygen gas in the compartment on the left. Because water is composed of two parts hydrogen and one part oxygen, there is twice as much hydrogen gas as there is oxygen gas.

Get It?
Explain the process of electrolysis.

The chemical symbols of the periodic table make it easy to write the formulas for chemical compounds. For example, table salt, which is called sodium chloride, is composed of one part sodium (Na) and one part chlorine (Cl), and its chemical formula is NaCl. Water is composed of two parts hydrogen (H) and one part oxygen (O), and its chemical formula is H_2O. The subscript 2 indicates that two hydrogen atoms combine with one oxygen atom to form water.

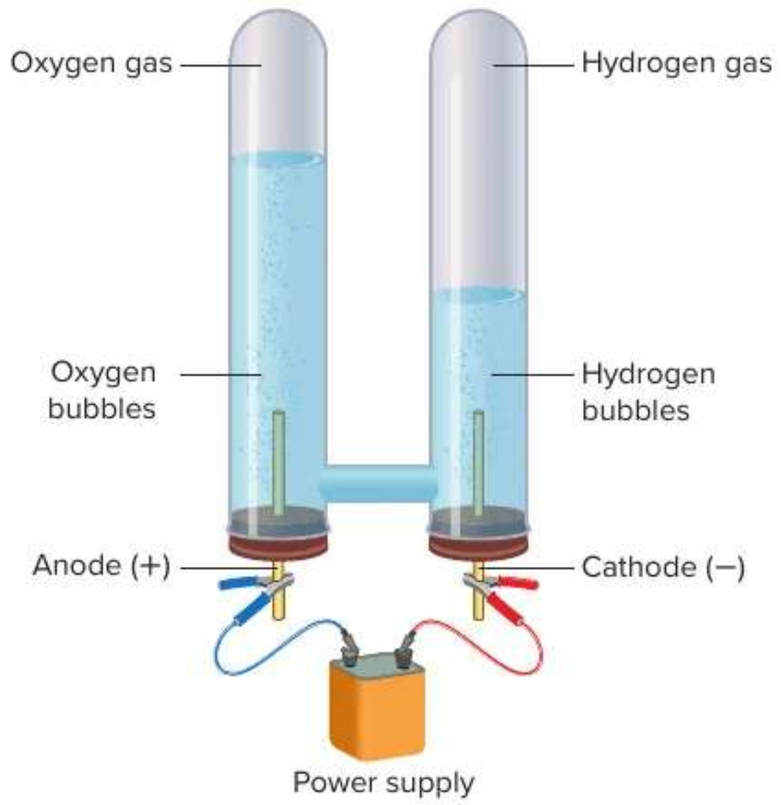


Figure 15 An electric current breaks down water into its components, oxygen and hydrogen.

Determine *What is the ratio between the amount of hydrogen and the amount of oxygen released during electrolysis?*



Potassium



Iodine



Potassium Iodide

Figure 16 When potassium and iodine react, they form potassium iodide, a compound with different properties.

(t)Andrew Lambert Photography/Science Source, (b)Charles D. Winters/Science Source, (r)Martyn F. Chillmaid/Science Source

Properties of compounds The properties of a compound are different from those of its component elements. The example of water in **Figure 15** illustrates this fact. Water is a stable compound that is liquid at room temperature. When water is broken down, its components, hydrogen and oxygen, are dramatically different than the liquid they form when combined. Oxygen and hydrogen are colorless, odorless gases that undergo vigorous chemical reactions with many elements. This difference in properties is a result of a chemical reaction between the elements.

Figure 16 shows the component elements—potassium and iodine—of the compound called potassium iodide. Note how different the properties of potassium iodide are from its component elements. Potassium is a light silvery metal that reacts with water. Iodine is a black solid that changes into a purple gas at room temperature. Potassium iodide is a white salt.

Get It?

Summarize how the properties of a compound and the properties of its component elements compare.

Law of Definite Proportions

An important characteristic of compounds is that the elements comprising them always combine in definite proportions by mass. This observation is so fundamental that it is summarized as the law of definite proportions. The **law of definite proportions** states that a compound is always composed of the same elements in the same proportion by mass, no matter how large or small the sample. The mass of the compound is equal to the sum of the masses of the elements that make up the compound.

The relative amounts of the elements in a compound can be expressed as percent by mass. The **percent by mass** is the ratio of the mass of each element to the total mass of the compound expressed as a percentage.

Percent by Mass

$$\text{percent by mass (\%)} = \frac{\text{mass of element}}{\text{mass of compound}} \times 100$$

Percent by mass is obtained by dividing the mass of the element by the mass of the compound and then by multiplying this ratio by 100 to express it as a percentage.

Get It?

State the law of definite proportions.

For example, consider the compound granulated sugar (sucrose). This compound is composed of three elements—carbon, hydrogen, and oxygen. The analysis of 20.00 g of sucrose from a bag of granulated sugar is given in **Table 3**. Note that the sum of the individual masses of the elements found in the sugar equals 20.00 g, which is the amount of the granulated sugar sample that was analyzed. This demonstrates the law of conservation of mass as applied to compounds: the mass of a compound is equal to the sum of the masses of the elements that make up the compound.

Table 3 Sucrose Analysis

	20.00 g of Granulated Sugar		500.0 g of Sugarcane	
Element	Analysis by Mass (g)	Percent by Mass (%)	Analysis by Mass (g)	Percent by Mass (%)
Carbon	8.44	$\frac{8.44\text{ g C}}{20.00\text{ g sucrose}} \times 100 = 42.20\%$	211.0	$\frac{211.0\text{ g C}}{500.0\text{ g sucrose}} \times 100 = 42.20\%$
Hydrogen	1.30	$\frac{1.30\text{ g H}}{20.00\text{ g sucrose}} \times 100 = 6.50\%$	32.5	$\frac{32.50\text{ g H}}{500.0\text{ g sucrose}} \times 100 = 6.500\%$
Oxygen	10.26	$\frac{10.26\text{ g O}}{20.00\text{ g sucrose}} \times 100 = 51.30\%$	256.5	$\frac{256.5\text{ g O}}{500.0\text{ g sucrose}} \times 100 = 51.30\%$
Total	20.00	100%	500.0	100%

Suppose you analyzed 500.0 g of sucrose from a sample of sugarcane. The analysis is shown in **Table 3**. The percent-by-mass values for the sugarcane are equal to the values obtained for the granulated sugar. According to the law of definite proportions, samples of a compound from any source must have the same mass proportions. Conversely, compounds with different mass proportions must be different compounds. Thus, you can conclude that samples of sucrose will always be composed of 42.20% carbon, 6.50% hydrogen, and 51.30% oxygen, no matter their sources.

PRACTICE Problems — ADDITIONAL PRACTICE

15. A 78.0-g sample of an unknown compound contains 12.4 g of hydrogen. What is the percent by mass of hydrogen in the compound?

16. 1.0 g of hydrogen reacts completely with 19.0 g of fluorine. What is the percent by mass of hydrogen in the compound that is formed?

17. If 3.5 g of element X reacts with 10.5 g of element Y to form the compound XY, what is the percent by mass of element X in the compound? The percent by mass of element Y?

18. Two unknown compounds are tested. Compound I contains 15.0 g of hydrogen and 120.0 g of oxygen. Compound II contains 2.0 g of hydrogen and 32.0 g of oxygen. Are the compounds the same? Explain your answer.

19. CHALLENGE All you know about two unknown compounds is that they have the same percent by mass of carbon. With only this information, can you be sure the two compounds are the same? Explain.

Law of Multiple Proportions

Compounds composed of different elements are obviously different compounds. However, different compounds can also be composed of the same elements. This happens when those different compounds have different mass compositions. The **law of multiple proportions** states that when different compounds are formed by a combination of the same elements, different masses of one element combine with the same fixed mass of the other element in a ratio of small whole numbers. Ratios compare the relative amounts of any items or substances. The comparison can be expressed using numbers separated by a colon or as a fraction. With regard to the law of multiple proportions, ratios express the relationship of elements in a compound.

Water and hydrogen peroxide The two distinct compounds water (H_2O) and hydrogen peroxide (H_2O_2) illustrate the law of multiple proportions. Each compound contains the same elements (hydrogen and oxygen). Water is composed of two parts hydrogen and one part oxygen. Hydrogen peroxide is composed of two parts hydrogen and two parts oxygen. Hydrogen peroxide differs from water in that it has twice as much oxygen. When you compare the mass of oxygen in hydrogen peroxide to the mass of oxygen in water, you get the ratio 2:1.

Get It?

State the law of multiple proportions in your own words.

Table 4 Analysis Data of Two Copper Compounds

Compound	% Cu	% Cl	Mass Cu (g) in 100.0 g of Compound	Mass Cl (g) in 100.0 g of Compound	Mass Ratio $\left(\frac{\text{mass Cu}}{\text{mass Cl}}\right)$
I	64.20	35.80	64.20	35.80	$\frac{1.793\text{ g Cu}}{1\text{ g Cl}}$
II	47.27	52.73	47.27	52.73	$\frac{0.8964\text{ g Cu}}{1\text{ g Cl}}$

Compounds made of copper and chlorine In another example, copper (Cu) reacts with chlorine (Cl) under different sets of conditions to form two different compounds. **Table 4** provides an analysis of their compositions. How can you tell that the compounds are different?

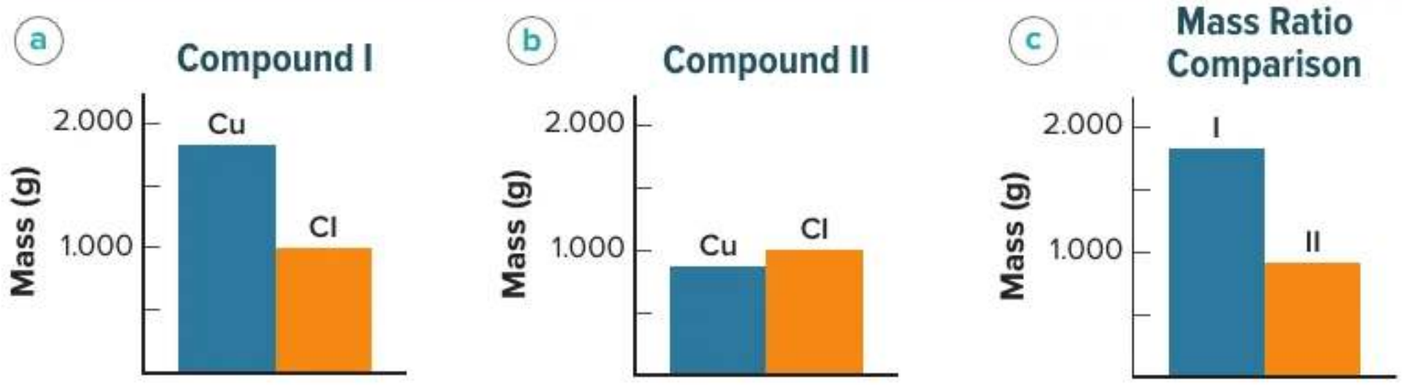
The two copper compounds must be different because they have different percents by mass. Compound I is composed of 64.20% copper, but Compound II contains only 47.27% copper. Compound I contains 35.80% chlorine, but Compound II contains 52.73% chlorine.

Using **Figure 17** and **Table 4**, compare the ratio of the mass of copper to the mass of chlorine for each compound. Notice that the mass ratio of copper to chlorine in Compound I (1.793) is exactly 2 times the mass ratio of copper to chlorine in Compound II (0.8964).

$$\frac{\text{mass ratio of Compound I}}{\text{mass ratio of Compound II}} = \frac{1.793\text{ g Cu/g Cl}}{0.8964\text{ g Cu/g Cl}} = 2.000$$

Get It?

Explain why the ratio of relative masses of copper in both compounds is 2:1.



Bar graph **a** compares the relative masses of copper and chlorine in Compound I.

Bar graph **b** compares the relative masses of copper and chlorine in Compound II.

Bar graph **c** shows a comparison between the relative masses of copper in both compounds. The ratio is 2:1.

Figure 17 Copper and chlorine can form different compounds.

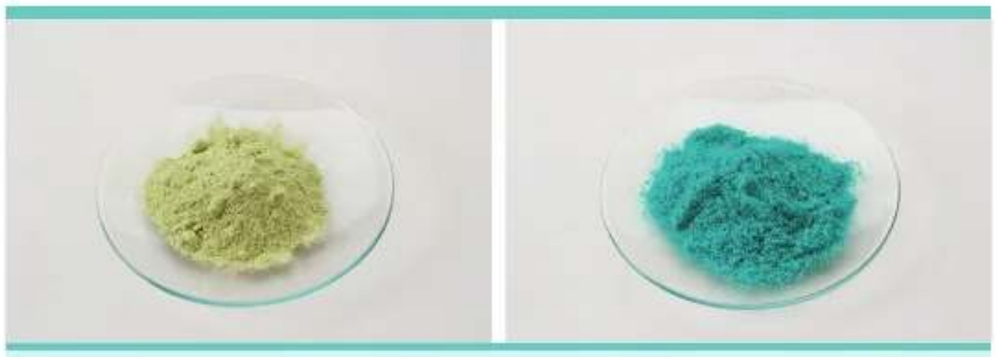


Figure 18 Different compounds are formed when different relative masses of each element are combined. Although they are both made of copper and chlorine, Compound I has a greenish color, whereas Compound II has a bluish color.

Figure 18 shows the two compounds formed by the combination of copper and chlorine and presented in **Table 4** and **Figure 17**. These compounds are called copper(I) chloride and copper(II) chloride. As the law of multiple proportions states, the different masses of copper that combine with a fixed mass of chlorine in the two different copper compounds can be expressed as a small whole-number ratio. In this case, the ratio is 2:1. Considering that there is a finite number of elements that exist today and an exponentially greater number of compounds that are composed of these elements under various conditions, it becomes clear how important the law of multiple proportions is in chemistry.

Check Your Progress

Summary

- Elements cannot be broken down into simpler substances.
- Elements are organized in the periodic table of the elements.
- Compounds are chemical combinations of two or more elements, and their properties differ from the properties of their component elements.
- The law of definite proportions states that a compound is always composed of the same elements in the same proportions.
- The law of multiple proportions states that if elements form more than one compound, those compounds will have compositions that are whole-number multiples of each other.

Demonstrate Understanding

20. **Compare and contrast** elements and compounds.
21. **Describe** the basic organizational feature of the periodic table of the elements.
22. **Explain** how the law of definite proportions applies to compounds.
23. **State** the type of compounds that are compared in the law of multiple proportions.
24. **Complete** the table. Then analyze the data to determine if Compounds I and II are the same compound. If the compounds are different, use the law of multiple proportions to show the relationship between them.

ANALYSIS DATA OF TWO IRON COMPOUNDS

Compound	Total Mass (g)	Mass Fe (g)	Mass O (g)	Mass Percent Fe	Mass Percent O
I	75.00	52.46	22.54		
II	56.00	43.53	12.47		

25. **Calculate** the mass percent of each element in water.
26. **Graph** Create a graph that illustrates the law of multiple proportions.

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Matt Meadows/McGraw-Hill Education

LESSON 4

MIXTURES OF MATTER

FOCUS QUESTION

Would the substance still burn blue if it were mixed with another substance?

Mixtures

You have already read that a pure substance has a uniform and unchanging composition. What happens when two or more substances are combined? A **mixture** is a combination of two or more pure substances in which each pure substance retains its individual chemical properties. The composition of mixtures is variable, and the number of mixtures that can be created by combining substances is infinite. Although much of the focus of chemistry is the behavior of substances, it is important to remember that most everyday matter occurs as mixtures. Substances tend to mix naturally; it is difficult to keep any substance pure.

Examine the mixtures in **Figure 19**. When oil, seasonings, and vinegar are mixed, you can still distinguish all of the components. If the mixture remains undisturbed long enough, the oil will form a layer on top of the vinegar. On the other hand, you cannot distinguish between the components of the mercury-silver mixture. You could, however, separate them by heating the mixture. The mercury will evaporate before the silver does, and you will obtain two separate substances: mercury vapor and solid silver. The mercury and silver physically mixed to form the mixture but did not chemically react with each other. They could be separated by the physical method of boiling. The vapor can be collected and condensed into liquid mercury.



Figure 19 There are different types of mixtures. The components of some mixtures, like this salad dressing, are visible. It is not possible to see the different components of some mixtures, such as this mercury-silver filling.

(t)McGraw-Hill Education, (b) Szasz-Fabian Jozsef/Shutterstock.com

3D THINKING DCI Disciplinary Core Ideas CCC Crosscutting Concepts SEP Science & Engineering Practices

COLLECT EVIDENCE

Use your Science Journal to record the evidence you collect as you complete the readings and activities in this lesson.

INVESTIGATE

GO ONLINE to find these activities and more resources.

Small-Scale Lab: Separation of Aspirin

Carry out an investigation to determine the scale, proportion, and quantity of the components of a common medication.

Identify Crosscutting Concepts

Create a table of the crosscutting concepts and fill in examples you find as you read.

Types of mixtures

The combinations of pure substances shown in **Figure 19** are both mixtures, despite their obvious visual differences. Mixtures can be defined in different ways and are classified as either heterogeneous or homogeneous.

A **heterogeneous mixture** is a mixture that does not blend smoothly throughout and in which the individual substances remain distinct. The salad dressing mixture is an example of a heterogeneous mixture. Its composition is not uniform—the substances have not blended smoothly and remain distinct. We can therefore say that the existence of two or more distinct areas indicates a heterogeneous mixture.

A **homogeneous mixture** is a mixture that has constant composition throughout; it always has a single phase. If you cut two pieces out of a silver-mercury amalgam, their compositions will be the same. They will contain the same relative amounts of silver and mercury, no matter the size of each piece.

Homogeneous mixtures are also referred to as **solutions.** You are probably most familiar with solutions in a liquid form, such as tea and lemonade, but solutions can be solids, liquids, or gases. They can be a mixture of a solid and a gas, a solid and a liquid, a gas and a liquid, and so on. **Table 5** lists the various types of solution systems and examples.

The solid-solid solution known as steel is called an alloy. An alloy is a homogeneous mixture of metals, or a mixture of a metal and a nonmetal in which the metal substance is the major component. For instance, steel is a mixture of iron and carbon. Adding carbon atoms increases the hardness of the metal.

Manufacturers combine the properties of various metals in an alloy to achieve greater strength and durability in their products. Jewelry is often made of alloys such as bronze, sterling silver, pewter, and 14-karat gold.

Get It?

List six examples of solutions that you encounter as you go through your daily activities. Indicate the system to which each belongs.

Table 5 Types of Solution Systems

System	Example
Gas-gas	Air in a scuba tank is primarily a mixture of nitrogen, oxygen, and argon gases.
Gas-liquid	Oxygen and carbon dioxide are dissolved in seawater.
Liquid-gas	Moist air exhaled by the scuba diver contains water droplets.
Liquid-liquid	When it is raining, fresh water mixes with seawater.
Solid-liquid	Solid salts are dissolved in seawater.
Solid-solid	The air tank is made of an alloy—a mixture of two metals.

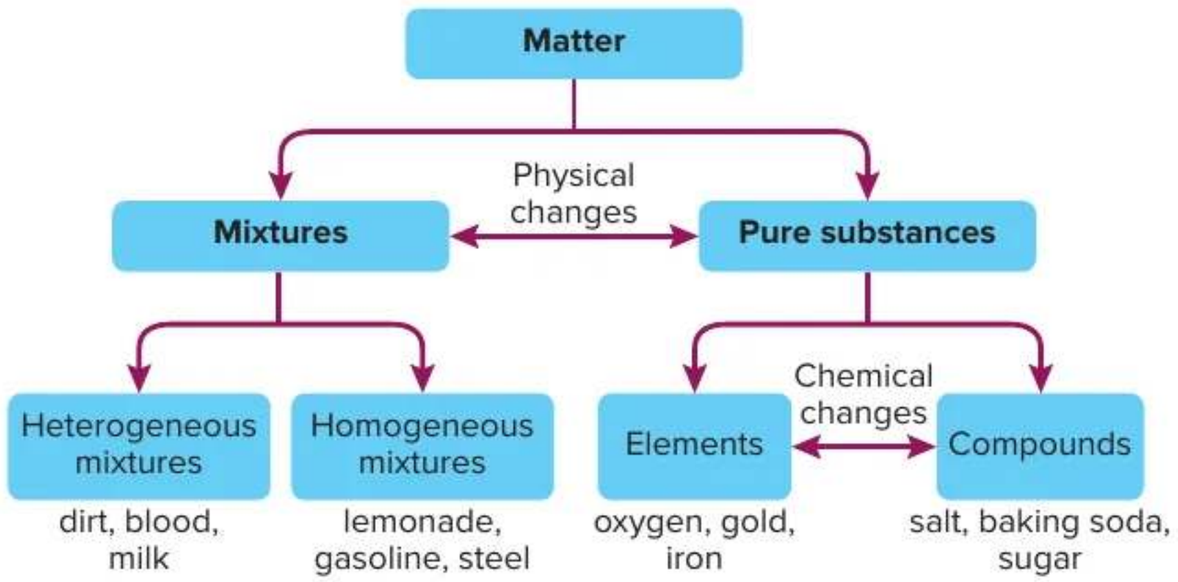


Figure 20 Matter can be classified into different categories that have defined properties.

Examine *How are mixtures and substances related? Elements and compounds?*

Recall what you have learned about the organization of matter. Matter is classified as pure substances and mixtures. An element is a pure substance that cannot be separated into simpler substances, whereas a compound is a chemical combination of two or more elements and can be separated into its components. A mixture can be homogeneous or heterogeneous. **Figure 20** summarizes the classification of matter and how these components are related to one another.

Get It?

Summarize the different types of matter and how they are related to each other.

Separating Mixtures

Most matter exists naturally in the form of mixtures. To gain a thorough understanding of matter, it is important to be able to separate mixtures into their component substances. Because the substances in a mixture are physically combined, the processes used to separate a mixture are physical processes that are based on differences in the physical properties of the substances. For instance, a mixture of iron and sand can be separated into its components with a magnet because a magnet will attract iron but not sand. Numerous techniques have been developed that take advantage of different physical properties in order to separate various mixtures.

STEM CAREER Connection

High School Chemistry Teacher

Do you enjoy helping others? Do you like chemistry? Are you able to effectively explain yourself when talking to others? You may wish to consider a career teaching high school chemistry. High school teachers prepare students for life after high school. They prepare students to be valuable members of society by teaching lessons and skills needed to be successful in post-secondary education or to enter the workforce.

WORD ORIGIN

mixture

from the Latin word *misceo,* meaning *to mix* or *blend*



Figure 21 As the mixture passes through the filter, the solids remain in the filter, while the filtrate (the remaining liquid) is collected in the beaker.

Filtration Heterogeneous mixtures composed of solids and liquids are easily separated by filtration. **Filtration** is a technique that uses a porous barrier to separate a solid from a liquid. As **Figure 21** shows, the mixture is poured through a piece of filter paper that has been folded into a cone shape. The liquid passes through, leaving the solids trapped in the filter paper.

Get It?

Predict the result of trying to use filtration to separate the components of salad dressing. imagine using salad dressing that is a mixture of oil, vinegar, and spices and a paper filter for the procedure.

Distillation Most homogeneous mixtures can be separated by distillation. **Distillation** is a physical separation technique that is based on differences in the boiling points of the substances involved. In distillation, a mixture is heated until the substance with the lowest boiling point boils to a vapor that can then be condensed into a liquid and collected. When precisely controlled, distillation can separate substances that have boiling points differing by only a few degrees.

Sublimation Mixtures can also be separated by **sublimation,** which is the process during which a solid changes to vapor without melting, i.e., without going through the liquid phase. Sublimation can be used to separate two solids present in a mixture when one of the solids sublimates but not the other.

Chromatography **Chromatography** is a technique that separates the components of a mixture dissolved in either a gas or a liquid (called the mobile phase) based on the ability of each component to travel or to be drawn across the surface of a fixed substrate (called the stationary phase).

For example, chromatography paper is a stationary phase with a solid substrate. During paper chromatography, the separation occurs because the various components of the mixture in the liquid mobile phase spread through the paper at different rates. Components with the strongest attraction for the paper travel slower.

Get It?

Compare and contrast the mobile phase with the stationary phase in chromatography.

Martyn F. Chillmaid/Science Source

Crystallization Making rock candy from a sugar solution is an example of separating a mixture by crystallization. **Crystallization** is a separation technique that results in the formation of pure solid particles of a substance from a solution containing the dissolved substance. When the solution contains as much dissolved substance as it can possibly hold, the addition of even a tiny amount more often causes the dissolved substance to come out of solution and collect as crystals on any available surface.

In the rock candy example, as water evaporates from the sugar-water solution, the solution becomes more concentrated. This is equivalent to adding more of the dissolved substance to the solution. As more water evaporates, the sugar forms a solid crystal on the string, as shown in **Figure 22**. Crystallization produces highly pure solids.

Get It?

Classify which techniques for separating mixtures depend on phase changes and identify the change used by each.



Figure 22 As the water evaporates from the water-sugar solution, the sugar crystals form on the string.

Ted Kinsman/Science Source

Check Your Progress

Summary

- A mixture is a physical blend of two or more pure substances in any proportion.
- Solutions are homogeneous mixtures.
- Mixtures can be separated by physical means. Common separation techniques include filtration, distillation, sublimation, chromatography, and crystallization.

Demonstrate Understanding

27. **Classify** each of the following as either a heterogeneous or a homogeneous mixture.
    a. tap water b. air c. raisin muffin
28. **Compare** mixtures and substances.
29. **Describe** the separation technique that could be used to separate each of the following mixtures.
    a. two colorless liquids
    b. a nondissolving solid mixed with a liquid
    c. red and blue marbles of the same size and mass
30. **Design** a concept map that summarizes the relationships among the categories of matter, elements, mixtures, compounds, pure substances, and homogeneous and heterogeneous mixtures.

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SCIENCE & SOCIETY

In Rare Form

What do smartphones, hybrid vehicles, and lasers have in common? Components used in all three are manufactured with rare earth elements (REE). Without REEs, many products could not function. Currently, the United States obtains REEs by trading with China, where most REE mines are located.



Rare earth elements are difficult to mine without negative effects on the environment.

High demand, low supply?

Rare earth elements (REEs) include scandium (SC-21), yttrium (Y-39), and 15 others. *Rare* is a misnomer; many (but not all) REEs are actually quite common. They are found in Earth's crust all over the world. However, it is not easy to find a high concentration of these elements in one place. Without a high concentration, mining is not cost-effective. In addition, it can be difficult to separate the elements from ore and from one another when they occur in the same area, and mining REEs in a way that does not harm the environment is complicated and expensive.

REEs are in demand because of their desirable qualities, which vary from element to element. These include magnetic, electrical, mechanical, chemical, and elastic properties. REEs are extremely important in the production of high-tech products, such as magnets, radar, and sonar, as well as everyday products, such as batteries and televisions. Many renewable energy technologies, from wind turbines to solar cells, also rely on REEs.

China mines the vast majority of the total world production of REEs. As China becomes more developed, it may need more REEs for its own use, and it might start exporting smaller amounts of REEs to other countries. Soon, worldwide demand for REEs may exceed the supply that China provides. However, other countries—including Australia, Russia, India, and Brazil—are mining REEs, although China's output is much greater than all the rest put together.

The United States also has REE deposits and is working to open its own mines. Until the U.S. can open its own mines, what can the country do to ensure its supply of REEs? Recycling REEs from e-waste is difficult and expensive, so it is not as common as it could be. New processes, however, have made the process easier and cheaper, and REE recycling will most likely be on the rise in the future.



ASK QUESTIONS TO CLARIFY

After reading this feature, brainstorm three questions about the topic of REEs. Use print or online resources to research answers to your questions. Share what you have learned with your class.

Barry Sweet/ZUMAPRESS/Newscom

MODULE 2
STUDY GUIDE

GO ONLINE to study with your Science Notebook.

Lesson 1 PROPERTIES OF MATTER

- Four states of matter are solid, liquid, gas, and plasma.
- Physical properties can be observed without altering a substance's composition.
- Chemical properties describe a substance's ability to combine with or change into one or more new substances.
- External conditions can affect both physical and chemical properties.

- states of matter
- solid
- liquid
- gas
- vapor
- physical property
- extensive property
- intensive property
- chemical property

Lesson 2 CHANGES IN MATTER

- A physical change alters the physical properties of a substance without changing its composition.
- A chemical change, also known as a chemical reaction, involves a change in a substance's composition.
- In a chemical reaction, reactants form products.
- The law of conservation of mass states that mass is neither created nor destroyed during a chemical reaction; it is conserved.

$$\text{mass}_{\text{reactants}} = \text{mass}_{\text{products}}$$

- physical change
- phase change
- chemical change
- law of conservation of mass

Lesson 3 ELEMENTS AND COMPOUNDS

- Elements cannot be broken down into simpler substances.
- Elements are organized in the periodic table of the elements.
- Compounds are chemical combinations of two or more elements, and their properties differ from the properties of their component elements.
- The law of definite proportions states that a compound is always composed of the same elements in the same proportions.

$$\text{percent by mass} = \frac{\text{mass of the element}}{\text{mass of the compound}} \times 100$$

- The law of multiple proportions states that if elements form more than one compound, those compounds will have compositions that are whole-number multiples of each other.

- element
- periodic table
- compound
- law of definite proportions
- percent by mass
- law of multiple proportions

Lesson 4 MIXTURES OF MATTER

- A mixture is a physical blend of two or more pure substances in any proportion.
- Solutions are homogeneous mixtures.
- Mixtures can be separated by physical means. Common separation techniques include filtration, distillation, sublimation, chromatography, and crystallization.

- mixture
- heterogeneous mixture
- homogeneous mixture
- solution
- filtration
- distillation
- sublimation
- chromatography
- crystallization

THREE-DIMENSIONAL THINKING

Module Wrap-Up



REVISIT THE PHENOMENON

Why is this volcano spewing bright blue flames?

CER Claim, Evidence, Reasoning

Explain Your Reasoning Revisit the claim you made when you encountered the phenomenon. Summarize the evidence you gathered from your investigations and research and finalize your Summary Table. Does your evidence support your claim? If not, revise your claim. Explain why your evidence supports your claim.

STEM UNIT PROJECT

Now that you've completed the module, revisit your STEM unit project. You will summarize your evidence and apply it to the project.

GO FURTHER

SEP Data Analysis Lab

How is compressed gas released?

Tanks of compressed gases are common in chemistry laboratories. For example, nitrogen is often flowed over a reaction in progress to displace other gases that might interfere with the experiment. Given what you know about gases, explain how the release of compressed nitrogen is controlled.

CER Analyze and Interpret Data

The particles of gases are far apart, and gases tend to fill their containers—even if the container is a laboratory room. Tanks of compressed gas come from the supplier capped to prevent the gas from escaping. In the lab, a chemist or technician attaches a regulator to the tank and secures the tank to a stable fixture.

1. **Claim** Explain why the flow of a compressed gas must be controlled for practical and safe use.
2. **Evidence, Reasoning** Predict what would happen if the valve on a full tank of compressed gas were suddenly opened all the way or if the tank were accidentally punctured. Explain your prediction.

Sopon Chienwittayakun/Moment/Getty Images

Credits

1. Module 01 A Physics Toolkit: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 1
2. Module 02 Representing Motion: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 27
3. Module 03 Accelerated Motion: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 52
4. Module 04 Forces in One Dimension: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 80
5. Module 05 Displacement and Force in Two Dimensions: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 107
6. Module 10 Energy and Its Conservation: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 132
7. Module 13 Vibrations and Waves: *Chapter from UAE Inspire Science Physics, Student Edition, 2024-25 by Zitzewitz, 2024* 175
8. Module 14 Solids, Liquids, and Gases: *Chapter from UAE Inspire Physical Science, Student Edition, 2024-25 by McLaughlin, Thompson, Zike, 2024* 198
9. Module 02 Matter-Properties and Changes: *Chapter from UAE Inspire Science Chemistry, Student Edition, 2024-25 by Buthelezi, 2024* 222

Student Notes

Student Notes

Student Notes

Student Notes

Student Notes

Student Notes

Student Notes