

2025-2026

MATH EOT3 – 12 General

Ms. Fatima Al Saeedi



اللهم إني أسألك فهم النبيين وحفظ المرسلين والملائكة المقربين.

رب أدخلني مدخل صدق، وأخرجني مخرج صدق، واجعل لي من لدنك سلطانا نصيراً.

اللَّهُمَّ لَا سَهْلَ إِلَّا مَا جَعَلْتَهُ سَهْلًا وَأَنْتَ تَجْعَلُ الْحَزْنَ سَهْلًا إِذَا شِئْتَ.

اللهم ارزقهم قوة الحفظ، وسرعة الفهم وصفاء الذهن، اللهم ألهمهم الصواب في الجواب.

اللهم انى توكلت عليك، وسلمت أمري إليك، لا ملجأ ولا منجى منك إلا إليك آمنت بكتابك الذي أنزلت ونبيك الذي أرسلت.

Reveal MATH

| | |
|---------------|-----------------------------|
| Academic Year | 2025/2026 |
| العام الدراسي | |
| Term | 3 |
| الفصل | |
| Subject | Mathematics/(Bridge,Reveal) |
| المادة | الرياضيات/(بريدج و ريفيل) |
| Grade | 12 |
| الصف | |
| Stream | General |
| المسار | العام |

| | |
|--|---------------------|
| Number of MCQ عدد الأسئلة الموضوعية | 30 |
| Marks of MCQ درجة الأسئلة الموضوعية | (3,4) |
| Type of All Questions نوع كافة الأسئلة | أسئلة موضوعية / MCQ |
| Maximum Overall Grade الدرجة القصوى الممكنة | 100 |
| Exam Duration - مدة الامتحان | 120 minutes |
| Mode of Implementation - طريقة التطبيق | SwiftAssess |
| Calculator | Allowed |
| الآلة الحاسبة | مسموحة |

MCQ (3-4 marks)

| | | | |
|---|---|------------------------------------|-----------|
| 1 | تبسيط التعابير النسبية | | |
| | Simplify rational expressions. | Example 2 & Check | 312 |
| | | | |
| 2 | تبسيط التعابير النسبية | | |
| | Simplify rational expressions by multiplying and dividing | Example 3 | 313 |
| | | | |
| 3 | تبسيط الكسور الجبرية المركبة المتضمنة تعابير نسبية | | |
| | Simplify rational expressions by multiplying and dividing | Example 5 & Check | 314 |
| | | | |
| 4 | جمع التعابير النسبية وطرحها | | |
| | Add and subtract rational expressions | Example 1 | 319 & 320 |
| | | | |
| 5 | جمع التعابير النسبية وطرحها | | |
| | Add and subtract rational expressions | Example 2 | 320 |
| | | | |
| 6 | تحديد خصائص دوال المقلوب | | |
| | Determine properties of reciprocal functions | Key Concept • Reciprocal Functions | 327 |

MCQ (3-4 marks)

| | | | |
|----|---|--|-----------|
| 7 | تحديد خصائص دوال المقلوب | | |
| | Determine properties of reciprocal functions. | (23,24) | 334 |
| 8 | تمثيل تحويلات دوال المقلوب بيانياً | | |
| | Graph transformations of reciprocal functions. | Example 4 & Check | 331 & 332 |
| 9 | تمثيل الدوال النسبية ذات خطوط التقارب الأفقية والرأسية بيانياً | | |
| | Graph rational functions with vertical and horizontal asymptotes. | Key Concept • Vertical and Horizontal Asymptotes | 337 |
| 10 | تمثيل الدوال النسبية ذات خطوط التقارب الأفقية والرأسية بيانياً | | |
| | Graph rational functions with vertical and horizontal asymptotes. | Example 1 | 337 & 338 |
| 11 | تمثيل الدوال النسبية ذات خط التقارب المائل ونقطة الانفصال بيانياً | | |
| | Graph rational functions with oblique asymptotes and point discontinuity. | Key Concept • Oblique Asymptotes | 340 |
| 12 | تمثيل الدوال النسبية ذات خط التقارب المائل ونقطة الانفصال بيانياً | | |
| | Graph rational functions with oblique asymptotes and point discontinuity. | Example 5 | 342 |

MCQ (3-4 marks)

| | | | |
|----|---|------------------------------------|-----------|
| 13 | حل المعادلات النسبية | | |
| | Solve rational equations . | Example 1 | 355 |
| 14 | حل المعادلات النسبية | | |
| | Solve rational equations . | Example 2 | 356 |
| 15 | رسم الزوايا في وضع قياسي وإيجادها | | |
| | Draw and find angles in standard position. | Example 1 & Example 2 & Check | 418 |
| 16 | رسم الزوايا في وضع قياسي وإيجادها | | |
| | Draw and find angles in standard position. | Example 3 | 418 |
| 17 | التحويل بين القياس بالدرجات والقياس بالراديان. | | |
| | Convert between degrees and radian measures and find arc lengths by using central angles. | Example 4 & Example 5 | 420 |
| 18 | التحويل بين القياس بالدرجات والقياس بالراديان. | | |
| | Convert between degrees and radian measures and find arc lengths by using central angles. | Key Concept • Arc Length & (63,64) | 419 & 422 |

MCQ (3-4 marks)

| | | | |
|----|---|-------------------|-----|
| 19 | إيجاد قيم النسب المثلثية | | |
| | Find values of trigonometric ratios. | Example 1 | 425 |
| 20 | إيجاد قيم النسب المثلثية | | |
| | Find values of trigonometric ratios. | Example 2 & Check | 426 |
| 21 | إيجاد قيم النسب المثلثية باستخدام زوايا المرجع | | |
| | Find values of trigonometric functions by using reference angles. | Example 5 | 429 |
| 22 | إيجاد قيم النسب المثلثية باستخدام زوايا المرجع | | |
| | Find values of trigonometric functions by using reference angles. | Example 6 & Check | 430 |
| 23 | التعرف على دائرة الوحدة والنسب المثلثية | | |
| | Identify the unit circle and trigonometric ratios. | Example 1 | 436 |
| 24 | استخدام خصائص الدوال الدورية لإيجاد قيمة الدوال المثلثية | | |
| | Use the properties of periodic functions to evaluate trigonometric functions. | (13-18) | 441 |

MCQ (3-4 marks)

| | | | |
|----|--|--|-----------|
| 25 | استخدام خصائص الدوال الدورية لإيجاد قيمة الدوال المثلثية Use the properties of periodic functions to evaluate trigonometric functions. | Example 5 & Check | 440 |
| 26 | وصف دوال الجيب (\sin) وجيب التمام (\cos) والظل (\tan) وتمثيلها بيانياً Describe and graph the sine, cosine, and tangent functions. | Key Concept • Sine and Cosine Functions | 445 |
| 27 | وصف دوال الجيب (\sin) وجيب التمام (\cos) والظل (\tan) وتمثيلها بيانياً Describe and graph the sine, cosine, and tangent functions. | (5-10) | 451 |
| 28 | وصف دوال الجيب (\sin) وجيب التمام (\cos) والظل (\tan) وتمثيلها بيانياً Describe and graph the sine, cosine, and tangent functions. | (24-26) | 452 |
| 29 | وصف دوال الجيب (\sin) وجيب التمام (\cos) والظل (\tan) وتمثيلها بيانياً Describe and graph the sine, cosine, and tangent functions. | Example 1 & Check | 455 & 456 |
| 30 | وصف الدوال المثلثية غير دوال الجيب (\sin) وجيب التمام (\cos) والظل (\tan) وتمثيلها بيانياً Describe and graph trigonometric functions other than sine, cosine, and tangent. | Key Concept • Graphs of Reciprocal Functions | 458 |

Simplify each expression.

$$\frac{(6x^2 - 5xy)(x + 2y)}{(x + y)(5y - 6x)}$$

A) $\frac{x(x+2y)}{x+y}$

B) $-\frac{x(x+2y)}{x+y}$

C) $\frac{x(x+y)}{x+y}$

D) $\frac{-x(x+2y)}{x-y}$

$$\frac{(7y - 3x)(5x - 1)}{(5x^3 + x^2)(3x - 7y)}$$

A) $-\frac{5x-1}{x^2(5x+1)}$

B) $\frac{(7y-3x)(5x-1)}{x^2(5x+1)(3x-7y)}$

C) $\frac{1}{x^2}$

D) $\frac{5x-1}{5x^3+x^2}$

$$\frac{9x^2 - x^3}{x^2 - 3x - 54}$$

A) $-\frac{x^2}{x+6}$

B) $\frac{x^2}{x+3}$

C) $\frac{x^2}{x+6}$

D) $-\frac{x^2}{x-3}$

$$\frac{16 - c^2}{c^2 + c - 20}$$

A) $\frac{c-4}{c+5}$

B) $-\frac{c+4}{c-2}$

C) $\frac{c-4}{c+2}$

D) $-\frac{c+4}{c+5}$

Simplify each expression.

| | | | |
|--|---|--|---|
| $\frac{3x}{8y} \cdot \frac{12x^2y}{9xy^3}$ | $\frac{10d^5}{6cd} \div \frac{30c^3d^2}{4c}$ | $\frac{14xy^2z^3}{21w^4x^2z} \cdot \frac{7wxyz}{12w^2y^3z}$ | $\frac{64a^2b^5}{35b^2c^3f^4} \div \frac{12a^4b^3c}{70abcf^2}$ |
| <p>A) $\frac{y^3}{2x^2}$</p> <p>B) $\frac{x^2}{8y^3}$</p> <p>C) $\frac{x}{72y^3}$</p> <p>D) $\frac{x^2}{2y^3}$</p> | <p>A) $\frac{d}{9c^3}$</p> <p>B) $\frac{2d^2}{9c^3}$</p> <p>C) $\frac{d^2}{9c^3}$</p> <p>D) $\frac{2d}{9c^2}$</p> | <p>A) $\frac{z^2}{18w^3}$</p> <p>B) $\frac{7z}{18w^5}$</p> <p>C) $\frac{7z^2}{18w^5}$</p> <p>D) $\frac{7z}{18w^2}$</p> | <p>A) $\frac{32b}{3ac^3f^2}$</p> <p>B) $\frac{32b}{ac^3f^2}$</p> <p>C) $\frac{32ba^{-1}}{3cf}$</p> <p>D) $\frac{32b}{3ac^3f^3}$</p> |

Simplify each expression.

$$\frac{\frac{3x}{x-y}}{\frac{6xy}{4x^2-4y^2}}$$

A) $\frac{(x+y)}{y}$

B) $\frac{(x-y)}{y}$

C) $\frac{2(x-y)}{y}$

D) $\frac{2(x+y)}{y}$

$$\frac{\frac{x^2-9y^2}{xy}}{\frac{2x+6y}{x^2}}$$

A) $\frac{x(x-y)}{y}$

B) $\frac{x(x+3y)}{2y}$

C) $\frac{x(x-3y)}{2y}$

D) $\frac{x(x-y)}{2y}$

$$\frac{\frac{y-x}{z^3}}{\frac{x-y}{6z^2}}$$

A) $-\frac{6}{z}$

B) $\frac{6}{z}$

C) $\frac{3}{z}$

D) $-\frac{3}{z}$

$$\frac{\frac{a^2-b^2}{b^3}}{\frac{b^2-ab}{a^2}}$$

A) $\frac{a^2(a+b)}{b^4}$

B) $\frac{a(a+b)}{b^3}$

C) $\frac{a+b}{b^3}$

D) $\frac{a-b}{b^4}$

Simplify each expression.

$$\frac{7a}{4b} + \frac{4c^2}{10}$$

$$\frac{2x}{9y} - \frac{7y}{6z}$$

$$\frac{3}{x} + \frac{5}{y}$$

$$\frac{3}{8p^2r} + \frac{5}{4p^2r}$$

A) $\frac{a+8bc^2}{22b}$

A) $\frac{2xz-21y^2}{18yz}$

A) $\frac{5x-3y}{xy}$

A) $\frac{13}{8p^2r}$

B) $\frac{35a+8bc}{20b^2}$

B) $\frac{4xz-21y^2}{18yz}$

B) $\frac{5x+3y}{xy}$

B) $\frac{12}{8p^2r}$

C) $\frac{35a+8bc^2}{20b}$

C) $\frac{2xz+21y^2}{18y^2z}$

C) $\frac{5x-3y}{x^2y}$

C) $\frac{12}{8pr}$

D) $\frac{35a-8c^2}{20b}$

D) $\frac{4xz-y^2}{18y^2z}$

D) $\frac{5x+3y}{xy^2}$

D) $\frac{13}{8p^2r^2}$

Simplify each expression.

$$\frac{2x + 1}{x^2 + 2x - 15} - \frac{7}{5x - 15}$$

A) $\frac{3x - 30}{5(x - 3)(x + 5)}$

B) $\frac{3x + 30}{(x - 3)(x + 5)}$

C) $\frac{3x - 30}{(x + 3)(x + 5)}$

D) $\frac{3x - 30}{5(x - 3)(x - 5)}$

$$\frac{3x}{4x^2 + 4} - \frac{2x^2}{x^4 - 1}$$

A) $\frac{3x^3 - 8x^2}{4(x^2 + 1)(x^2 - 1)}$

B) $\frac{3x^3 - 8x^2}{4(x + 1)(x - 1)}$

C) $\frac{3x^3 - 8x^2 - 3x}{4(x + 1)(x - 1)}$

D) $\frac{3x^3 - 8x^2 - 3x}{4(x^2 + 1)(x^2 - 1)}$

$$\frac{3t}{2 - x} + \frac{5}{x - 2}$$

A) $\frac{5 - 3t}{x - 2}$

B) $\frac{5 + 3t}{x - 2}$

C) $\frac{3 + 5t}{x + 2}$

D) $\frac{3 - 5t}{x - 2}$

$$\frac{n}{n - 3} + \frac{2n + 2}{n^2 - 2n - 3}$$

A) $\frac{n + 2}{n + 3}$

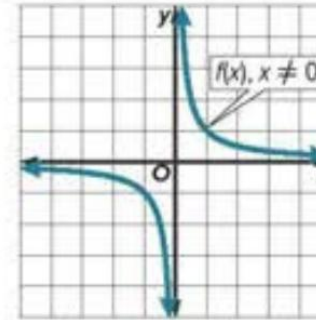
B) $\frac{n - 2}{n - 3}$

C) $\frac{n + 2}{n - 3}$

D) $\frac{n - 2}{n + 3}$

Key Concept • Reciprocal Functions

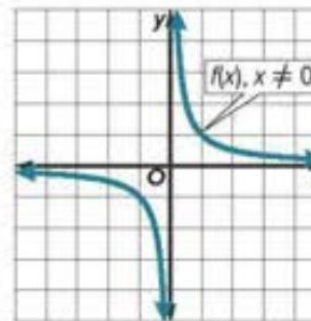
| | |
|------------------|--------------------------|
| Parent function | $f(x) = \frac{1}{x}$ |
| Type of graph | hyperbola |
| Domain and range | all nonzero real numbers |
| Asymptotes | $x = 0$ and $f(x) = 0$ |
| Intercepts | none |
| Not defined | $x = 0$ |



| What is the parent reciprocal function? | What type of graph does $f(x) = \frac{1}{x}$ produce? | Which value is NOT in the domain of $f(x) = \frac{1}{x}$? | Which value is NOT in the range of $f(x) = \frac{1}{x}$? |
|---|---|--|---|
| A) x^2 | A) Parabola | A) 3 | A) 3 |
| B) $\frac{1}{x}$ | B) Circle | B) $\frac{1}{3}$ | B) $\frac{1}{3}$ |
| C) $ x $ | C) Hyperbola | C) 2 | C) 2 |
| D) \sqrt{x} | D) Line | D) 0 | D) 0 |

Key Concept • Reciprocal Functions

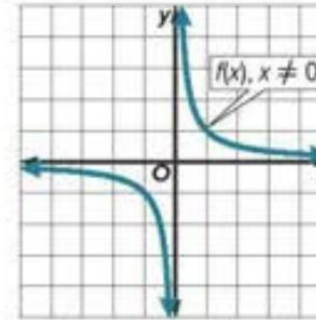
| | |
|------------------|--------------------------|
| Parent function | $f(x) = \frac{1}{x}$ |
| Type of graph | hyperbola |
| Domain and range | all nonzero real numbers |
| Asymptotes | $x = 0$ and $f(x) = 0$ |
| Intercepts | none |
| Not defined | $x = 0$ |



| What is the vertical asymptote of $f(x) = \frac{1}{x}$? | What is the horizontal asymptote of $f(x) = \frac{1}{x}$? | Which statement is true about the x-intercept of $f(x) = \frac{1}{x}$? | Which statement is true about the y-intercept of $f(x) = \frac{1}{x}$? |
|---|---|--|--|
| <p>A) $x = 0$</p> <p>B) $y = 1$</p> <p>C) $y = 0$</p> <p>D) $x = 1$</p> | <p>A) $x = 0$</p> <p>B) $y = 1$</p> <p>C) $y = 0$</p> <p>D) $x = 1$</p> | <p>A) One x-intercept</p> <p>B) Two x-intercepts</p> <p>C) Infinitely many x-intercepts</p> <p>D) None</p> | <p>A) One y-intercept</p> <p>B) Two y-intercepts</p> <p>C) Infinitely many y-intercepts</p> <p>D) None</p> |

Key Concept • Reciprocal Functions

| | |
|------------------|--------------------------|
| Parent function | $f(x) = \frac{1}{x}$ |
| Type of graph | hyperbola |
| Domain and range | all nonzero real numbers |
| Asymptotes | $x = 0$ and $f(x) = 0$ |
| Intercepts | none |
| Not defined | $x = 0$ |



| In which quadrants does the graph of $f(x) = \frac{1}{x}$ lie? | What happens to $f(x) = \frac{1}{x}$ as $x \rightarrow \infty$? | What happens to $f(x) = \frac{1}{x}$ as $x \rightarrow 0^+$? | Which statement is true about the domain and range? |
|--|--|--|--|
| <p>A) I and II</p> <p>B) II and III</p> <p>C) I and III</p> <p>D) III and IV</p> | <p>A) $f(x) \rightarrow \infty$</p> <p>B) $f(x) \rightarrow 0$</p> <p>C) $f(x) \rightarrow 1$</p> <p>D) $f(x) \rightarrow -\infty$</p> | <p>A) $f(x) \rightarrow \infty$</p> <p>B) $f(x) \rightarrow 0$</p> <p>C) $f(x) \rightarrow 1$</p> <p>D) $f(x) \rightarrow -\infty$</p> | <p>A) Domain includes 0</p> <p>B) Range includes 0</p> <p>C) Both exclude 0</p> <p>D) Both include 0</p> |

Determine the values of x for which $f(x)$ is undefined.

$$f(x) = \frac{2}{-2x + 5}$$

$$f(x) = \frac{-12}{-3x - 7}$$

$$f(x) = \frac{5}{x}$$

$$f(x) = \frac{10}{x - 3}$$

A) $x = \frac{5}{2}$

A) $x = \frac{5}{2}$

A) $x = \frac{5}{2}$

A) $x = \frac{5}{2}$

B) $x = 3$

B) $x = 3$

B) $x = 3$

B) $x = 3$

C) $x = 0$

C) $x = 0$

C) $x = 0$

C) $x = 0$

D) $x = -\frac{7}{3}$

D) $x = -\frac{7}{3}$

D) $x = -\frac{7}{3}$

D) $x = -\frac{7}{3}$

Key Concept • Reciprocal Functions

| | | |
|-------------------------|--------------------------|--|
| Parent function | $f(x) = \frac{1}{x}$ | |
| Type of graph | hyperbola | |
| Domain and range | all nonzero real numbers | |
| Asymptotes | $x = 0$ and $f(x) = 0$ | |
| Intercepts | none | |
| Not defined | $x = 0$ | |

Dilation Vertically
Expand (Stretch) $|a| > 1$
Compress $0 < |a| < 1$

Reflected
 about x-axis
 $-f(x)$

$$g(x) = \frac{a}{x - h} + k$$

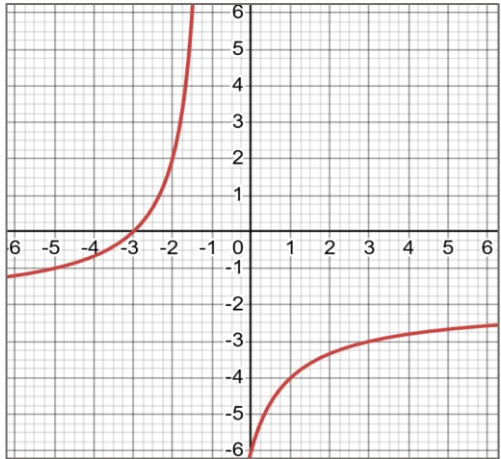
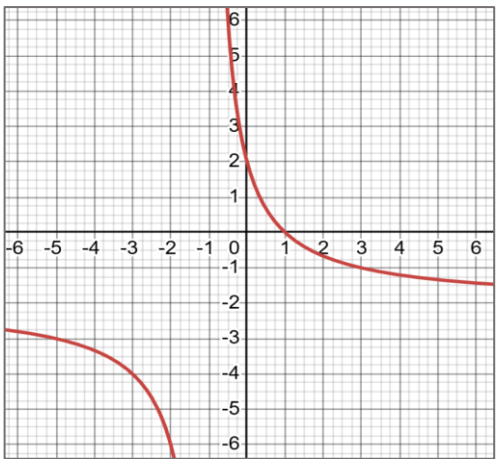
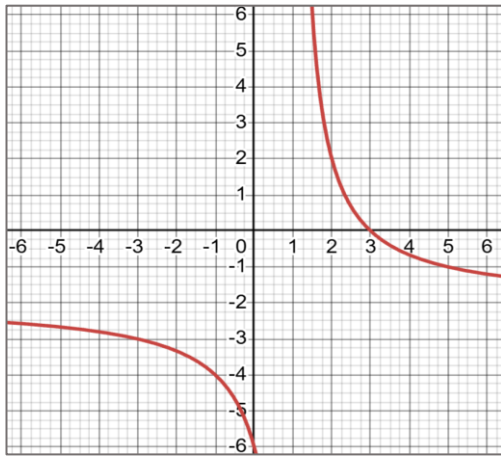
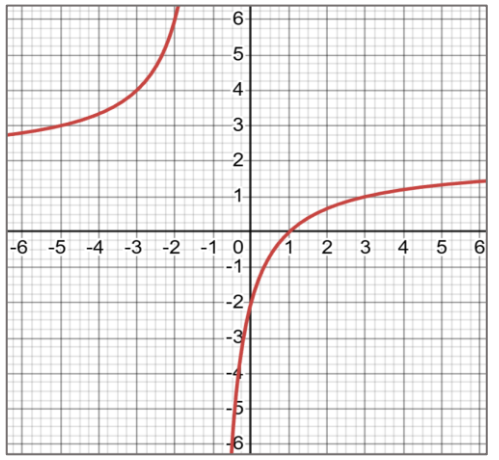
Horizontal translation
 (+) shift Left
 (-) shift Right
 (vertical asymptote)

Vertical translation
 (+) shift UP
 (-) shift Down
 (horizontal asymptote)

Horizontal asymptote: For $f(x) = \frac{n}{b(x)}$, the horizontal asymptote is $f(x) = 0$. For a reciprocal function of the form $f(x) = \frac{n}{b(x)} + k$, where k is a constant, the horizontal asymptote is $f(x) = k$.

Vertical asymptote: For $f(x) = \frac{n}{b(x)}$, the vertical asymptote is found where $b(x) = 0$. For example, for $f(x) = \frac{1}{x}$, the vertical asymptote is $x = 0$.

Graph $g(x) = \frac{-4}{x+1} - 2$. State the domain and range.

| Graph | | Transformations | Domain and Range |
|---|--|---|--|
| A)  | B)  | A) $a = -4$, reflected about x-axis and stretched vertically. $h = -1$, translated left 1 unit $k = -2$, translated down 2 units | A) Domain $\{x \mid x \neq 2\}$ Range $\{y \mid y \neq -1\}$ B) Domain $\{x \mid x \neq -1\}$ Range $\{y \mid y \neq 2\}$ |
| C)  | D)  | B) $a = -4$, reflected about x-axis and compressed vertically. $h = 1$, translated left 1 unit $k = -2$, translated down 2 units C) $a = -4$, reflected about x-axis and compressed vertically. $h = 1$, translated right 1 unit $k = -2$, translated up 2 units | C) Domain $\{x \mid x \neq -1\}$ Range $\{y \mid y \neq -2\}$ D) Domain $\{x \mid x \neq -2\}$ Range $\{y \mid y \neq -1\}$ |
| | | | Vertical and Horizontal Asymptotes |
| | | | A) V.A: $x = 2$ H.A $y = -1$ B) V.A: $x = -1$ H.A $y = -2$ C) V.A: $x = -1$ H.A $y = 2$ D) V.A: $x = -2$ H.A $y = -1$ |

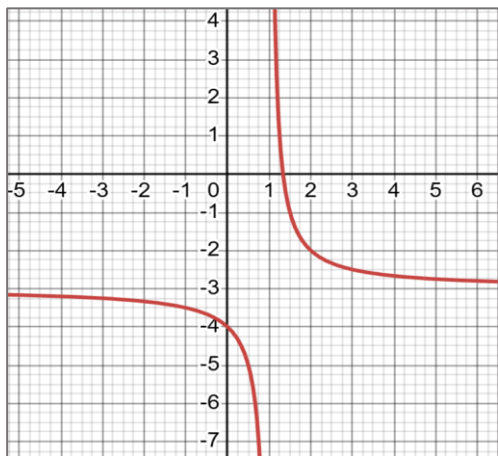
$$\text{Graph } g(x) = \frac{1}{x-1} + 3.$$

Graph

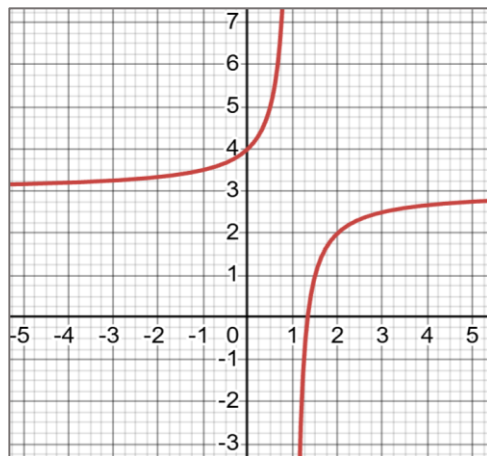
Transformations

Domain and Range

A)



B)



A) $h = -1$, translated left 1 unit
 $k = -3$, translated down 3 units

B) $h = 1$, translated left 1 unit
 $k = 3$, translated down 3 units

C) $h = 1$, translated right 1 unit
 $k = 3$, translated up 3 units

D) $h = 1$, translated right 1 unit
 $k = -3$, translated down 3 units

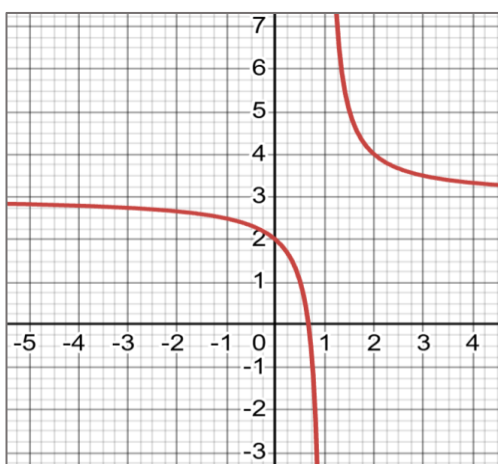
A) Domain $\{x \mid x \neq 1\}$
 Range $\{y \mid y \neq 3\}$

B) Domain $\{x \mid x \neq -1\}$
 Range $\{y \mid y \neq 3\}$

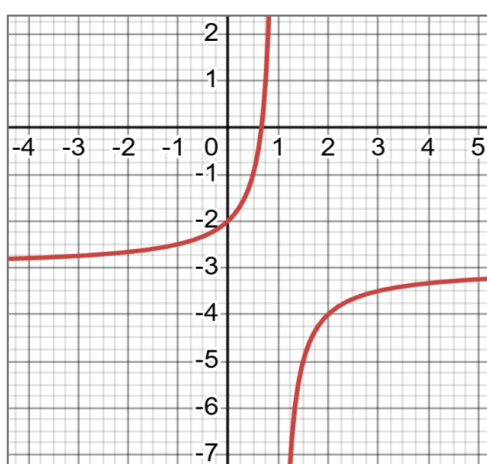
C) Domain $\{x \mid x \neq -1\}$
 Range $\{y \mid y \neq -3\}$

D) Domain $\{x \mid x \neq 3\}$
 Range $\{y \mid y \neq -1\}$

C)



D)



Vertical and Horizontal Asymptotes

A) V.A: $x = 3$
 H.A $y = 1$

B) V.A: $x = 1$
 H.A $y = 3$

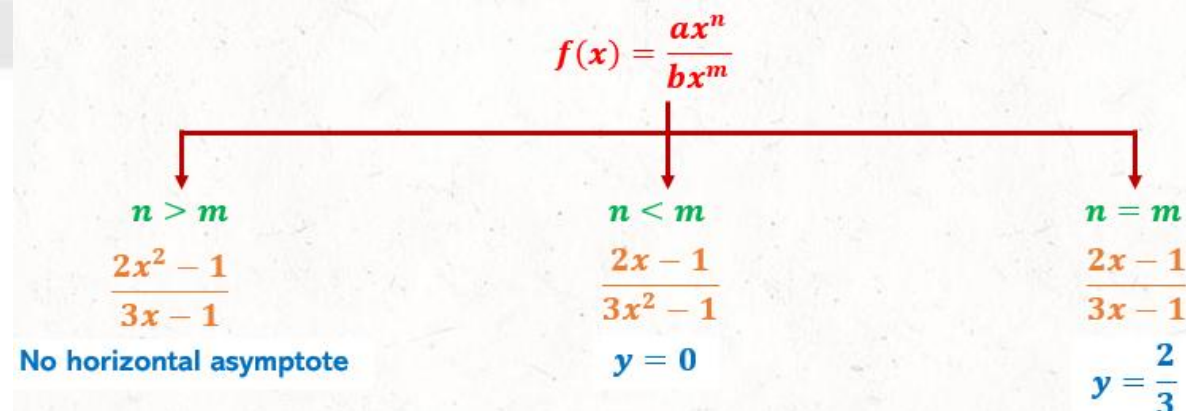
C) V.A: $x = -1$
 H.A $y = 3$

D) V.A: $x = 3$
 H.A $y = -1$

Key Concept • Vertical and Horizontal Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

- $f(x)$ has a vertical asymptote whenever $b(x) = 0$.
- $f(x)$ has at most one horizontal asymptote.

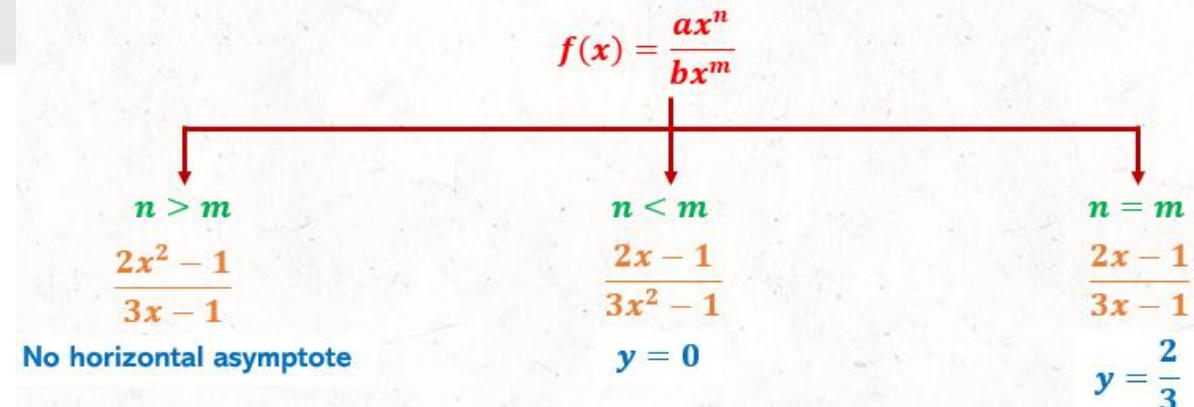


| For a rational function $f(x) = \frac{a(x)}{b(x)}$, a vertical asymptote occurs when: | How many horizontal asymptotes can a rational function have at most? | If the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is: | If the degree of the numerator is greater than the degree of the denominator, then: |
|--|--|---|---|
| A) $a(x) = 0$ | A) None | A) $x = 0$ | A) $x = 0$ |
| B) $b(x) = 0$ | B) One | B) $y = 1$ | B) $y = 1$ |
| C) $a(x) = b(x)$ | C) Two | C) $y = 0$ | C) $y = 0$ |
| D) Degree of numerator is greater | D) Infinite | D) No horizontal asymptote | D) No horizontal asymptote |

Key Concept • Vertical and Horizontal Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

- $f(x)$ has a vertical asymptote whenever $b(x) = 0$.
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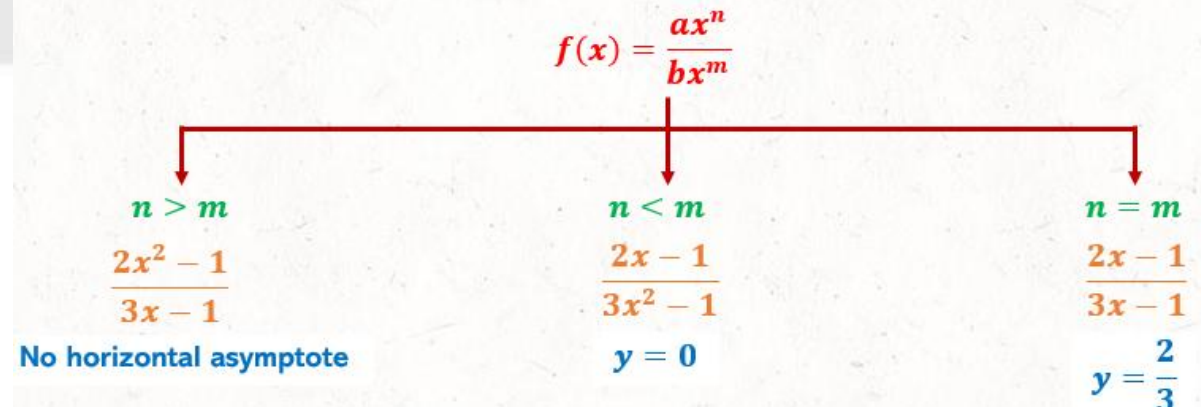


| If the degree of the numerator equals the degree of the denominator, the horizontal asymptote is: | Find the horizontal asymptote of $f(x) = \frac{3x^2+1}{5x^2-2}$ | Find the horizontal asymptote of $f(x) = \frac{2x+1}{x^3-4}$ | Find the horizontal asymptote of $f(x) = \frac{x^4+2}{3x^2+1}$ |
|---|---|--|--|
| A) Ratio of leading coefficients | A) $y = 0$ | A) $y = 0$ | A) $y = 0$ |
| B) Product of leading coefficients | B) $y = \frac{5}{3}$ | B) $y = \frac{5}{3}$ | B) $y = \frac{5}{3}$ |
| C) $y = 0$ | C) $y = \frac{3}{5}$ | C) $y = \frac{3}{5}$ | C) $y = \frac{3}{5}$ |
| D) No asymptote | D) No horizontal asymptote | D) No horizontal asymptote | D) No horizontal asymptote |

Key Concept • Vertical and Horizontal Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

- $f(x)$ has a vertical asymptote whenever $b(x) = 0$.
- $f(x)$ has at most one horizontal asymptote.



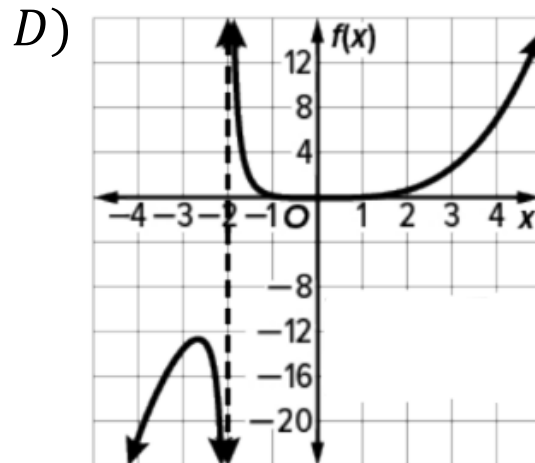
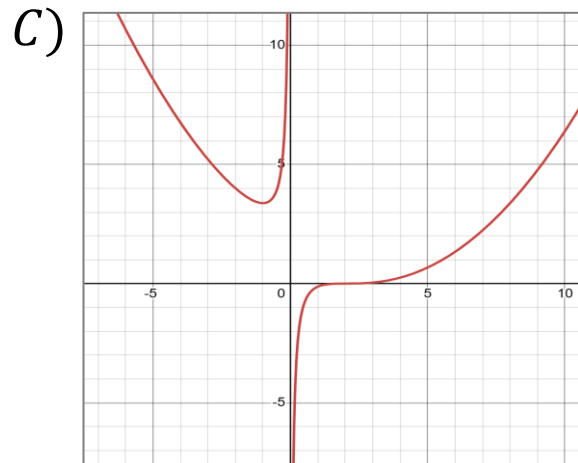
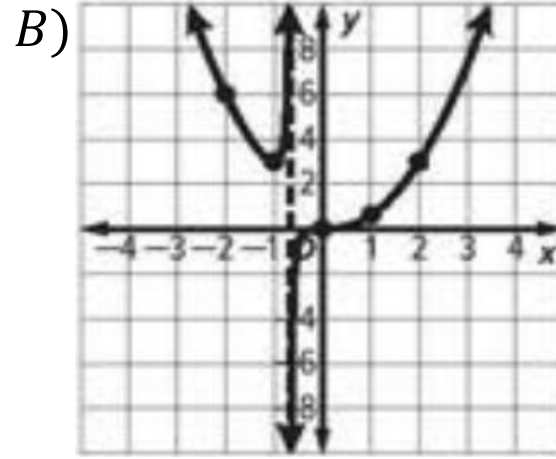
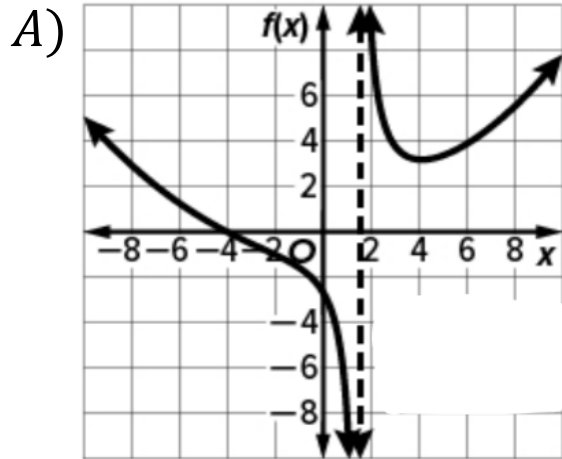
| What is the vertical asymptote of $f(x) = \frac{1}{x-4}$ | What is the vertical asymptote of $f(x) = \frac{x+1}{x^2-9}$ | Which function has horizontal asymptote $y = 1$ | Which function has no horizontal asymptote? |
|---|---|---|--|
| <p>A) $y = 4$</p> <p>B) $x = 3$</p> <p>C) $x = \pm 3$</p> <p>D) $y = 0$</p> | <p>A) $y = 4$</p> <p>B) $x = 3$</p> <p>C) $x = \pm 3$</p> <p>D) $y = 0$</p> | <p>A) $\frac{x+1}{x+2}$</p> <p>B) $\frac{1}{x}$</p> <p>C) $\frac{x}{x^2+1}$</p> <p>D) $\frac{x^3}{x-1}$</p> | <p>A) $\frac{2x+1}{x^2+3}$</p> <p>B) $\frac{x^3+1}{x+2}$</p> <p>C) $\frac{x^2}{x^2+1}$</p> <p>D) $\frac{5}{x}$</p> |

$$\text{Graph } f(x) = \frac{x^3}{x + \frac{2}{3}}$$

Graph

Find zeros

Find the asymptotes



A) $x = 3$

B) $x = \frac{2}{3}$

C) $x = -\frac{2}{3}$

D) $x = 0$

A) V.A: $x = -\frac{2}{3}$

H.A: none

B) V.A: $x = -\frac{2}{3}$

H.A $y = 0$

C) V.A: $x = \frac{2}{3}$

H.A: none

D) V.A: $x = 0$

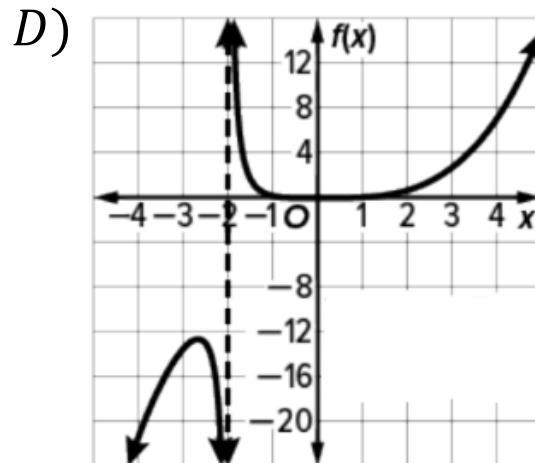
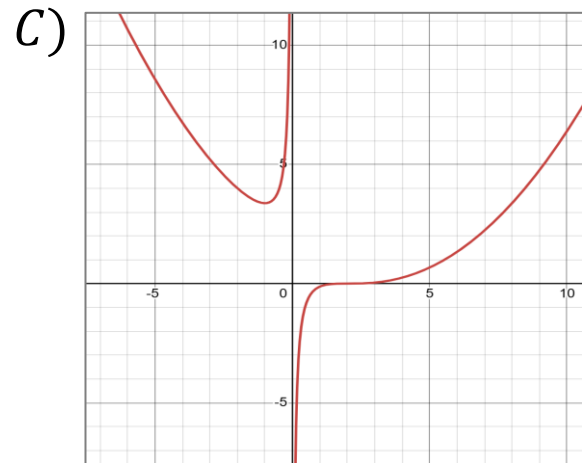
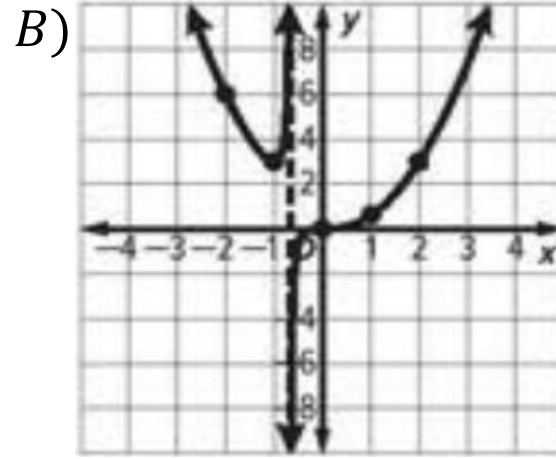
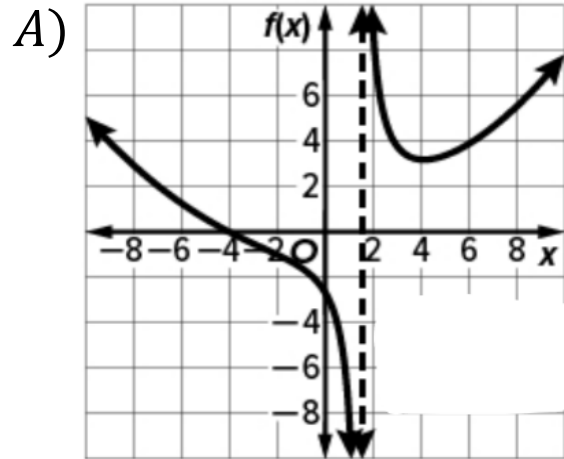
H.A $y = 1$

Consider $g(x) = \frac{(0.5x - 1)^3}{x}$

Graph

Find zeros

Find the asymptotes



A) $x = 0.5$

B) $x = 1$

C) $x = -0.5$

D) $x = 2$

A) V.A: $x = 0$

H.A: none

B) V.A: $x = 0$

H.A $y = 0$

C) V.A: $x = 2$

H.A: none

D) V.A: $x = 2$

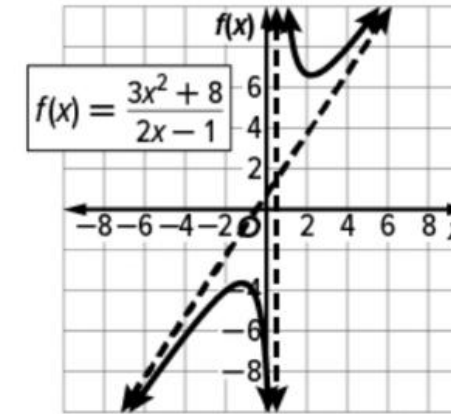
H.A $y = 0.5$

An **oblique asymptote**, or slant asymptote, is neither horizontal nor vertical.

Key Concept • Oblique Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1.

The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.



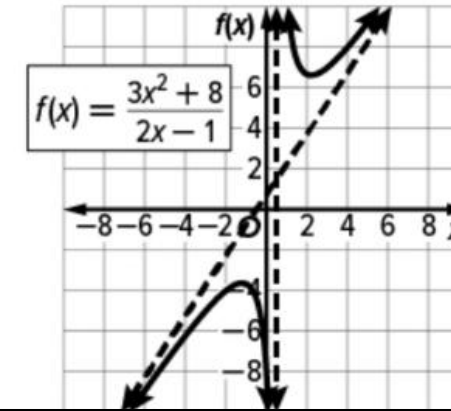
| What is another name for an oblique asymptote? | An oblique asymptote is: | A rational function has an oblique asymptote when: | Which function has an oblique asymptote? |
|--|--|--|--|
| <p>A) Vertical asymptote</p> <p>B) Horizontal asymptote</p> <p>C) Slant asymptote</p> <p>D) Curved asymptote</p> | <p>A) Vertical</p> <p>B) Horizontal</p> <p>C) Neither horizontal nor vertical</p> <p>D) Parabola</p> | <p>A) Degree of numerator – degree of denominator = 1</p> <p>B) Degree of denominator is greater</p> <p>C) Degree of denominator – degree of numerator = 1</p> <p>D) Degree of numerator = degree of denominator</p> | <p>A) $\frac{x^2-1}{x^2-4}$</p> <p>B) $\frac{x^2+3x+1}{x+1}$</p> <p>C) $\frac{x+2}{x^3+1}$</p> <p>D) $\frac{5}{x-2}$</p> |

An **oblique asymptote**, or slant asymptote, is neither horizontal nor vertical.

Key Concept • Oblique Asymptotes

If $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1.

The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.



| | | | |
|---|---|--|---|
| What method is used to find an oblique asymptote? | The equation of the oblique asymptote is: | Determine the type of asymptote for $f(x) = \frac{x^2+2}{x-2}$ | If the degree of the numerator is 3 and the degree of the denominator is 2, the function has: |
| <p>A) Long/ Synthetic division</p> <p>B) Graphing only</p> <p>C) Factoring</p> <p>D) Substitution</p> | <p>A) The remainder only</p> <p>B) The denominator</p> <p>C) The x-intercept</p> <p>D) The quotient without the remainder</p> | <p>A) Horizontal</p> <p>B) Oblique</p> <p>C) Vertical</p> <p>D) None</p> | <p>A) A vertical asymptote</p> <p>B) An oblique asymptote</p> <p>C) A horizontal asymptote</p> <p>D) No asymptote</p> |

Graph each function. Find the point discontinuity.

Key Concept • Point Discontinuity

If $f(x) = \frac{a(x)}{b(x)}$, $b(x) \neq 0$, and $x - c$ is a factor of both $a(x)$ and $b(x)$, then there is a point discontinuity at $x = c$.

$$f(x) = \frac{x^2 - 4}{x + 2}$$

$$f(x) = \frac{(x + 3)^2}{x + 3}$$

$$f(x) = \frac{x^2 - 2x - 8}{x - 4}$$

$$f(x) = \frac{x^2 - 64}{x - 8}$$

A) $x = -3$

A) $x = -3$

A) $x = -3$

A) $x = -3$

B) $x = 4$

B) $x = 4$

B) $x = 4$

B) $x = 4$

C) $x = -2$

C) $x = -2$

C) $x = -2$

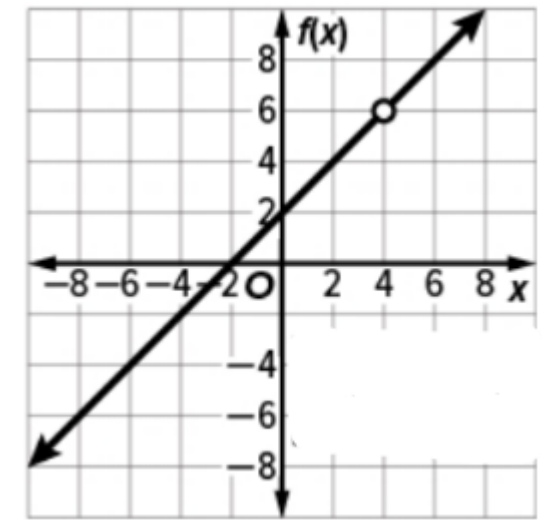
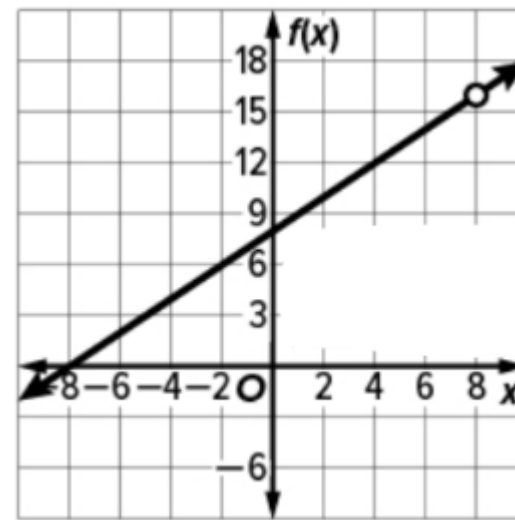
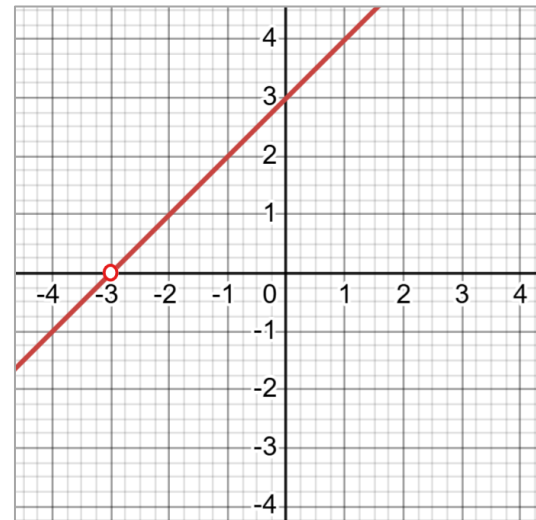
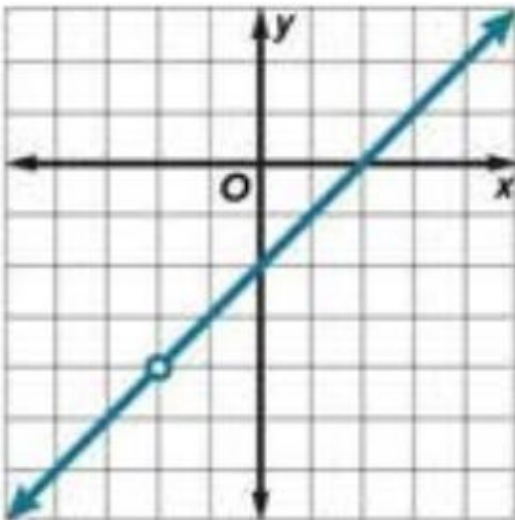
C) $x = -2$

D) $x = 8$

D) $x = 8$

D) $x = 8$

D) $x = 8$



Solve each equation. Check your solutions.

$$\frac{7}{12} + \frac{9}{x-4} = \frac{55}{48}$$

A) $x = \frac{1}{2}$ and $\frac{7}{4}$

B) $x = 20$

C) $x = 7$

D) $x = \frac{1}{2}$

$$\frac{5}{6} - \frac{2}{4x+1} = \frac{x}{3}$$

A) $x = \frac{1}{2}$ and $\frac{7}{4}$

B) $x = 20$

C) $x = 7$

D) $x = \frac{1}{2}$

$$\frac{14}{x-2} - \frac{18}{x+1} = \frac{22}{x^2-x-2}$$

A) $x = \frac{1}{2}$ and $\frac{7}{4}$

B) $x = 20$

C) $x = 7$

D) $x = \frac{1}{2}$

$$\frac{2}{a+2} + \frac{10}{a+5} = \frac{36}{a^2+7a+10}$$

A) $x = \frac{1}{2}$ and $\frac{7}{4}$

B) $x = 20$

C) $x = 7$

D) $x = \frac{1}{2}$

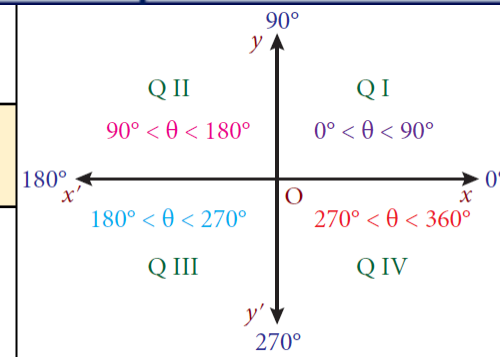
Solve each equation. Check your solutions.

| | | | |
|--|--|--|--|
| $\frac{2m}{m-4} - \frac{m^2+7m+4}{3m^2-18m+24} = \frac{4m}{3m-6}$ | $\frac{3x}{x-4} - \frac{x^2-7x-4}{x^2-16} = \frac{5}{x+4}$ | $\frac{x}{2x-1} + \frac{3}{x+4} = \frac{21}{2x^2+7x-4}$ | $\frac{2}{y-5} + \frac{y-1}{2y+1} = \frac{2}{2y^2-9y-5}$ |
| <p>A) $x = 1$ B) $x = 4$ C) $x = -1$ D) $x = -1$ and 4</p> | <p>A) $x = -3$ B) $x = -4$ C) $x = -3$ and -4 D) $x = 3$</p> | <p>A) $x = -2$ B) $x = -12$ and -2 C) $x = -12$ D) $x = 2$</p> | <p>A) $x = -5$ B) $x = 5$ C) $x = 2$ D) \emptyset</p> |

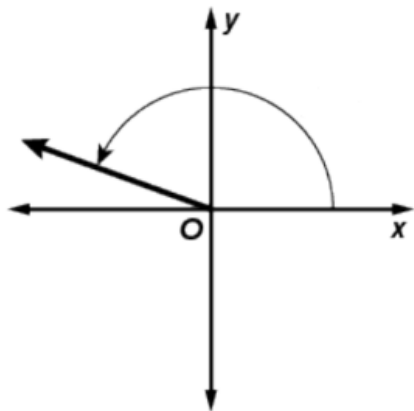
Draw an angle with the given measure in standard position.

200°

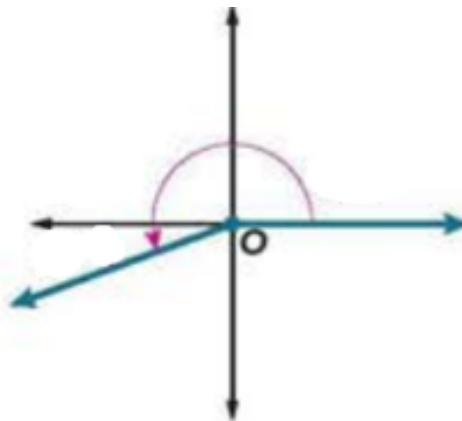
475°



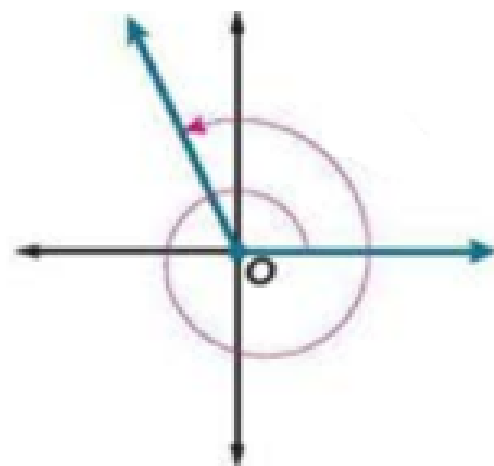
A)



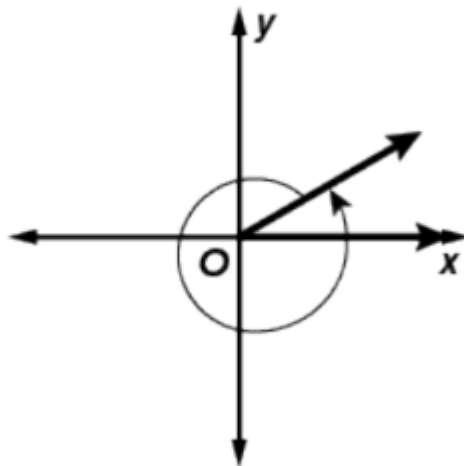
B)



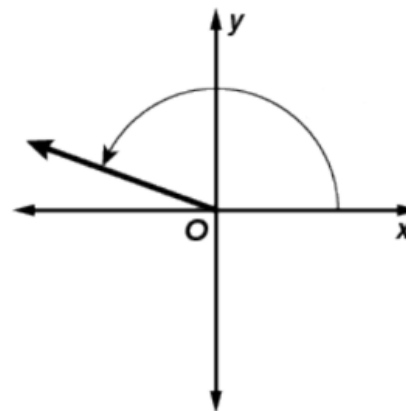
C)



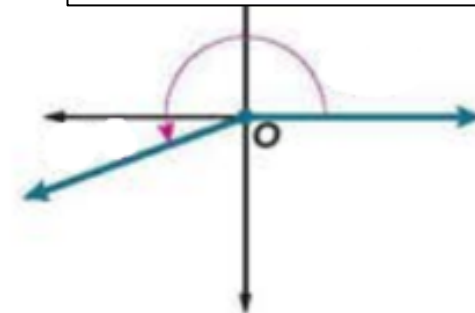
D)



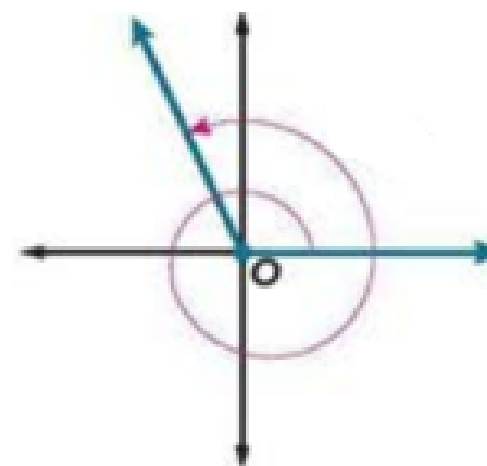
A)



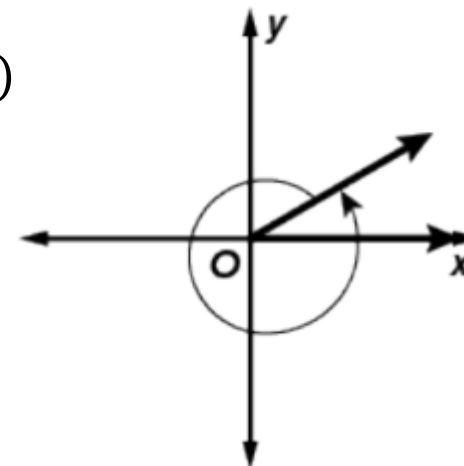
B)



C)



D)



| | | | |
|---|---|---|---|
| An angle of 200° in standard position has its terminal side in: | An angle of 475° in standard position has its terminal side in: | An angle of -400° in standard position has its terminal side in: | An angle of -320° in standard position rotates: |
| <p>A) Quadrant I</p> <p>B) Quadrant II</p> <p>C) Quadrant III</p> <p>D) Quadrant IV</p> | <p>A) Quadrant I</p> <p>B) Quadrant II</p> <p>C) Quadrant III</p> <p>D) Quadrant IV</p> | <p>A) Quadrant I</p> <p>B) Quadrant II</p> <p>C) Quadrant III</p> <p>D) Quadrant IV</p> | <p>A) Quadrant I</p> <p>B) Quadrant II</p> <p>C) Quadrant III</p> <p>D) Quadrant IV</p> |
| An angle of 200° in standard position rotates: | An angle of 475° in standard position rotates: | An angle of -400° in standard position rotates: | An angle of -320° in standard position rotates: |
| <p>A) 475° counterclockwise</p> <p>B) 475° clockwise</p> <p>C) 200° clockwise</p> <p>D) 200° counterclockwise</p> | <p>A) 475° counterclockwise</p> <p>B) 475° clockwise</p> <p>C) 200° clockwise</p> <p>D) 200° counterclockwise</p> | <p>A) -320° counterclockwise</p> <p>B) -400° clockwise</p> <p>C) -320° clockwise</p> <p>D) -400° counterclockwise</p> | <p>A) -320° counterclockwise</p> <p>B) -400° clockwise</p> <p>C) -320° clockwise</p> <p>D) -400° counterclockwise</p> |

| | | | |
|--|--|--|--|
| Find a positive coterminal angle for 35° . | Find a negative coterminal angle for 35° . | Find a positive coterminal angle for -50° . | Find a negative coterminal angle for -50° . |
| A) -325° B) 395° C) 310° D) -410° | A) -325° B) 395° C) 310° D) -410° | A) -325° B) 395° C) 310° D) -410° | A) -325° B) 395° C) 310° D) -410° |
| A 450° angle is coterminal with: | A -210° angle is coterminal with: | Which angle is coterminal with 810° ? | Which angle is coterminal with 1110° ? |
| A) 30° B) 90° C) 150° D) 270° | A) 30° B) 90° C) 150° D) 270° | A) 30° B) 90° C) 150° D) 270° | A) 30° B) 90° C) 150° D) 270° |

Key Concept • Convert Between Degrees and Radians

Degrees to Radians

To convert a degree measure to radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180^\circ}$.

Radians to Degrees

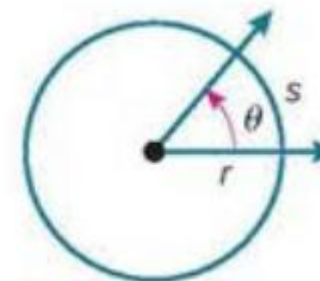
To convert a radian measure to degrees, multiply the number of radians by $\frac{180^\circ}{\pi \text{ radians}}$.

| Rewrite -100° in radians | Rewrite -220° in radians | Rewrite $\frac{11\pi}{4}$ in degrees | Rewrite $\frac{7\pi}{6}$ in degrees |
|---|---|--|--|
| <p>A) $-\frac{11\pi}{9}$ B) $\frac{11\pi}{9}$</p> <p>C) $-\frac{5\pi}{9}$ D) $\frac{5\pi}{9}$</p> | <p>A) $-\frac{11\pi}{9}$ B) $\frac{11\pi}{9}$</p> <p>C) $-\frac{5\pi}{9}$ D) $\frac{5\pi}{9}$</p> | <p>A) 210°</p> <p>B) 495°</p> <p>C) 120°</p> <p>D) 60°</p> | <p>A) 210°</p> <p>B) 495°</p> <p>C) 120°</p> <p>D) 60°</p> |

Key Concept • Arc Length

Words: For a circle with radius r and central angle θ (in radians), the arc length s equals the product of r and θ .

Symbols: $s = r\theta$



Which formula is used for arc length when θ is in radians?

A) $s = r\theta$

B) $s = \pi r^2$

C) $s = 2\pi r$

D) $s = \frac{\theta}{r}$

A wheel has radius 10 cm and rotates through $\frac{3\pi}{5}$ radians. Find s .

A) $2\pi\text{ cm}$

B) $15\pi\text{ cm}$

C) $3\pi\text{ cm}$

D) $6\pi\text{ cm}$

If $s = 15\pi\text{ cm}$ and $r = 5\text{ cm}$, what is θ in radians?

A) $2\pi\text{ cm}$

B) $15\pi\text{ cm}$

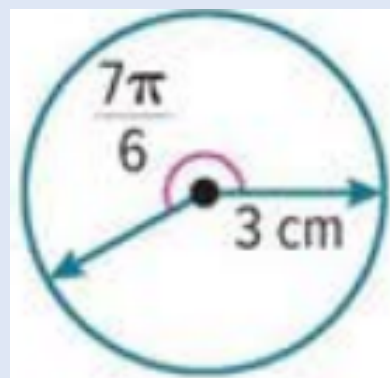
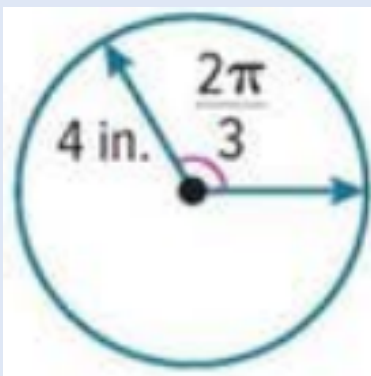
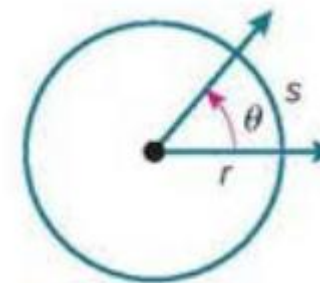
C) $3\pi\text{ cm}$

D) $6\pi\text{ cm}$

Key Concept • Arc Length

Words: For a circle with radius r and central angle θ (in radians), the arc length s equals the product of r and θ .

Symbols: $s = r\theta$



Find the arc length when
 $d = 12\text{ cm}$ and $\theta = \frac{\pi}{3}$ radians.

- A) 12.4 in
- B) 11.0 cm
- C) 6.28 cm
- D) 8.4 in.

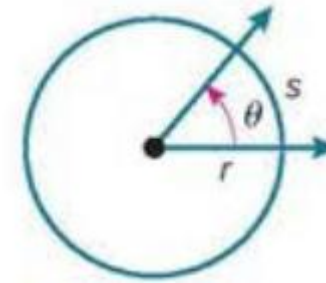
- A) 12.4 in
- B) 11.0 cm
- C) 6.28 cm
- D) 8.4 in.

- A) 12.4 in
- B) 11.0 cm
- C) 6.28 cm
- D) 8.4 in.

Key Concept • Arc Length

Words: For a circle with radius r and central angle θ (in radians), the arc length s equals the product of r and θ .

Symbols: $s = r\theta$



A traffic circle, or roundabout, is a circular roadway at the intersection of two or more streets that allows cars to travel through more continuously than a traffic light or stop sign. The diameter of a traffic circle is 160 feet. How far does a car travel in the roundabout if it goes three-fourths of the way around?

- A) 120 ft
- B) 377 ft
- C) 377 m
- D) 120 m

Key Concept – Trigonometric Functions in Right Triangles

If θ is the measure of an acute angle of a right triangle, then the trigonometric functions involving the **opposite** side *opp*, the **adjacent** side *adj*, and the **hypotenuse** *hyp* are defined as follows.

Reciprocal Functions

sine: $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

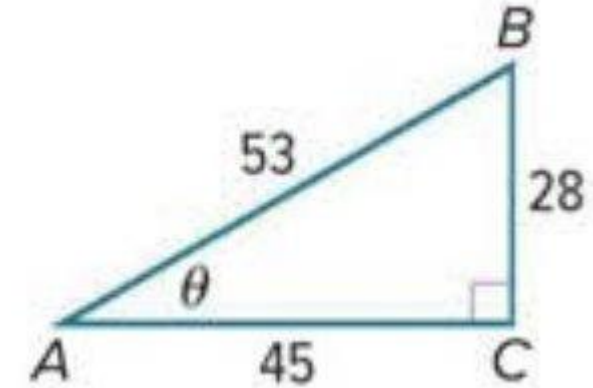
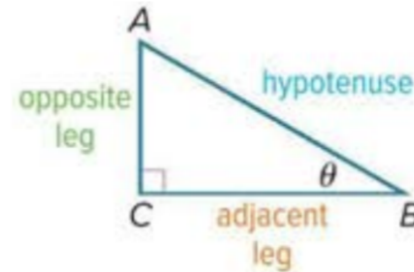
cosecant: $\csc \theta = \frac{\text{hyp}}{\text{opp}}$

cosine: $\cos \theta = \frac{\text{adj}}{\text{hyp}}$

secant: $\sec \theta = \frac{\text{hyp}}{\text{adj}}$

tangent: $\tan \theta = \frac{\text{opp}}{\text{adj}}$

cotangent: $\cot \theta = \frac{\text{adj}}{\text{opp}}$



Find $\sin \theta$

Find $\cos \theta$

Find $\tan \theta$

A) $\frac{28}{53}$

B) $\frac{28}{45}$

A) $\frac{28}{53}$

B) $\frac{28}{45}$

A) $\frac{28}{53}$

B) $\frac{28}{45}$

C) $\frac{53}{45}$

D) $\frac{45}{53}$

C) $\frac{53}{45}$

D) $\frac{45}{53}$

C) $\frac{53}{45}$

D) $\frac{45}{53}$

Key Concept – Trigonometric Functions in Right Triangles

If θ is the measure of an acute angle of a right triangle, then the trigonometric functions involving the **opposite** side *opp*, the **adjacent** side *adj*, and the **hypotenuse** *hyp* are defined as follows.

Reciprocal Functions

sine: $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

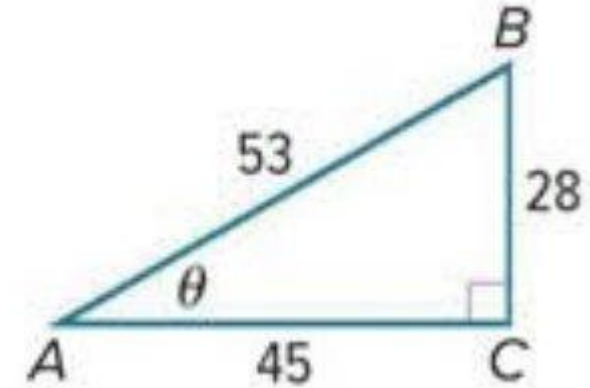
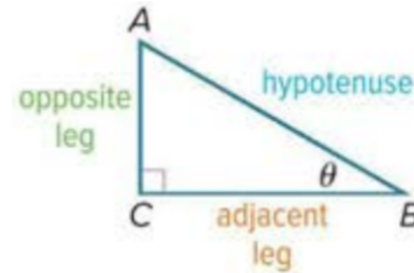
cosecant: $\csc \theta = \frac{\text{hyp}}{\text{opp}}$

cosine: $\cos \theta = \frac{\text{adj}}{\text{hyp}}$

secant: $\sec \theta = \frac{\text{hyp}}{\text{adj}}$

tangent: $\tan \theta = \frac{\text{opp}}{\text{adj}}$

cotangent: $\cot \theta = \frac{\text{adj}}{\text{opp}}$



Find $\csc \theta$

Find $\sec \theta$

Find $\cot \theta$

A) $\frac{45}{28}$

B) $\frac{28}{45}$

A) $\frac{45}{28}$

B) $\frac{28}{45}$

A) $\frac{45}{28}$

B) $\frac{28}{45}$

C) $\frac{53}{28}$

D) $\frac{53}{45}$

C) $\frac{53}{28}$

D) $\frac{53}{45}$

C) $\frac{53}{28}$

D) $\frac{53}{45}$

Key Concept – Trigonometric Functions in Right Triangles

If θ is the measure of an acute angle of a right triangle, then the trigonometric functions involving the **opposite side *opp***, the **adjacent side *adj***, and the **hypotenuse *hyp*** are defined as follows.

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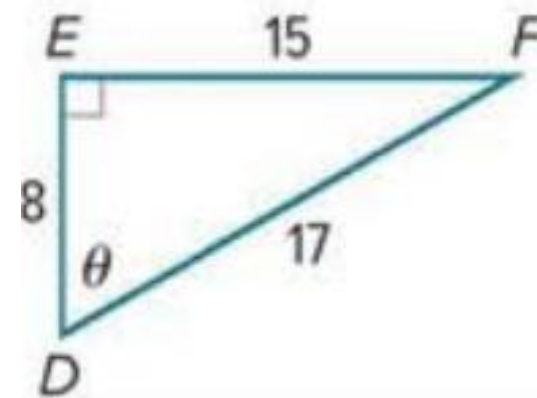
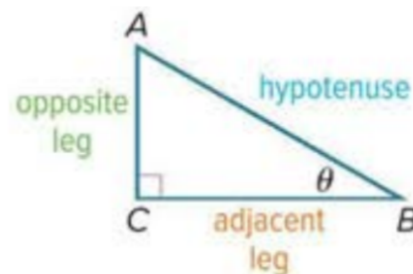
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cosine: $\cos \theta = \frac{\text{adj}}{\text{hyp}}$

secant: $\sec \theta = \frac{\text{hyp}}{\text{adj}}$

tangent: $\tan \theta = \frac{\text{opp}}{\text{adj}}$

cotangent: $\cot \theta = \frac{\text{adj}}{\text{opp}}$



Find $\sin \theta$

Find $\cos \theta$

Find $\tan \theta$

A) $\frac{15}{8}$

B) $\frac{15}{17}$

A) $\frac{15}{8}$

B) $\frac{15}{17}$

A) $\frac{15}{8}$

B) $\frac{15}{17}$

C) $\frac{8}{17}$

D) $\frac{8}{15}$

C) $\frac{8}{17}$

D) $\frac{8}{15}$

C) $\frac{8}{17}$

D) $\frac{8}{15}$

Key Concept – Trigonometric Functions in Right Triangles

If θ is the measure of an acute angle of a right triangle, then the trigonometric functions involving the **opposite side** *opp*, the **adjacent side** *adj*, and the **hypotenuse** *hyp* are defined as follows.

Reciprocal Functions

sine: $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

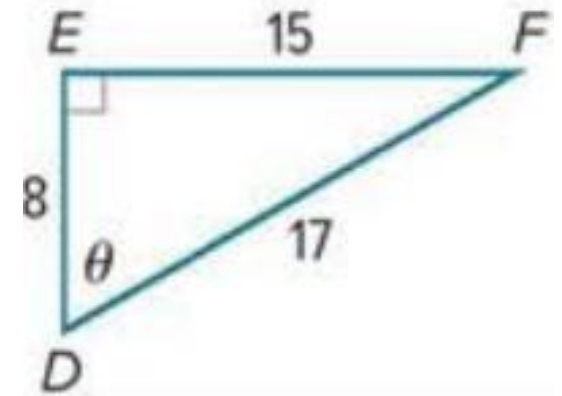
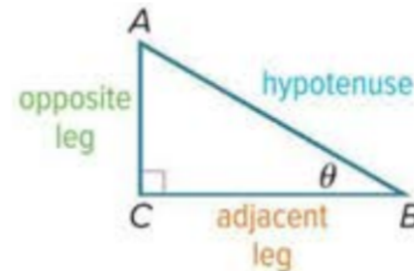
cosecant: $\csc \theta = \frac{\text{hyp}}{\text{opp}}$

cosine: $\cos \theta = \frac{\text{adj}}{\text{hyp}}$

secant: $\sec \theta = \frac{\text{hyp}}{\text{adj}}$

tangent: $\tan \theta = \frac{\text{opp}}{\text{adj}}$

cotangent: $\cot \theta = \frac{\text{adj}}{\text{opp}}$



Find $\csc \theta$

Find $\sec \theta$

Find $\cot \theta$

A) $\frac{17}{15}$

B) $\frac{17}{8}$

A) $\frac{17}{15}$

B) $\frac{17}{8}$

A) $\frac{17}{15}$

B) $\frac{17}{8}$

C) $\frac{8}{17}$

D) $\frac{8}{15}$

C) $\frac{8}{17}$

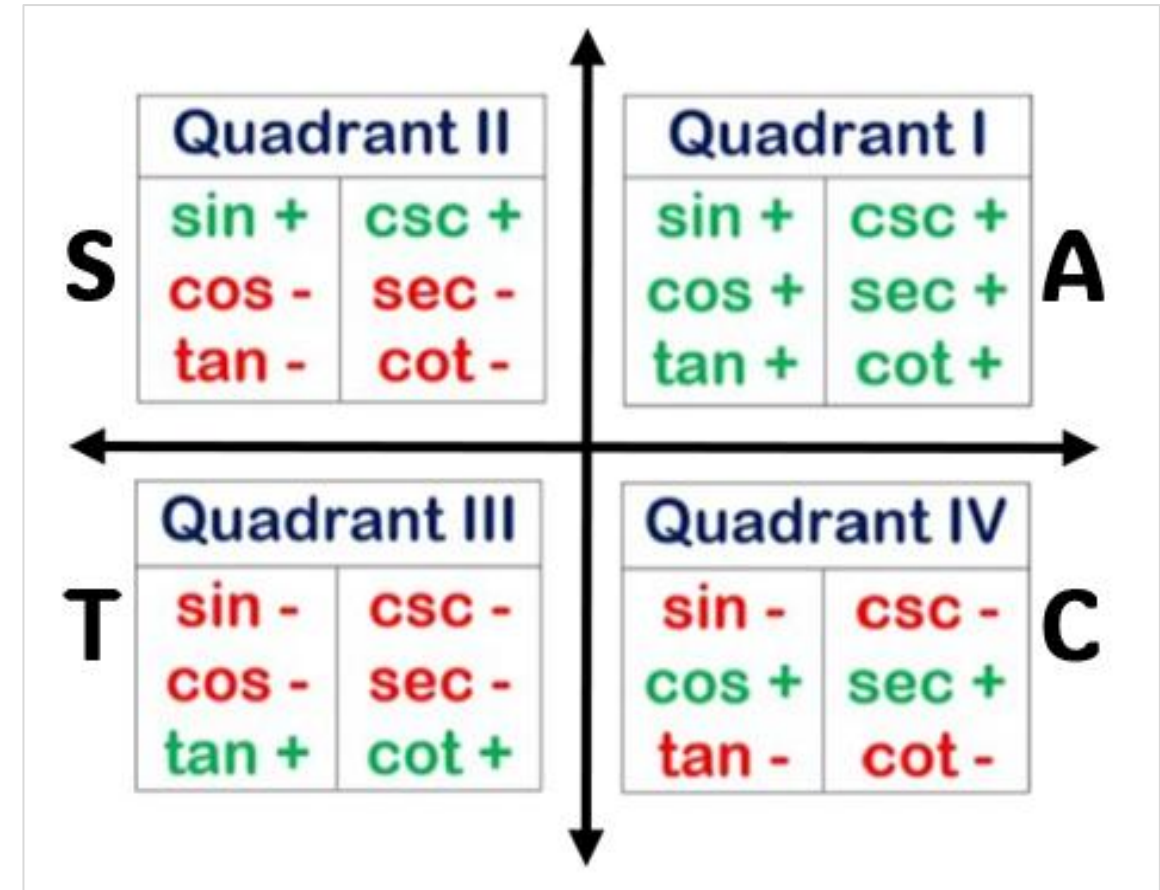
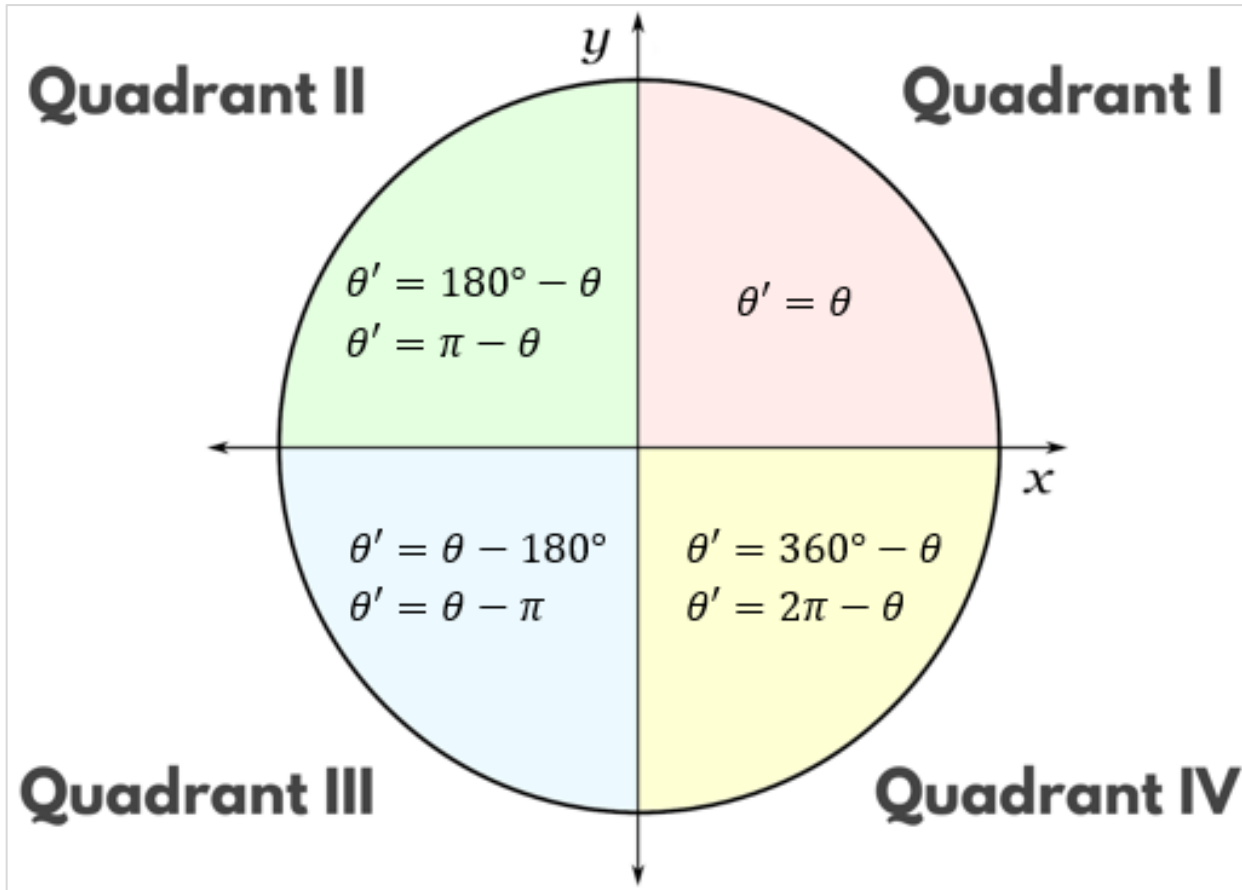
D) $\frac{8}{15}$

C) $\frac{8}{17}$

D) $\frac{8}{15}$

| | | | |
|--|---|--|--|
| If $\sin \theta = \frac{3}{5}$, then $\csc \theta =$ | If $\cos \theta = \frac{8}{17}$, then $\sec \theta =$ | If $\tan \theta = \frac{5}{12}$, then $\cot \theta =$ | If $\cos A = \frac{9}{13}$, then $\sin A =$ |
| A) $\frac{17}{8}$ B) $\frac{2\sqrt{22}}{13}$ C) $\frac{5}{3}$ D) $\frac{12}{5}$ | A) $\frac{17}{8}$ B) $\frac{2\sqrt{22}}{13}$ C) $\frac{5}{3}$ D) $\frac{12}{5}$ | A) $\frac{17}{8}$ B) $\frac{2\sqrt{22}}{13}$ C) $\frac{5}{3}$ D) $\frac{12}{5}$ | A) $\frac{17}{8}$ B) $\frac{2\sqrt{22}}{13}$ C) $\frac{5}{3}$ D) $\frac{12}{5}$ |
| If $\cos A = \frac{9}{13}$, then $\tan A =$ | If $\cos A = \frac{9}{13}$, then $\cot A =$ | If $\cos A = \frac{9}{13}$, then $\csc A =$ | If $\sec B = \frac{11}{3}$, then $\sin B =$ |
| A) $\frac{2\sqrt{22}}{9}$ B) $\frac{9\sqrt{22}}{44}$ C) $\frac{13}{2\sqrt{22}}$ D) $\frac{13}{9}$ | A) $\frac{2\sqrt{22}}{9}$ B) $\frac{9\sqrt{22}}{44}$ C) $\frac{13\sqrt{22}}{44}$ D) $\frac{13}{9}$ | A) $\frac{2\sqrt{22}}{9}$ B) $\frac{9\sqrt{22}}{44}$ C) $\frac{13}{2\sqrt{22}}$ D) $\frac{13}{9}$ | A) $\frac{4\sqrt{7}}{3}$ B) $\frac{11\sqrt{7}}{28}$ C) $\frac{4\sqrt{7}}{11}$ D) $\frac{3\sqrt{7}}{28}$ |

| | | | |
|--|--|--|--|
| If $\sec B = \frac{11}{3}$, then $\tan B =$ | If $\sec B = \frac{11}{3}$, then $\csc B =$ | If $\sec B = \frac{11}{3}$, then $\cot B =$ | If $\tan C = 3$, then $\sin C =$ |
| <p>A) $\frac{4\sqrt{7}}{3}$ B) $\frac{11}{4\sqrt{7}}$</p> <p>C) $\frac{4\sqrt{7}}{11}$ D) $\frac{3\sqrt{7}}{28}$</p> | <p>A) $\frac{4\sqrt{7}}{3}$ B) $\frac{11}{4\sqrt{7}}$</p> <p>C) $\frac{4\sqrt{7}}{11}$ D) $\frac{3\sqrt{7}}{28}$</p> | <p>A) $\frac{4\sqrt{7}}{3}$ B) $\frac{11}{4\sqrt{7}}$</p> <p>C) $\frac{4\sqrt{7}}{11}$ D) $\frac{3\sqrt{7}}{28}$</p> | <p>A) $\frac{3}{\sqrt{10}}$ B) $\sqrt{10}$</p> <p>C) $\frac{\sqrt{10}}{10}$ B) $\frac{\sqrt{10}}{3}$</p> |
| If $\tan C = 3$, then $\cos C =$ | If $\tan C = 3$, then $\sec C =$ | | If $\tan C = 3$, then $\csc C =$ |
| <p>A) $\frac{3}{\sqrt{10}}$ B) $\sqrt{10}$</p> <p>C) $\frac{\sqrt{10}}{10}$ B) $\frac{\sqrt{10}}{3}$</p> | <p>A) $\frac{3}{\sqrt{10}}$ B) $\sqrt{10}$</p> <p>C) $\frac{\sqrt{10}}{10}$ B) $\frac{\sqrt{10}}{3}$</p> | | <p>A) $\frac{3}{\sqrt{10}}$ B) $\sqrt{10}$</p> <p>C) $\frac{\sqrt{10}}{10}$ B) $\frac{\sqrt{10}}{3}$</p> |



Sketch each angle. Then find the measure of its reference angle.

| | | | | |
|--|--|--|--|--|
| 155° | 45° | -150° | -60° | |
| A) 45° B) 30° C) 25° D) 60° | A) 45° B) 30° C) 25° D) 60° | A) 45° B) 30° C) 25° D) 60° | A) 45° B) 30° C) 25° D) 60° | |
| $\frac{11\pi}{6}$ | $\frac{4\pi}{9}$ | $-\frac{8\pi}{3}$ | $-\frac{11\pi}{9}$ | $\frac{23\pi}{6}$ |
| A) $\frac{2\pi}{9}$ B) $\frac{\pi}{6}$ C) $\frac{4\pi}{9}$ D) $\frac{\pi}{3}$ | A) $\frac{2\pi}{9}$ B) $\frac{\pi}{6}$ C) $\frac{4\pi}{9}$ D) $\frac{\pi}{3}$ | A) $\frac{2\pi}{9}$ B) $\frac{\pi}{6}$ C) $\frac{4\pi}{9}$ D) $\frac{\pi}{3}$ | A) $\frac{2\pi}{9}$ B) $\frac{\pi}{6}$ C) $\frac{4\pi}{9}$ D) $\frac{\pi}{3}$ | A) $\frac{2\pi}{9}$ B) $\frac{\pi}{6}$ C) $\frac{4\pi}{9}$ D) $\frac{\pi}{3}$ |

Find the exact value of each trigonometric function.

$$\tan \frac{7\pi}{4}$$

$$A) -1 \quad B) \sqrt{2}$$

$$C) -\frac{\sqrt{2}}{2} \quad D) \frac{12}{5}$$

$$\cos \left(-\frac{11\pi}{4}\right)$$

$$A) -1 \quad B) \sqrt{2}$$

$$C) -\frac{\sqrt{2}}{2} \quad D) \frac{12}{5}$$

$$\sin (-150^\circ)$$

$$A) -\frac{1}{2} \quad B) \sqrt{3}$$

$$C) -\sqrt{2} \quad D) \frac{1}{2}$$

$$\csc \frac{\pi}{4}$$

$$A) -1 \quad B) \sqrt{2}$$

$$C) -\frac{\sqrt{2}}{2} \quad D) \frac{12}{5}$$

$$\sec 225^\circ$$

$$A) -\frac{1}{2} \quad B) \sqrt{3}$$

$$C) -\sqrt{2} \quad D) \frac{1}{2}$$

$$\cot 30^\circ$$

$$A) -\frac{1}{2} \quad B) \sqrt{3}$$

$$C) -\sqrt{2} \quad D) \frac{1}{2}$$

The terminal side of angle θ in standard position intersects the unit circle at each point P .

Find $\cos \theta$ and $\sin \theta$.

$$P(\cos \theta, \sin \theta)$$

$$P\left(-\frac{12}{13}, \frac{5}{13}\right)$$

$$A) \cos \theta = \frac{5}{13}$$

$$\sin \theta = \frac{12}{13}$$

$$B) \cos \theta = \frac{5}{13}$$

$$\sin \theta = -\frac{12}{13}$$

$$C) \cos \theta = -\frac{12}{13}$$

$$\sin \theta = \frac{5}{13}$$

$$D) \cos \theta = -\frac{12}{13}$$

$$\sin \theta = -\frac{5}{13}$$

$$P\left(-\frac{4}{5}, -\frac{3}{5}\right)$$

$$A) \cos \theta = -\frac{3}{5}$$

$$\sin \theta = -\frac{4}{5}$$

$$B) \cos \theta = -\frac{4}{5}$$

$$\sin \theta = -\frac{3}{5}$$

$$C) \cos \theta = -\frac{5}{4}$$

$$\sin \theta = -\frac{3}{5}$$

$$D) \cos \theta = \frac{3}{5}$$

$$\sin \theta = \frac{4}{5}$$

$$P(0, -1)$$

$$A) \cos \theta = 0$$

$$\sin \theta = 1$$

$$B) \cos \theta = -1$$

$$\sin \theta = 0$$

$$C) \cos \theta = 1$$

$$\sin \theta = 0$$

$$D) \cos \theta = 0$$

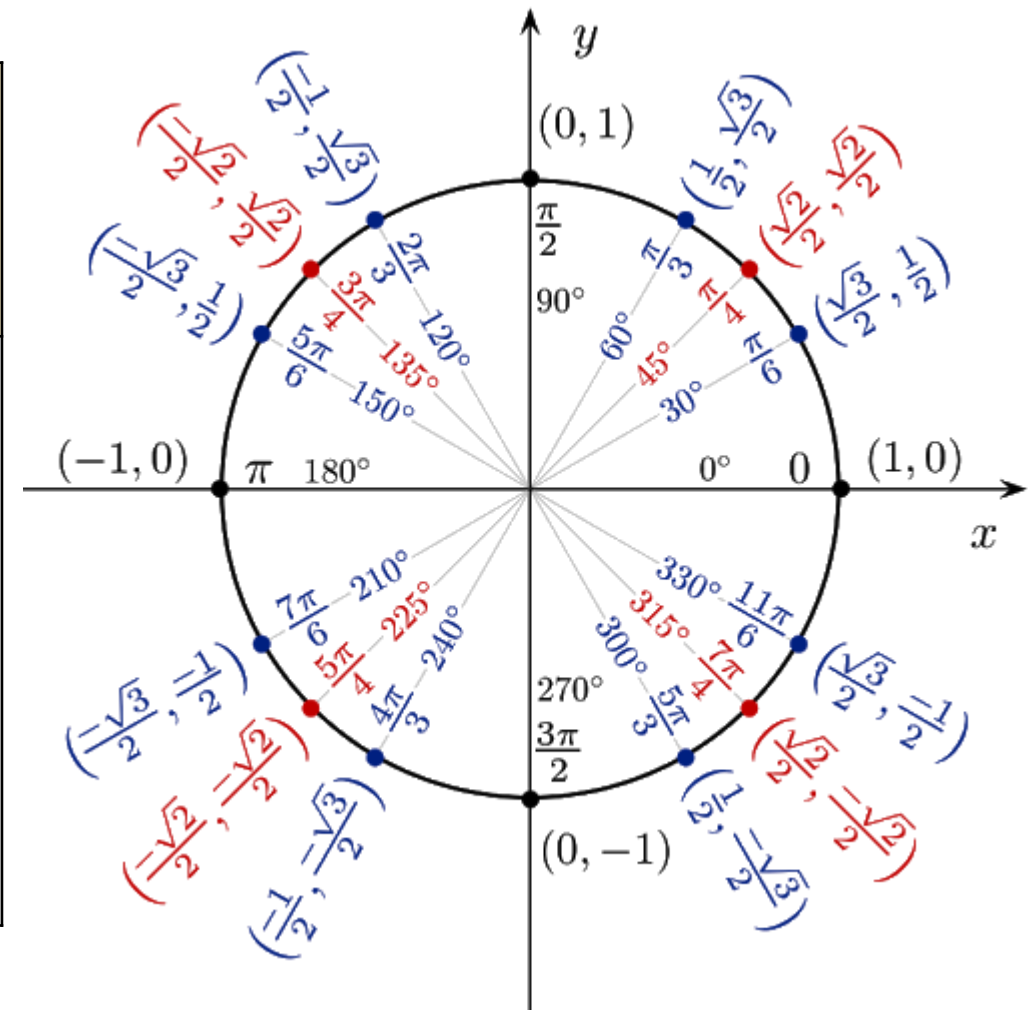
$$\sin \theta = -1$$

The point $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ lies in which quadrant?

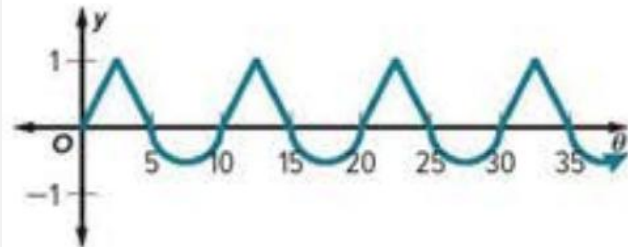
- A) Quadrant I
- B) Quadrant II
- C) Quadrant III
- D) Quadrant IV

Which angle has unit-circle point $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$?

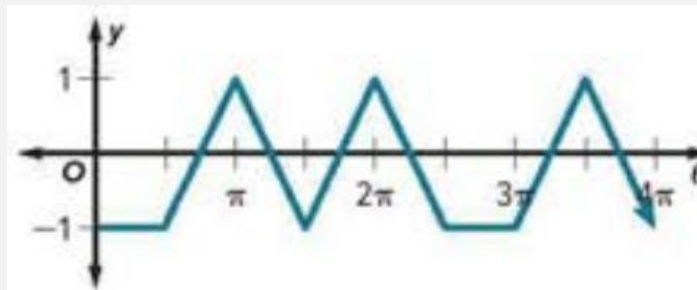
- A) 60°
- B) 120°
- C) 240°
- D) 300°



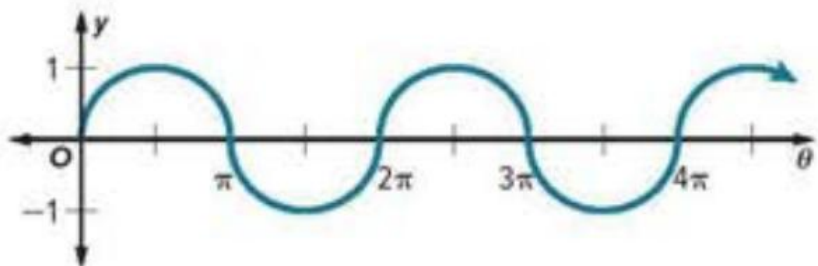
Determine the period of the function.



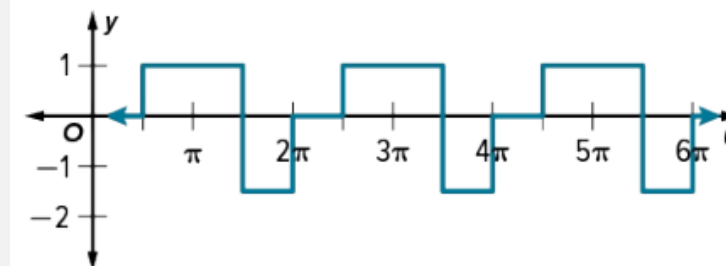
- A) 10 B) 2π C) $\frac{5\pi}{2}$ D) 5



- A) 10 B) 2π C) $\frac{5\pi}{2}$ D) 5

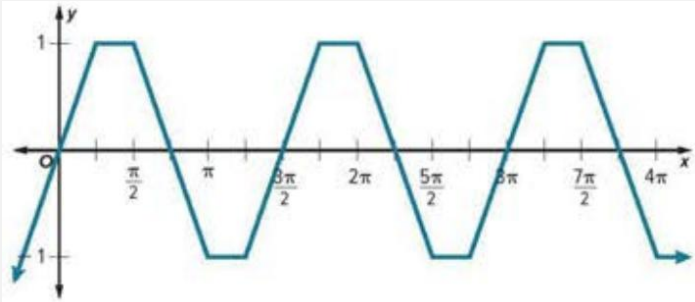


- A) 10 B) 2π C) $\frac{5\pi}{2}$ D) 5

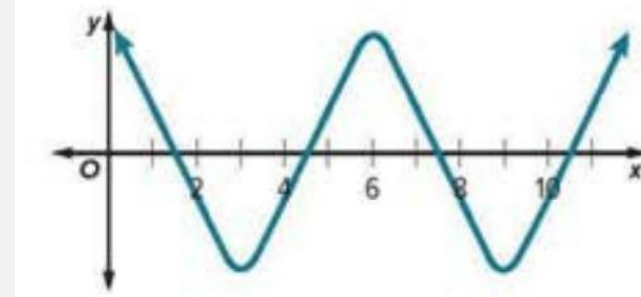


- A) 10 B) 2π C) $\frac{5\pi}{2}$ D) 5

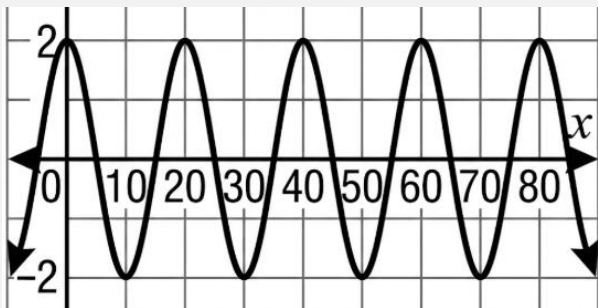
Determine the period of the function.



- A) 6 B) 4 C) $\frac{3\pi}{2}$ D) 20



- A) 6 B) 4 C) $\frac{3\pi}{2}$ D) 20



- A) 6 B) 4 C) $\frac{3\pi}{2}$ D) 20



- A) 6 B) 4 C) $\frac{3\pi}{2}$ D) 20

Find the exact value of each expression.

$$\cos \frac{10\pi}{3}$$

$$A) -\frac{1}{2} \quad B) \frac{\sqrt{3}}{2}$$

$$C) -\frac{\sqrt{2}}{2} \quad D) \frac{1}{2}$$

$$\sin \frac{21\pi}{4}$$

$$A) -\frac{1}{2} \quad B) \frac{\sqrt{3}}{2}$$

$$C) -\frac{\sqrt{2}}{2} \quad D) \frac{1}{2}$$

$$\cos \frac{11\pi}{3}$$

$$A) -\frac{1}{2} \quad B) \frac{\sqrt{3}}{2}$$

$$C) -\frac{\sqrt{2}}{2} \quad D) \frac{1}{2}$$

$$\cos 60^\circ$$

$$A) \frac{4}{5} \quad B) -\frac{\sqrt{3}}{2}$$

$$C) \frac{1}{2} \quad D) \frac{1}{5}$$

$$\sin 600^\circ$$

$$A) \frac{4}{5} \quad B) -\frac{\sqrt{3}}{2}$$

$$C) \frac{1}{2} \quad D) \frac{1}{5}$$

$$\sin \frac{8\pi}{3}$$

$$A) -\frac{1}{2} \quad B) \frac{\sqrt{3}}{2}$$

$$C) -\frac{\sqrt{2}}{2} \quad D) \frac{1}{2}$$

Sine and Cosine Functions

| Function | $y = a \sin bx$ | $y = a \cos bx$ |
|--------------|--|--|
| Parent graph | | |
| Domain | All real numbers | All real numbers |
| Range | $\{y \mid y_{Min} \leq y \leq y_{Max}\}$ | $\{y \mid y_{Min} \leq y \leq y_{Max}\}$ |
| Amplitude | $ a $ OR $\frac{y_{Max} - y_{Min}}{2}$ | $ a $ OR $\frac{y_{Max} - y_{Min}}{2}$ |
| Midline | $y = 0$ | $y = 0$ |
| Period | $\frac{360^\circ}{b}$ | $\frac{360^\circ}{b}$ |
| Oscillation | Between Minimum point and Maximum point | Between Minimum point and Maximum point |

| | | | |
|---|--|---|---|
| What is the amplitude of the parent function $y = \sin x$? | What is the period of $y = \cos x$? | What is the domain of $y = \sin x$? | What is the range of $y = \cos x$? |
| <p>A) 0</p> <p>B) 1</p> <p>C) 360</p> <p>D) -1</p> | <p>A) 90°</p> <p>B) 180°</p> <p>C) 360°</p> <p>D) 720°</p> | <p>A) $-1 \leq y \leq 1$</p> <p>B) All real numbers</p> <p>C) $0 \leq x \leq 360$</p> <p>D) Positive numbers only</p> | <p>A) $0 \leq y \leq 1$</p> <p>B) All real numbers</p> <p>C) $-1 \leq y \leq 1$</p> <p>D) $-360 \leq y \leq 360$</p> |
| What is the midline of the parent sine function? | Between which values do the sine and cosine graphs oscillate? | Which function starts at $(0,1)$? | What is the maximum value of the parent sine function? |
| <p>A) $y = 0$</p> <p>B) $x = 0$</p> <p>C) $y = 1$</p> <p>D) $x = 1$</p> | <p>A) 0 and 1</p> <p>B) -360 and 360</p> <p>C) -2 and 2</p> <p>D) -1 and 1</p> | <p>A) $y = \cos x$</p> <p>B) $y = \sin x$</p> <p>C) $y = \tan x$</p> <p>D) $y = x^2$</p> | <p>A) -1</p> <p>B) 0</p> <p>C) 1</p> <p>D) 2</p> |

Find the amplitude of each function.

Amplitude

|a|

$$y = 2 \cos \theta$$

A) $\frac{3}{4}$ B) 2 C) 1

D) $\frac{1}{2}$ E) 3

$$y = 2 \sin \theta$$

A) $\frac{3}{4}$ B) 2 C) 1

D) $\frac{1}{2}$ E) 3

$$y = \cos \frac{1}{2} \theta$$

A) $\frac{3}{4}$ B) 2 C) 1

D) $\frac{1}{2}$ E) 3

$$y = \frac{3}{4} \cos \theta$$

A) $\frac{3}{4}$ B) 2 C) 1

D) $\frac{1}{2}$ E) 3

$$y = \frac{1}{2} \sin 2\theta$$

A) $\frac{3}{4}$ B) 2 C) 1

D) $\frac{1}{2}$ E) 3

$$y = 3 \cos 2\theta$$

A) $\frac{3}{4}$ B) 2 C) 1

D) $\frac{1}{2}$ E) 3

Find the period of each function.

Period

 $\frac{360^\circ}{b}$

$$y = 2 \cos \theta$$

$$y = 2 \sin \theta$$

$$y = \cos \frac{1}{2} \theta$$

- A) 90° B) 720°
 C) 180° D) 360°

- A) 90° B) 720°
 C) 180° D) 360°

- A) 90° B) 720°
 C) 180° D) 360°

$$y = \frac{3}{4} \cos \theta$$

$$y = \frac{1}{2} \sin 2\theta$$

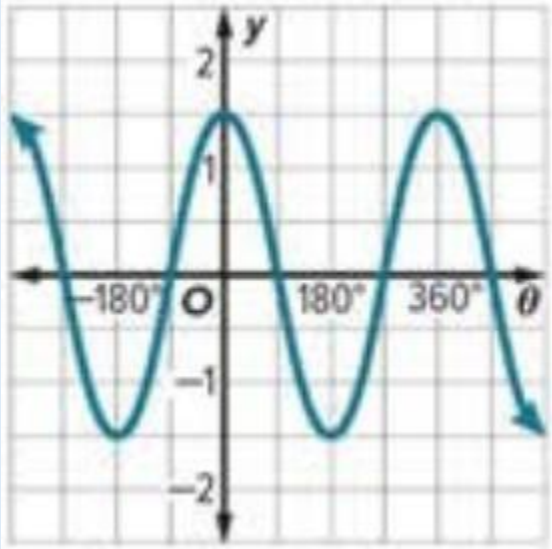
$$y = 3 \cos 2\theta$$

- A) $\frac{\pi}{2}$ B) 4π
 C) π D) 2π

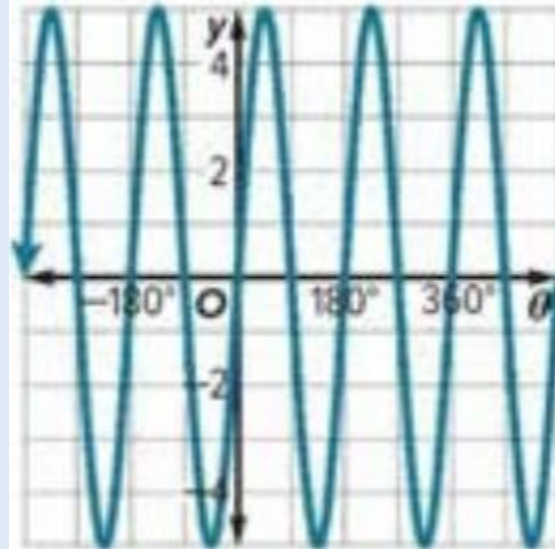
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 C) π D) 2π

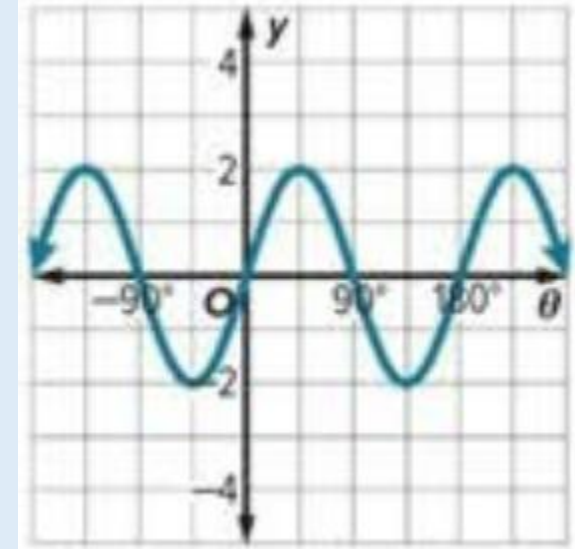
For each graph, identify the period



- A) 90°
- B) 360°
- C) 180°
- D) 270°



- A) 90°
- B) 360°
- C) 180°
- D) 270°



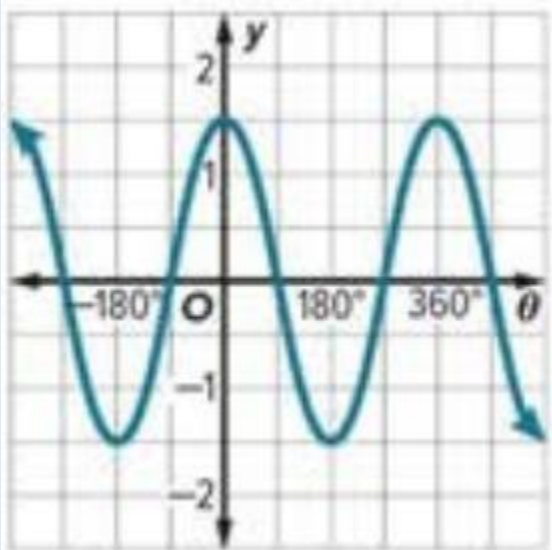
- A) 90°
- B) 360°
- C) 180°
- D) 270°

For each graph, write an equation.

Function

$y = a \sin bx$

$y = a \cos bx$

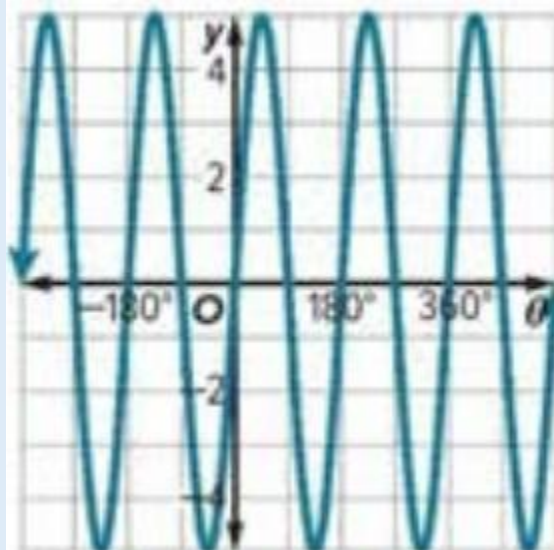


A) $y = 2 \sin 2x$

B) $y = \frac{3}{2} \cos x$

C) $y = 5 \sin 2x$

D) $y = 2 \cos 2x$

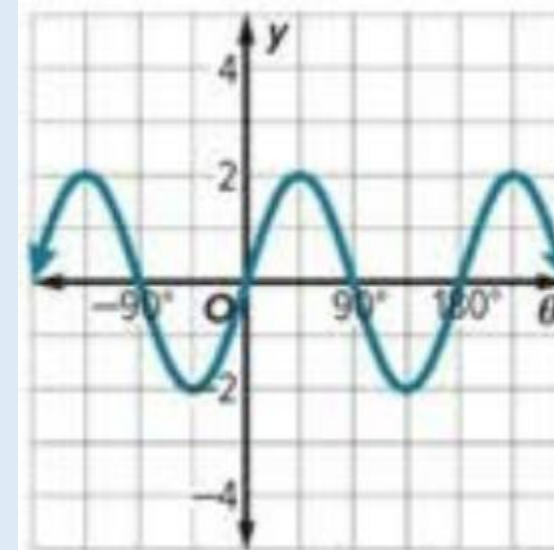


A) $y = 2 \sin 2x$

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C) $y = 5 \sin 2x$

D) $y = 2 \cos 2x$



A) $y = 2 \sin 2x$

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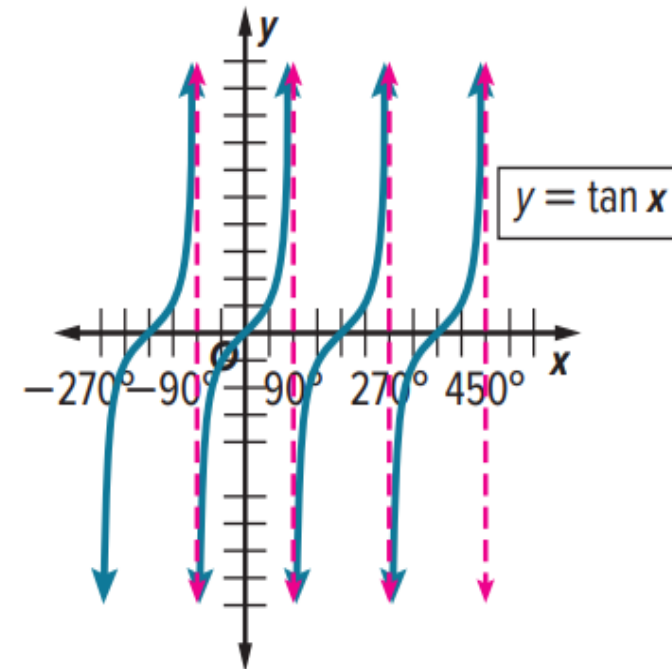
C) $y = 5 \sin 2x$

D) $y = 2 \cos 2x$

Graphs of Tangent Functions

| | |
|--------------|---|
| Function | $y = a \tan bx$ |
| Domain | $\left\{x \mid x \neq \frac{(90 + 180n)^\circ}{ b }, n \text{ is an integer}\right\}$ |
| Range | All real numbers |
| Amplitude | undefined |
| Period | $\frac{180^\circ}{ b }$ OR $\frac{\pi}{ b }$ |
| x-intercepts | $\frac{180n}{ b }$ or $\frac{\pi n}{ b }$, where n is an integer. |
| Midline | $y = 0$ |
| Asymptotes | $x = \frac{(90+180n)^\circ}{ b }$ or $\frac{(\frac{\pi}{2}+\pi n)^\circ}{ b }$, where n is an integer. |

Graph



Find the period, asymptotes, x-intercepts, midline, and transformations of each tangent function. Then graph the function.

$$y = \tan 3x$$

$$\frac{180^\circ}{|b|} \text{ OR } \frac{\pi}{|b|} \quad x = \frac{(90+180n)^\circ}{|b|} \text{ or } \frac{(\frac{\pi}{2}+\pi n)^\circ}{|b|} \quad \frac{180n}{|b|} \text{ or } \frac{\pi n}{|b|}$$

| Find the period | Find the asymptotes | Find the x-intercepts | Find the midline |
|-----------------|-------------------------|------------------------|------------------|
| A) 360° | A) $x = -30, 0, 60$ | A) $x = 0, 30, 60$ | A) $y = 1$ |
| B) 45° | B) $x = -30, 0, 30, 60$ | B) $x = 0, 60, 120$ | B) $y = 0$ |
| C) 720° | C) $x = -30, 0, 30$ | C) $x = -60, 0, 30$ | C) $y = 2$ |
| D) 60° | D) $x = 0, 60, 120$ | D) $x = -60, 60, -120$ | D) $y = -1$ |

Find the period, asymptotes, x-intercepts, midline, and transformations of each tangent function. Then graph the function.
 $y = \tan 3x$

Transformations

A) $a = 3$, stretched vertically.

B) $a = 3$, compressed horizontally.

C) $b = 3$, compressed horizontally.

D) $b = 3$, stretched vertically.

$$y = a \tan bx$$

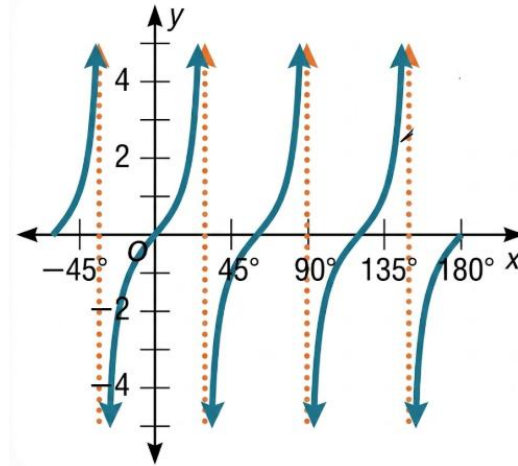
Reflected about
the x -axis $-f(x)$

Vertical Dilation
Stretch: $|a| > 1$
Compress: $0 < |a| < 1$

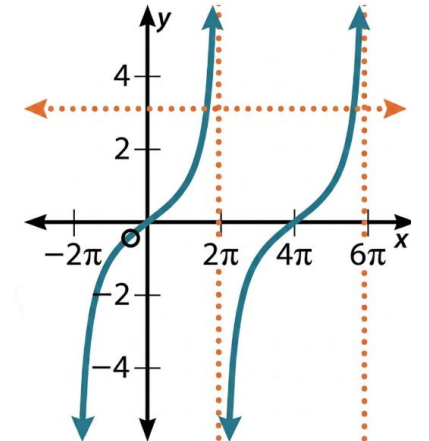
Horizontal Dilation
Stretch: $0 < |b| < 1$
Compress: $|b| > 1$

Graph

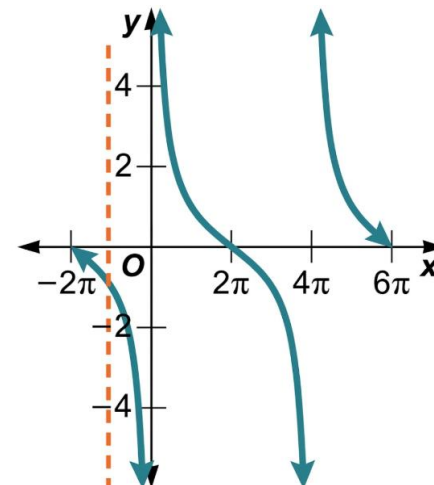
A)



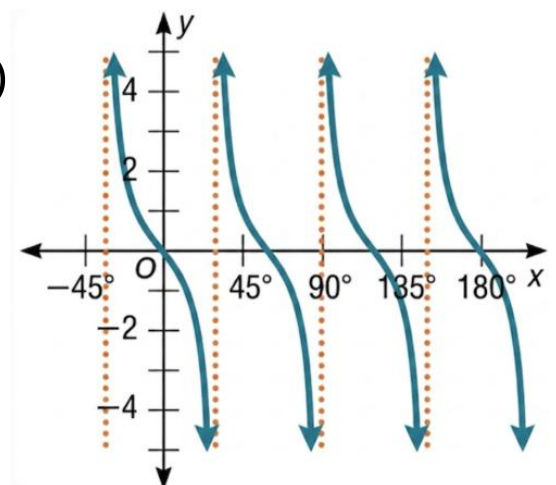
B)



C)



D)



Find the period, asymptotes, x-intercepts, midline, and transformations of each tangent function. Then graph the function.

$$y = \tan 0.25x$$

$$\frac{180^\circ}{|b|} \text{ OR } \frac{\pi}{|b|} \quad x = \frac{(90+180n)^\circ}{|b|} \text{ or } \frac{(\frac{\pi}{2}+\pi n)^\circ}{|b|} \quad \frac{180n}{|b|} \text{ or } \frac{\pi n}{|b|}$$

| Find the period | Find the asymptotes | Find the x-intercepts | Find the midline |
|--------------------|-------------------------|-------------------------|------------------|
| A) π | A) $x = -2\pi, 2\pi$ | A) $x = -2\pi, 2\pi$ | A) $y = 1$ |
| B) $\frac{\pi}{4}$ | B) $x = -4\pi, 0, 4\pi$ | B) $x = -4\pi, 0, 4\pi$ | B) $y = 0$ |
| C) 4π | C) $x = -2\pi, 0, 2\pi$ | C) $x = -2\pi, 0, 2\pi$ | C) $y = 2$ |
| D) $\frac{\pi}{3}$ | D) $x = -4\pi, 0, 2\pi$ | D) $x = -4\pi, 0, 2\pi$ | D) $y = -1$ |

Find the period, asymptotes, x-intercepts, midline, and transformations of each tangent function. Then graph the function.
 $y = \tan 0.25x$

Transformations

- A) $a = 0.25$, stretched vertically.
 B) $a = 0.25$, compressed vertically.
 C) $b = 0.25$, compressed horizontally.
 D) $b = 0.25$, stretched horizontally.

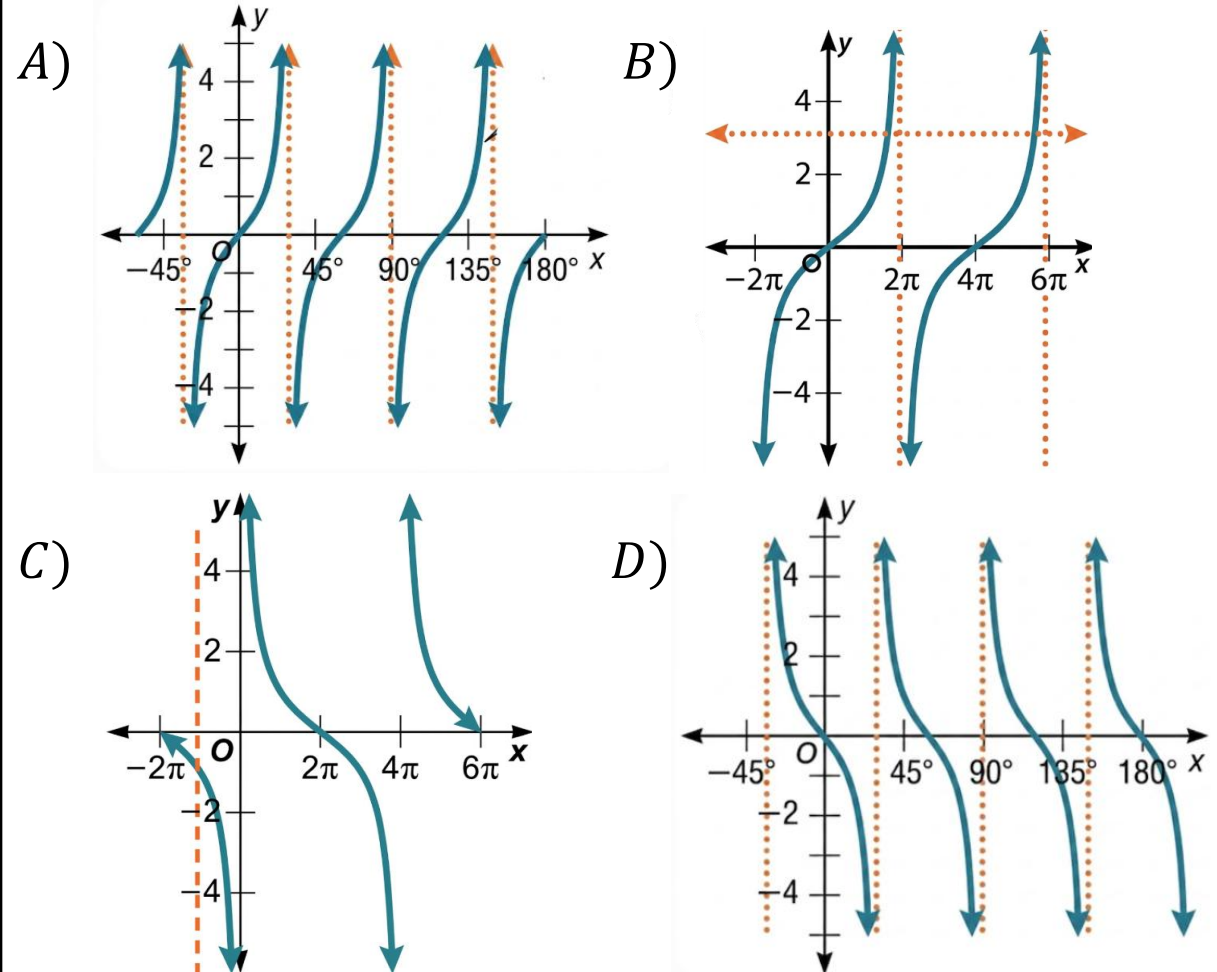
$$y = a \tan bx$$

Reflected about
the x -axis $-f(x)$

Vertical Dilation
Stretch: $|a| > 1$
Compress: $0 < |a| < 1$

Horizontal Dilation
Stretch: $0 < |b| < 1$
Compress: $|b| > 1$

Graph



Graphs of Reciprocal Functions

| Function | $y = acsc bx$ | $y = asec bx$ | $y = acot bx$ |
|------------|---|--|---|
| Graph | | | |
| Domain | $\{x \mid x \neq 180n^\circ, n \text{ is an integer}\}$ | $\{x \mid x \neq (90 + 180n)^\circ, n \text{ is an integer}\}$ | $\{x \mid x \neq 180n^\circ, n \text{ is an integer}\}$ |
| Range | $\{y \mid y \leq -1 \text{ or } y \geq 1\}$ | $\{y \mid y \leq -1 \text{ or } y \geq 1\}$ | all real numbers |
| Amplitude | undefined | undefined | undefined |
| Period | $\frac{360^\circ}{ b }$ or $\frac{2\pi}{ b }$ | $\frac{360^\circ}{ b }$ or $\frac{2\pi}{ b }$ | $\frac{180^\circ}{ b }$ or $\frac{\pi}{ b }$ |
| Asymptotes | $\frac{180n}{ b }$ or $\frac{\pi n}{ b }$ | $\frac{90+180n}{ b }$ or $\frac{\frac{\pi}{2}+\pi n}{ b }$ | $\frac{180n}{ b }$ or $\frac{\pi n}{ b }$ |
| Midline | $y = 0$ | $y = 0$ | $y = 0$ |

| | | | |
|--|---|--|---|
| What is the period of the parent function $y = \cot x$? | Which reciprocal function has the domain $\{x \mid x \neq (90 + 180n)^\circ, n \text{ is an integer}\}$? | What is the range of $y = \csc x$? | What is the amplitude of $y = \sec x$? |
| A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) 2π | A) $y = \sec x$ B) $y = \csc x$ C) $y = \cot x$ D) $y = \tan x$ | A) $y \leq -1$ or $y \geq 1$ B) $-1 \leq y \leq 1$ C) 360 D) All real numbers | A) 1 B) 0 C) 360 D) Undefined |
| Which reciprocal function has all real numbers as its range? | What is the midline of the reciprocal parent functions shown? | For $y = \csc 3x$, what is the maximum value? | For $y = 3\sec x$, what is the minimum value? |
| A) $y = \sec x$ B) $y = \csc x$ C) $y = \cot x$ D) $y = \tan x$ | A) $x = 0$ B) $y = 1$ C) $y = 0$ D) Undefined | A) $(-60, 1)$ B) $(-30, -1)$ C) $(30, -1)$ D) $(60, -1)$ | A) $(3, 0)$ B) $(180, -3)$ C) $(-90, 3)$ D) $(0, 3)$ |



Done

اللهم وفقني وافتح على قلبي ونور بصيرتي ولا تضيع
لي يا الله تعباً وسخر لي من حيث لا أحتسب عوناً

♡ بالتوفيق والنجاح إن شاء الله ♡

